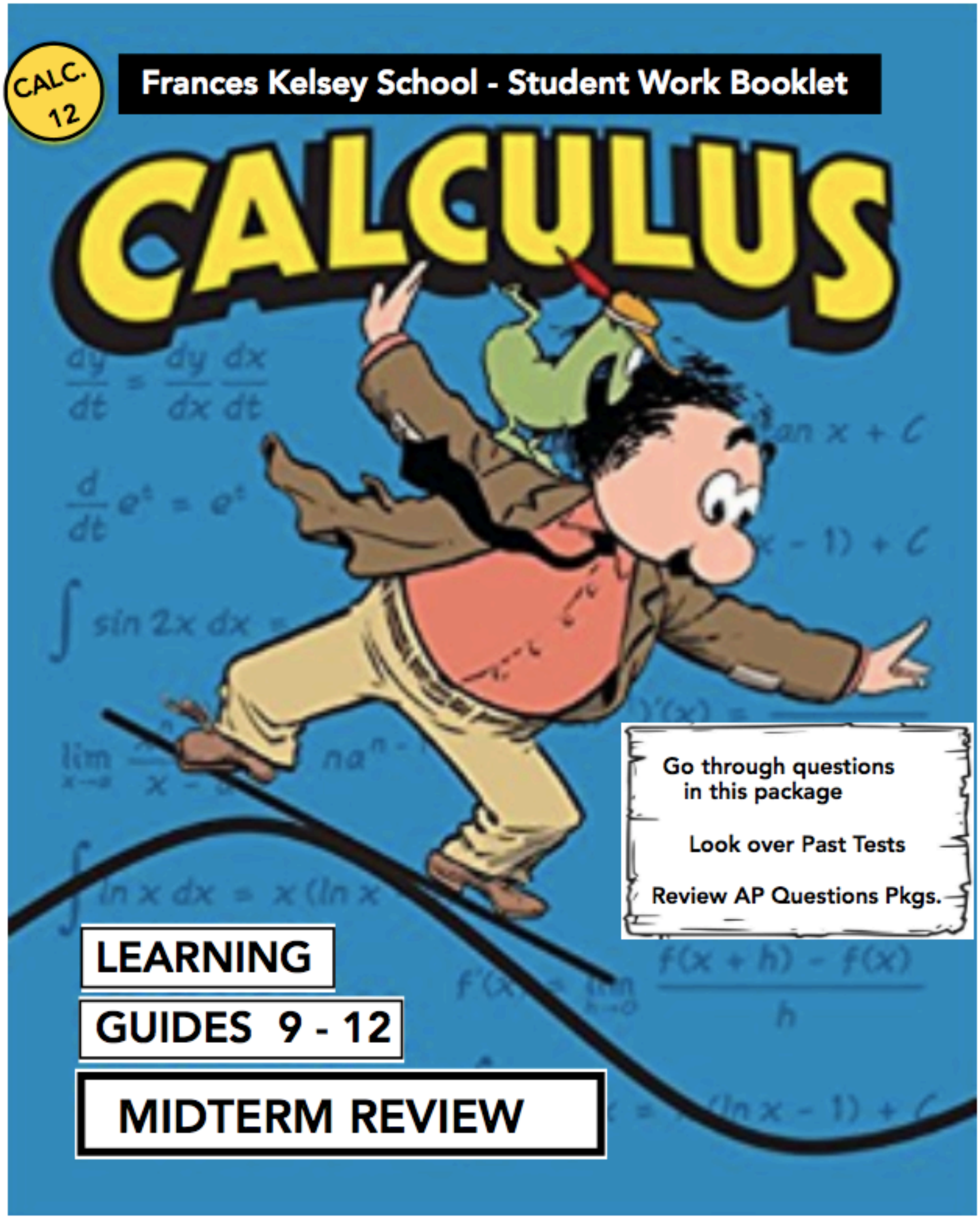
CALC. 12
Frances Kelsey School - Student Work Booklet
CALCULUS
Go through questions in this package
Look over Past Tests
Review AP Questions Pkgs.
LEARNING
GUIDES 9 - 12
MIDTERM REVIEW

LG 9 – 12 ***Worksheet Package***

Part A:

1. The Calculus 12 LG 9-12 test is a cumulative exam (the Calculus 12 Midterm) based on the concepts in LG's 1-8. As it is a cumulative exam it may only be written once!

2. Go over your tests for LG 1-3, 4-6 and 7-8 paying special attention to your mistakes and questions that you feel that you did not fully understand.

Part B:

1. Find the first derivative of each of the following functions:

$a)\ y = 8x^5 - 3x^7 + \frac{2}{3}x^6 - 5x$

$b)\ f(x) = 4\pi x^5 - 2\pi^3 + 6\pi^2 x^5$

$c)\ h(x) = ax^2 + b^2x^3 + c^3$

$d)\ y = \frac{5}{x^2} - 6x^{-3} + \frac{2}{x} - 5$

$e)\ y = 6\sqrt{x} - \frac{5}{\sqrt{x}} + \frac{2}{\sqrt[3]{x}} - \frac{3}{4\sqrt{x}}$

$f)\ p(x) = \frac{-2x^3}{x - 3x^2}$

$g)\ y = \left(2x^2 + 3\right)\left(4x - 5\right)$

$h)\ \left(x^2 - 5x + 7\right)^3$

2. If $y = 8x^6 - 7x^2 + 5x$ find:

a) $\dfrac{d^2y}{dx^2}$ b) $\left(\dfrac{dy}{dx}\right)^2$ c) y'' d) $\dfrac{d^3y}{dx^3}$

a) $\dfrac{dy}{dx}$ b) $\dfrac{dy}{dK}$ c) $\dfrac{dy}{dp}$ d) $\dfrac{dy}{dm}$

3. If $y = 5K^4x^3 - 2mx^2 + 6K$ find:

a) $\dfrac{dy}{dx}$ b) $\dfrac{dy}{dK}$ c) $\dfrac{dy}{dp}$ d) $\dfrac{dy}{dm}$

4. Find the first derivative of each of the following:

a) $y = \sqrt{5x^2 - 7x}$

b) $y = \left(x^2 - 1\right)^3 \left(2x + 5\right)^2$

c) $f(x) = \dfrac{4x^2}{\left(1 - 3x^3\right)^2}$

d) $y = \dfrac{1}{\sqrt{4 - u^2}}$

e) $y = 4m^2\left(3m-2\right)^5$

f) $h(x) = \left(x + \frac{1}{\sqrt{x}} - \frac{2}{\sqrt[3]{x}}\right)$

g) $p(x) = \sqrt{x}\left(2x - 3x^2\right)^4$

h) $y = \left(\frac{x^2-2}{x^2+2}\right)^2$

5. *If* $s(t) = 6t^4 - 8t + 9$ *is a position function describing motion in a straight line, find the velocity and acceleration as functions of time t.*

6. *If* $x^2 + y^2 = z^2$ *and* $\frac{dx}{dt} = -3,\ \frac{dy}{dt} = 5,\ x = 3$ *and* $y = -4$, *find* $\frac{dz}{dt}$.

7. *How fast is the radius of a circular oil slick changing when the radius is 10 m and the area is increasing at a rate of 100 m^2/s ?*

8. *Find the first derivative of each of the following functions:*

$a)\ \ y = 4x^2 + 2y^2 + 8$

$b)\ \ y = 6xy^2$

$c)\ \ f(x) = \cos\left(2x^4\right)$

$d)\ \ h(x) = \sin^4\left(2x^2 - 5x\right)$

$e)\ \ m(x) = x\sin x^2$

$f)\ \ y = \ln\left(2x^2 - 5x\right)$

$g)\ \ g(x) = \left(x\ln x\right)^3$

$h)\ \ f(x) = 3^{2x-5}$

9. *Find* $\frac{dy}{dx}$ *for each of the following.*

$a)\ \ y = x^{\pi} + e^x + e^5 + \pi^6$

$b)\ \ y = e^{\pi x} - e^{\pi} + e^x - x^e$

$c)\ \ y = xe^x + \pi e^{\pi}$

$d)\ \ y = e^{6x} - 6^x + 6e^{6x}$

10. *Find* $\frac{dy}{dx}$ *for each of the following.*

$a)\ \ y = \dfrac{\cos 3x}{\sin 3x}$

$b)\ \ y = \tan(x^3) - \tan^3(x)$

$c)\ \ y = \sqrt{x}\ e^{\sqrt{x}}$

$d)\ \ y = Ax^3 - 2Bx^2 + 3c^4x$

$e)\ \ y + e^x = 3y^2$

$f)\ \ y = 5x - \dfrac{2}{x^4} + \dfrac{6}{\sqrt{x}} - \dfrac{2}{5\sqrt{x}}$

$g)\ \ y = \ln\left(x^3 - 2x + 7\right)$

$h)\ \ y = \left(x^2 - 3x^5 + 2\right)^{-3}$

$i)\ \ 6y - 4x^2 = 3xy$

$j)\ \ y = 6^{x^3}$

$k)\ \ y = \sin(xy)$

$l)\ \ y = 5x\sin 4x + \sin^3 x - \sin x^3$

Part C:

1. *Find the equation of the tangent line to the curve* $y = 5x^3 - 2x + 6$ *at the point (-1, 3).*

2. *Find the equation of the normal line to the curve* $y = 6x^4 + 8x^3 - 30$ *when x = -2.*

3. *If* $y = 6 + 2u^2, \quad u = 4v^2 - 3v, \quad v = -3 + 2x^2, \quad find\ \frac{dy}{dx} \quad at\ x = -1.$

4. *Use Calculus to find the critical points of each of the following functions. Then use Calculus to decide which of the critical points, if any, are turning points? Show a thumbnail sketch.*

$a)\ \ y = 2x^2 - 12x + 10$

$b)\ \ y = x^3 - 12x$

5. *Find the local extrema of each of the following functions using the first derivative test. Show a thumbnail sketch.*

$a)\ \ y = 2x^3 + 3x^2 - 12x$

$b)\ \ y = -4x^3 + 48x - 10$

6. *Use Calculus to find the inflection points of each of the following functions. Show a thumbnail sketch.*

$a)\ \ y = 2x^2 - 12x + 10$

$b)\ \ y = x^3 - 12x$

7. *Use the second derivative test to find and classify all local extrema. Show a thumbnail sketch.*

$a)\ \ y = 2x^2 - 12x + 10$

$b)\ \ y = x^3 - 12x$

8. *Find the general antiderivative of each of the following functions:*

$a)\ 6x^3 - 2x^2 + \frac{5}{x^2}$

$b)\ 3e^{4x} - 5e^x$

$c)\ \sin(3x) + 3x$

$d)\ (\cos x)(\sin^3 x)$

$e)\ x^2 \sec^2(x^3)$

$f)\ (3x^2 + 2x)e^{(x^3+x^2)}$

9. *Find the position function(s) for an object with acceleration function a(t) = t – 2, initial velocity v(0) = 2 and initial position s(0) = 10.*

10. *Simplify:*

$a)\ \int x^3\ dx$

$b)\ \int \pi x^2\ dx$

$c)\ \int \frac{1}{1-x}\ dx$

$d)\ \int e^{4x} - x^2\ dx$

$e)\ \int 5y^2\ dx$

$f)\ \int \sin(\pi x)\ dx$

* use u-substitution for *g* to *j*

$g)\ \int (2x+3)^7\ dx\ ,\ u = 2x+3$

$h)\ \int \frac{1}{(3x-2)^8}\ dx\ ,\ u = 3x-2$

$i)\ \int x^3\sqrt{2-3x^4}\ dx$

$j)\ \int \frac{3x}{4x^2-3}\ dx$

11. *Find the* <u>*exact*</u> *value of each of the following:*

$a)\ \int_{1}^{\sqrt{2}} 3x \ \ dx$

$b)\ \int_{\pi}^{4\pi} 2e^{x} \ \ dx$

$c)\ \int_{1}^{6} 3m \ \ dx$

$d)\ \int_{0}^{\frac{\pi}{2}} 2\cos x \ \ dx$

$e)\ \int_{-2}^{1} xe^{x^2} \ \ dx$

$f)\ \int_{e}^{4} \frac{\ln x}{x} \ \ dx$

12. *Simplify:*

$a)\ \int_{a}^{b} 4x^2 \ \ dx$

$b)\ \int_{\pi}^{3} \frac{5}{x} \ \ dx$

13. *Find the exact area between y = 4x and the x-axis over the interval* $-2 \le x \le 4$.

14. *Find the exact area between* $y = 4x^2$ *and the y-axis over the interval* $0 \le x \le 3$.

15. *Find the exact area between the two curves*
$f(x) = x - 3$ and $g(x) = -x^2 - 6$ over the interval $-2 \le x \le 5$.

16. *Find the exact area enclosed by the two curves* $f(x) = 6x$ and $g(x) = x^2$.

17. *Use the Algorithm for Extreme Values to find the maximum and minimum values of* $f(x)=2x^3-6x^2$ over the interval $-1\le x\le 6$.

18. *Simplify:*

$a)\ \int_a^b 4x\ \ dx$

$b)\ \int_a^2 e^x\ \ dx$

$c)\ \int_{2m}^{5m} y\ \ dx$

$d)\ \int_{-2a}^{5a} k^2\ \ dx$

19. *Solve for x:*

$a)\ \int_1^x 2m\ \ dm = 8$

$b)\ \int_x^5 2t+3\ \ dt = 40$

Part D:

1. Evaluate each of the following limits:

$a)\ \lim_{x\to -1} 2x^2 - 5x + 3$

$b)\ \lim_{x\to e} \left(x^3 - 2x\right)$

$c)\ \lim_{x\to a} \frac{(x+a)^2}{\left(x^2 + 2a^2\right)}$

$d)\ \lim_{x\to 3} \frac{x^2 - 9}{x - 3}$

$e)\ \lim_{x\to 2} \frac{x-2}{x^3 - 8}$

$f)\ \lim_{x\to 0} \frac{2\cos x - 2}{2x}$

$g)\ \lim_{x\to 0} \frac{\cos x - 1}{2e^x - 2}$

$h)\ \lim_{x\to 4} \frac{\sqrt{x} - 2}{x - 4}$

2. If $\lim_{x\to 2} f(x) = 1$, use the properties of limits to evaluate each limit:

$a)\ \lim_{x\to 2} \frac{x^2 - 2}{4f(x)}$

$b)\ \lim_{x\to 2} \sqrt{\left[f(x)\right]^2 + 10 - x}$

3. *Use the definition of the derivative to find* $\frac{dy}{dx}$:

$a)\ y=\frac{1}{x+3}$

$b)\ y=x^2-3x$

4. *Use the fundamental trigonometric limit to evaluate each of the following:*

$a)\ \lim_{h\to 0}\frac{3\sin 4h}{2h}$

$b)\ \lim_{h\to 0}\frac{6\sin h}{h}$

5. *Use the fundamental limit involving e to evaluate each of the following:*

$a)\ \lim_{h\to 0}(1+h)^{\frac{4}{h}}$

$b)\ \lim_{n\to +\infty}\left(1+\frac{5}{n}\right)^n$

6. *Evaluate each of the following limits:*

$a)\ \lim_{x\to\pi} 2x^2+5\pi x-3\pi^2$

$b)\ \lim_{x\to 4} \frac{x-4}{\sqrt{x}-2}$

$c)\ \lim_{x\to 0} \frac{\sqrt{4+x}-2}{x}$

$d)\ \lim_{x\to 0} \frac{e^x-1}{\sin x}$

$e)\ \lim_{h\to 0} \frac{2\sin 3h}{5h}$

$f)\ \lim_{h\to 0} (1+h)^{\frac{6}{h}}$