FOUNDATIONS & PRE-CALCULUS 10

Seminar Notes

Learning Guides 10 & 11

Frances Kelsey Secondary School – 2019/20

Topic 1

Representing Relations

Example 1

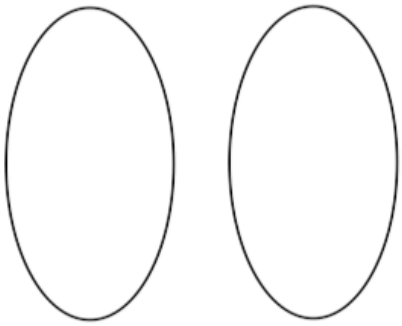
3 Ways you can represent a Relation:

1. Table

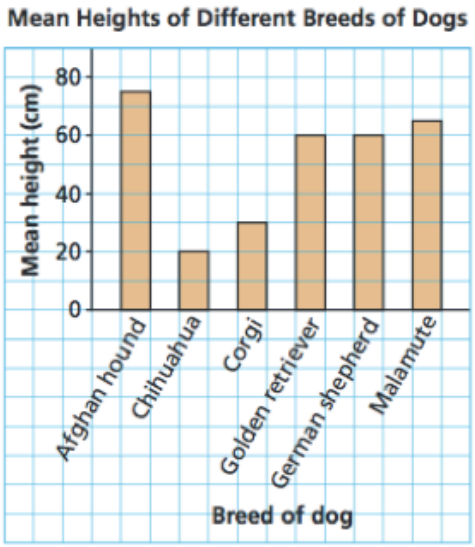
Fruit	Colour

2. Ordered Pairs

3. Arrow Diagram



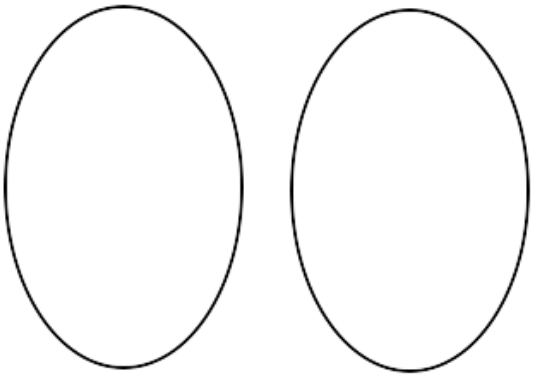
Example 2



Different breeds of dogs can be associated with their mean heights. Consider the relation represented by this graph. Represent the relation:

a) as a table

b) as an arrow diagram

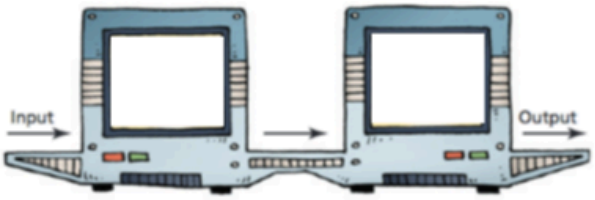


Topic 2

Properties of Functions

Given:

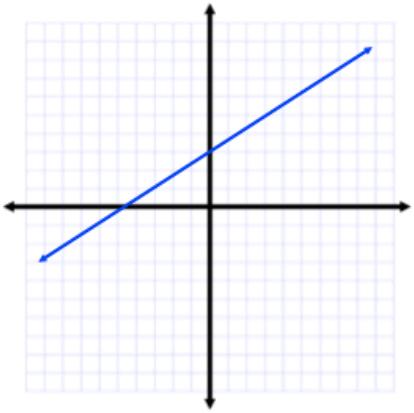
Input	Output
1	5
2	7
	9
4	
	13



Make Connections

$y = 2x - 1$

x	y



Topic 3

Identifying Functions

What's a Functions or Not a Function?

Vertical Line Test for a Function

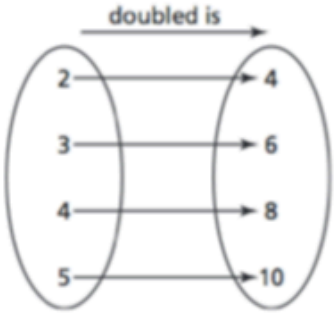
A graph represents a function when no two points on the graph lie on the same vertical line.
Place a ruler vertically on a graph, then slide the ruler across the graph. If one edge of the ruler always intersects the graph at no more than one point, the graph represents a function.

Exercises

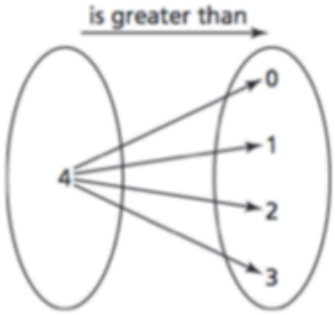
A

Which diagram/graph represents a function?

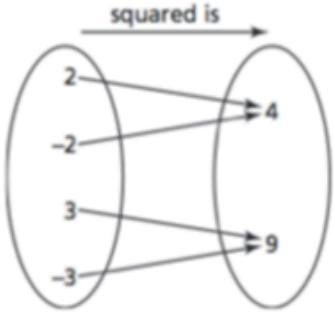
a)



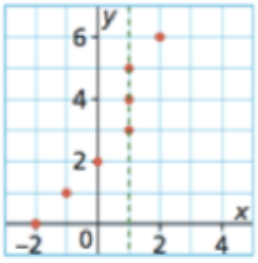
b)



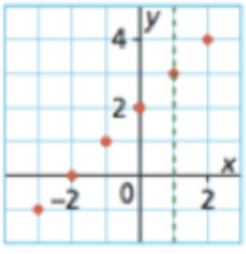
c)



d)



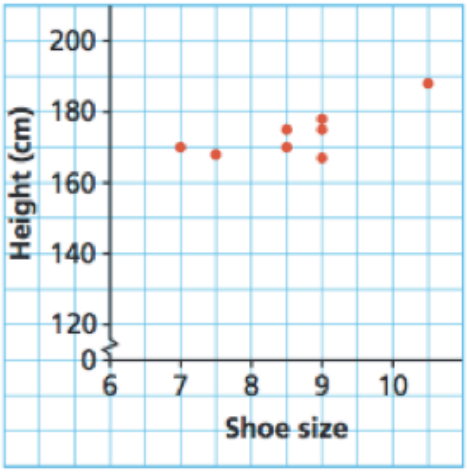
e)



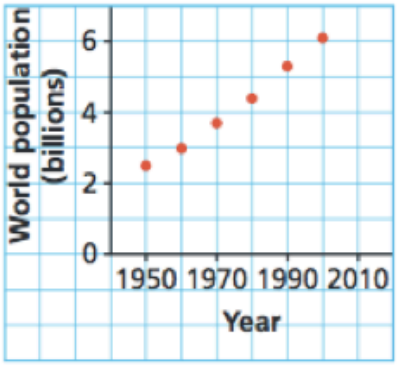
Identifying whether a Graph Represents a Function

Which of these graphs represents a function? Justify the answer.

a) Height against Shoe Size



b) World Population



In the workplace, a person's gross pay, P dollars, often depends on the number of hours worked, h.
So, we say P is the *dependent variable*. Since the number of hours worked, h, does not depend on the gross pay, P, we say that h is the *independent variable*.

independent variable → | dependent variable ←

Hours Worked, h	Gross Pay, P ($)
1	12
2	24
3	36
4	48
5	60

domain | range

A table of values usually represents a sample of the ordered pairs in a relation.

The values of the independent variable are listed in the first column of a table of values. These elements belong to the domain.

The values of the dependent variable are listed in the second column of a table of values. These elements belong to the range.

Number of Marbles, n	Mass of Marbles, m (g)
1	1.27
2	2.54
3	3.81
4	5.08
5	6.35
6	7.62

a) Why is this relation also a function?

b) Identify the independent variable and the dependent variable. Justify the choices.

c) Write the domain and range.

5. Which sets of ordered pairs represent functions? Identify the domain and range of each set of ordered pairs.

a) $\{(1, 3), (2, 6), (3, 9), (4, 12)\}$

b) $\{(1, 0), (0, 1), (-1, 0), (0, -1)\}$

c) $\{(2, 3), (4, 5), (6, 7), (8, 9)\}$

d) $\{(0, 1), (0, 2), (1, 2), (0, 3), (1, 3), (2, 3)\}$

Topic 4

Using Function Notation to Determine Values

The equation $V = -0.08d + 50$ represents the volume, V litres, of gas remaining in a vehicle's tank after travelling d kilometres. The gas tank is not refilled until it is empty.

a) Describe the function.
Write the equation in function notation.

b) Determine the value of $V(600)$.
What does this number represent?

c) Determine the value of d when $V(d) = 26$.
What does this number represent?

CHECK YOUR UNDERSTANDING

Try: The equation $C = 25n + 1000$ represents the cost, C dollars, for a feast following an Arctic sports competition, where n is the number of people attending.

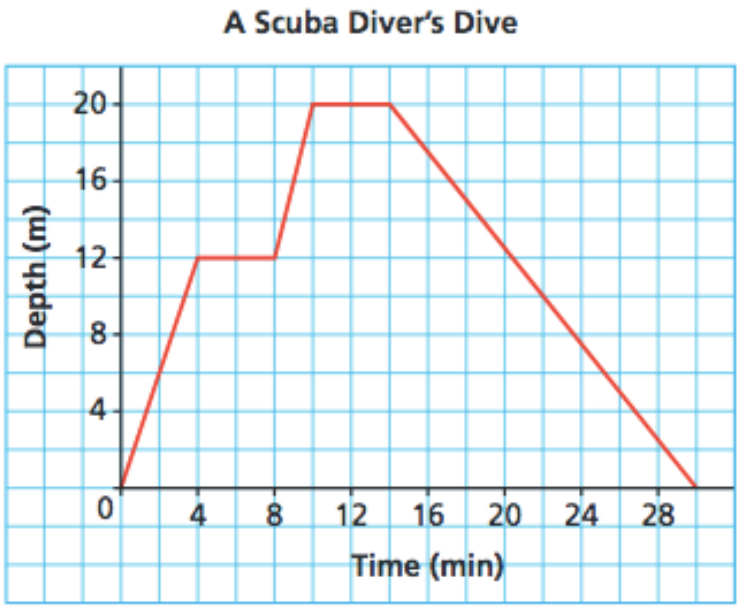
a) Describe the function.
Write the equation in function notation.

LEARNING GUIDE 11

Topic 1

Interpreting and Sketching Graphs

Example 1 Interpreting a Graph



How many minutes did the dive last?

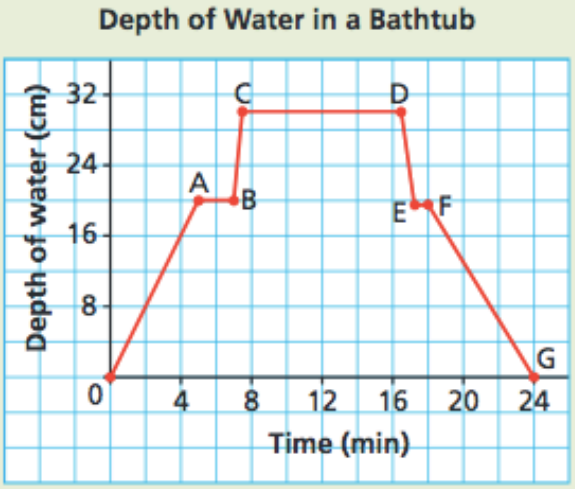
At what times did the diver stop her descent?

What was the greatest depth the diver reached? For how many minutes was the diver at that depth?

TRY THIS

Work with a partner.
You will need grid paper.

This graph shows the depth of water in a bathtub as a function of time.



A. What does each segment of the graph represent? Compare your description with that of your partner. Are both your stories the same? Should they be? Explain.

B. Sketch a graph to represent this situation:

You put the plug in the bath and turn on the taps.
You leave the bathroom and return to discover that the bath has overflowed.
You turn off the taps and pull out the plug to let out some water. You put the plug back in.

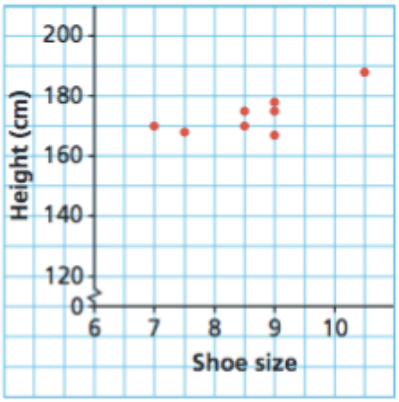
C. Compare your graph with that of your partner. How are the graphs the same? How are they different?

Topic 2 Graphs of Relations and Functions

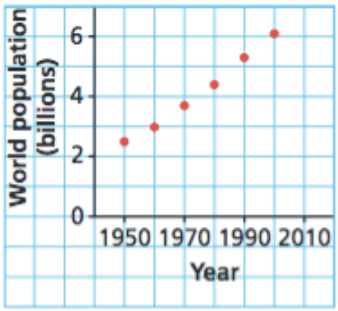
Example 1 Identifying whether a Graph Represents a Function

Which of these graphs represents a function? Justify the answer.

a) Height against Shoe Size



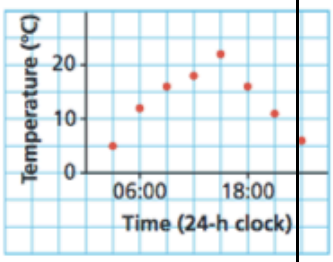
b) World Population



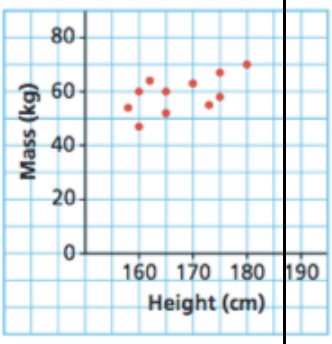
CHECK YOUR UNDERSTANDING

Try: Which of these graphs represents a function? Justify your answer.

a) Outside Temperature over a 24-h Period



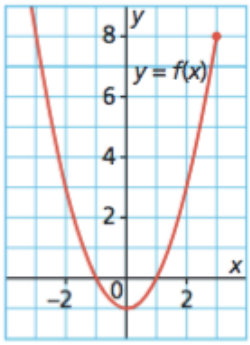
b) Masses of Students against Height



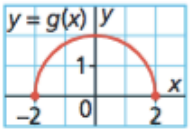
Example 2 Determining the Domain and Range of the Graph of a Function

Determine the domain and range of the graph of each function.

a)



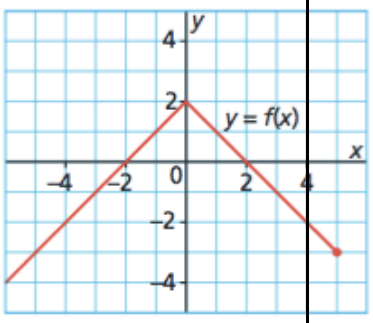
b)



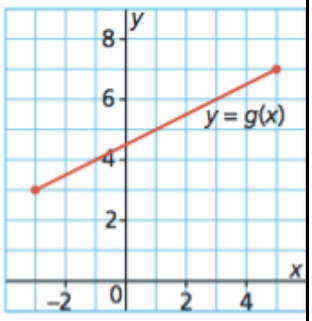
CHECK YOUR UNDERSTANDING

Try: Determine the domain and range of the graph of each function.

a)



b)



Two ways you can write the Domain and Range.

Set Notation	Interval
$-1 \le x \le 5$	$[-1,5]$
$-1 \le y \le 2$	$[-1,2]$

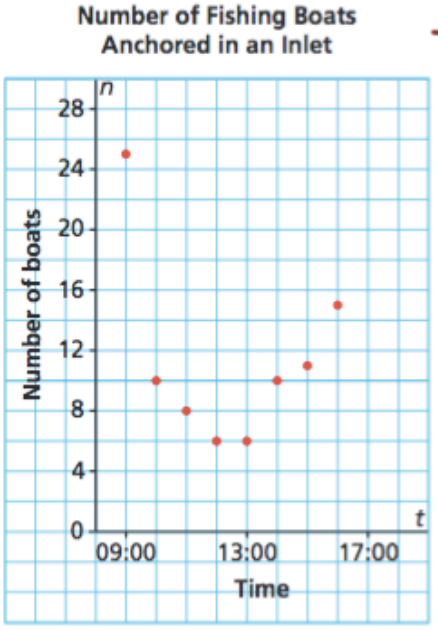
Example 3 Determining the Domain and Range of the Graph of a Situation

This graph shows the number of fishing boats, n, anchored in an inlet in the Queen Charlotte Islands as a function of time, t.

a) Identify the dependent variable and the independent variable. Justify the choices.

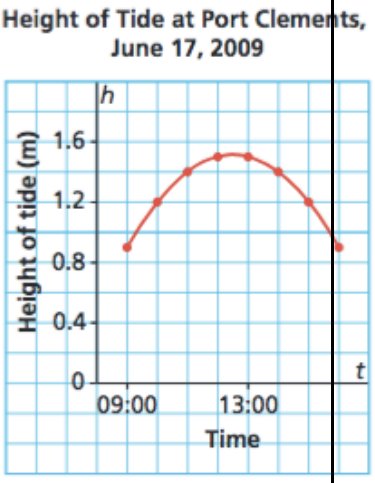
b) Why are the points on the graph not connected? Explain.

c) Determine the domain and range of the graph.



CHECK YOUR UNDERSTANDING

Try: This graph shows the approximate height of the tide, h metres, as a function of time, t, at Port Clements, Haida Gwaii on June 17, 2009.



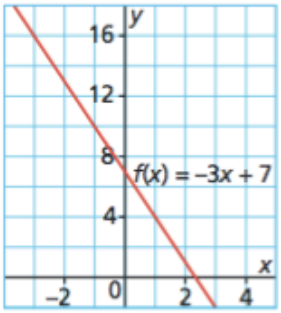
a) Identify the dependent variable and the independent variable. Justify your choices.

b) Why are the points on the graph connected? Explain.

c) Determine the domain and range of the graph.

Example 4 Determining Domain Values and Range Values from the Graph of a Function

Here is a graph of the function $f(x) = -3x + 7$.

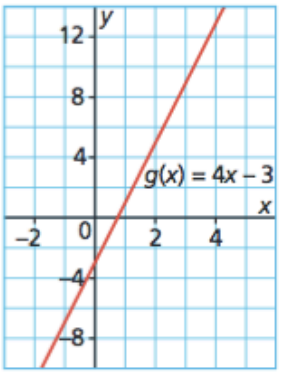


a) Determine the range value when the domain value is -2.

b) Determine the domain value when the range value is 4.

CHECK YOUR UNDERSTANDING

Try: Here is a graph of the function $g(x) = 4x - 3$.



a) Determine the range value when the domain value is 3.

b) Determine the domain value when the range value is -7.

Topic 3 Properties of Linear Relations

Construct Understanding

The cost for a car rental is $60, plus $20 for every 100 km driven.
The independent variable is the distance driven and the dependent variable is the cost.
We can identify that this is a linear relation in different ways.

- a table of values

Independent variable →

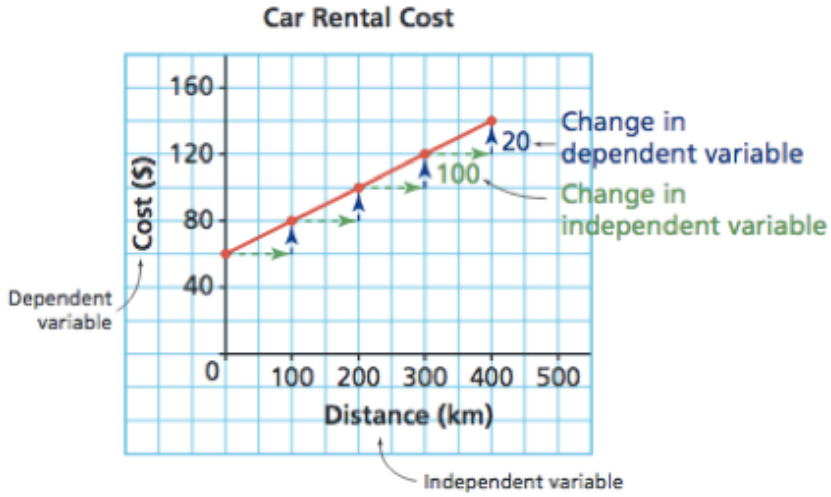
Distance (km)	Cost ($)
0	60
100	80
200	100
300	120
400	140

← Dependent variable

+100, +100, +100, +100 (Distance)

+20, +20, +20, +20 (Cost)

- a graph



We can use each representation above to calculate the **rate of change**.

The rate of change can be expressed as a fraction:

$$\frac{\text{change in dependent variable}}{\text{change in independent variable}} = \frac{\$20}{100 \text{ km}}$$

$$= \$0.20/\text{km}$$

An equation for this linear function is:

$C = 0.20d + 60$

- initial amount
- independent variable
- rate of change
- dependent variable

Example 1 Determining whether a Table of Values Represents a Linear Relation

Which table of values represents a linear relation? Justify the answer.

a) The relation between temperature in degrees Celsius, C, and temperature in degrees Fahrenheit, F

C	F
0	32
5	41
10	50
15	59
20	68

b) The relation between the current, I amps, and power, P watts, in an electrical circuit

I	P
0	0
5	75
10	300
15	675
20	1200

CHECK YOUR UNDERSTANDING

Try: Which table of values represents a linear relation? Justify your answer.

a) The relation between the number of bacteria in a culture, n, and time, t minutes.

t	n
0	1
20	2
40	4
60	8
80	16
100	32

b) The relation between the amount of goods and services tax charged, T dollars, and the amount of the purchase, A dollars

A	T
60	3
120	6
180	9
240	12
300	15

Example 2 Determining whether an Equation Represents a Linear Relation

a) Graph each equation.

i) $y = -3x + 25$ **ii)** $y = 2x^2 + 5$

iii) $y = 5$ **iv)** $x = 1$

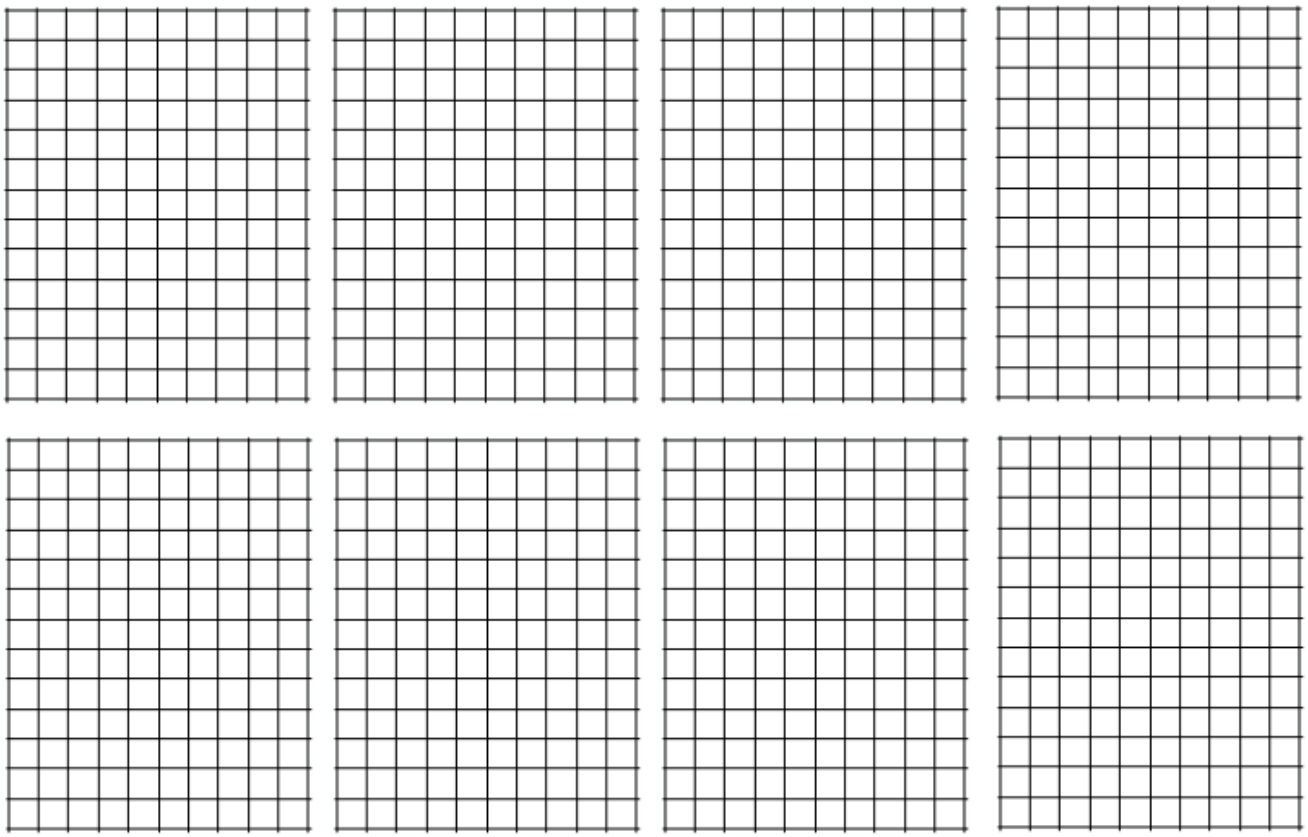
b) Which equations in part a represent linear relations? How do you know?

CHECK YOUR UNDERSTANDING

Try: **a)** Graph each equation.

i) $x = -2$
ii) $y = x + 25$
iii) $y = 25$
iv) $y = x^2 + 25$

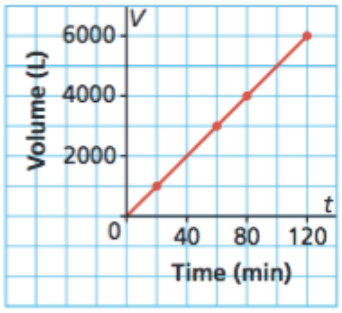
b) Which equations in part a represent linear relations? How do you know?



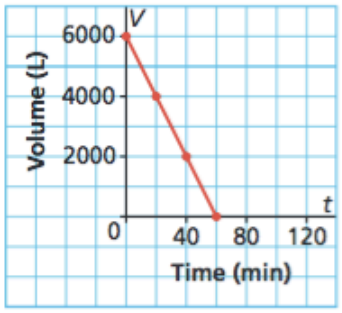
Example 4 Determining the Rate of Change of a Linear Relation from Its Graph

A water tank on a farm near Swift Current, Saskatchewan, holds 6000 L.
Graph A represents the tank being filled at a constant rate.
Graph B represents the tank being emptied at a constant rate.

Graph A
Filling a Water Tank



Graph B
Emptying a Water Tank



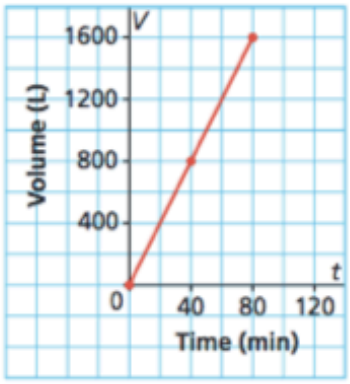
a) Identify the independent and dependent variables.

b) Determine the rate of change of each relation, then describe what it represents.

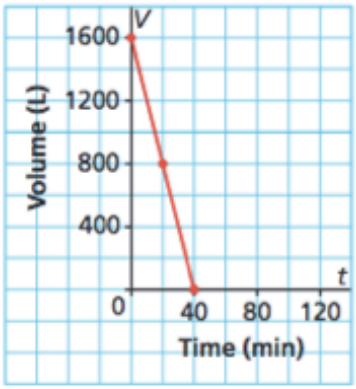
CHECK YOUR UNDERSTANDING

Try: A hot tub contains 1600 L of water. Graph A represents the hot tub being filled at a constant rate. Graph B represents the hot tub being emptied at a constant rate.

Graph A
Filling a Hot Tub



Graph B
Emptying a Hot Tub

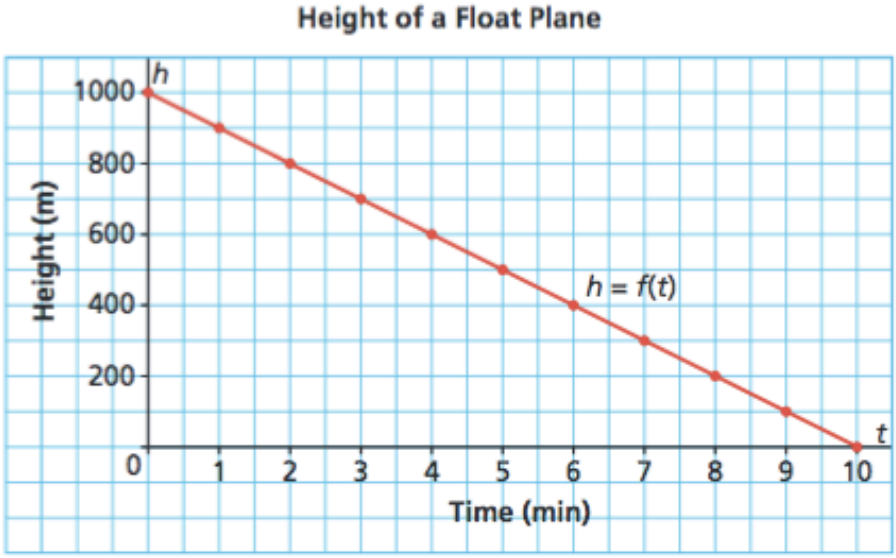


a) Identify the dependent and independent variables.

b) Determine the rate of change of each relation, then describe what it represents.

Topic 4 Interpreting Graphs of Linear Functions

Construct Understanding



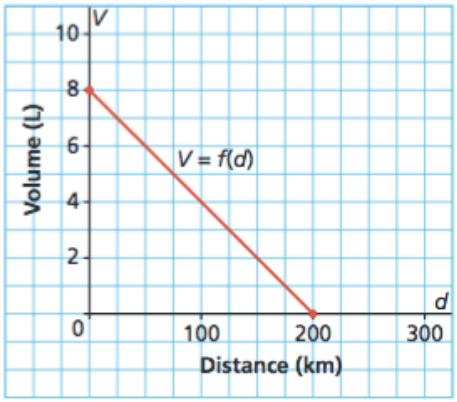
Where does the graph intersect the vertical axis? What does this point represent?

Where does the graph intersect the horizontal axis?
What does this point represent?

What is the rate of change for this graph? What does it represent?

Example 1 Determining Intercepts, Domain, and Range of the Graph of a Linear Function

Volume of Gas in a Scooter



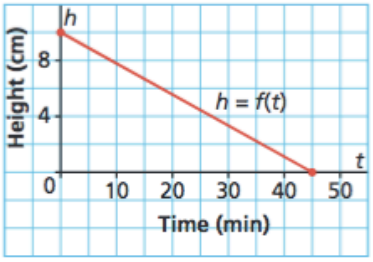
a) Write the coordinates of the points where the graph intersects the axes. Determine the vertical and horizontal intercepts. Describe what the points of intersection represent.

b) What are the domain and range of this function?

CHECK YOUR UNDERSTANDING

Try: This graph shows how the height of a burning candle changes with time.

Height of a Burning Candle

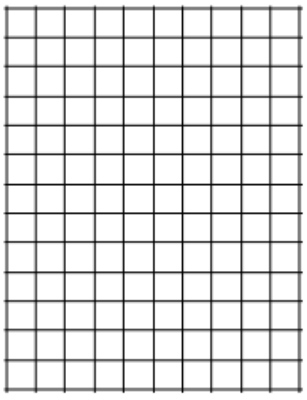


a) Write the coordinates of the points where the graph intersects the axes. Determine the vertical and horizontal intercepts. Describe what the points of intersection represent.

b) What are the domain and range of this function?

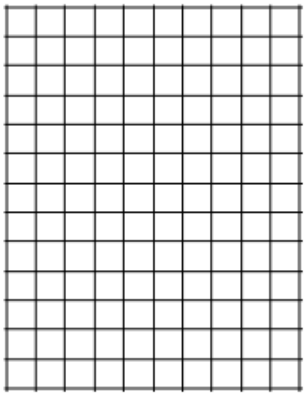
Example 2 Sketching a Graph of a Linear Function in Function Notation

Sketch a graph of the linear function $f(x) = -2x + 7$.



CHECK YOUR UNDERSTANDING

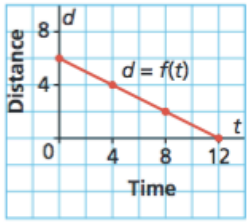
Try: Sketch a graph of the linear function $f(x) = 4x - 3$.



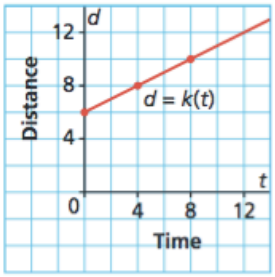
Example 3 Matching a Graph to a Given Rate of Change and Vertical Intercept

Which graph has a rate of change of $\frac{1}{2}$ and a vertical intercept of 6? Justify the answer.

a)



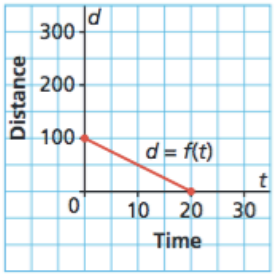
b)



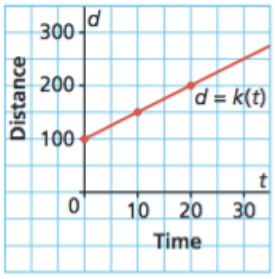
CHECK YOUR UNDERSTANDING

Try: Which graph has a rate of change of −5 and a vertical intercept of 100? Justify your answer.

a)

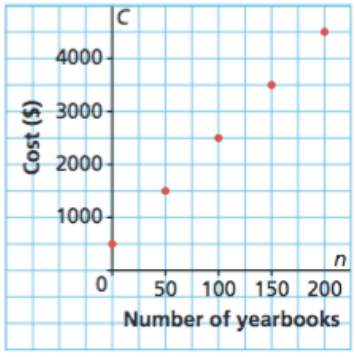


b)



Example 4 Solving a Problem Involving a Linear Function

Cost of Publishing a Yearbook

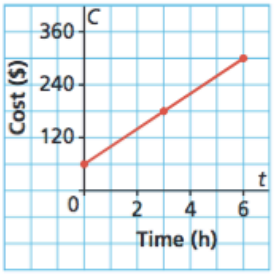


The budget for publishing costs is $4200. What is the maximum number of books that can be printed?

CHECK YOUR UNDERSTANDING

Try: This graph shows the total cost for a house call by an electrician for up to 6 h work.

Cost of an Electrician's House Call



The electrician charges $190 to complete a job. For how many hours did she work?