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FOUNDATIONS & PRE-CALCULUS 10

Seminar Notes

Learning Guides 5

Frances Kelsey Secondary School – 2019/20

Sequences

Arithmetic

Has a Common Difference

$$d = t_2 - t_1$$
$$where:$$
$$t_2 - t_1 = t_3 - t_2$$

Geometric

Has a Common Ratio

$$r = \frac{t_2}{t_1}$$
$$where: \ \frac{t_2}{t_1} = \frac{t_3}{t_2}$$

Part 1. Arithmetic Sequences

Is where a list of terms have a common difference, d.
For example: **2, 5, 8, 11** - is an arithmetic sequence with a common difference of **3**.

Try: State yes or no if the following sequences are arithmetic. If yes, state the common difference.

a) -5, 0, 5, 10, 15
b) 1, 3, 6, 10, 15
c) 9, 5, 1, -3, -7

Arithmetic Sequences General Terms

$$t_n = t_1 + (n - 1)d$$

t_1 =

n =

d =

t_n =

Increasing Arithmetic Sequence	Decreasing Arithmetic Sequence
* Common difference is positive!	* Common difference is negative!
5 , 9 , 13 , 17 , ... +4 +4 +4	20 , 17 , 14 , 11 , ... -3 -3 -3

Topic 1 Finding General Term

To find the general term t_n you will need the common difference d and the first term t_1. The general term of a arithmetic sequence is given by the following formula:

$$t_n = t_1 + (n - 1)d$$

- write the general term formula
- substitute t_1 and d
- simplify general term

Example 1

a) Write a general term t_n for
1, -3, -7, -11, ...

b) Write a general term t_n for
2, 7, 12, 17, ...

Topic 2 Finding A Specific Term

To find a specific term you will need the general term of a arithmetic sequence given by the following formula:

$$t_n = t_1 + (n - 1)d$$

- write the general term formula
- substitute t_1 and d
- simplify general term
- substitute n to find desired value

Example 2

a) Write a general term t_n for
$1, 5, 9, 13, \ldots t_{17}$

b) Write a general term t_n for
$1, 5, 9, 13, \ldots$
What is term t_{25}?

C) A visual and performing arts group wants to hire a community events leader. The person will be paid $12 for the first hour of work, $19 for two hours of work, $26 for three hours of work, and so on.

i) Write the general term that you could use to determine the pay for any number of hours worked.

ii) What will the person get paid for 6 h of work?

D) What is the charge for 10 h if the furnace technician charges $45 for the house call plus $46 per hour?

Topic 3 Finding Number of Terms

To find the number of terms n you will need the common difference d, the first term t_1, and the t_n. The general term of a arithmetic sequence is given by the following formula:

$$t_n = t_1 + (n - 1)d$$

- ☛ write the general term formula
- ☛ substitute t_1, t_n and d
- ☛ simplify to find desired value

Example 1

a) For 1, 5, 9, 13, ...which term is 153?

b) For 2, 6, 10, 14, … , which term is 13122?

Topic 4 Finding The Arithmetic Mean

The arithmetic mean is the point(s) between 2 numbers that would form a arithmetic sequence of all 3 numbers (they would have a common difference).

To find the arithmetic mean you will need the number of terms **n**, the first term t_1, and the t_n. The general term of a arithmetic sequence is given by the following formula:

$$t_n = t_1 + (n - 1)d$$

- ☛ write the general term formula
- ☛ substitute t_1, t_n and n
- ☛ simplify to find d
- ☛ use d to find mean

Example 1

a) What are the 2 arithmetic means between 2 and 17?

2 , ☐ , ☐ , 17

b) In a arithmetic sequence, $t_3 = 24$ and $t_8 = 59$, Find the first two terms.

☐, ☐, 24, ☐, ☐, ☐, ☐, 59

Topic 5

General Formula for Sum of an Arithmetic Series

$$S_n = \frac{n}{2}\left[2t_1 + (n-1)d\right]$$

$$S_n = \frac{n}{2}\left[t_1 + t_n\right]$$ * use if t is known

Example 1

a) Find the sum of the 1st 9 terms of 2+5+8+…..

Topic 6 Finding Arithmetic Sum

(given t_1, t_n, n)

$$S_n = \frac{n}{2}\left[t_1 + t_n\right]$$

- substitute t_1, t_n, n into formula
- simplify to find desired value

Example 1

Find the sum of 2 + 6 + 10 + ... + 202

Example 2

b) Find the sum of the series:

3+6+9+12+15+18+21+24+27+30+33

H.M.P.

1. The first three terms of the an arithmetic sequence are given by x, $(3x - 8)$, 11.5

   a) Determine the first term and the common difference.

   b) Determine the 18th term.

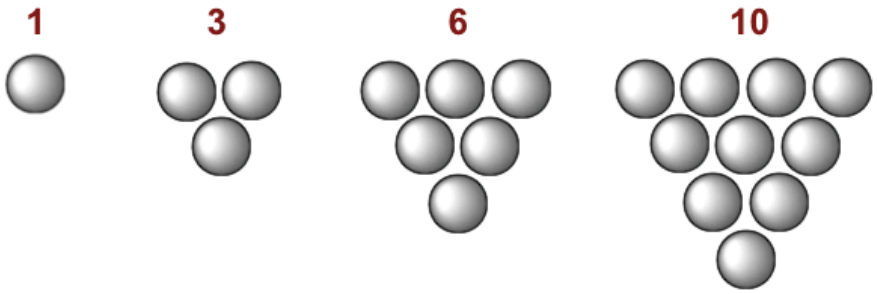
   c) Determine the sum of the first 18 terms.

2. **The sum of the first n terms of an arithmetic series is $S_n = 3n^2 + 4n$**

a) Determine the first three terms of this series.

b) Determine the sum of the first 20 terms of this series.

3. Given the diagram:



a) Find the 10th triangular number.