WORKSHEET #3 - Factoring Review

Factoring Using GCF:

To factor using a GCF, take the greatest common factor (GCF), for the numerical coefficient. When choosing the GCF for the variables, if all terms have a common variable, take the ones with the lowest exponent.

Example: $9x^4 + 3x^3 + 12x^2$ GCF: Coefficients = 3
                                                      Variables (x) = x^2

$GCF = 3x^2$
Next, you just divide each monomial by the GCF!
Answer = $3x^2(3x^2 + x + 4)$
Then, check by using the distributive property!

Exercises:
Factor each of the following using the GCF and check by using the distributive property:

1) $2a + 2b$ 2) $5x^2 + 5$ 3) $18c - 27d$ 4) $hb + hc$

5) $6x - 18$ 6) $3a^2 - 9$ 7) $4x^2 - 4y$ 8) $p + prt$

9) $10x - 15x^3$ 10) $2x - 4x^3$ 11) $8x - 12$ 12) $8 - 4y$

13) $3ab^2 - 6a^2b$ 14) $10xy - 15x^2y^2$ 15) $21r^3s^2 - 14r^2s$ 16) $2x^2 + 8x + 4$

17) $6c^3d - 12c^2d^2 + 3cd$ 18) $3x^2 - 6x - 30$ 19) $ay - 4aw - 12a$

20) $c^3 - c^2 + 2c$ 21) $2ma + 4mb + 2mc$ 22) $9ab^2 - 6ab - 3a$

23) $15x^3y^3z^3 - 5xyz$ 24) $24x^{11} + 4x^{10} - 6x^9 + 2x^8$

25) $26x^4y - 39x^3y^2 + 52x^2y^3 - 13xy^4$ 26) $16x^5 + 12xy - 9y^5$

Factoring Simple Trinomials (Case I):

Case I is when there is a coefficient of 1 in front of your variable^2 term (x^2).

You have two hints that will help you:
1) When the last sign is addition, both signs are the same and match the middle term.
2) When the last sign is subtraction, both signs are different and the larger number goes with the sign of the middle term.

Examples:

Hint #1:   Hint #2:

$x^2 - 5x + 6$ $x^2 + 5x - 36$

$(x -)(x -)$ $(x -)(x +)$

Find factors of 6, w/ sum of 5.   Find factors of 36 w/ difference of 5.

$(x - 3)(x - 2)$ $(x - 4)(x + 9)$

CHECK USING FOIL   CHECK USING FOIL

Exercises:
Factor each trinomial into two binomials and check using FOIL:

1) $a^2 + 3a + 2$ 2) $c^2 + 6c + 5$ 3) $x^2 + 8x + 7$ 4) $r^2 + 12r + 11$

5) $m^2 + 5m + 4$ 6) $y^2 + 12y + 35$ 7) $x^2 + 11x + 24$ 8) $a^2 + 11a + 18$

9) $16 + 17c + c^2$ 10) $x^2 + 2x + 1$ 11) $z^2 + 10z + 25$ 12) $a^2 - 8a + 7$

13) $a^2 - 6a + 5$ 14) $x^2 - 5x + 6$ 15) $x^2 - 11x + 10$ 16) $y^2 - 6y + 8$

17) $15 - 8y + y^2$ 18) $x^2 - 10x + 24$ 19) $c^2 - 14c + 40$ 20) $x^2 - 16x + 48$

21) $x^2 - 14x + 49$ 22) $x^2 - x - 2$ 23) $x^2 - 6x - 7$ 24) $y^2 + 4y - 5$

25) $z^2 - 12z - 13$ 26) $c^2 - 2c - 15$ 27) $c^2 + 2c - 35$ 28) $x^2 - 7x - 18$

29) $z^2 + 9z - 36$ 30) $x^2 - 13x - 48$ 31) $x^2 - 16x + 64$ 32) $x^2 - 11x - 42$

33) $x^2 - 9$ 34) $x^2 - 36$ 35) $x^2 - 121$ 36) $64x^2 - 81$

37) $9x^2 - 25$ 38) $144x^2 - 49$ 39) $x^2 - 225$ 40) $x^2 + 100$

Factoring Trinomials (Case II):

Decomposition: Factoring $ax^2 + bx + c$ when a $\neq$ 1

Example: Factor $3x^2 + 11x + 6$

we look for 2 #'s that add to give the middle term (11)

2#'s that multiply to give the product of the first and last term (3)(6) = 18

add to get 11 and multiply to get 18   would be 9 and 2

DECOMPOSITION

Step 1: Find the two terms

The 2 terms would be 2x and 9x

Step 2: We RE-WRITE our original expression with the factors in it

$3x^2 + 11x + 6$ = $3x^2 + 9x + 2x + 6$

 Step 3: Now we common factor by the first 2 terms

$3x^2 + 9x + 2x + 6$

= $3x^2 + 9x$ + 2x + 6

= 3x(x + 3)   + 2x + 6

Step 3: Figure out the factors of the last two terms

We know the last two terms MUST have a factor that is the SAME as the 1st two terms

3x(x + 3)   + ? (x + 3)   SO what to multiply (x + 3) by to get +2x + 6 ?????

the answer is 2 since   2(x + 3) = +2x + 6

Step 4: Re-write as two factors

3x(x + 3)   + 2x + 6

= 3x(x + 3)   + 2 (x + 3)

= (3x + 2)(x + 3)

> We can do this if we think of 3a + 4a = (3 + 4)a
>
> Thus, 3(x + y) + 4(x + y) = (3 + 4)(x + y)
>
> Thus a(x + y) + b (x + y) = (a + b)(x + y)

$$(3x + 4)(2x - 1)$$

Step 4: Foil Check

Date _____   Name _____

Factoring Trinomials (Case II):

Triple Play: Factoring $ax^2 + bx + c$ when a ≠1

Example: Factor $3x^2 + 11x + 6$

Step 1: Setup $(3x(\quad))(3x(\quad))$
                          over 3

+18

1	18	= 19
2	9	= 11 middle #
3	6	= 9

Step 2: Now multiply the first # by the last # , then set-up a T chart
                          3 X 6 = +18

         Put the 2 numbers into the top bracket slots with correct signs
         ☆ Remember, these two numbers must add to the middle # and multiply to last #

Step 3: Divide out denominator $\dfrac{(3x + 2)(3x + 9)}{3}$

Answer: $(3x + 2)(x + 3)$

☆ Watch out, sometimes you must divide n conquer to get rid of the denominator

Step 4: Foil Check

Exercises:
Factor each of the following:

1) $2x^2 + 15x + 7$ 2) $3x^2 - 5x - 12$ 3) $9x^2 + 11x + 2$ 4) $7x^2 - 22x + 3$

5) $18x^2 - 9x - 2$ 6) $4x^2 + 7x - 2$ 7) $2x^2 + 13x + 21$ 8) $11x^2 - 98x - 9$

9) $3x^2 - 20x - 63$ 10) $3x^2 - 20x - 7$ 11) $8x^2 + 13x - 6$ 12) $4x^2 - 17x - 42$

13) $2x^2 - 9x - 18$ 14) $6x^2 + 17x - 14$ 15) $3x^2 + 5x - 12$ 16) $2x^2 + 9x + 4$

Factoring Completely:

When asked to factor completely, you will have to use a combination of the methods that we have used previously. **GCF, GCF, GCF!**

Exercises:
Factor Completely:

1) $4x^2 + 20x + 24$ 2) $10x^2 - 80x + 150$ 3) $9x^2 + 90x - 99$

4) $3x^3 + 27x^2 + 60x$ 5) $12x^6 + 54x^5 + 60x^4$ 6) $8x^9 + 16x^8 + 192x^7$

ANSWERS: To Exercise on Factoring Using GCF
1) $2(a + b)$ 2) $5(x^2 + 1)$ 3) $9(2c - 3d)$ 4) $h(b + c)$
5) $6(x - 3)$ 6) $3(a^2 - 3)$ 7) $4(x^2 - y)$ 8) $p(1 + rt)$
9) $5x(2 - 3x^2)$ 10) $2x(1 - 2x^2)$ 11) $4(2x - 3)$ 12) $4(2 - y)$
13) $3ab(b - 2a)$ 14) $5xy(2 - 3xy)$ 15) $7r^2s(3rs - 2)$ 16) $2(x^2 + 4x + 2)$
17) $3cd(2c^2 - 4cd + 1)$ 18) $3(x^2 - 2x - 10)$ 19) $a(y - 4w - 12)$
20) $c(c^2 - c + 2)$ 21) $2m(a + 2b + c)$ 22) $3a(3b^2 - 2b - 1)$
23) $5xyz(3x^2y^2z^2 - 1)$ 24) $2x^8(12x^3 + 2x^2 - 3x + 1)$
25) $13xy(2x^3 - 3x^2y + 4xy^2 - y^3)$ 26) $prime\ or\ C.N.B.F.$

ANSWERS: To Exercise on Factoring Simple Trinomials (Case I):
1) $(a + 2)(a + 1)$ 2) $(c + 5)(c + 1)$ 3) $(x + 7)(x + 1)$ 4) $(r + 11)(r + 1)$
5) $(m + 4)(m + 1)$ 6) $(y + 7)(y + 5)$ 7) $(x + 8)(x + 3)$ 8) $(a + 9)(q + 2)$
9) $(16 + c)(1 + c)$ 10) $(x + 1)(c + 1)$ 11) $(z + 5)(z + 5)$ 12) $(a - 7)(a - 1)$
13) $(a - 5)(a - 1)$ 14) $(x - 6)(x + 1)$ 15) $(x - 10)(x - 1)$ 16) $(y - 2)(y - 4)$
17) $(5 - y)(3 - y)$ 18) $(x - 6)(x - 4)$ 19) $(c - 10)(c - 4)$ 20) $(x - 12)(x - 4)$
21) $(x - 7)(x - 7)$ 22) $(x - 2)(x + 1)$ 23) $(x - 7)(x + 1)$ 24) $(y + 5)(y - 1)$
25) $(z - 13)(z + 1)$ 26) $(c - 5)(c + 3)$ 27) $(c + 7)(x - 5)$ 28) $(x - 9)(x + 2)$
29) $(z - 3)(z + 12)$ 30) $(x + 3)(c - 16)$ 31) $(x - 8)(x - 8)$ 32) $(x + 3)(r - 14)$
33) $(x + 3)(x - 3)$ 34) $(x + 6)(x - 6)$ 35) $(x + 11)(x - 11)$ 36) $(8x + 9)(8x - 9)$
37) $(3x + 5)(3x - 5)$ 38) $(12x + 7)(12x - 7)$ 39) $(x + 15)(x - 15)$ 40) $prime$

ANSWERS: To Exercise on Factoring Trinomials (Case II)
1) $(2x + 1)(x + 7)$ 2) $(3x + 4)(x - 3)$ 3) $(9x + 2)(x + 1)$ 4) $(7x - 1)(x - 3)$
5) $(6x + 1)(3x - 2)$ 6) $(4x + 1)(x - 2)$ 7) $(2x + 7)(x + 3)$ 8) $(11x + 1)(x - 9)$
9) $(3x + 7)(x - 9)$ 10) $(3x + 1)(x - 7)$ 11) $(8x - 3)(x + 2)$ 12) $(4x + 7)(x - 6)$
13) $(2x + 3)(x - 6)$ 14) $(3x - 2)(2x + 7)$ 15) $(3x - 4)(x + 3)$ 16) $(2x + 1)(x + 4)$

ANSWERS: To Exercise on Factoring Completely:
1) $4(x + 2)(x + 3)$ 2) $10(x - 5)(x - 3)$ 3) $9(x + 11)(x - 1)$
4) $3x(x + 5)(x + 4)$ 5) $6x^4(2x + 5)(x + 2)$ 6) $8x^7(x + 6)(x - 4)$