1.1 - PRACTICE QUESTIONS

1. *Find the first derivative of each function with respect to* x:

a) $y = 3x^2 - 5$

$6x$

b) $y = 8x - 2$

8

c) $f(x) = 6x^2 - 3x + 2$

$12x - 3$

d) $y = -x^2 + 6$

$-2x$

e) $g(x) = \pi x^3 + 6x - 3$

$3\pi x^2 + 6$

f) $h(x) = 5\pi^2 x^4 + 6x^2 - 3\pi^4$

$20\pi^2 x^3 + 12x$

g) $k(x) = \frac{1}{4}x^8 - \frac{2}{3}x^6 + \frac{2}{5}x^4 - \frac{3}{4}$

$2x^7 - 4x^5 + \frac{8}{5}x^3$

h) $y = 6\pi^3 - 8\pi^2 + 24$

0

2. *Given* y, *find* $\frac{dy}{dx}$:

a) $y = 4x^3 - 2x + 6$

$12x^2 - 2$

b) $y = \frac{1}{5}x^5 + \frac{2}{3}x^3 + \frac{1}{2}x^2 + 1$

$x^4 + 2x^2 + x$

c) $y = x^4 - \pi^4$

$4x^3$

d) $y = \pi^3 x^3 - 3\pi x$

$3\pi^3 x^2 - 3\pi$

3. *Solve:*

a) *if* $p = 4q^3 + 2q^2 - 5$ *find* $\frac{dp}{dq} =$ $12q^2 + 4q$

b) *if* $g(t) = 4t^3 - 3t^2 + 6t$ *find* $g'(t) =$ $12t^2 - 6t + 6$

c) *if* $y = 2x^7 - 5x + 3$ *find* $y' =$ $14x^6 - 5$

4. If $y=2x^3-3x+7$ find: $y' = 6x^2 - 3$

a) y' at $x=-2$

$y'(-2) = 6(-2)^2 - 3 = 21$

b) y' at $(1, 5)$

$y'(1) = 6(1)^2 - 3 = 3$

c) $f(0)$ and $f'(0)$

$f(0) = 2(0)^3 - 3(0) + 7 = 7$

$f'(0) = 6(0)^2 - 3 = -3$

5. Find y' with respect to x if:

a) $y = ax^3 + bx^2 + d$

$3ax^2 + 2bx$

b) $y = ax^4 - ax^2 + bx$

$4ax^3 - 2ax + b$

c) $y = 4ax^5 + kx^3 - Cx + D$

$20ax^4 + 3kx^2 - C$

d) $y = D^2x^3 + 5M^3x^2 - 7$

$3Dx^2 + 10M^3x$

6. Find $\frac{dy}{dp}$ if:

a) $y = 4p^3 - 2p^2 + 6$

$12p^2 - 4p$

b) $y = -5p^4 + 6p - \frac{2}{3}$

$-20p^3 + 6$

c) $y = 4mp^4 + 16p^2 - 6c$

$16mp^3 + 32p$

d) $y = 6a^4 + 8a^3 - 2p^2$

$-4p$

7. If $y = 6x^5 - 2x^2 + 9x - 3$ find:

a) $y' = 30x^4 - 4x + 9$

b) $y'' = 120x^3 - 4$

c) $y''' = 360x^2$

d) $\frac{dy}{dx} = 30x^4 - 4x + 9$

e) $\frac{d^2y}{dx^2} = 120x^3 - 4$

f) $\frac{d^3y}{dx^3} = 360x^3$

g) $\left(\frac{dy}{dx}\right)^2 = (30x^4 - 4x + 9)^2$

h) $\left(\frac{d^2y}{dx^2}\right)^3 = (120x^3 - 4)^3$

i) $\frac{d^8y}{dx^8} = 0$ (eigth derivative)

8. If $f(x)=2x^3+6x-7$ find:

a) $f'(x) = 6x^2+6$

b) $f''(x) = 12x$

c) $(f(x))^2 = (2x^3+6x-7)^2$

d) $(f'(x))^2 = (6x^2+6)^2$

e) $(f''(x))^2 = (12x)^2$

f) $\left(f^{(3)}(x)\right)^2 = (12)^2 = 144$

9. Find $\frac{dy}{dx}$ if:

a) $y = Ax^3 - Bx^2 - C$

$3Ax^2 - 2Bx$

b) $y = 5Ax^3 - 6Bx^2 - Cx$

$15Ax^2 - 12Bx - C$

c) $y = 5A^2x^2 - 6B^4x^2 - C^5x$

$10A^2x - 12B^4x - C^5$

10. If $y = 2Dx^5 - 3k^2x^3 + 2$

a) $\frac{dy}{dx} = 10Dx^4 - 9k^2x^2$

b) $\frac{dy}{dD} = 2x^5$

c) $\frac{dy}{dk} = -6kx^3$

d) $\frac{dy}{dz} = 0$

11. *Simplify each expression first, and then find* y'.

a) $y = (2x+1)(3x-5)$

$6x^2 - 10x + 3x - 5$

$y = 6x^2 - 7x - 5$

$y' = 12x - 7$

b) $y = (2x-3)^2(2x-3)$

$y = 4x^2 - 12x + 9$

$y' = 8x - 12$

c) $y = (4x)^3(4x)(4x)$

$y = 64x^3$

$y' = 192x^2$

d) $y = x^2(x^3 - 6)$

$y = x^5 - 6x^2$

$y' = 5x^4 - 12x$

e) $y = (\pi x)^3 - 3\pi x$

$y = \pi^3 x^3 - 3\pi x$

$y' = 3\pi^3 x^2 - 3\pi$

f) $y = \dfrac{x^2 - 5x + 4}{x - 1} \Rightarrow \dfrac{(x-4)\cancel{(x-1)}}{\cancel{(x-1)}}$

$y = x - 4$

$y' = 1$

12. *Rewrite each rational expression using exponents to remove quotients first, and then find the first derivative.*

a) $y = \dfrac{5}{x^2} \Rightarrow y = 5x^{-2}$

$y' = -10x^{-3}$ or $\dfrac{-10}{x^3}$

b) $y = -\dfrac{6}{x^3} \Rightarrow -6x^{-3}$

$y' = 18x^{-4}$ or $\dfrac{18}{x^4}$

c) $y = \dfrac{2}{x^4} - \dfrac{3}{x^2} + \dfrac{5}{x} - 7x$

$y = 2x^{-4} - 3x^{-2} + 5x^{-1} - 7x$

$y' = -8x^{-5} + 6x^{-3} - 5x^{-2} - 7$

or

$y' = \dfrac{-8}{x^5} + \dfrac{6}{x^3} - \dfrac{5}{x^2} - 7$

d) $y = 4x^3 - \dfrac{2}{x^2} + 7x^{-5} - \dfrac{3}{x^{-4}}$

$y = 4x^3 - 2x^{-2} + 7x^{-5} - 3x^4$

$y' = 12x^2 - 4x^{-3} - 35x^{-6} - 12x^3$

or

$y' = 12x^2 - \dfrac{4}{x^3} - \dfrac{35}{x^6} - 12x^3$

***BE SURE TO DO MULTIPLE CHOICE QUESTIONS ONLINE **VERSION 7** #1 – 22, #48 – 55
https://moodle.sd79.bc.ca/pluginfile.php/1871/mod_resource/content/5/AB%20Calculus%20Version%207.pdf

1.2 - PRACTICE QUESTIONS

$FS' + F'S$

1. *Use the **Product Rule** to find the first derivative.*

a) $y = (3x + 1)(2x - 5)$

$F \quad S$

$(3x+1)(2) + (3)(2x-5)$

or

$6x + 2 + 6x - 15$

Simplified → $y' = 12x - 13$

b) $y = 3x^2(8x - 3)$

$(3x^2)(8) + (6x)(8x-3)$

$24x^2 + 48x^2 - 18x$

or $y' = 72x^2 - 18x$

c) $y = (2x + 1)(4x^2 - 4x + 1)$

$(2x+1)(8x-4) + (2)(4x^2 - 4x + 1)$

$16x^2 \cancel{-8x} \cancel{+8x} - 4 + 8x^2 - 8x + 2$

or $y' = 24x^2 - 8x - 2$

d) $y = (3x^3 - 2x^2)(3x^3 + 2x^2)$

$(3x^3 - 2x^2)(9x^2 + 4x) + (9x^2 - 4x)(3x^3 + 2x^2)$

* note: DON'T SIMPLIFY

2. Find $\frac{dy}{dx}$ at the given value of x.

a) $y = (2 + 7x)(x - 3);\ x = 2$

$\frac{dy}{dx} = 9$

b) $y = (1 + 2x)(1 - 2x);\ x = \frac{1}{2}$

$\frac{dy}{dx} = -4$

$\frac{BT' - B'T}{B^2}$

3. *Use the **Quotient Rule** to find the first derivative.*

a) $y = \dfrac{x^2}{2x+1}$

$$\frac{(2x+1)(2x) - (2)(x^2)}{(2x+1)^2}$$

or

Simplified $y' = \dfrac{2x(x+1)}{(2x+1)^2}$

b) $y = \dfrac{4x^2}{1-6x^3}$

$$\frac{(1-6x^3)(8x) - (-18x^2)(4x^2)}{(1-6x^3)^2}$$

or

$$y' = \frac{8x(3x^3+1)}{(1-6x^3)^2}$$

c) $y = \dfrac{x^2-4x}{x+2}$

$$\frac{(x+2)(2x-4) - (1)(x^2-4x)}{(x+2)^2}$$

or

$$y' = \frac{x^2+4x-8}{(x+2)^2}$$

d) $y = \dfrac{x^2-9}{x^2+9}$

$$\frac{(x^2+9)(2x) - (2x)(x^2-9)}{(x^2+9)^2}$$

or

$$y' = \frac{36x}{(x^2+9)^2}$$

e) $y = \dfrac{x^3}{8-x^3}$

$$\frac{(8-x^3)(3x^2) - (-3x^2)(x^3)}{(8-x^3)^2}$$

or

$$y' = \frac{24x^2}{(8-x^3)^2}$$

f) $y = \dfrac{4-x^2}{3x}$

$$\frac{(3x)(-2x) - (3)(4-x^2)}{(3x)^2}$$

or

$$y' = \frac{-3(x^2+4)}{9x^2}$$

4. *Find* $\frac{dy}{dx}$ *at the given value of* x.

a) $y = \frac{x+1}{2x^2-1}$, $x = 0$

$\frac{dy}{dx} = -1$

b) $y = \frac{x^2-1}{x^2+1}$, $x = 1$

$\frac{dy}{dx} = 1$

c) $y = \frac{x^3}{8-x^3}$, $x = -1$

$\frac{dy}{dx} = \frac{3}{27}$

d) $y = \frac{2+x^2}{3x}$, $x = -2$

$\frac{dy}{dx} = \frac{1}{6}$

5. *Use the **Chain Rule** to find the first derivative.*

a) $y = (6x^2)^5$

$5(6x^2)^4(12x)$

or

$y' = 60x(6x^2)^4$

b) $y = (-3x^4)^5 + 6x^2 - 7x$

$5(-3x^4)^4(-12x^3) + 12x - 7$

or

$y' = -60x^3(-3x^4)^4 + 12x - 7$

c) $y = (p^2 - 3p + 1)^4$

$4(p^2 - 3p + 1)^3(2p - 3)$

or

$y' = (8p - 12)(p^2 - 3p + 1)^3$

★ DON'T FORGET THIS BRACKET

d) $y = (x^2 - 1)^3(2x - 1)^4$

F S

$(x^2-1)^3 \cdot 4(2x-1)^3(2) + 3(x^2-1)^2(2x)(2x-1)^4$

or

$y' = 8(x^2-1)^3(2x-1)^3 + 6x(x^2-1)^2(2x-1)^4$

e) $y = \dfrac{6x}{(x^2+1)^4}$ $\dfrac{BT' - B'T}{B^2}$

$$\frac{(x^2+1)^4(6) - 4(x^2+1)^3(2x)(6x)}{(x^2+1)^8}$$

or

$y' = 6(x^2+1)^4 - 48x^2(x^2+1)^3$

f) $y = (2x^4 + 8)^{\frac{1}{2}}$

$\frac{1}{2}(2x^4+8)^{-1/2}(8x^3)$

or

$y' = \dfrac{4x^3}{(2x^4+8)^{1/2}}$ or $\dfrac{4x^3}{\sqrt{(2x^4+8)}}$

g) $y = (3t^4 - 2t)^{\frac{1}{4}}$

$\frac{1}{4}(3t^4 - 2t)^{-3/4}(12t^3 - 2)$

or

$y' = \dfrac{6t^3 - 1}{2(3t^4 - 2t)^{3/4}}$

h) $y = \sqrt{5x + 7}$ RW $(5x+7)^{1/2}$

$\frac{1}{2}(5x+7)^{-1/2}(5)$

or

$y' = \dfrac{5}{2(5x+7)^{1/2}}$

6. *Use the* ***Chain Rule*** *to find the first derivative. **Continued*

$i)\ y = \dfrac{1}{\sqrt{4+t^2}}$ RW $(4+t^2)^{-1/2}$

$-\frac{1}{2}(4+t^2)^{-3/2}(2t)$

or

$\dfrac{-t}{(4+t^2)^{3/2}}$

$j)\ y = \left(1 + u^{\frac{1}{3}}\right)^6$

$6(1+u^{1/3})^5(\frac{1}{3}u^{-2/3})$

or

$\dfrac{2(1+u^{1/3})^5}{u^{2/3}}$

$k)\ y = (1 + \sqrt[3]{u})^6$ RW $(1+u^{1/3})^6$

$6(1+u^{1/3})^5(\frac{1}{3}u^{-2/3})$

or

$\dfrac{2(1+u^{1/3})^5}{u^{2/3}}$

$l)\ y = \left(1 + \frac{1}{\sqrt[3]{x}}\right)^6$ RW $(1+x^{-1/3})^6$

$6(1+x^{-1/3})^5(-\frac{1}{3}x^{-4/3})$

or

$\dfrac{-2(1+x^{-1/3})^5}{x^{4/3}}$

$m)\ y = (\pi x)^3 + 2\pi^2 x + 6\pi x$

$3(\pi x)^2(\pi) + 2\pi^2 + 6\pi$

or

$\pi\,(3(\pi x)^2 + 2\pi + 6)$

$n)\ y = (2x^3 + x)^4$

$4(2x^3+x)^3(6x^2+1)$

or

$(24x^2+4)(2x^3+x)^3$

$o)\ y = \dfrac{6\pi x}{(x^3-\pi)^2}$ Bt

$\dfrac{(x^3-\pi)^2(6\pi) - 2(x^3-\pi)(3x^2)(6\pi x)}{(x^3-\pi)^4}$

$p)\ y = \underset{f}{4x^2}\underset{s}{(2x-5)^3}$ FS

$(4x^2)(3(2x-5)^2(2)) + (8x)(2x-5)^3$

7. *Find the first derivative of each expression below.*

a) $y = \pi x + (5\pi x)^3$

$\pi + 3(5\pi x)^2(5\pi)$

or

$\pi\left(1 + 15(5\pi x^2)^2\right)$

b) $y = (1 - x + 2x^2 - 3x^3)^4$

$4(1-x+2x^2-3x^3)^3(-1+4x-9x^2)$

or

$(-36x^2+16x-4)(1-x+2x^2-3x^3)^3$

c) $y = ((2x)^4 + (16 - x)^3)^2$

$2\left((2x)^4+(16-x)^3\right)\left(4(2x)^3(2)+3(16-x)^2(-1)\right)$

d) $y = \dfrac{(2x-1)^2}{(x-2)^3}$

$\dfrac{(x-2)^3\left(2(2x-1)(2)\right)-3(x-2)^2(1)(2x-1)^2}{(x-2)^6}$

or

$\dfrac{(8x-4)(x-2)-3(2x-1)^2}{(x-2)^4}$

e) $y = (2x - 1)^{-3}$

$-3(2x-1)^{-4}(2)$

or

$\dfrac{-6}{(2x-1)^4}$

f) $y = \dfrac{\pi x}{(x^3 - \pi)^2}$

$\dfrac{(x^3-\pi)^2(\pi)-2(x^3-\pi)(3x^2)(\pi x)}{(x^3-\pi)^4}$

or

$\dfrac{\pi(x^3-\pi)-6x^3\pi}{(x^3-\pi)^3}$

g) $y = \sqrt{x}(1 - 2x)^5$

$x^{1/2}\left(5(1-2x)^4(-2)\right)+\frac{1}{2}\left(x^{-1/2}\right)(1-2x)^5$

h) $y = \left(\dfrac{x^2-1}{x^2+1}\right)^2$

$2\left(\dfrac{x^2-1}{x^2+1}\right)\cdot\dfrac{(x^2+1)(2x)-(2x)(x^2-1)}{(x^2+1)^2}$

8. Use the CHAIN RULE to find $\frac{dy}{dx}$ at the indicated value x:

a) $y = 2u^2 + 5$ $\quad u = 3x \quad x = 1$

$= 36$

b) $y = \frac{5}{u+2}$ $\quad u = 3x - 2 \quad x = 1$

$= \frac{-5}{3}$

c) $y = \sqrt{u^2 + 3}$ $\quad u = 2x^2 - 1 \quad x = 1$

$= 2$

d) $y = 2u^2$ $\quad u = 3v \quad v = 2x + 1 \quad x = 0$

$= 72$

e) $y = 4u^3 - 3u^2$ $\quad u = 2v^2 + 4v \quad v = 1 - 2x^2 \quad x = -1$

$= 0$

EXTENDED QUESTIONS

Find the derivative of each of the following functions and simplify.

1. $f(x) = 4x^3 - 3x^2 + 2x - \pi$

   $f'(x) = 12x^2 - 6x + 2$

2. $f(x) = \frac{x^2}{3} - \frac{3}{x^2}$ $\quad f'(x) = \frac{2}{3}x + \frac{6}{x^2} \Rightarrow \frac{2x^3+18}{3x^2}$

3. $f(x) = -3(2x^2 - 5x + 1)$

   $f'(x) = -12x + 15$

4. $f(x) = \sqrt{x} - \frac{1}{\sqrt{x}}$ $\quad f'(x) = \frac{1}{2\sqrt{x}} + \frac{1}{2x\sqrt{x}} \Rightarrow \frac{x+1}{2x\sqrt{x}}$

5. $f(x) = \frac{x+1}{x-2}$ $\quad f'(x) = \frac{-3}{(x-2)^2}$

6. $f(x) = \frac{x^2-2}{x^2}$ $\quad f'(x) = \frac{4}{x^3}$

7. $f(x) = \frac{x^2}{x^2-2}$ $\quad f'(x) = \frac{-4x}{(x^2-2)^2}$

8. $f(x) = \sqrt{x}(x^2+1)$ $\quad f'(x) = \frac{5x\sqrt{x}}{2} + \frac{1}{2\sqrt{x}} \Rightarrow \frac{5x^2+2}{2\sqrt{x}}$

10. $f(x) = \frac{2}{\sqrt{x}} + \frac{\sqrt{x}}{2}$ $\quad f'(x) = \frac{-1}{x\sqrt{x}} + \frac{1}{4\sqrt{x}} \Rightarrow \frac{x-4}{4x\sqrt{x}}$

11. $f(x)=\frac{2x}{x-1}$ $\quad f'(x) = \frac{-2}{(x-1)^2}$

12. $f(x)=(3x-2)(2x+1)$ $\quad f'(x) = 12x-1$

13. $y=5x^2-5\sqrt{x}-\frac{3}{x}$ $\quad y' = 10x - \frac{5}{2\sqrt{x}} + \frac{3}{x^2} \Rightarrow \frac{20x^3\sqrt{x} - 5x^2 + 6\sqrt{x}}{2x^2\sqrt{x}}$

14. $y=\frac{\sqrt{x}}{\sqrt{x}-1}$ $\quad y' = \frac{-1}{2\sqrt{x}(\sqrt{x}-1)^2}$

16. $y=6x^{\frac{-3}{2}}+7x^{\frac{1}{5}}+1$ $\quad y' = -9x^{-5/2} + \frac{7}{5}x^{-4/5} \Rightarrow \frac{-9}{x^{5/2}} + \frac{7}{5x^{4/5}}$

17. $y=\frac{-7}{1-x^3}$ $\quad y' = \frac{-21x^2}{(1-x^3)^2}$

18. $y=\frac{4}{3}x^{\left(\frac{3}{4}-\pi\right)}$ $\quad y' = \left(1-\frac{4}{3}\pi\right)x^{-1/4-\pi}$

19. $y=\frac{1}{7x}$ $\quad y' = \frac{-1}{7x^2}$

1.3 - "Simplifying Completely" using Chain Rule

$$y=\frac{6x}{\left(x^2+1\right)^4} \qquad\qquad y=\frac{\left(x+4\right)^{\frac{1}{2}}}{\left(x-4\right)^{\frac{1}{2}}}$$

1.3 - PRACTICE QUESTIONS

Find the derivative for each of the following and simplify.

1. $g(t)=\frac{(t+3)^4}{\left(t^2+5\right)^{1/2}}$

$$\frac{(t^2+5)^{1/2}\cdot 4(t+3)^3-\frac{1}{2}(t^2-5)^{-1/2}(2t)(t+3)^4}{(t^2+5)^1}$$

tidy up

$$\frac{4(t^2+5)^{1/2}(t+3)^3-t(t^2+5)^{-1/2}(t+3)^4}{(t^2+5)^1}$$

Factor

$$\frac{(t^2+5)^{-1/2}(t+3)^3\left[4(t^2+5)-t(t+3)\right]}{(t^2+5)^1}$$

$$\frac{(t+3)^3\left[4t^2+20-t^2-3t\right]}{(t^2+5)^{1/2}(t^2+5)^1} \Rightarrow \boxed{\frac{(t+3)^3\left[3t^2-3t+20\right]}{(t^2+5)^{3/2}}}$$

2. $f(x)=x^4(5x-1)^3$

$$x^4\cdot 3(5x-1)^2(5)+4x^3(5x-1)^3$$

$$15x^4(5x-1)^2+4x^3(5x-1)^3$$

$$x^3(5x-1)^2\left[15x+4(5x-1)\right]$$

$$x^3(5x-1)^2\left[15x+20x-4\right] \Rightarrow \boxed{x^3(5x-1)^2\left[35x-4\right]}$$

3. $y = x^2\sqrt{x^3+1}$ RW $\overset{f}{x^2} \cdot \overset{s}{(x^3+1)^{1/2}}$

$(x^2)\left(\frac{1}{2}(x^3+1)^{-1/2}(3x^2)\right) + 2x(x^3+1)^{1/2}$ Tidy up

$\frac{3}{2}x^4(x^3+1)^{-1/2} + 2x(x^3+1)^{1/2}$ Factor

$x(x^3+1)^{-1/2}\left[\frac{3x^3}{2} + \frac{2(x^3+1)}{1}\right]$

$x(x^3+1)^{-1/2}\left[\frac{3x^3+4(x^3+1)}{2}\right]$

$x(x^3+1)^{-1/2}\left[\frac{3x^3+4x^3+4}{2}\right]$

$\frac{x(7x^3+4)}{2(x^3+1)^{1/2}} \Rightarrow \boxed{\frac{7x^4+4x}{2(x^3+1)^{1/2}}}$

4. $f(x) = \frac{4x^4-4x^2+5}{2x^{5/3}+3}$ BT

$\frac{\left[(2x^{5/3}+3)(16x^3-8x)\right] - \left[\left(\frac{10}{3}x^{2/3}\right)(4x^4-4x^2+5)\right]}{(2x^{5/3}+3)^2}$

distribute

$\frac{32x^{14/3} - 16x^{8/3} + 48x^3 - 24x - \frac{40}{3}x^{14/3} + \frac{40}{3}x^{8/3} - \frac{50}{3}x^{2/3}}{(2x^{5/3}+3)^2}$

Collect like Term

$3\left[\frac{\frac{56}{3}x^{14/3} - \frac{8}{3}x^{8/3} - 24x + 48x^3 - \frac{50}{3}x^{2/3}}{(2x^{5/3}+3)^2}\right]$

now multiply top & bottom by 3 to get rid of Complex fractions

$\Rightarrow \boxed{\frac{56x^{14/3} - 8x^{8/3} - 72x + 144x^3 - 50x^{2/3}}{3(2x^{5/3}+3)^2}}$

Find $f'(x)$ and $f''(x)$, simplified for the following.

1. $f(x)=\dfrac{1}{x^2+1}$

$$f'(x) = \frac{-2x}{(x^2+1)^2}$$

$$f''(x) = \frac{2(3x^2-1)}{(x^2+1)^3}$$

2. $f(x)=\sqrt{4x-x^2}$

$$f'(x) = \frac{2-x}{(4x-x^2)^{1/2}} \quad \text{or} \quad \frac{2-x}{\sqrt{4x-x^2}}$$

$$f''(x) = \frac{-4}{(4x-x^2)^{3/2}} \quad \text{or} \quad \frac{-4}{(\sqrt{4-x^2})^3}$$

1.4 - PRACTICE QUESTIONS

1. *Each position function below describes motion in a straight line. Find the velocity and acceleration as functions of time (t).*

a) $s(t) = 5t^2 - 2t + 7$

$v(t) = 10t - 2$

$a(t) = 10$

b) $s(t) = 4t^4 - \frac{1}{2}t^2 + 3$

$v(t) = 16t^3 - t$

$a(t) = 48t^2 - 1$

c) $s(t) = 6t - 8$

$v(t) = 6$

$a(t) = 0$

d) $s(t) = t - 8 + \frac{6}{t}$

$v(t) = 1 - 6t^{-2}$

$a(t) = 12t^{-3}$

e) $s(t) = t(t-3)^2$

RW $\Rightarrow$ $s(t) = t^3 - 6t^2 + 9t$

$v(t) = 3t^2 - 12t + 9$

$a(t) = 6t - 12$

f) $s(t) = t + \frac{4t}{t+2}$

$v(t) = 1 + \frac{8}{(t+2)^2}$

$a(t) = \frac{-16}{(x+2)^3}$

1.5 - PRACTICE QUESTIONS

note : use y' inplace of $\frac{dy}{dx}$, easier for isolating

1. Use ***IMPLICIT DIFFERENTIATION*** *to find* $\frac{dy}{dx}$ *in terms of x and y.*

a) $4x^2 + y^2 = 8$ find derivative

$8x + 2yy' = 0$ isolate y'

$\frac{2y y'}{2y} = \frac{-8x}{2y}$

$y' = \frac{dy}{dx} = \frac{-4x}{y}$

b) $3x - 4y^2 = 2$

$\frac{dy}{dx} = \frac{3}{8y}$

c) $x^2 + 5y^2 + y = 10$

$\frac{dy}{dx} = \frac{-2x}{10y+1}$

d) $xy^2 = 4$

$\frac{dy}{dx} = \frac{-y}{2x}$

e) $x^2 + 2\overset{f\,s}{xy} - y^2 = 13$

$\frac{dy}{dx} = \frac{-x-y}{x-y}$

f) $y^3 + y = 4x$

$\frac{dy}{dx} = \frac{4}{3y^2+1}$

g) $y(x^2 + 3) = y^4 + 1$

$\frac{dy}{dx} = \frac{2xy}{4y^3 - x^2 - 3}$

h) $xy^3 + x^3y = 2$

$\frac{dy}{dx} = \frac{-y^3 - 3x^2y}{3xy^2 + x^3}$

Note: now use $\frac{dx}{dt}$ and $\frac{dy}{dt}$ $\Rightarrow$ with respect to time

2. *If* $x^2+y^2=8$ *and* $\frac{dx}{dt}=3$, *find* $\frac{dy}{dt}$ at (-2, 2).

$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$$

$$2(-2)(3)+2(2)\frac{dy}{dt}=0$$

$$-12+4\frac{dy}{dt}=0$$

$$\frac{\cancel{4}dy}{\cancel{4}dt}=12 \Rightarrow \frac{dy}{dt}=3$$

3. *If* $x^2+y^2=z^2$ *and* $\frac{dx}{dt}=-2$, $\frac{dy}{dt}=-1$, $x=1$ *and* $y=-3$, *find* $\frac{dz}{dt}$.

$$\frac{dz}{dt}=\frac{1}{\sqrt{10}} \quad \text{or} \quad \frac{\sqrt{10}}{10}$$

***BE SURE TO DO MULTIPLE CHOICE QUESTIONS ONLINE #179 – 189
https://moodle.sd79.bc.ca/pluginfile.php/1871/mod_resource/content/5/AB%20Calculus%20Version%207.pdf

EXTENDED QUESTIONS

For each problem, use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y.

1) $2x^3=2y^2+5$

$$\frac{dy}{dx}=\frac{3x^2}{2y}$$

2) $3x^2+3y^2=2$

$$\frac{dy}{dx}=\frac{-x}{y}$$

3) $5y^2=2x^3-5y$

$$\frac{dy}{dx}=\frac{6x^2}{10y+5}$$

4) $4x^2=2y^3+4y$

$$\frac{dy}{dx}=\frac{4x}{3y^2+2}$$

5) $5x^3 = -3xy + 2$

$$\frac{dy}{dx} = \frac{-y - 5x^2}{x}$$

6) $1 = 3x + 2x^2y^2$

$$\frac{dy}{dx} = \frac{-3 - 4xy^2}{4x^2y}$$

7) $3x^2y^2 = 4x^2 - 4xy$

$$\frac{dy}{dx} = \frac{4x - 2y - 3xy^2}{3x^2y + 2x}$$

8) $5x^3 + xy^2 = 5x^3y^3$

$$\frac{dy}{dx} = \frac{15x^2y^3 - 15x^2 - y^2}{2xy - 15x^3y^2}$$

9) $2x^3 = (3xy + 1)^2$

$$\frac{dy}{dx} = \frac{-3xy^2 - y + x^2}{3x^2y + x}$$

10) $x^2 = (4x^2y^3 + 1)^2$

$$\frac{dy}{dx} = \frac{-32x^2y^6 - 8y^3 + 1}{48x^3y^5 + 12xy^2}$$

11) $\sin 2x^2y^3 = 3x^3 + 1$

$$\frac{dy}{dx} = \frac{9x - 4y^3 \cos 2x^2y^3}{6xy^2 \cos 2x^2y^3}$$

12) $3x^2 + 3 = \ln 5xy^2$

$$\frac{dy}{dx} = \frac{6x^2y - y}{2x}$$

For each problem, use implicit differentiation to find $\frac{d^2y}{dx^2}$ in terms of x and y.

13) $4y^2 + 2 = 3x^2$

$$\frac{d^2y}{dx^2} = \frac{12y^2 - 9x^2}{16y^3}$$

14) $5 = 4x^2 + 5y^2$

$$\frac{d^2y}{dx^2} = \frac{-20y^2 - 16x^2}{25y^3}$$

1.6 - PRACTICE QUESTIONS

1. If A is the area of a circle of radius r, find $\frac{dA}{dt}$ in terms of $\frac{dr}{dt}$.

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

2. The area of a circular oil slick on the surface of the sea is increasing at the rate of 150 m^2/s. How fast is the radius changing when:

$A = \pi r^2$

a) the radius 25 m.

$150 = 2\pi(25)\frac{dr}{dt} \Rightarrow \frac{150}{50\pi} = \frac{50\pi}{50\pi}\frac{dr}{dt}$

$\frac{dr}{dt} = \frac{3}{\pi}$ or $0.955\ m/s$

b) the area is 1000 m^2

first find radius

$1000 = \pi r^2$

$r = \frac{10\sqrt{10\pi}}{\pi}$ or 17.841

EXACT

$150 = 2\pi\left(\frac{10\sqrt{10\pi}}{\pi}\right)\frac{dr}{dt}$

$\frac{150}{20\sqrt{10\pi}} = \frac{20\sqrt{10\pi}}{20\sqrt{10\pi}}\frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{15}{2\sqrt{10\pi}}$ or $1.338\ m/s$

EXACT

3. How fast is the side of a square shrinking when the length of the side is 2 m and the area is decreasing at 0.25 m^2/s ?.

$\frac{ds}{dt} = \frac{-1}{16}\ m/s$

4. The hypotenuse of a right triangle is of fixed length but the lengths of the other two sides x and y depend on time. How fast is y changing when $\frac{dx}{dt} = 4$ and $x = 8$ if the length of the hypotenuse is 17?

$\frac{dy}{dt} = \frac{-32}{15}$

5. A spherical balloon is inflated so that the volume is increasing at the rate of 5 m^3/min.

a) at what rate is the diameter increasing when the radius is 6 m?

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$5 = 4\pi(6)^2 \frac{dr}{dt}$$

$$\frac{5}{144\pi} = \frac{\cancel{144\pi}}{\cancel{144\pi}} \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{5}{144\pi} \times 2$$

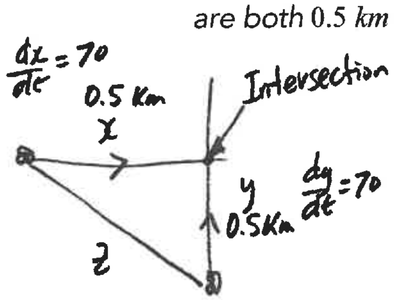
diameter

$$\Rightarrow \frac{dd}{dt} = \frac{5}{72\pi} \text{ or } 0.022 \text{ m/min}$$

b) at what rate is the diameter increasing when the volume is 36 m^3?

$$\frac{dd}{dt} = 0.189 \text{ m/min}$$

6. Two cars approach an intersection, one traveling east and the other north. If both cars are traveling at 70 km/h, how fast are they approaching each other when they are both 0.5 km from the intersection?



$$x^2 + y^2 = z^2$$

$$(0.5)^2 + (0.5)^2 = z^2$$

$$z = \frac{\sqrt{2}}{2}$$

$$x^2 + y^2 = z^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

$$2(0.5)(70) + 2(0.5)(70) = \cancel{2}\left(\frac{\sqrt{2}}{\cancel{2}}\right)\frac{dz}{dt}$$

$$\frac{140}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}}\frac{dz}{dt}$$

$$\frac{dz}{dt} = 70\sqrt{2} \text{ Km/hr}$$

EXTENDED QUESTIONS

1) A spherical snowball melts in such a way that the instant at which its radius is 20 cm, its radius is decreasing at 3 cm/min. At what rate is the volume of the ball of snow changing at that instant?

$V = \frac{4}{3}\pi r^3$

$\frac{dr}{dt} = -3 \quad @\, r = 20$

$\frac{dv}{dt} = ?$

$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dv}{dt} = 4\pi(20)^2(-3) = -4800\pi$ or decreasing at $4800\pi\ cm^3/min$

2) A spherical snowball is melting. Its radius decreases at a constant rate of 2 cm per minute from an initial value of 70 cm. How fast is the volume decreasing half an hour later?

$V = \frac{4}{3}\pi r^3$

$\frac{dr}{dt} = -2$

$r = 70 - 2m$

$r = 70 - 2(30) \therefore r = 10$

$\frac{dv}{dt} = ?$

$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dv}{dt} = 4\pi(10)^2(-2) = -800\pi$ or decreasing at $800\pi\ cm^3/min$

3) A 3 meter ladder stands against a wall. The foot of the ladder moves outward at a speed of .1 meters/sec. when the foot is 1 meter from the wall. At that moment, how fast is the top of the ladder falling? What if the foot has been 2 meters from the wall?

$x^2 + y^2 = 3^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2y\frac{dy}{dt} = -2x\frac{dx}{dt}$

$\frac{dx}{dt} = 0.1$

$\frac{dy}{dt} = ?$ a) when $x = 1 \therefore y = \sqrt{9-1} = \sqrt{8}$

b) when $x = 2 \therefore y = \sqrt{9-4} = \sqrt{5}$

a) $2(\sqrt{8})\frac{dy}{dt} = -2(1)(0.1)$

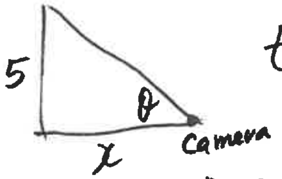
$\frac{\cancel{2\sqrt{8}}}{\cancel{2\sqrt{8}}}\frac{dy}{dt} = \frac{-0.2}{2\sqrt{8}}$

a) $\frac{dy}{dt} = \frac{-\sqrt{2}}{40}$ m/sec

b) $2\sqrt{5}\frac{dy}{dt} = -2(2)(.1)$

$\frac{dy}{dt} = \frac{0.2}{\sqrt{5}} = \frac{\sqrt{5}}{25}$ m/sec

4) An airplane flying at 450 km/hr at a constant altitude of 5 km, is approaching a camera mounted on the ground. Let θ be the angle of elevation above the ground at which the camera is pointed. When $\theta = \pi/3$, how fast does the camera have to rotate in order to keep the plane in view?



$\tan\theta = \frac{5}{x}$

$\sec^2\theta \frac{d\theta}{dt} = \frac{-5}{x^2}\frac{dx}{dt}$

$\theta = \pi/3$

$\frac{d\theta}{dt} = ?$

$\frac{H}{A}$

$(2)^2\frac{d\theta}{dt} = \frac{-5}{\left(\frac{5}{\sqrt{3}}\right)^2}(-450)$

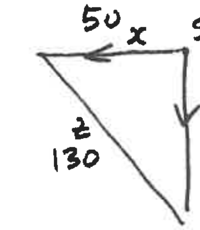
$x \Rightarrow \tan\theta = \frac{5}{x}$

$\sqrt{3} = \frac{5}{x} \Rightarrow x = \frac{5}{\sqrt{3}}$

$\frac{d\theta}{dt} = 67.5$ rad/hr

dx

5) Two cars start moving from the same point. One travels south at 60 mph and the other travels west at 25 mph. At what rate is the distance between the cars increasing two hours later?



$\frac{dx}{dt} = 25$

$\frac{dy}{dt} = 60$

This is distance between the cars → $\frac{dz}{dt} = ?$

2 hrs later. $\therefore x = 50$, $y = 120$

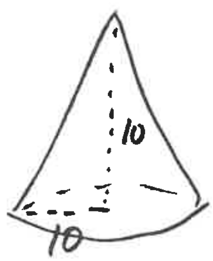
$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$2(50)(25) + 2(120)(60) = 2(130)\frac{dz}{dt}$

$\frac{dz}{dt} = 65$ mph

6) Gravel is being dumped from a conveyor belt at a rate of 30 ft³/min and its coarseness is such that it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 10 ft high?



$\frac{dV}{dt} = 30$

$\frac{dh}{dt} = ?$

$d = h$

$2r = h$

$r = \frac{h}{2}$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$

$V = \frac{\pi}{12} h^3$

$\frac{dV}{dt} = \frac{\pi}{4} h^2 \frac{dh}{dt}$

$30 = \frac{\pi}{4}(10)^2 \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{6}{5\pi}$ ft/min

or

0.382 ft/min

8. A hot-air balloon rising straight up from a level field is tracked by a range finder 500 ft from the lift-off point. At the moment the range finder's elevation angle is $\frac{\pi}{4}$, the angle is increasing at the rate of 0.14 rad/min. How fast is the balloon rising at that moment?

$\frac{d\theta}{dt} = 0.14$

$\sec\frac{\pi}{4} = \sqrt{2}$

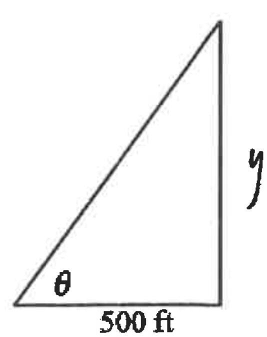
$\frac{dy}{dt} = ?$

$\tan\theta = \frac{y}{500}$

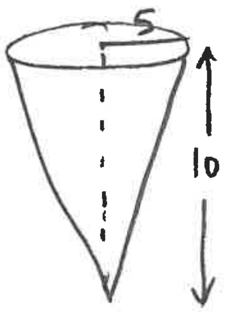
$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{500}\frac{dy}{dt}$

$(\sqrt{2})^2(0.14) = \frac{1}{500}\frac{dy}{dt}$

$\frac{dy}{dt} = 140$ ft/min



9. Water runs into a conical tank at the rate of 9 ft³/min. The tank stands point down and has a height of 10ft and a base radius of 5ft. How fast is the water level rising when the water is 6ft deep?



$$\frac{dv}{dt} = 9$$

$$\frac{dh}{dt} = ? \text{ at } 6$$

$$r = \frac{1}{2}h$$

$$\frac{r}{h} = \frac{5}{10}$$

$$10r = 5h$$

$$r = \frac{1}{2}h$$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi \left(\frac{1}{2}h\right)^2 h$$

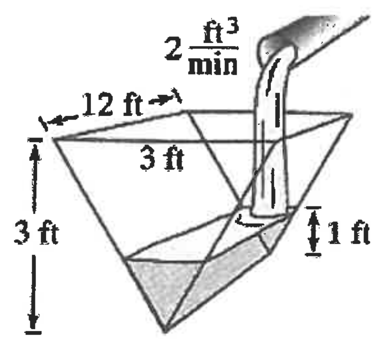
$$V = \frac{\pi}{12} h^3$$

$$\frac{dv}{dt} = \frac{\pi}{4} h^2 \frac{dh}{dt}$$

$$9 = \frac{\pi}{4}(6)^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{1}{\pi} \text{ ft/min}$$

10. A trough 3ft wide and 12ft long is being filled at a rate of 2 cubic feet per minute. The ends of the trough are isosceles triangles with altitudes 3ft. How fast is the water level rising when the depth is 1ft?



$$\frac{dv}{dt} = 2$$

$$L = 12$$

$$b = h$$

$$\frac{dh}{dt} = ? \text{ at } h = 1$$

* h = depth

$$V = \frac{1}{2} bhL$$

$$V = \frac{1}{2} bh(12)$$

$$V = 6h \cdot h$$

$$V = 6h^2$$

$$\frac{dv}{dt} = 12h \frac{dh}{dt}$$

$$2 = 12(1) \cdot \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{1}{6} \text{ ft/min}$$

You Try: Find the derivative of the following.

$y = \sin^4(3x^2 - 5x)$

$y = 8\csc(4x^3)$

$y = x^3 \cos(x)$

1.7 - PRACTICE QUESTIONS

1. *Find* $\frac{dy}{dx}$:

a) $y = 3\cos(4x)$

$-3\sin(4x)(4)$

or

$-12\sin(4x)$

b) $y = \cos\left(3x + \frac{\pi}{2}\right)$

$-\sin\left(3x + \frac{\pi}{2}\right)(3)$

or

$-3\sin\left(3x + \frac{\pi}{2}\right)$

c) $y = \cos(2x^3)$

$-\sin(2x^3)(6x^2)$

or

$-6x^2\sin(2x^3)$

d) $y = \cos^3(2x)$ RW $(\cos(2x))^3$

$3(\cos(2x))^2(-\sin(2x))(2))$

or

$-6\cos^2(2x)\sin(2x)$

1. *Continued - Find* $\frac{dy}{dx}$:

e) $y = cos(x^2 + x)$

$-\text{Sin}(x^2+x)(2x+1)$

or

$-(2x+1)\text{Sin}(x^2+x)$

f) $y = (x + \cos(x))^2$

$2(x+\text{Cos}(x))'(1-\text{Sin}(x))$

or

$2(1-\text{Sin}(x))(x+\text{Cos}(x))$

g) $y = 2sin(\pi x) + x^2$

$2\text{Cos}(\pi x)(\pi)+2x$

or

$2\pi\text{Cos}(\pi x)+2x$

h) $y = 3sin(x^2 - 1)$

$3\text{Cos}(x^2-1)(2x)$

or

$6x\text{Cos}(x^2-1)$

i) $y = (\sin(2x) + \cos(x))^2$

$2[\text{Sin}(2x)+\text{Cos}(x))'(\text{Cos}(2x)(2)+-\text{Sin}(x)(1)]$

2. *Differentiate each function:*

a) $f(x) = x\cos(x)$

f s

$x(-\text{Sin}(x))+(1)(\text{Cos}(x))$

or

$-x\text{Sin}(x)+\text{Cos}(x)$

b) $g(x) = x^3\sin(2x)$

f s

$x^3(\text{Cos}(2x)(2)+3x^2(\text{Sin}(2x))$

or

$2x^3\text{Cos}(2x)+3x^2\text{Sin}(2x)$

2. Continued - Differentiate each function:

c) $k(x) = x^3 cos(3x^2)$
f s

$x^3(-\sin(3x^2)(6x) + 3x^2(\cos(3x^2))$

or

$-6x^4 \sin(3x^2) + 3x^2 \cos(3x^2)$

d) $h(x) = sin(\cos(\pi x))$

$\cos(\cos(\pi x))(-\sin(\pi x))(\pi)$

or

$-\pi \sin(\pi x) \cos(\cos(\pi x))$

e) $m(x) = \sin(x)\cos(x)$

$\sin(x)(-\sin(x)) + \cos(x)(\cos(x))$

or

$-\sin^2(x) + \cos^2(x)$

f) $p(x) = \dfrac{\sin(2x)}{cos(2x)}$ BT

$\dfrac{(\cos(2x))(\cos(2x))(2) - (-\sin(2x)(2))(\sin(2x))}{\cos^2(2x)}$

or

$\dfrac{2\cos^2(2x) + 2\sin^2(2x)}{\cos^2(2x)} = 2 + \dfrac{2\sin^2(2x)}{\cos^2(2x)}$

g) $g(x) = sin^2\left(x^{\frac{1}{2}}\right)$ RW $\left(\sin(x^{1/2})\right)^2$

$2(\sin(x^{1/2}))(\cos(x^{1/2}))(\tfrac{1}{2}x^{-1/2})$

or

$\dfrac{\sin(\sqrt{x})\cos(\sqrt{x})}{\sqrt{x}}$

h) $k(x) = \sin\left(\dfrac{1}{x}\right)$

RW $\sin(x^{-1})$

$\cos(x^{-1})(-x^{-2})$

or

$\dfrac{-\cos(\frac{1}{x})}{x^2}$

3. Find $\dfrac{dy}{dx}$ in each case where A, B, m and n are constants:

a) $y = \cos(Ax + B)$

$-\sin(Ax+B)(A)$ or

$-A\sin(Ax+B)$

b) $y = A\cos^n(Bx)$ RW $(A\cos(Bx))^n$

$n(A\cos(Bx))^{n-1}(-A\sin(Bx))(B)$

c) $y = \sin^m(x^n)$

$m(\sin(x^n))^{m-1}(\cos(x^n))(nx^{n-1})$

d) $y = Ax^n \sin^m(Bx)$
f s

$(Ax^n)(m(\sin(Bx))^{m-1}(\cos(Bx))(B) + (nAx^{n-1})(\sin^m(Bx))$

4. *Find* $\frac{dy}{dx}$ *in each case:*

a) $y = 2\tan x - \tan(2x)$

$2\sec^2(x) - \sec^2(2x)(2)$

b) $y = 3\sec(2x^2+1)$

$3\sec(2x^2+1)\tan(2x^2+1)(4x)$

c) $y = 3\sec(5x)$

$3\sec(5x)\tan(5x)(5)$

d) $y = \sqrt{x^2 + \sec^2 x}$

$\frac{1}{2}(x^2+\sec^2 x)^{-1/2}(2x + (2\sec x)(\sec x \tan x))$

e) $y = \frac{x^2}{\tan x}$

$\frac{(\tan x)(2x) - (\sec^2 x)(x^2)}{\tan^2 x}$

f) $y = \tan(x^2) - \tan^2 x$

$\sec^2(x^2)(2x) - (2\tan x)(\sec^2 x)$

g) $y = \sqrt{x}\csc\sqrt{x}$

$(x^{1/2})(-\csc x^{1/2}\cot x^{1/2})(\frac{1}{2}x^{-1/2}) + (\frac{1}{2}x^{-1/2})(\csc x^{1/2})$

h) $y = x^2 \tan\left(\frac{1}{x}\right)$

$(x^2)(\sec^2 x^{-1})(-1x^{-2}) + (2x)(\tan x^{-1})$

i) $y = \sin(\tan x)$

$\cos(\tan x)(\sec^2 x)$

5. *Find* $\frac{dy}{dx}$ *in each case. Watch for the need for Implicit Differentiation!*

a) $y = \cot(2x) + \csc(2x)$

$-\csc^2(2x)(2) + (-\csc(2x)\cot(2x)(2)$

b) $y = 2x^3\cot(x)$

$(2x^3)(-\csc^2 x) + (6x^2)(\cot x)$

5. Continued - Find $\frac{dy}{dx}$ in each case. *Watch for the need for Implicit Differentiation!*

c) $y = (x + \csc(x))^2$

$2(x+\csc(x))^{1}(1-\csc(x)\cot(x))$

d) $y = \sqrt{\pi^2 + csc^2(x)}$

$\frac{1}{2}(\pi^2+\csc^2 x)^{-1/2}(2\csc x)(-\csc x \cot x)$

e) $y = \frac{\cot(x)}{1 + csc^2(x)}$

$$\frac{(1+\csc^2 x)(-\csc^2 x)-(2\csc x)(-\csc x)(\cot x)(\cot x)}{(1+\csc^2 x)^2}$$

f) $y = \sqrt{x}\csc(x)$

$(x^{1/2})(-\csc x \cot x)+(\tfrac{1}{2}x^{-1/2})(\csc x)$

IMP

g) $y = \overset{fs}{\sin}(xy)$

$y' = \boxed{\cos(xy)}(xy'+y)$

$y' = xy'\square + y\square$

$y' - xy'\square = y\square$

$\frac{y'(1-x\square)}{1-x\square} = \frac{y\square}{1-x\square}$

$y' = \frac{y\cos(xy)}{1-x\cos(xy)}$

IMP

h) $y = \cot(x+y)$

$y' = \boxed{-\csc^2(x+y)}(1+y')$

$y' = 1\square + y'\square$

$y' - y'\square = \square$

$\frac{y'(1-\square)}{1-\square} = \frac{\square}{1-\square}$

$y' = \frac{-\csc^2(x+y)}{1+\csc^2(x+y)}$

1.8 - PRACTICE QUESTIONS

1. *Differentiate each function:*

a) $f(x)=5e^{2x}$

$5e^{2x}(2)$

or

$10e^{2x}$

b) $h(x)=2e^{x^2-x}$

$2e^{x^2-x}(2x-1)$

or

$(4x-2)e^{x^2-x}$

c) $k(x)=3e^{2\sin x}$

$3e^{2\sin x}(2\cos x)$

or

$6e^{2\sin x}\cos x$

d) $p(x)=x^2e^x$

+3

$x^2e^x+2xe^x$

e) $q(x)=x^2e^{-3x}$

$(x^2e^{-3x}(-3))+(2x)e^{-3x}$

or

$2xe^{-3x}-3x^2e^{-3x}$

f) $m(x)=\left(e^{2x}-e^{-2x}\right)^2$

$2[(e^{2x}-e^{-2x})((e^{2x})(2)-(e^{-2x})(-2))]$

or

$2(e^{2x}-e^{-2x})(2e^{2x}+2e^{-2x})$

g) $g(x)=\sqrt{x}\ e^{\sqrt{x}}$

$x^{1/2}e^{x^{1/2}}(\frac{1}{2}x^{-1/2})+\frac{1}{2}x^{-1/2}e^{x^{1/2}}$

h) $f(x)=\ln\left(\pi+e^{2x}\right)$

$\frac{1}{\pi+e^{2x}}(e^{2x})(2)$

i) $m(x)=\dfrac{e^{2x}}{1+e^{2x}}$

$\dfrac{(1+e^{2x})(2e^{2x})-(2e^{2x})(e^{2x})}{(1+e^{2x})^2}$

j) $g(x)=e^x\ln\sqrt{x}$

$(e^x)(\frac{1}{x^{1/2}})(\frac{1}{2}x^{-1/2})+(e^x)(\ln x^{1/2})$

k) $w(x)=\ln\left(e^x+e^{-x}\right)$

$(\frac{1}{e^x+e^{-x}})(e^x-e^{-x})$

l) $r(x)=\dfrac{e^x}{\ln x}$

$\dfrac{(\ln x)(e^x)-(\frac{1}{x})(e^x)}{(\ln x)^2}$

2. *If y defined implicitly as a function of x by the given equation, find* $\frac{dy}{dx}$:

a) $x + y\ln x = 2$

$1 + y\cdot\frac{1}{x} + y'\ln x = 0$

$\frac{y'\ln x}{\ln x} = \frac{-1-\frac{y}{x}}{\ln x}$

$y' = \frac{dy}{dx} = \frac{-1-\frac{y}{x}}{\ln x}$

$y' = \frac{-x-y}{x\ln x}$ or

b) $y - e^{xy} = 5$

$y' - \boxed{e^{xy}}(xy' + y) = 0$

$y' - xy'\Box - y\Box = 0$

$y' - xy'\Box = y\Box$

$\frac{y'(1 - x\Box)}{1 - x\Box} = \frac{y\Box}{1 - x\Box}$

$y' = \frac{dy}{dx} = \frac{ye^{xy}}{1 - xe^{xy}}$

c) $e^{\sin 2y} + 2x = 4y$

$e^{\sin 2y}(\cos 2y)(2y') + 2 = 4y'$

$2 = 4y' - 2y'e^{\sin 2y}(\cos 2y)$

$\frac{2}{4-2e^{\sin 2y}\cos 2y} = \frac{y'(4 - 2e^{\sin 2y}\cos 2y)}{4-2e^{\sin 2y}\cos 2y}$

$y' = \frac{dy}{dx} = \frac{2}{4 - 2e^{\sin 2y}\cos 2y}$ or $\frac{1}{2 - e^{\sin 2y}\cos 2y}$

3. *Find* $\frac{dy}{dx}$:

a) $y = \ln|x^2 - 1|$

$\frac{1}{|x^2-1|}(2x)$

b) $y = \ln|x^3 - 7x + 1|$

$\frac{1}{|x^3-7x+1|}(3x^2-7)$

c) $y = (\ln|x|)^3$

$3(\ln|x|)^2\left(\frac{1}{|x|}\right)$

d) $y = \ln|\tan x|$

$\frac{1}{|\tan x|}(\sec^2 x)$

e) $y = \cos x \ln|\cos x|$

$\cos x\left(\frac{1}{|\cos x|}\right)(-\sin x) + (-\sin x)(\ln|\cos x|)$

f) $y = \sin(\ln|x|)$

$\cos(\ln|x|)\left(\frac{1}{x}\right)$

***BE SURE TO DO MULTIPLE CHOICE QUESTIONS ONLINE **VERSION 7**.
#96, 98, 99, 100, 101,103, 105, 106, 109, 122, 141, 142, 143, 145
https://moodle.sd79.bc.ca/pluginfile.php/1871/mod_resource/content/5/AB%20Calculus%20Version%207.pdf

1.9 - PRACTICE QUESTIONS

1. *Differentiate each function:*

a) $y = 2^x$

$\ln(2)(2^x)$

b) $y = 10^{x^3}$

$\ln(10)(10^{x^3})(3x^2)$

c) $y = 2^{sinx}$

$\ln(2)(2^{\sin x})(\cos x)$

d) $y = \pi^{x^2}$

$\ln(\pi)(\pi^{x^2})(2x)$

e) $y = 3^{x^2+3x}$

$\ln(3)(3^{x^2+3x})(2x+3)$

f) $y = (2x^3)(3^{2x})$

$\quad\;\; f \quad s$

$(2x^3)(\ln(3)(3^{2x})(2)) + \ln(2)(2x^3)(3x^2)(3^{2x})$

2. *Differentiate each function with respect to x.*

a) $y = log_3(3x^2)$

$$\frac{6x}{3x^2 \ln(3)}$$

b) $y = log_2(4x^2)$

$$\frac{8x}{4x^2 \ln(2)}$$

c) $y = [log_3 (3x^5 + 5)]^5$

$$5[\log_3(3x^5+5)]^4 \cdot \frac{15x^4}{(3x^5+5)\ln 3}$$

d) $y = log_5 (-5x^3 - 2)^2$

RW

$2\log_5(-5x^3-2)$

$$\frac{2}{(-5x^3-2)\ln 5} \cdot (-15x^2)$$

or

$$\frac{-30x^2}{(-5x^3-2)\ln 5}$$

1.10 - PRACTICE QUESTIONS

1. *Two functions, $f(x)$ and $g(x)$, are continuous and differentiable for all real numbers. Some values of the functions and their derivatives are given in the following table.*

x	0	1	2	3	4
$f(x)$	$\frac{1}{2}$	$\frac{1}{3}$	1	-1	3
$g(x)$	-2	1	$-\frac{1}{2}$	2	$-\frac{1}{3}$
$f'(x)$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{1}{4}$	0	$-\frac{4}{5}$
$g'(x)$	-1	$\frac{2}{3}$	-4	-3	$-\frac{1}{3}$

Based on the table, calculate the following:

a) $\frac{d}{dx}(f(x)+g(x))$, evaluated at $x=4$

$f'(4)+g'(x)$

$\frac{-4}{5} + -\frac{1}{3}$

$= \frac{-17}{15}$

b) $\frac{d}{dx}(\overset{f}{f(x)}\overset{s}{g(x)})$, evaluated at $x=1$

$fs'+f's$

$f(1)\cdot g'(1) + f'(1)\cdot g(1)$

$\frac{1}{3}\cdot\frac{2}{3} + \frac{5}{3}\cdot(1)$

$\frac{2}{9} + \frac{5}{3}$

$= \frac{17}{9}$

c) $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)$, evaluated at $x=0$

$\frac{BT'-B'T}{B^2}$

$\frac{g(0)\cdot f'(0) - g'(0)\cdot f(0)}{(g(0))^2}$

$\frac{(-2)\cdot\frac{3}{2} - (-1)\cdot\frac{1}{2}}{(-2)^2}$

$= \frac{-5}{8}$

d) $\frac{d}{dx}(f(g(x)))$, evaluated at $x=3$ Chain Rule

$f'(g(3))\cdot g'(3)$

$f'(2)\cdot g'(3)$

$\frac{1}{4}\cdot -3$

$= \frac{-3}{4}$

2.1 - PRACTICE QUESTIONS

1. *Find the* ***GENERAL ANTIDERIVATIVE*** *of each of the following functions:*
   Verify your answers using differentiation.

a) $4x^3 - 3x^2$

$x^4 - x^3 + C$

b) $2x^2 - 8x + 6$

$\frac{2}{3}x^3 - 4x^2 + 6x + C$

c) $3x^5 + 4x^3 - 7$

$\frac{1}{2}x^6 + x^4 - 7x + C$

d) $x^4 + \frac{1}{x}$

$\frac{1}{5}x^5 + \ln x + C$

e) $x^4 + \frac{1}{x^2} + \ln 5 \; 5^x$

$\frac{1}{5}x^5 - x^{-1} + 5^x + C$

f) $\frac{3}{x^5} + 2x - 7$

$-\frac{3}{4}x^{-4} + x^2 - 7x + C$

g) $\frac{10}{x} + x^e$

$10 \ln x + \frac{1}{e+1}x^{e+1} + C$

h) $x^{\frac{2}{3}} - x^3$

$\frac{3}{5}x^{5/3} - \frac{1}{4}x^4 + C$

i) $2\cos x - 3x$

$2 \sin x - \frac{3}{2}x^2 + C$

j) $\cos(2x) + 4x$

$\frac{1}{2}\sin(2x) + 2x^2 + C$

k) $3\sqrt{x} - \frac{1}{\sqrt{x}}$

$2x^{3/2} - 2x^{1/2} + C$

l) $10e^x - 2e^{2x}$

$10e^x - e^{2x} + C$

m) $\frac{3}{x} - \frac{4}{x^2} + 2\pi$

$3 \ln x + 4x^{-1} + 2\pi x + C$

n) $\frac{x}{x^2+1} + 6$

$\frac{1}{2}\ln(x^2+1) + 6x + C$

o) $x^2 \cos(x^3)$

$\frac{1}{3}\sin(x^3) + C$

p) $2e^{\pi x} + x^{-2} - 5$

$\frac{2}{\pi}e^{\pi x} - x^{-1} - 5x + C$

q) $\frac{5x^2}{x^3 - 1}$

$\frac{5}{3}\ln(x^3 - 1) + C$

r) $e^{\sqrt{3}\,x} - x^2 e^{x^3}$

$\frac{1}{\sqrt{3}}e^{\sqrt{3x}} - \frac{1}{3}e^{x^3}$

2. If:

a) $\dfrac{dy}{dx} = 7e^{-2x}$, *then* $y =$ $-\frac{7}{2}e^{-2x} + c$

b) $\dfrac{dy}{dx} = 3x^2 + 5x - 2$, *then* $y =$ $x^3 + \frac{5}{2}x^2 - 2x + c$

c) $\dfrac{dy}{dT} = \sin T + \sin 2T + \sin 3T$, *then* $y =$ $-\cos T - \frac{1}{2}\cos 2T - \frac{1}{3}\cos 3T + c$

d) $\dfrac{dy}{du} = \dfrac{1}{u} - \dfrac{1}{u-1}$, *then* $y =$ $\ln u - \ln(u-1) + c$

e) $\dfrac{dp}{dx} = e^x + x^e$, *then* $p =$ $e^x + \frac{1}{e+1}x^{e+1} + c$

f) $\dfrac{df}{dt} = 5\sec^2 3t$, *then* $f =$ $\frac{5}{3}\tan 3t + c$

3. If $p = 4x^2 - 3x + 2$ find:

a) *the general antiderivative of* p

$\frac{4}{3}x^3 - \frac{3}{2}x^2 + 2x + c$

b) y *if* $\dfrac{dy}{dx} = p$ Same as a)

c) y *if* $y' = p$

Same as a)

d) y *if the first derivative of* $y = p$

Same as a)

4. Find:

a) the general antiderivative of $x^2 + x - 8$

$\frac{1}{3}x^3 + \frac{1}{2}x^2 - 8x + C$

b) if $\frac{dy}{dx} = x^2 + x - 8$ then $y =$

Same as a)

c) all the solutions to the differential equation $\frac{dy}{dx} = x^2 + x - 8$

Same as a)

d) the unique solution to $\frac{dy}{dx} = x^2 + x - 8$ satisfying the given initial condition y(0) = 6

$\frac{1}{3}x^3 + \frac{1}{2}x^2 - 8x + 6$

e) the equations of all curves y = f(x) whose tangent line has a slope of $x^2 + x - 8$

Same as a)

f) the rate of change/growth or the decay rate of change of y with respect to x is given by the expression $x^2 + x - 8$. Find an expression for y.

Same as a)

5. Find the position function s(t) for an object with velocity function v(t):

a) $v(t) = 2t^2 - 3t$

$s(t) = \frac{2}{3}t^3 - \frac{3}{2}t^2 + c$

b) $v(t) = t^3 + 4t + 6$

$s(t) = \frac{1}{4}t^4 + 2t^2 + 6t + c$

c) $v(t) = t^2 - 5t$

$s(t) = \frac{1}{3}t^3 - \frac{5}{2}t^2 + c$

6. *Find the position function s(t) for an object with velocity function v(t) and initial position s(0):*

a) $v(t) = 3t - t^2, \quad s(0) = 5$

$s(t) = \frac{3}{2}t^2 - \frac{1}{3}t^3 + 5$

b) $v(t) = 6t, \quad s(0) = 7$

$s(t) = 3t^2 + 7$

$v(t) = t^2 + 2t\ , \quad s(0) = 4$

$s(t) = \frac{1}{3}t^3 + t^2 + 4$

7. *Find the velocity function v(t) for an object with acceleration function a(t) and initial velocity v(0):*

a) $a(t) = 5, \ v(0) = 10$

$v(t) = 5t + 10$

b) $a(t) = t - 1, \ v(0) = 1$

$v(t) = \frac{1}{2}t^2 - t + 1$

c) $a(t) = t^2 + t, \ v(0) = 0$

$v(t) = \frac{1}{3}t^3 + \frac{1}{2}t^2$

8. *Find the position function s(t) for an object with acceleration function a(t), initial velocity v(0) and initial position s(0).*

a) $a(t) = 5, \ v(0) = 10$
$s(0) = 20$

$s(t) = \frac{5}{2}t^2 + 10t + 20$

b) $a(t) = t - 1, \ v(0) = 1$
$s(0) = 0$

$s(t) = \frac{1}{6}t^3 - \frac{1}{2}t^2 + t$

c) $a(t) = t^2 + t, \ v(0) = 0$
$s(0) = 0$

$s(t) = \frac{1}{12}t^4 + \frac{1}{6}t^3$

9. *Find each of the following* ***INDEFINITE INTEGRALS****. Check your answers by differentiation.*

a) $\int x^2\,dx$

$\frac{1}{3}x^3 + C$

b) $\int \pi\,dx$

$\pi x + C$

c) $\int x^{-3}\,dx$

$-\frac{1}{2}x^{-2} + C$

d) $\int 8x^3\,dx$

$2x^4 + C$

e) $\int \frac{1}{x-1}\,dx$

$\ln(x-1) + C$

f) $\int \frac{1}{1-x}\,dx$

$-\ln(1-x) + C$

g) $\int e^{4y}\,dy$

$\frac{1}{4}e^{4y} + C$

h) $\int \cos\pi y\,dy$

$\frac{1}{\pi}\sin\pi y + C$

i) $\int e^y + y^e\,dy$

$e^y + \frac{1}{e+1}y^{e+1} + C$

EXTENDED QUESTIONS – go to website to do.
https://mathcs.clarku.edu/~djoyce/ma120/integralpractice1.pdf

2.2 - PRACTICE QUESTIONS

1. *Find the antiderivative by* ***U-Substitution***.

a) $\int (3x-5)^{17}\,dx$

$$\boxed{\begin{aligned} u &= 3x-5 \\ \frac{du}{3} &= \frac{3\,dx}{3} \\ dx &= \frac{du}{3} \end{aligned}}$$

replace dx with

$$\int u^{17}\,\frac{du}{3}$$

$$\frac{1}{3}\int u^{17}\,du$$

$$\frac{1}{3}\cdot\frac{1}{18}u^{18}$$

$$=\frac{1}{54}(3x-5)^{18}+C$$

b) $\int \frac{1}{(4x+7)^6}\,dx$

$u = 4x+7$

$$=-\frac{1}{20}(4x+7)^{-5}+C$$

c) $\int x\sqrt{x^2+9}\,dx$

$u = x^2+9$

$$\frac{du}{2x} = \frac{2x\,dx}{2x}$$

$$dx = \frac{du}{2x}$$

$$\int \cancel{x}\sqrt{u}\,\frac{du}{2\cancel{x}}$$

$$\frac{1}{2}\int u^{1/2}\,du$$

$$\frac{1}{2}\cdot\frac{2}{3}u^{3/2}$$

$$\frac{2}{6}u^{3/2} = \frac{1}{3}(x^2+9)^{3/2}+C$$

d) $\int x^2\sqrt{2x^3-4}\,dx$

$u = 2x^3-4$

$$=\frac{1}{9}(2x^3-4)^{3/2}+C$$

e) $\int \frac{(\ln x)^{10}}{x}\,dx$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$dx = x\,du$

$\int \frac{u^{10}}{\cancel{x}} \cdot \cancel{x}\,du$

$\int u^{10}\,du$

$\frac{1}{11}u^{11}$

$= \frac{1}{11}(\ln x)^{11} + C$

f) $\int \frac{5x}{5+2x^2}\,dx$

$= \frac{5}{4}\ln(5+2x^2) + C$

g) $\int \frac{4x}{\sqrt{x^2+1}}\,dx$

$= 4\sqrt{x^2+1} + C$

h) $\int e^{t^2+\ln(t)}\,dt$

RW

$e^{t^2} \cdot \cancel{e^{\ln t}}\ t$

$\int t e^{t^2}\,dt$

$u = t^2$

$du = 2t\,dt$

$dt = \frac{du}{2t}$

$\int \cancel{t} e^{u} \frac{du}{2\cancel{t}}$

$\frac{1}{2}\int e^{u}\,du$

$\frac{1}{2}e^{u}$

$= \frac{1}{2}e^{t^2} + C$

EXTENDED QUESTIONS

Integration by "Double Substitution" - Evaluate each indefinite integral.

1. $\int x\sqrt{1+x}\,dx$

$u = 1+x \Rightarrow x = u-1$

$du = 1\,dx$

$\int x\sqrt{u}\,du$

$\int (u-1)(u^{1/2})\,du$

$\int u^{3/2} - u^{1/2}\,du$

$\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}$

$= \frac{2}{5}(1+x)^{5/2} - \frac{2}{3}(1+x)^{3/2} + C$

2. $\int \frac{x}{\sqrt{x+1}}\,dx$

$= \frac{2}{3}(x+1)^{3/2} - 2\sqrt{x+1} + C$

3. $\int \frac{x^3}{x^2-3}\,dx$

$= \frac{3}{2}\ln(x^2-3) + \frac{1}{2}x^2 + C$

4. $\int \frac{13x^7}{\sqrt{3x^4-5}}\,dx$

$u = 3x^4 - 5 \Rightarrow x^4 = \frac{u+5}{3}$

$du = 12x^3\,dx$

$= 13\int \frac{\cancel{x^7}}{u^{1/2}} \cdot \frac{du}{12\cancel{x^3}}$

$13\left[\frac{1}{12}\int \frac{x^4}{u^{1/2}}\,du\right]$

$= \frac{13}{54}(3x^4-5)^{3/2} + \frac{65}{18}(3x^4-5)^{1/2} + C$

2.3 - PRACTICE QUESTIONS

1. *Evaluate each indefinite integral using* **Integration by Parts**.

a) $\int x \sin(x)\, dx$

$u = x \qquad V = -\cos x$

$du = 1 \qquad dV = \sin x$

$-x\cos x - \int -\cos x\, dx$

$-x\cos x + \int \cos x\, dx$

$= -x\cos x + \sin x + c$

b) $\int x \cos(4x)\, dx$

$= \frac{1}{4}x\sin(4x) + \frac{1}{16}\cos(4x) + C$

c) $\int 2x^2 e^x\, dx$

$u = 2x^2 \qquad V = e^x$

$du = 4x \qquad dv = e^x$

$2x^2e^x - \int 4xe^x\, dx$

$2x^2e^x - 4\boxed{\int xe^x\, dx}$ IBP again

$u = x \qquad V = e^x$

$du = 1 \qquad dv = e^x$

$xe^x - \int e^x dx$

$\boxed{xe^x - e^x}$

$2x^2e^x - 4(xe^x - e^x)$

$= 2x^2e^x - 4xe^x - 4e^x + C$

d) $\int x^2 \ln|x|\, dx$

$= \frac{1}{3}x^3\ln x - \frac{1}{9}x^3 + C$

EXTENDED QUESTIONS

Evaluate each indefinite integral using integration by parts. u and dv are provided.

1) $\int xe^x\,dx;\; u = x,\; dv = e^x\,dx$

$xe^x - e^x + C$

2) $\int x\cos x\,dx;\; u = x,\; dv = \cos x\,dx$

$x\sin x + \cos x + C$

3) $\int x \cdot 2^x\,dx;\; u = x,\; dv = 2^x\,dx$

$\dfrac{x2^x}{\ln 2} - \dfrac{2^x}{(\ln 2)^2} + C$

4) $\int \sqrt{x}\ln x\,dx;\; u = \ln x,\; dv = \sqrt{x}\,dx$

$\dfrac{2x^{3/2}\ln x}{3} - \dfrac{4x^{3/2}}{9} + C$

Evaluate each indefinite integral.

5) $\int xe^{-x}\,dx$

$\dfrac{-x-1}{e^x} + C$

6) $\int x^2\cos 3x\,dx$

$\dfrac{x^2\sin 3x}{3} + \dfrac{2x\cos 3x}{9} - \dfrac{2\sin 3x}{27} + C$

7) $\int \dfrac{x^2}{e^{2x}}\,dx$

$\dfrac{-2x^2-2x-1}{4e^{2x}} + C$

8) $\int x^2e^{5x}\,dx$

$\dfrac{x^2e^{5x}}{5} - \dfrac{2xe^{5x}}{25} + \dfrac{2e^{5x}}{125} + C$

9) $\int \ln(x+3)\,dx$

$(x+3)\ln(x+3) - x + 3 + C$

10) $\int \cos 2x \cdot e^{-x}\,dx$

$\dfrac{2\sin 2x - \cos 2x}{5e^x} + C$

2.4 - PRACTICE QUESTIONS

1. Find the **EXACT VALUE** of each of the following DEFINITE INTERGRALS.

a) $\int_{1}^{3} 2x \, dx$

8

b) $\int_{-1}^{1} x^2 \, dx$

$\frac{2}{3}$

c) $\int_{-2}^{-1} x-6 \, dx$

$-\frac{15}{2}$

d) $\int_{1}^{\sqrt{2}} x \, dx$

$\frac{1}{2}$

e) $\int_{\pi}^{2\pi} e^x \, dx$

$e^{2\pi}-e^{\pi}$

f) $\int_{-2}^{-1} x^2+6 \, dx$

$\frac{25}{3}$

g) $\int_{1}^{4} 3y \, dy$

$\frac{45}{2}$

h) $\int_{1}^{4} 3y \, dm$

$9y$

i) $\int_{e}^{4} \frac{\ln t}{t} \, dt$

$\frac{1}{2}(\ln 4)^2-\frac{1}{2}$

j) $\int_{0}^{\frac{\pi}{2}} \cos t \, dt$

1

k) $\int_{2}^{5} e^x \, dr$

$3e^x$

l) $\int_{-3}^{-1} xe^{x^2} \, dm$

$2xe^{x^2}$

2. Simplify:

a) $\int_{a}^{b} 2x \, dx$

b^2-a^2

b) $\int_{a}^{b} 3x^2 \, dx$

b^3-a^3

c) $\int_{0}^{b} x^2 \, dx$

$\frac{b^3}{3}$

d) $\int_{a}^{b} 2x \, dm$

$2xb-2xa$

e) $\int_{0}^{b} e^p \, dp$

e^b-1

f) $\int_{\pi}^{4} \frac{2}{t} \, dt$

$2\ln 4-2\ln\pi$

3. *Simplify:*

a) $\int_a^5 2x\,dx$

$25-a^2$

b) $\int_0^b x^3\,dx$

$\frac{b^4}{4}$

c) $\int_a^0 e^x\,dx$

$1-e^a$

d) $\int_{4m}^{4m} \frac{y}{y^3+1}\,dy$

0

e) $\int_{2p}^{4p} y\,dt$

$2py$

f) $\int_{-4a}^{6a} c\,dm$

$10ac$

4. *Solve for x:*

a) $\int_1^x 2m\,dm = 8$

± 3

b) $\int_x^1 4\,dy = 16$

-3

c) $\int_x^5 2t+3\,dt = 40$

0 or -3

d) $\int_2^x 4k\,dy = 8x$

$\frac{2k}{k-2}$

e) $\int_1^4 x\,dq = 15$

5

f) $\int_2^x r\,dr = 1$

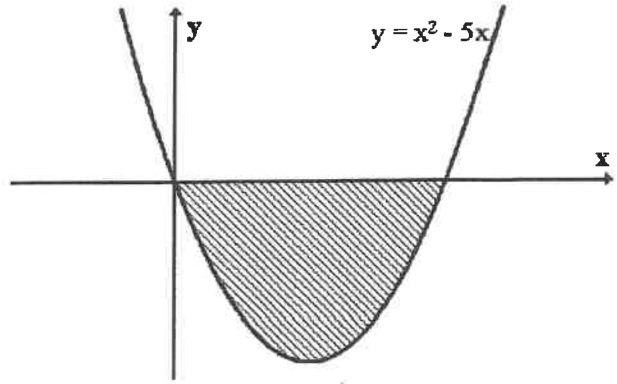
$\pm\sqrt{6}$

2.5 - PRACTICE QUESTIONS

1. The diagram opposite shows the graph of $y = x^2 - 5x$.

   Calculate the shaded area.

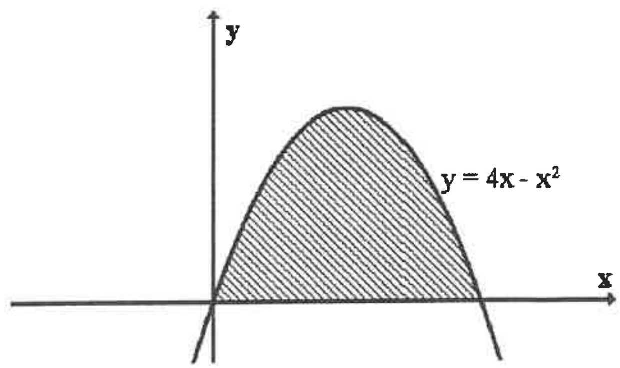
   20.83 or $\frac{125}{6}$



2. The diagram shows the graph of $y = 4x - x^2$.

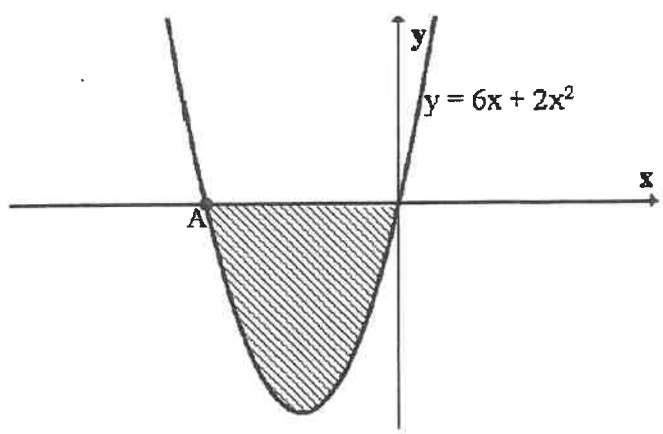
   Calculate the area between the curve and the x-axis.

   10.667 or $\frac{32}{3}$



3. The diagram shows part of the graph of $y = 6x + 2x^2$.

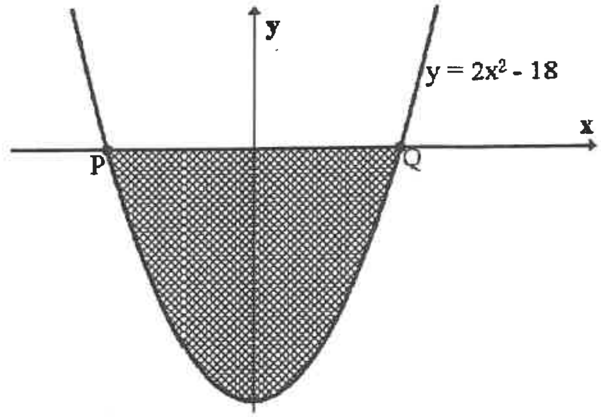
   (a) Find the coordinates of A. (- 3, 0)
   (b) Calculate the shaded area.
   9



4. The dagram shows part of the graph of $y = 2x^2 - 18$.

(a) Calculate the coordinates of P and Q. $(3,0)$
(b) Find the shaded area. $(-3,0)$

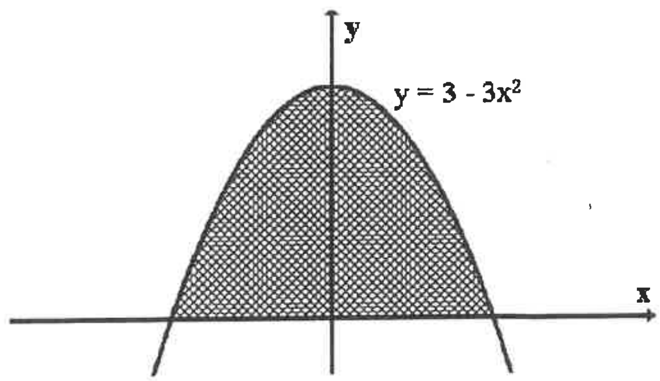
72



5. The diagram shows part of the graph of $y = 3 - 3x^2$.

Calculate the shaded area.

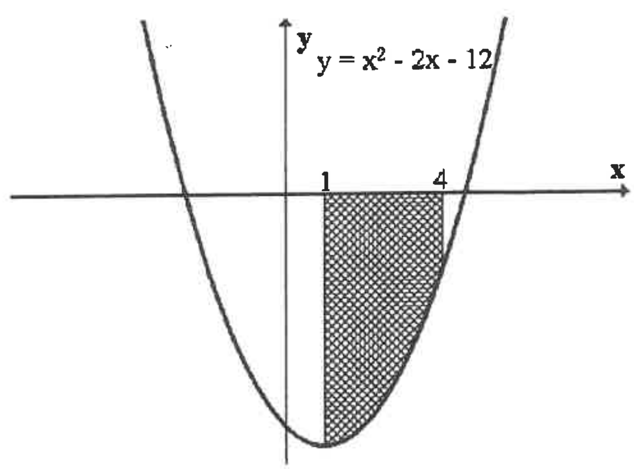
4



6. The diagram shows the graph of $y = x^2 - 2x - 12$.

Calculate the shaded area.

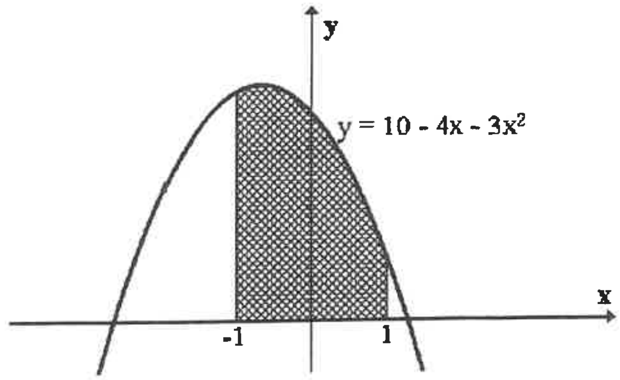
30



7. The diagram shows part of the graph of $y = 10 - 4x - 3x^2$

Calculate the shaded area.

18



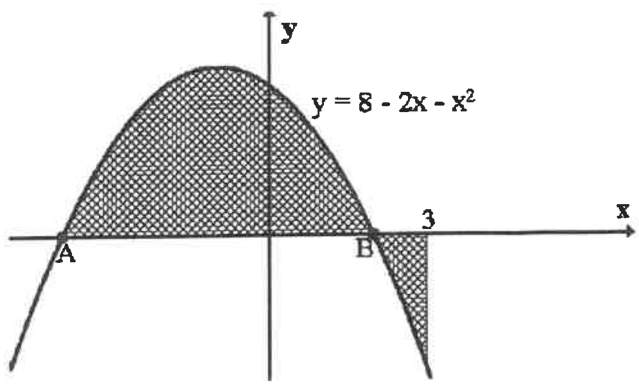
8. The diagram shows the graph of $y = 8 - 2x - x^2$.

(-4,0) (2,0)

(a) Find the coordinates of A and B.

(b) Calculate the shaded area.

39.33 or $\frac{118}{3}$



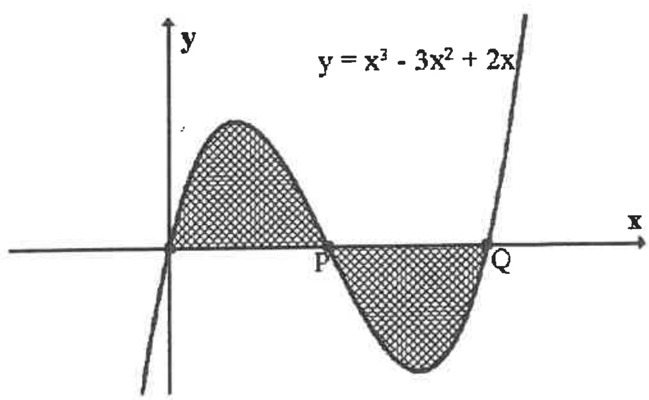
9. The diagram opposite shows part of the graph of $y = x^3 - 3x^2 + 2x$.

(1,0) (2,0)

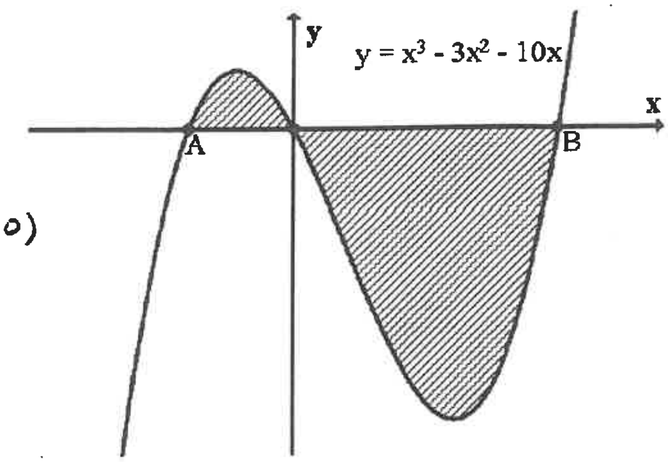
(a) Find the coordinates of P and Q.

(b) Calculate the shaded area.

0.5 or $\frac{1}{2}$



10. The diagram shows the graph of $y = x^3 - 3x^2 - 10x$.



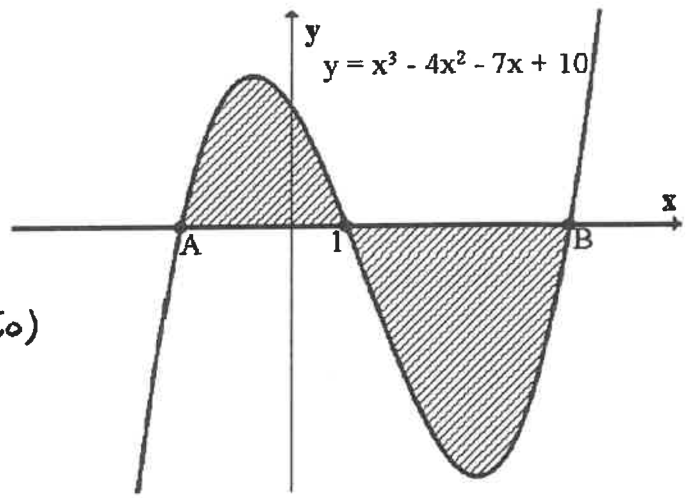
(-2,0) (5,0)

(a) Find the coordinates of A and B.

(b) Calculate the shaded area.

101.75 or $\frac{407}{4}$

11. The diagram shows the graph of $y = x^3 - 4x^2 - 7x + 10$.



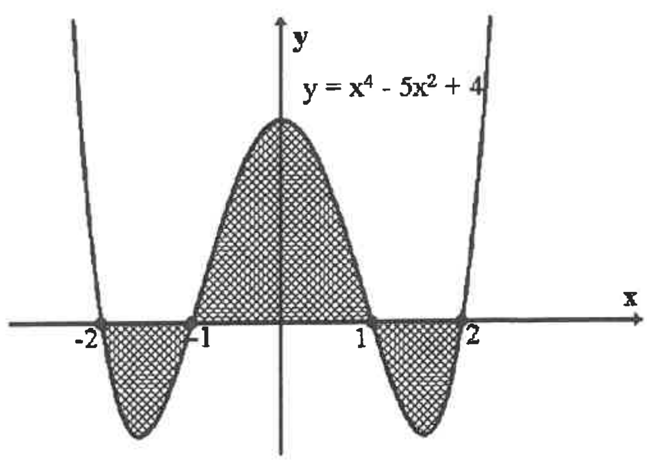
(-2,0) (5,0)

(a) Find the coordinates of A and B.

(b) Calculate the shaded area.

78.083 or $\frac{937}{12}$

12. The diagram shows the graph of $y = x^4 - 5x^2 + 4$.



Calculate the shaded area.

8

2.6 - PRACTICE QUESTIONS

1. *Find the Exact Area between the given curve and the y-axis over the given interval.*

a) $y = 3x, \quad 0 \le x \le 4$

24

b) $y = x^2, \quad 0 \le x \le 3$

18

c) $y = 4x^3, \quad 0 \le x \le 3$

243

d) $y = \cos x, \quad 0 \le x \le \dfrac{\pi}{2}$

1

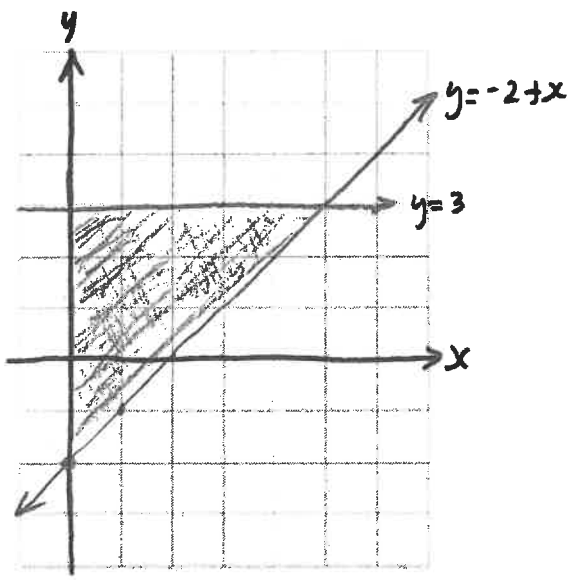
e) $y = e^x, \quad 0 \le x \le 2$

$e^2 + 1$

f) $y = -4x, \quad 0 \le x \le 4$

32

2. *Find the area of the region bounded by* $y = -2 + x$, $x = 0$ *and* $y = 3$.



$= \dfrac{25}{2}$

2.7 - PRACTICE QUESTIONS

Determine the shaded area the region enclosed for each:

1. *Find the* ***exact area*** *between the two curves f(x) and g(x) over the given interval.*

a) $f(x) = 2,\ g(x) = -3;\quad -1 \leq x \leq 4$

25

b) $f(x) = -2x + 1,\ g(x) = -4x - 5;\quad 0 \leq x \leq 2$

16

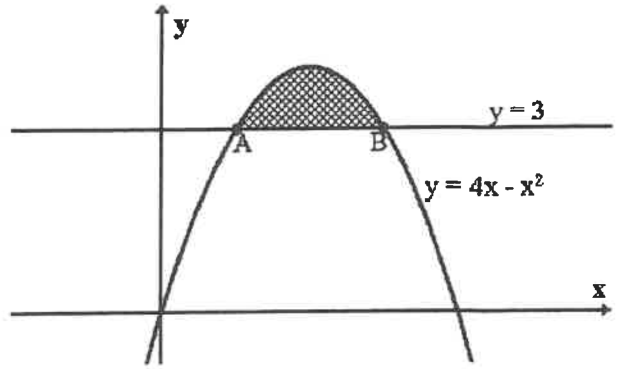
a) $f(x) = e^x,\ g(x) = \frac{1}{2}x;\quad -2 \leq x \leq 0$

$2 - e^{-2}$

b) $f(x) = \sin x,\quad g(x) = -2x;\quad 0 \leq x \leq \pi$

$2 + \pi^2$

2.



The diagram opposite shows the curve $y = 4x - x^2$ and the line $y = 3$.

(1,3) (3,3)

(a) Find the coordinates of A and B.

(b) Calculate the shaded area.

a) $3 = 4x - x^2$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$x = 3 \quad x = 1$

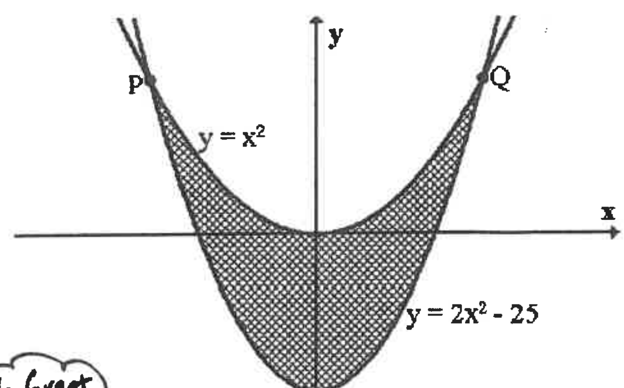
Top - Bottom

$\int_1^3 4x - x^2 - 3$ RW $\int_1^3 -x^2 + 4x - 3\,dx$

$-\frac{1}{3}x^3 + 2x^2 - 3x \Big|_1^3$

$\left[-\frac{1}{3}(3)^3 + 2(3)^2 - 3(3)\right] - \left[-\frac{1}{3}(1)^3 + 2(1)^2 - 3(1)\right] = \frac{4}{3}$

3.



The curves with equations $y = x^2$ and $y = 2x^2 - 25$ intersect at P and Q.

Calculate the area enclosed between the curves.

$x^2 = 2x^2 - 25$

$0 = x^2 - 25$

$(x+5)(x-5) = 0$

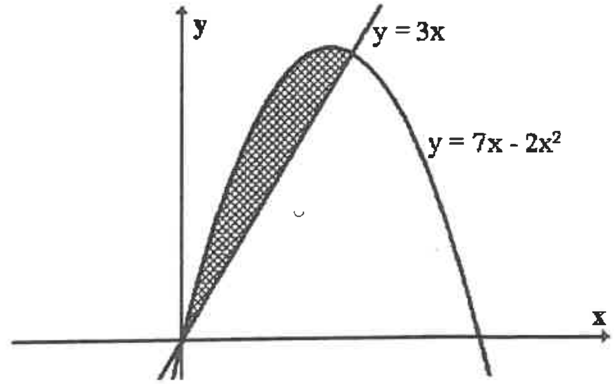
$x = -5 \quad x = 5$

don't forget to tidy up Before you integrate

$\int_{-5}^{5} x^2 - (2x^2 - 25)\,dx$

$= \frac{500}{3}$

4.



The diagram opposite shows the curve $y = 7x - 2x^2$ and the line $y = 3x$.

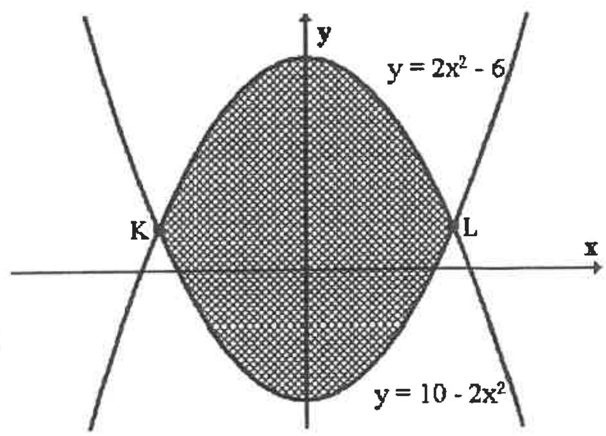
Calculate the shaded area.

Tidy up

$$\int_0^2 7x - 2x^2 - 3x \, dx$$

$$= \frac{8}{3}$$

5.



The curves with equations $y = 2x^2 - 6$ and $y = 10 - 2x^2$ intersect at K and L.

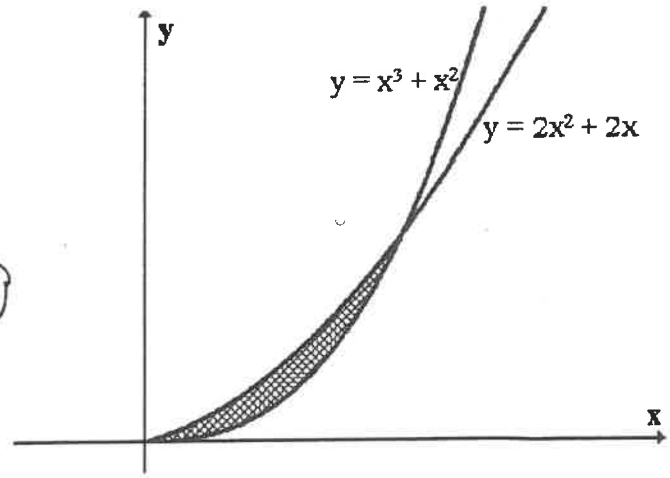
Calculate the area enclosed by these two curves.

Tidy up

$$\int_{-2}^{2} 2x^2 - 6 - (10 - 2x^2) \, dx$$

$$= \frac{128}{3}$$

6.



The diagram opposite shows part of the curves $y = x^3 + x^2$ and $y = 2x^2 + 2x$.

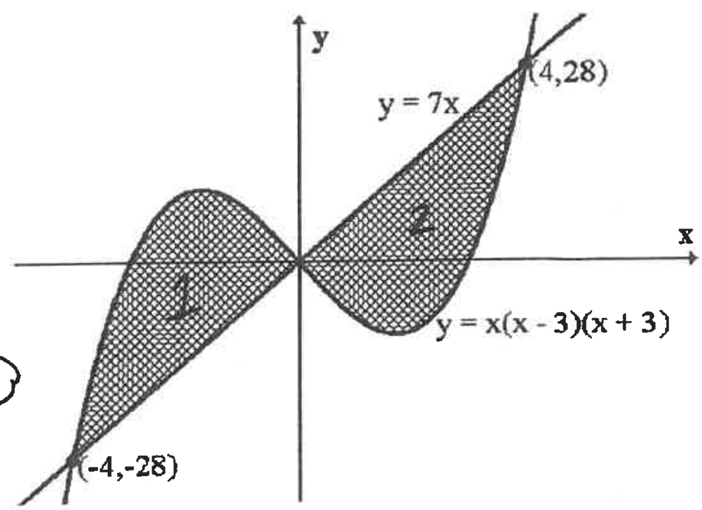
Calculate the shaded area.

Tidy up

$$\int_0^2 2x^2 + 2x - (x^3 + x^2)\,dx$$

$$= \frac{8}{3}$$

7.



. The curve $y = x(x - 3)(x + 3)$ and the line $y = 7x$ intersect at the points (0,0); (-4,-28) and (4,28).

Calculate the area enclosed by the curve and the line.

Tidy up

Region 1 $\int_{-4}^{0} x^3 - 9x - 7x\,dx$

$= 64$

Tidy up

Region 2 $\int_0^4 7x - (x^3 - 9x)\,dx$

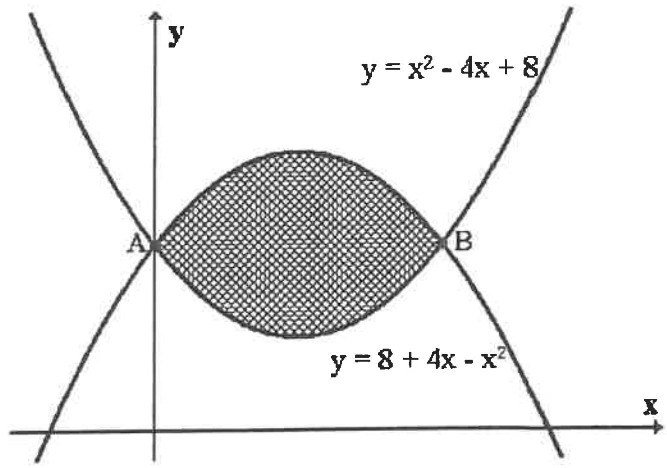
$= 64$

$= 128$

8.

The parabolas $y = x^2 - 4x + 8$ and $y = 8 + 4x - x^2$ intersect at A and B.

(a) Find the coordinates of A and B.
(b) Calculate the shaded area.

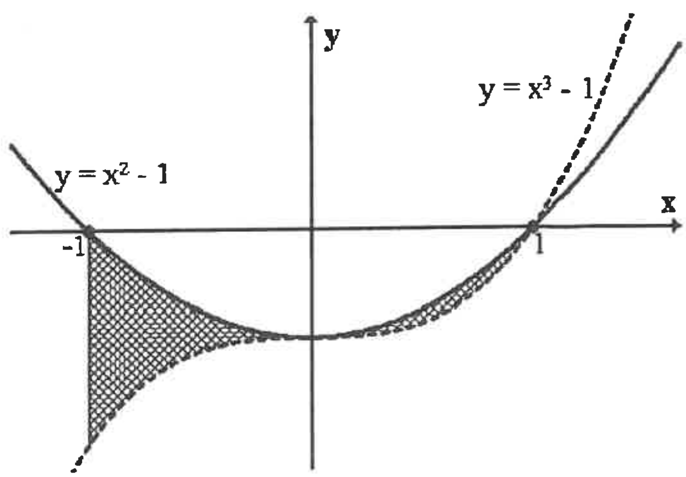


$\frac{64}{3}$

9.

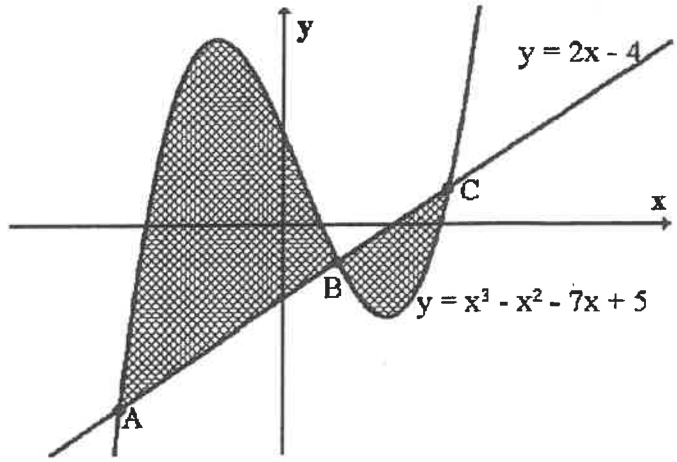
The diagram shows parts of the curves $y = x^3 - 1$ and $y = x^2 - 1$.

Calculate the shaded area.



$\frac{2}{3}$

10.

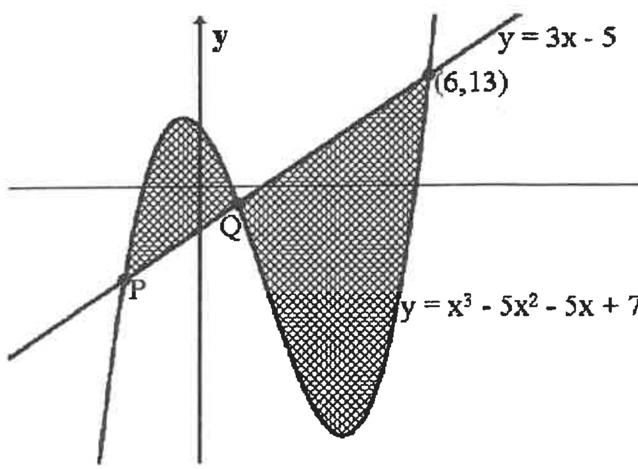


The curve $y = x^3 - x^2 - 7x + 5$ and the line $y = 2x - 4$ are shown opposite.

(a) B has coordinates (1,-2). Find the coordinates of A and C.
(b) Hence calculate the shaded area.

$\frac{148}{3}$

11.



. The diagram shows the line $y = 3x - 5$ and the curve $y = x^3 - 5x^2 - 5x + 7$.

(a) Find the coordinates of P and Q.
(b) Calculate the shaded area.

$\frac{863}{6}$

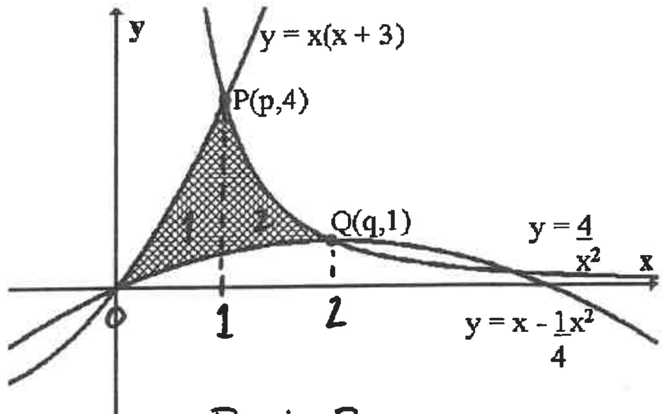
EXTENDED QUESTION

The diagram opposite shows an area enclosed by 3 curves:

$y = x(x+3)$, $y = \frac{4}{x^2}$ and $y = x - \frac{1}{4}x^2$

(a) P and Q have coordinates (p,4) and (q,1). Find the values of p and q.

(b) Calculate the shaded area.



Point P

$4 = x(x+3)$

$4 = x^2 + 3x$

$0 = x^2 + 3x - 4$

$0 = (x-1)(x+4)$

$x = 1 \quad x = -4$

$P(1,4)$

Point Q

$1 = \frac{4}{x^2}$

$x^2 = 4$

$x^2 - 4 = 0$

$(x-2)(x+2) = 0$

$x = 2 \quad x = -2$

$Q(2,1)$

Region 1

$$\int_0^1 x^2 + 3x - \left(x - \frac{x^2}{4}\right) dx$$

$\frac{17}{12}$

Region 2

$$\int_1^2 4x^{-2} - \left(x - \frac{1}{4}x^2\right) dx$$

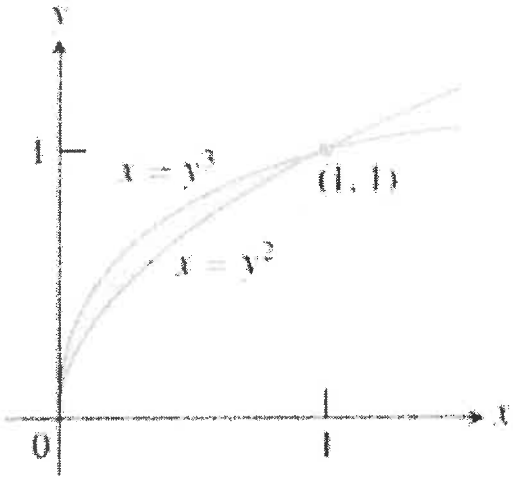
$\frac{13}{12}$

$= \frac{5}{2}$

2.8 - PRACTICE QUESTIONS

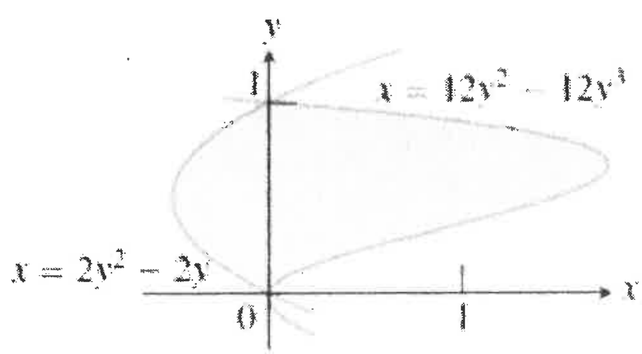
Find the area of the shaded region between two curves – y-axis

1)



$= \frac{1}{12}$

2)



$= \frac{4}{3}$

3. Find the area of the region(s) enclosed by the graphs of $x - y^2 = 0$ and $x + 2y^2 = 3$.

$= 4$

4. Find the area of the region enclosed by the following curves: $x = y^2 - 2$, *and* $x = y$.

$= \frac{9}{2}$

5. Find the area of the region enclosed by the following curves: $x = \frac{1}{2}y^2 - 3$, *and* $y = x - 1$.

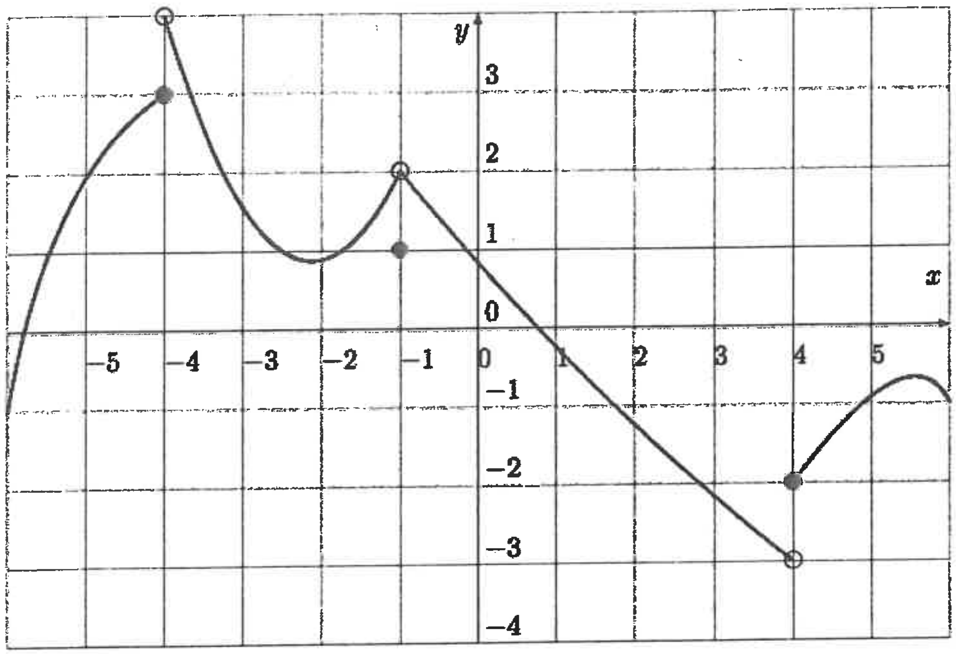
$= 18$

6. Find the area of the region enclosed by the following curves: $x = y^3 - y$, *and* $x = 1 - y^4$.

$= \frac{8}{5}$

3.1 - PRACTICE QUESTIONS

1. Consider the following function defined by its graph:

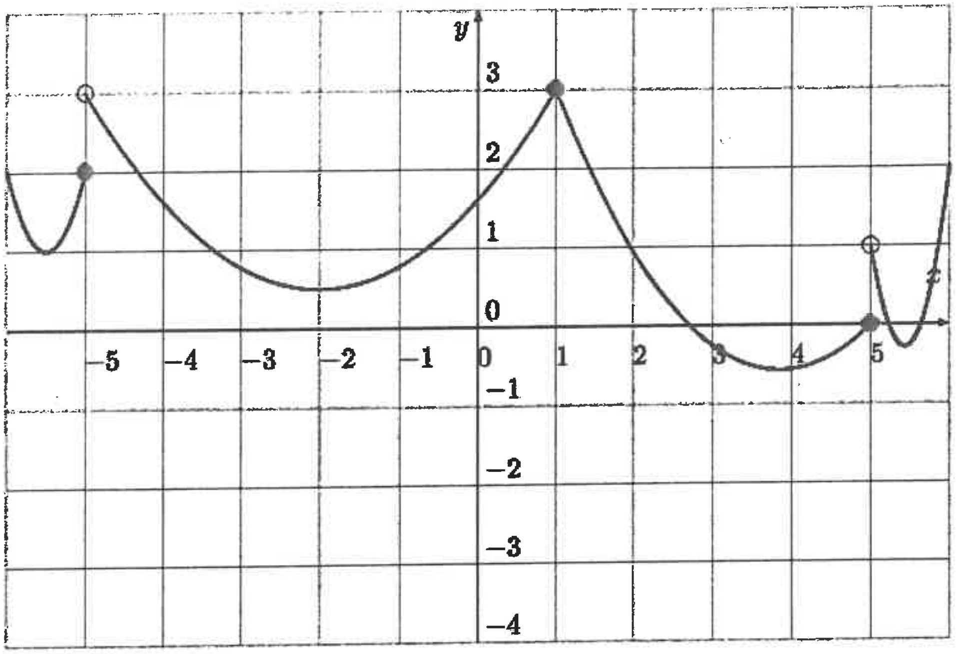


Find the following limits:

a) $\lim_{x \to -1^-} f(x)$ b) $\lim_{x \to -1^+} f(x)$ c) $\lim_{x \to -1} f(x)$ d) $\lim_{x \to -4} f(x)$ e) $\lim_{x \to 4} f(x)$

2 2 2 DNE DNE

2. Consider the following function defined by its graph:

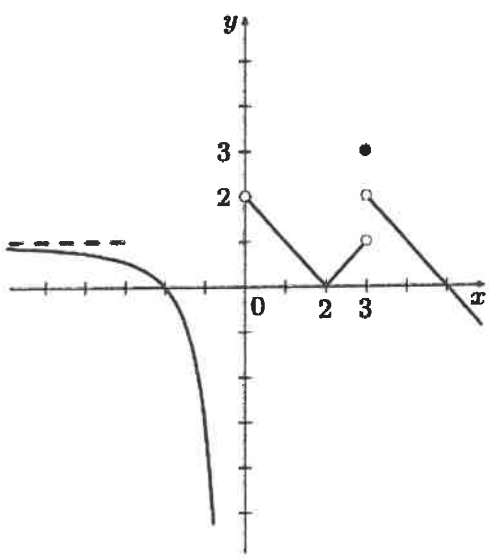


Find the following limits:

a) $\lim_{x\to 1^-} f(x)$ b) $\lim_{x\to 1^+} f(x)$ c) $\lim_{x\to 1} f(x)$ d) $\lim_{x\to -5} f(x)$ e) $\lim_{x\to 5} f(x)$

3 3 3 DNE DNE

3. *Use the graph of the function f(x) to answer each question.*
Use $\infty, -\infty$ or DNE where appropriate.



(a) $f(0) =$ DNE

(b) $f(2) =$ 0

(c) $f(3) =$ 3

(d) $\lim_{x\to 0^-} f(x) =$ $-\infty$

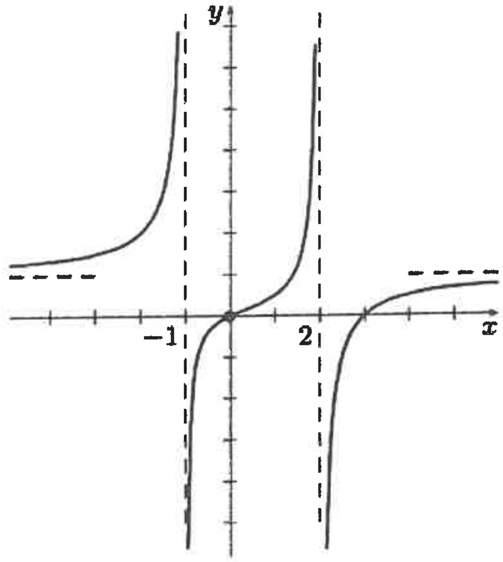
(e) $\lim_{x\to 0} f(x) =$ DNE

(f) $\lim_{x\to 3^+} f(x) =$ 2

(g) $\lim_{x\to 3} f(x) =$ DNE

(h) $\lim_{x\to -\infty} f(x) =$ 1

4. *Use the graph of the function f(x) to answer each question.*
Use $\infty, -\infty$ or DNE where appropriate.



(a) $f(0) =$ 0

(b) $f(2) =$ DNE or UNDEFINED

(c) $f(3) =$ 0

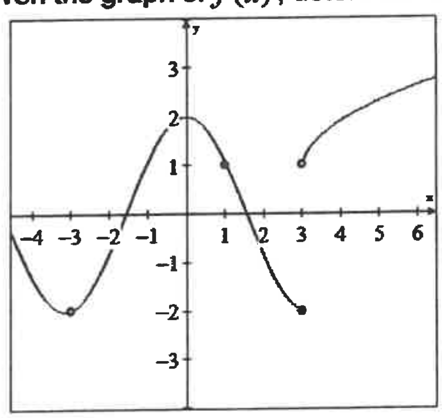
(d) $\lim_{x \to -1} f(x) =$ DNE

(e) $\lim_{x \to 0} f(x) =$ 0

(f) $\lim_{x \to 2^+} f(x) =$ $-\infty$

(g) $\lim_{x \to \infty} f(x) =$ 1

5. **Given the graph of $f(x)$, determine the following.**



a) $\lim_{x \to -3^-} f(x)$ -2 b) $\lim_{x \to -3^+} f(x)$ -2 c) $\lim_{x \to -3} f(x)$ -2

d) $\lim_{x \to 1^+} f(x)$ 1 e) $\lim_{x \to 1^-} f(x)$ 1 f) $\lim_{x \to 1} f(x)$ 1

g) $\lim_{x \to 3^-} f(x)$ -2 h) $\lim_{x \to 3^+} f(x)$ 1 i) $\lim_{x \to 3} f(x)$ DNE

j) $f(-3)$ UNDEFINED k) $f(1)$ 1 l) $f(3)$ -2

6. *Identify the points of discontinuity, holes, vertical asymptotes, and horizontal asymptote of each.*

a) $f(x) = -\dfrac{4}{x^2 - 3x}$

Disc = 0, 3	VA = 0, 3
Holes = none	HA = 0

b) $f(x) = \dfrac{x - 4}{-4x - 16}$

Disc = −4	VA = −4
Holes = none	HA = −1/4

c) $f(x) = \dfrac{3x^2 - 12x}{x^2 - 2x - 3}$

Disc = 3, −1	VA = 3, −1
Holes = none	HA = 3

d) $f(x) = \dfrac{x^2 + x - 6}{-4x^2 - 16x - 12}$

Disc = −1, −3	VA = −1
Holes = −3	HA = −1/4

7. *MULTIPLE CHOICE. Find the slant asymptote, if any, of the graph of the rational function.*

1) $f(x) = \frac{x^2 + 3x - 6}{x - 3}$

A) $y = x + 6$ B) $y = x$
C) $y = x + 3$ D) no slant asymptote

2) $f(x) = \frac{x^2 - 4x + 9}{x + 5}$

A) $y = x - 9$ B) $x = y + 4$
C) $y = x + 13$ D) no slant asymptote

3) $f(x) = \frac{x^2 - 8x + 9}{x + 4}$

A) $y = x - 12$ B) $x = y + 8$
C) $y = x + 17$ D) no slant asymptote

4) $f(x) = \frac{x^2 - 2x + 2}{x + 2}$

A) $x = y + 2$ B) $y = x - 4$
C) $y = x + 4$ D) no slant asymptote

8. *Identify the horizontal asymptotes of each using limits to ∞.*

a) $\lim_{x\to\infty} \frac{3x + 5}{x - 4}$

$$\frac{\frac{3\cancel{x}}{\cancel{x}} + \frac{5}{x}}{\frac{\cancel{x}}{\cancel{x}} - \frac{4}{x}} = \frac{3 + \frac{5}{x}}{1 - \frac{4}{x}}$$

$$\lim_{x\to\infty} \frac{3 + \frac{5}{\infty}}{1 - \frac{4}{\infty}} = 3$$

b) $\lim_{t\to\infty} \frac{t^2 + 2}{t^3 + t^2 - 1}$

$$\frac{\frac{t^2}{t^3} + \frac{2}{t^3}}{\frac{t^3}{t^3} + \frac{t^2}{t^3} - \frac{1}{t^3}} = \frac{\frac{1}{t} + \frac{2}{t^3}}{1 + \frac{1}{t} - \frac{1}{t^3}}$$

$$\lim_{x\to\infty} \frac{0 + 0}{1 + 0 - 0} = \frac{0}{1} = 0$$

c) $\lim_{t\to\infty} \frac{t^3 + t^2 - 1}{t^2 + 2}$

$$\lim_{t\to\infty} \frac{\frac{t^3}{t^2} + \frac{t^2}{t^2} - \frac{1}{t^2}}{\frac{t^2}{t^2} + \frac{2}{t^2}} = \frac{t + 1 - \frac{1}{t^2}}{1 - \frac{2}{t^2}}$$

$$\lim_{t\to\infty} \frac{\infty + 1 - 0}{1 + 0} = \infty$$

d) $\lim_{x\to\infty} \frac{x + 2}{\sqrt{9x^2 + 1}}$ ... $\sqrt{x^2} = x$

This is the same as dividing by x

$$\frac{\frac{x}{x} + \frac{2}{x}}{\frac{1}{x}\sqrt{9x^2 + 1}} = \frac{1 + \frac{2}{x}}{\sqrt{9 + \frac{1}{x^2}}} =$$

Same

$$\sqrt{\frac{1}{x^2}} \cdot \sqrt{9x^2 + 1}$$

$$\lim_{x\to\infty} \frac{1 + \frac{2}{\infty}}{\sqrt{9 + \frac{1}{\infty}}} = \frac{1}{\sqrt{9}} = \frac{1}{3}$$

9. **Calculate the following limits.**

$\lim_{x\to\infty} f(x) =$ 1

$\lim_{x\to-\infty} f(x) =$ 1

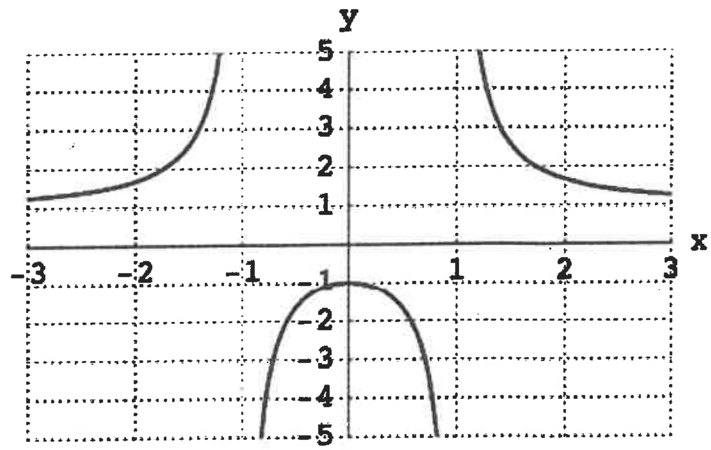
$\lim_{x\to-1^+} f(x) =$ $-\infty$

$\lim_{x\to-1^-} f(x) =$ ∞

$\lim_{x\to1^+} f(x) =$ ∞

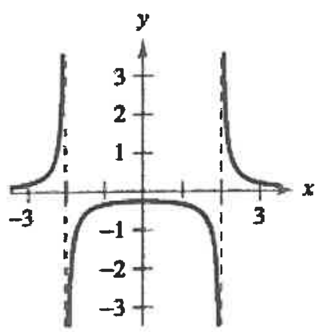
$\lim_{x\to1^-} f(x) =$ $-\infty$

$$f(x) = \frac{x^2+1}{x^2-1}$$



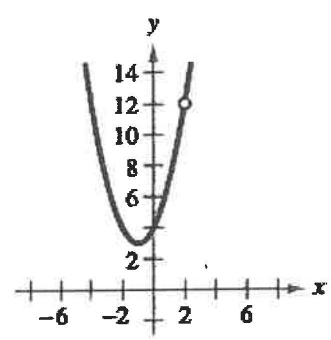
10. Describe the interval(s) on which the function is continuous on the entire real line.

a) $f(x) = \dfrac{1}{x^2-4}$



$(-\infty,-2)(-2,2)(2,\infty)$

b) $f(x) = \dfrac{x^3-8}{x-2}$



$(-\infty,2)(2,\infty)$

11. *Describe the continuity of the function on the closed interval. If there are any discontinuities, determine whether they are removable.*

Function	Interval
a) $f(x) = \frac{1}{x-2}$	$[1, 4]$

Discontin. $x = 2$

Function	Interval
b) $f(x) = \frac{x}{x^2 - 4x + 3}$	$[0, 4]$

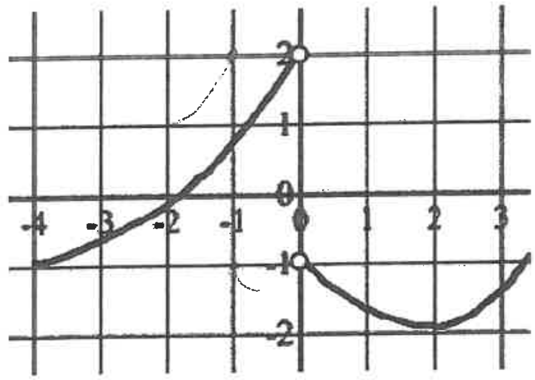
Discontin $x = 3, 1$

Function	Interval
c) $f(x) = \frac{x^2 - 16}{x - 4}$	$[1, 5]$

Discontin and removable $x = 4$

EXTENDED QUESTIONS

Consider the function g defined by the graph below.



$y = g(x)$

Circle the best choice:

1) Find $\lim_{x \to -1^-} g(x+1)$

(A) -4 (B) -3 (C) -2 (D) -1 (E) 0 (F) 1 (G) 2 (H) 3 (I) 4 (j) DNE

2) Find $\lim_{x \to 0^-} g(x^2)$

(A) -4 (B) -3 (C) -2 (D) -1 (E) 0 (F) 1 (G) 2 (H) 3 (I) 4 (j) DNE

3) Find $\lim_{x \to 1^+} (g(x-1))^2$

(A) -4 (B) -3 (C) -2 (D) -1 (E) 0 (F) 1 (G) 2 (H) 3 (I) 4 (j) DNE

4.

Sketch the graph of a function that satisfies all of the following properties at once.

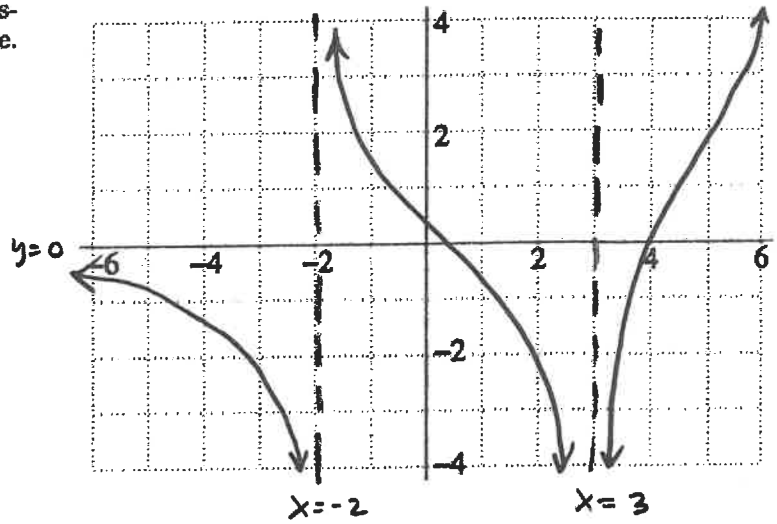
(a) $\lim_{x\to -2^+} f(x) = \infty$

(b) $\lim_{x\to -2^-} f(x) = -\infty$

(c) $\lim_{x\to \infty} f(x) = \infty$

(d) $\lim_{x\to -\infty} f(x) = 0$

(e) $\lim_{x\to 3} f(x) = -\infty$



5. Calculate the following limits. **Cannot use L'H

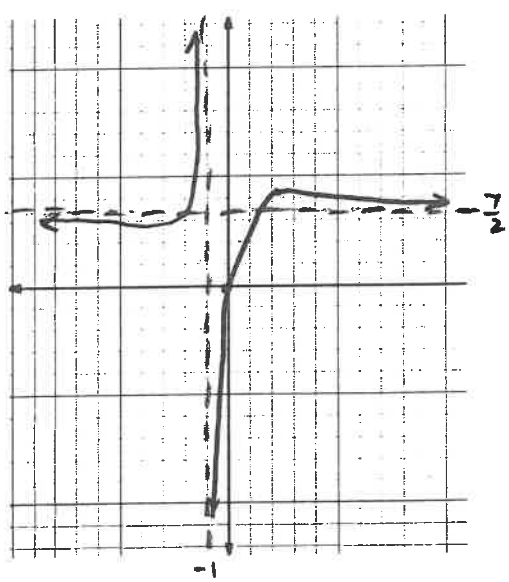
(a) $\lim_{x\to\infty} \frac{7x^3+4x}{2x^3-x^2+3}$

xint = 0
VA: x = -1
HA: y = 3.5

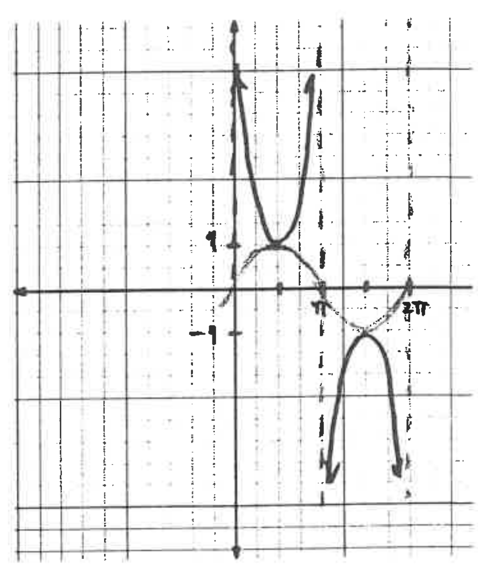
$$\frac{\frac{7x^3}{x^3}+\frac{4x}{x^3}}{\frac{2x^3}{x^3}-\frac{x^2}{x^2}+\frac{3}{x^3}} = \lim_{x\to\infty}\frac{7+\cancel{\frac{4}{x^2}}}{2-\cancel{\frac{1}{x}}+\cancel{\frac{3}{x^3}}} = \frac{7}{2}$$

(d) $\lim_{t\to\pi^+} \csc t$ $\frac{1}{\sin t}$ $= -\infty$

Sketch the above function.



Sketch the above function.



6.

$$\lim_{x\to\infty}\frac{(2x-1)(3-x)}{(x-1)(x+3)} \text{ is}$$

(A) -3 (B) -2 (C) 2 (D) 3 (E) nonexistent

7.

$$\lim_{x\to 0}\frac{5x^4+8x^2}{3x^4-16x^2} \text{ is}$$

(A) $-\frac{1}{2}$ (B) 0 (C) 1 (D) $\frac{5}{3}+1$ (E) nonexistent

8.

Let f be the function defined above. Which of the following statements about f are true?

I. f has a limit at $x = 2$.

II. f is continuous at $x = 2$.

III. f is differentiable at $x = 2$.

(A) I only

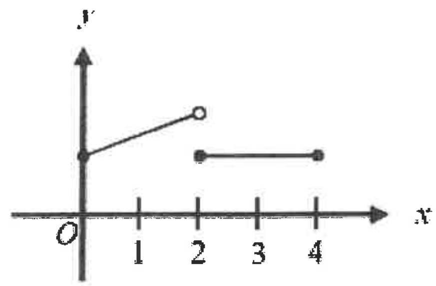
(B) II only

(C) III only

(D) I and II only

(E) I, II, and III

9.



Graph of f

The figure above shows the graph of a function f with domain $0 \le x \le 4$. Which of the following statements are true?

I. $\lim_{x\to 2^-} f(x)$ exists.

II. $\lim_{x\to 2^+} f(x)$ exists.

III. $\lim_{x\to 2} f(x)$ exists.

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

10.

For $x \geq 0$, the horizontal line $y = 2$ is an asymptote for the graph of the function f. Which of the following statements must be true?

(A) $f(0) = 2$

(B) $f(x) \neq 2$ for all $x \geq 0$

(C) $f(2)$ is undefined.

(D) $\lim_{x\to 2} f(x) = \infty$

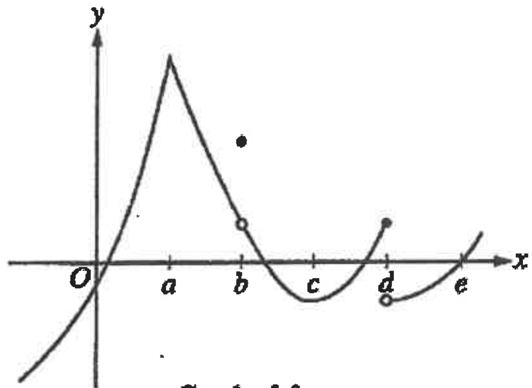
(E) $\lim_{x\to\infty} f(x) = 2$

11.

$$\lim_{x\to\infty} \frac{x^3 - 2x^2 + 3x - 4}{4x^3 - 3x^2 + 2x - 1} =$$

(A) 4 (B) 1 (C) $\frac{1}{4}$ (D) 0 (E) -1

12.



Graph of f

. The graph of a function f is shown above. At which value of x is f continuous, but not differentiable?

(A) a (B) b (C) c (D) d (E) e

13.

$$f(x) = \begin{cases} x+2 & \text{if } x \le 3 \\ 4x-7 & \text{if } x > 3 \end{cases}$$

Let f be the function given above. Which of the following statements are true about f ?

I. $\lim_{x \to 3} f(x)$ exists.

II. f is continuous at $x = 3$.

III. f is differentiable at $x = 3$.

(A) None

(B) I only

(C) II only

(D) I and II only

(E) I, II, and III

14.

Let f be a function defined by $f(x) = \begin{cases} 1 - 2\sin x & \text{for } x \le 0 \\ e^{-4x} & \text{for } x > 0. \end{cases}$

(a) Show that f is continuous at $x = 0$.

$f(x) = 1 - 2\sin x \quad x \le 0$ LEFT SIDE

$$\lim_{x \to 0^-} f(x) = 1 - 2\sin(0) = 1$$

$f(x) = e^{-4x} \quad x > 0$ RIGHT SIDE

$$\lim_{x \to 0^+} f(x) = e^{-4(0)} = e^0 = 1$$

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = 1 \quad \therefore F(x)$ is continuous at $x = 0$

15.

A 12,000-liter tank of water is filled to capacity. At time $t = 0$, water begins to drain out of the tank at a rate modeled by $r(t)$, measured in liters per hour, where r is given by the piecewise-defined function

$$r(t) = \begin{cases} \dfrac{600t}{t+3} & \text{for } 0 \le t \le 5 \\ 1000e^{-0.2t} & \text{for } t > 5 \end{cases}$$

(a) Is r continuous at $t = 5$? Show the work that leads to your answer.

Because left-hand and right-hand limits are not equal, r is not continuous at $t = 5$.

16.

Let f be the function given by $f(x) = \frac{\ln x}{x}$ for all $x > 0$. The derivative of f is given by $f'(x) = \frac{1 - \ln x}{x^2}$.

Find $\lim_{x \to 0^+} f(x)$. $= -\infty$ or DNE.

17. Let f be the function defined by

$$f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \le x \le 3 \\ 5 - x & \text{for } 3 < x \le 5. \end{cases}$$

(a) Is f continuous at $x = 3$? Explain why or why not.

No, because the derivative of :

$f(x) = \sqrt{x+1} = \frac{1}{2}$

$f(x) = 5 - x = -1$

Not $\underline{\underline{\text{equal}}}$ $\therefore$ Not Continuous

18.

Given the function $f(x) = \dfrac{x^3 + 2x^2 - 3x}{3x^2 + 3x - 6}$.

(a) What are the zeros of $f(x)$? $(-3,0) \div (0,0) \div (1,0)$

(b) What are the vertical asymptotes of $f(x)$? $x = -2$

(c) The end behavior model of $f(x)$ is the function $g(x)$. What is $g(x)$?

$g(x) = \frac{1}{3}x + \frac{1}{3}$

(d) What is $\lim_{x \to \infty} f(x)$? What is $\lim_{x \to \infty} \dfrac{f(x)}{g(x)}$?

$\lim_{x \to \infty} \dfrac{f(x)}{g(x)} = 1$

$\lim_{x \to \infty} f(x) = \infty$

3.2 - PRACTICE QUESTIONS

1. *Evaluate each limit:*

a) $\lim_{x \to 3} 2x - 8$

-2

b) $\lim_{x \to -2} x^2 - 1$

3

c) $\lim_{x \to -2} \left(x^2 - 2x - 3\right)$

5

d) $\lim_{x \to \pi} 2x^2 + 2x - 1$

$2\pi^2 + 2\pi - 1$

e) $\lim_{x \to \sqrt{2}} \left(x^2 - 1\right)$

1

f) $\lim_{x \to -\sqrt{2}} x^2 - 2x - 1$

$1 + 2\sqrt{2}$

2. *By evaluating one-sided limits, find the indicated limit if it exists:*

a) $f(x) = \begin{cases} x + 2 & \text{where } x < -1 \\ -x + 2 & \text{where } x \geq -1 \end{cases}$ $\lim_{x \to -1} f(x)$

Does not exist
or DNE

b) $f(x) = \begin{cases} x & \text{where } x \leq 1 \\ -x + 3 & \text{where } x > 1 \end{cases}$ $\lim_{x \to 1} f(x)$

DNE

c) $g(x) = \begin{cases} -x + 4 & \text{where } x \leq 2 \\ -2x + 6 & \text{where } x > 2 \end{cases}$ $\lim_{x \to 2} g(x)$

2

d) $k(x) = \begin{cases} 4 - x^2 & \text{where } x < 1 \\ x & \text{where } x \geq 1 \end{cases}$ $\lim_{x \to 1} k(x)$

DNE

e) $h(x) = \begin{cases} 4x & \text{where } x \geq \frac{1}{2} \\ \frac{1}{x} & \text{where } x < \frac{1}{2} \end{cases}$ $\lim_{x \to \frac{1}{2}} h(x)$

2

f) $m(x) = \begin{cases} x + 3 & \text{where } x < 2 \\ \frac{4}{x} & \text{where } x \geq 2 \end{cases}$ $\lim_{x \to 2} m(x)$

DNE

3. *Evaluate each of the following limits:*

a) $\lim_{x\to 2} \frac{3x}{x^2+2}$ 1

b) $\lim_{x\to -1} x^4+x^3+x^2$ 1

c) $\lim_{x\to 3} \sqrt{x^3+\frac{27}{x-1}}$ $\frac{9}{\sqrt{2}}$ or $\frac{9\sqrt{2}}{2}$

d) $\lim_{x\to 4} \left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2$ $\frac{25}{4}$

e) $\lim_{x\to 2\pi} \left(x^3+\pi^2 x-5\pi^3\right)$ $5\pi^3$

f) $\lim_{x\to a} \frac{(x+a)^2}{x^2+a^2}$ 2

g) $\lim_{x\to 0} \sqrt{1+\sqrt{1+x}}$ $\sqrt{2}$

h) $\lim_{h\to 0} \frac{1}{\sqrt{x}+\sqrt{x+h}}$ $\frac{1}{2\sqrt{x}}$ or $\frac{\sqrt{x}}{2x}$

i) $\lim_{x\to 4} \left(\sqrt{x}+2\right)^3$ 64

4. *If* $\lim_{x\to 2} f(x)=3$, *use the properties of limits to evaluate each of the limits below:*

a) $\lim_{x\to 2} \frac{x^2+5}{f(x)}$ 3

b) $\lim_{x\to 2} \sqrt{[f(x)]^2+x^4}$ 5

c) $\lim_{x\to 2} \sqrt{3f(x)-2x}$ $\sqrt{5}$

5. *Evaluate each limit.*

a) $\lim_{x \to 1^+} \frac{x^2 + x + 2}{x - 1}$

pick a number super close
to 1 on the high side
ex 1.00001, then plug in
for every x -> see output

$$\frac{(1.00001^2 + 1.00001 + 2)}{(1.00001 - 1)} = 400003 \therefore +\infty$$

output really larger number
-> if you made the number much
closer to 1 like 1.000000000001,
you will get a even larger
number $\therefore$ answer is $\underline{+\infty}$

b)

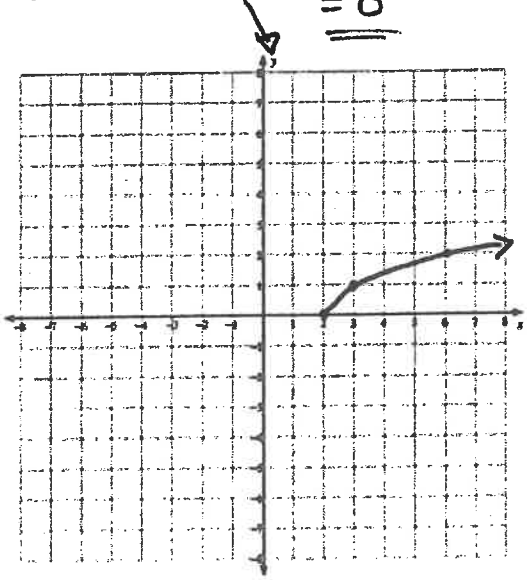
$$\lim_{x \to -3^-} \frac{x+7}{x+3}$$

$= \underline{-\infty}$

c) $\lim_{x \to 2^+} \sqrt{x - 2}$

TWO WAY TO SOLVE
① VISUALLY

$= \underline{0}$

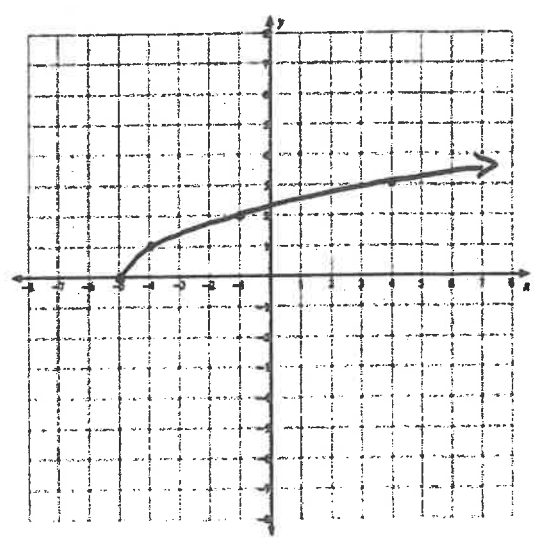


② PLUG IN NUMBER REALLY CLOSE
TO 2 ON HIGH SIDE LIKE 2.000001

$\sqrt{2.0000001 - 2} = 3.16 \times 10^{-4}$
0.000316 $\therefore$
$= 0$

d) $\lim_{x \to -5^-} \sqrt{x + 5}$

$= \underline{\text{DNE}}$



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6. *Evaluate each of the following limits using* ***L'HOPITAL'S RULE:***

a) $\lim_{x\to 2} \frac{x^2-4}{x-2}$

4

b) $\lim_{x\to -2} \frac{4-x^2}{2+x}$

4

c) $\lim_{x\to 1} \frac{x^3-1}{x-1}$

3

d) $\lim_{x\to -1} \frac{2x^2+5x+3}{x+1}$

1

e) $\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4}$

$\frac{1}{4}$

f) $\lim_{x\to 0} \frac{\sqrt{4+x}-2}{x}$

$\frac{1}{4}$

g) $\lim_{x\to 0} \frac{\cos x-1}{x}$

0

h) $\lim_{x\to 0} \frac{3\sin 2x}{2x}$

3

i) $\lim_{x\to 0} \frac{e^x-1}{2e^x-2}$

$\frac{1}{2}$

j) $\lim_{x\to 1} \frac{x-1}{\ln x}$

1

k) $\lim_{x\to 0} \frac{\cos x-1}{e^x-1}$

0

l) $\lim_{x\to 1} \frac{2\ln x}{4x^2-4}$

$\frac{1}{4}$

Evaluate each limit using ***L'HOPITAL'S RULE::***

WATCH OUT FOR NEGATIVE SIGNS

7) $\lim\limits_{x\to 1} -\dfrac{x^2-1}{x-1}$

-2

8) $\lim\limits_{x\to 5} -\dfrac{x^2-5x}{x-5}$

-5

9) $\lim\limits_{x\to 2} -\dfrac{x^2-x-2}{x-2}$

-3

10) $\lim\limits_{x\to -5} \dfrac{x^2+3x-10}{x+5}$

-7

11) $\lim\limits_{x\to 0} \dfrac{\dfrac{1}{-4+x}+\dfrac{1}{4}}{x}$

$-\dfrac{1}{16}$

12) $\lim\limits_{x\to -3} \dfrac{x}{\dfrac{1}{3+x}-\dfrac{1}{3}}$

0

13) $\lim\limits_{x\to 5} \dfrac{x-5}{\sqrt{x+4}-3}$

6

14) $\lim\limits_{x\to 3} \dfrac{\sqrt{x+6}-3}{x-3}$

$\dfrac{1}{6}$

15. *Evaluate each limit using factoring and simplifying. Check your answers using L'Hospital's Rule:*

a) $\lim_{x \to 1} \frac{x^2-1}{x-1}$

2

b) $\lim_{x \to 2} \frac{4-x^2}{x-2}$

-4

c) $\lim_{x \to -1} \frac{x^2-x-2}{x^2+4x+3}$

$-\frac{3}{2}$

16. *Evaluate each limit by rationalizing the numerator. Check your answers using L'Hospital's Rule:*

a) $\lim_{x \to 1} \frac{\sqrt{x}-1}{x-1}$

$\frac{1}{2}$

b) $\lim_{x \to 2} \frac{\sqrt{2}-\sqrt{x}}{2-x}$

$\frac{1}{2\sqrt{2}}$ or

$\frac{\sqrt{2}}{4}$

c) $\lim_{x \to 0} \frac{\sqrt{x+4}-2}{x}$

$\frac{1}{4}$

***BE SURE TO DO MULTIPLE CHOICE QUESTIONS ONLINE **VERSION 13**

#39 – 49, #184 – 188

https://moodle.sd79.bc.ca/pluginfile.php/64324/mod_resource/content/4/AB%20Calculus%20Version%2013.pdf

3.3 - PRACTICE QUESTIONS

1. Use the **DEFINITION OF THE DERIVATIVE** to find $\frac{dy}{dx}$. Check your answers by differentiation.

a) $y = 2x + 5$

$$\lim_{h \to 0} \frac{2(x+h)+5-(2x+5)}{h}$$

$$\lim_{h \to 0} \frac{\cancel{2x}+2h\cancel{+5}-\cancel{2x}\cancel{-5}}{h}$$

$$\lim_{h \to 0} \frac{2\cancel{h}}{\cancel{h}}$$

$$=2$$

b) $y = 3 - 2x$

-2

c) $y = x^2$

$2x$

d) $y = x^2 + 2$

$$\lim_{h \to 0} \frac{(x+h)^2+2-(x^2+2)}{h}$$

$$(x+h)(x+h)+2-x^2-2$$

$$\lim_{h \to 0} \frac{\cancel{x^2}+2xh+h^2\cancel{+2}\cancel{-x^2}\cancel{-2}}{h}$$

$$\lim_{h \to 0} \frac{2xh+h^2}{h}$$

$$\lim_{h \to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}}$$

$$\lim_{h \to 0} 2x+h$$

$$= 2x$$

e) $y = x^2 + 2x$

$2x+2$

f) $y = 2x^2 - 6x + 1$

$4x-6$

1. Continued ...Use the **DEFINITION OF THE DERIVATIVE** to find $\frac{dy}{dx}$.

g) $y = \frac{1}{x}$

$$\lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$\lim_{h\to 0} \frac{x - (x+h)}{hx(x+h)}$$

$$\lim_{h\to 0} \frac{\cancel{x}-\cancel{x}-h}{hx(x+h)}$$

$$\lim_{h\to 0} \frac{-\cancel{h}}{\cancel{h}x(x+h)}$$

$$\lim_{h\to 0} \frac{-1}{x(x+h)} = \frac{-1}{x^2}$$

h) $y = \frac{1}{x+2}$

$$\frac{-1}{(x+2)^2}$$

i) $y = \frac{1}{3x}$

$$\frac{-1}{3x^2}$$

j) $y = \sqrt{x}$

$$\frac{1}{2x^{1/2}}$$

or

$$\frac{1}{2\sqrt{x}}$$

k) $y = \sqrt{x+4}$

$$\lim_{h\to 0} \frac{(\sqrt{x+h+4} - \sqrt{x+4})}{h} \frac{(\sqrt{x+h+4} + \sqrt{x+4})}{(\sqrt{x+h+4} + \sqrt{x+4})}$$

$$\lim_{h\to 0} \frac{x+h+4-(x+4)}{h(\sqrt{x+h+4} + \sqrt{x+4}}$$

$$\lim_{h\to 0} \frac{\cancel{x}+h\cancel{+4}-\cancel{x}\cancel{-4}}{h(\sqrt{x+h+4} + \sqrt{x+4})}$$

$$\lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h+4} + \sqrt{x+4}}$$

$$\lim_{h\to 0} \frac{1}{\sqrt{x+h+4} + \sqrt{x+4}}$$

$$\frac{1}{\sqrt{x+4} + \sqrt{x+4}}$$

$$= \frac{1}{2\sqrt{x+4}}$$

l) $y = \sqrt{2x}$

$$\frac{1}{\sqrt{2x}}$$

3.4 - PRACTICE QUESTIONS

1. *Use the fundamental trigonometric limit to evaluate each of the following:*

NOTE: all of these can be done by l'Hopital Rul see

let $3h=x$ $\therefore h=\frac{x}{3}$

a) $\lim_{h\to 0}\frac{\sin 3h}{h}$

$\sin\frac{x\cdot 3}{x} \Rightarrow 3\cdot\boxed{\frac{\sin x}{x}}$ 1

$=3$

b) $\lim_{h\to 0}\frac{\sin 5h}{h}$

$=5$

c) $\lim_{h\to 0}\frac{\sin 6h}{6h}$

$=1$

d) $\lim_{h\to 0}\frac{4\sin h}{h}$

$=4$

e) $\lim_{h\to 0}\frac{3\sin 5h}{2h}$

$\frac{15}{2}$

f) $\lim_{h\to 0}\frac{3\sin 3h}{5h}$

$\frac{9}{5}$

g) $\lim_{n\to 0}\frac{\sin 7n}{\sin 2n}$ First find LCM $14n$

$\left[\frac{14n\cdot\sin 7n}{14n\cdot\sin 2n}\right]$

$\left[\frac{2n\,(7)\sin 7n}{(2)\,7n\,\sin 2n}\right]$

$\frac{\sin 7n}{7n}\cdot\frac{2n}{\sin 2n}\cdot\frac{7}{2}$

$1\cdot 1\cdot\frac{7}{2} = \frac{7}{2}$

h) $\lim_{x\to 0}\frac{2\sin 4x}{\sin 3x} \Rightarrow \frac{\frac{d}{dx}2\sin 4x}{\frac{d}{dx}\sin 3x} \Rightarrow \frac{8\cos 4x}{3\cos 3x}$

$\lim_{x\to 0}\frac{8\cos 4(0)}{3\cos 3(0)} = \frac{8\cos(0}{3\cos(0}$

$=\frac{8\cdot 1}{3\cdot 1}$

2. *Use the fundamental limit involving e to evaluate each of the following:*

a) $\lim_{h\to 0}(1+h)^{\frac{3}{h}}$

e^3

b) $\lim_{x\to 0}\left(1+\frac{x}{5}\right)^{\frac{1}{x}}$ ew $\left(1+\frac{h}{5}\right)^{\frac{1}{h}}$

$\frac{h}{5}=x$ $\therefore$ $h=5x$

$(1+x)^{\frac{1}{5x}}$

$\boxed{(1+x)^{\frac{1}{x}}}^{\frac{1}{5}} = e^{\frac{1}{5}}$

c) $\lim_{u\to 0}(1+3u)^{\frac{1}{u}}$

e^3

d) $\lim_{y\to 0}(1+4y)^{\frac{2}{y}}$

e^8

e) $\lim_{t\to 0}(1+\sin t)^{\frac{1}{\sin t}}$

e

f) $\lim_{h\to 0}(1+2h)^{\frac{1}{h}}$

e^2

3. *Use the fundamental limit involving e to evaluate each of the following:*

a) $\lim_{n\to+\infty}\left(1+\frac{2}{n}\right)^n$

e^2

b) $\lim_{n\to+\infty}\left(1+\frac{3}{n}\right)^n$

e^3

c) $\lim_{m\to+\infty}\left(1+\frac{1}{m^2}\right)^{m^2}$

e

3.5 - PRACTICE QUESTIONS

Use the piecewise functions to find the given values. Sketch below to help.

1.

$$g(x) = \begin{cases} \sqrt{5-x}, & x < -4 \\ x^2 - 5, & -4 \le x < 2 \\ x - 3, & x \ge 2 \end{cases}$$

a. $\lim_{x \to 2^-} g(x) = -1$

b. $\lim_{x \to -4^+} g(x) = 11$

c. $g(2) = -1$

d. $\lim_{x \to -4^-} g(x) = 3$

e. $\lim_{x \to 2^+} g(x) = -1$

f. $\lim_{x \to 2} g(x) = -1$

g. $\lim_{x \to -4} g(x) = \text{DNE}$

h. $g(-4) = 11$

2.

$$w(\theta) = \begin{cases} \sin\theta, & \theta \le \pi \\ \cos\theta, & \pi < \theta < 2\pi \\ \tan\theta, & \theta > 2\pi \end{cases}$$

a. $\lim_{x \to \pi^-} w(\theta) = 0$

b. $w(\pi) = 0$

c. $\lim_{x \to \pi^+} w(\theta) = -1$

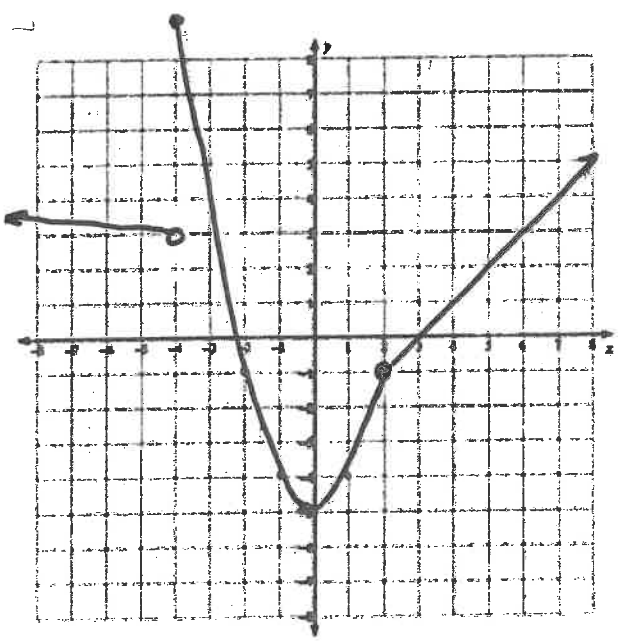
d. $\lim_{x \to 2\pi^-} w(\theta) = 1$

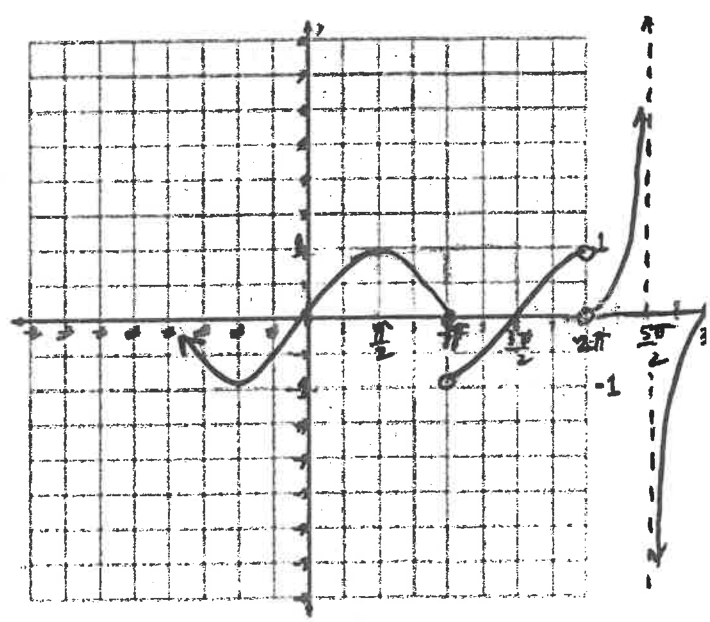
e. $\lim_{x \to \pi} w(\theta) = \text{DNE}$

f. $\lim_{x \to 2\pi^+} w(\theta) = 0$

g. $\lim_{x \to 2\pi} w(\theta) = \text{DNE}$

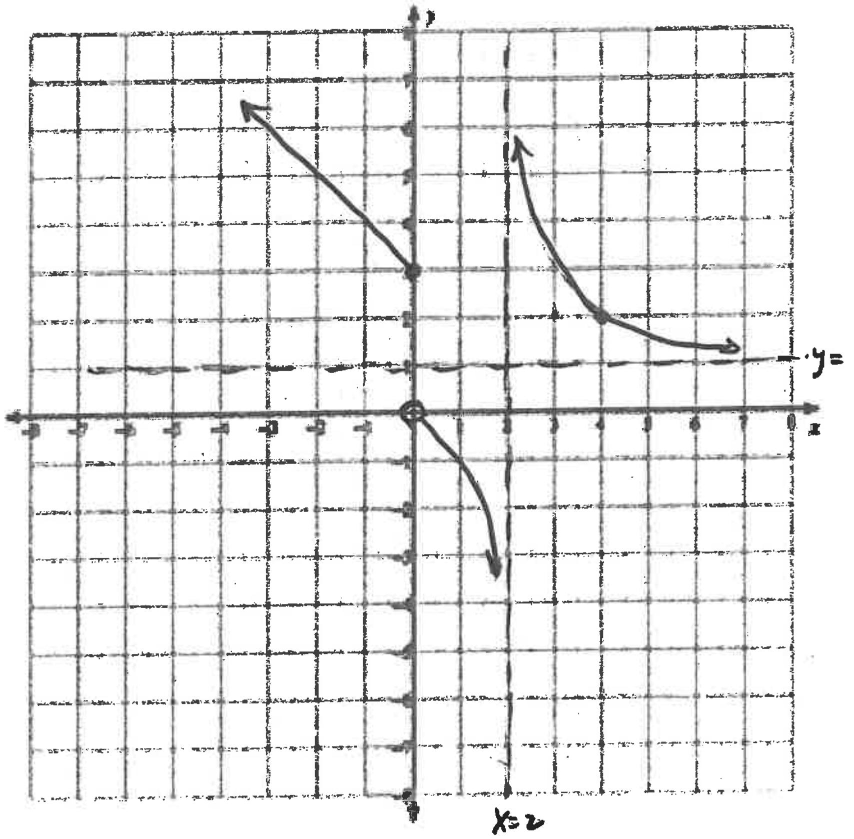
h. $w(2\pi) = \text{DNE}$





3. *Create your own piece-wise, non-continuous limit question with the following criteria:*
   > *oblique line (negative slope)*
   > *rational function (VA: x=2)*

Answer will vary

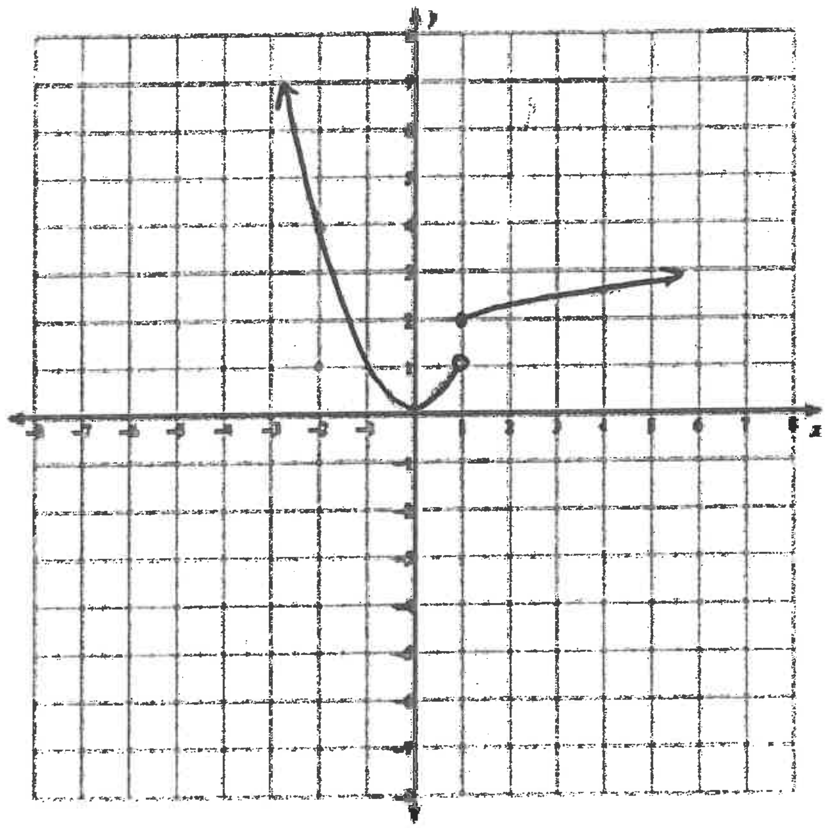


$$f(x)\begin{cases} 3-x & x \le 0 \\ \dfrac{x}{x-2} & x > 0 \end{cases}$$

$\lim_{x \to 0^-} f(x) = 3$ $\lim_{x \to 0} f(x) = \text{DNE}$

$\lim_{x \to 2^+} f(x) = +\infty$ $f(0) = 3$

4. *Create your own piece-wise, non-continuous limit question with the following criteria:*
   > parabola
   > radical function



$$g(x) \begin{cases} x^2 & x < 1 \\ \sqrt{x+3} & x \geq 1 \end{cases}$$

$\lim_{x \to -2} g(x) = 4$ $\qquad$ $\lim_{x \to 1^+} g(x) = 2$

$\lim_{x \to 1} g(x) =$ DNE

4.1 - PRACTICE QUESTIONS

1. *Find the slope, x-intercept and y-intercept of each of the following lines:*

a) $2x-4y+8=0$

m= ½
xint = -4
yint= 2

b) $\frac{2}{3}x-\frac{1}{4}y=2$

m = 8/3
xint = 3
yint = -8

c) $\frac{x}{2}-\frac{y}{5}=4$

m = 5/2
xint = 8
yint = -20

2. *Find the slope of a line parallel to and the slope of a line perpendicular to each of the lines in question #1.*

a) $\parallel \frac{1}{2}$
$\perp -2$

b) $\parallel 8/3$
$\perp -3/8$

c) $\parallel 5/2$
$\perp -2/5$

3. *Find the equation in slope-intercept form of the line passing through:*

a) $(-1, 2)$ *with slope of* $-\frac{1}{2}$

$y=mx+b$
$2=-\frac{1}{2}(-1)+b$
$b=\frac{3}{2}$ $\therefore$ $y=-\frac{1}{2}x+\frac{3}{2}$

b) the pts $(3, 1)$ *and* $(-2, -5)$

$y=\frac{6}{5}x-\frac{13}{5}$

c) $(2, -1)$ *and parallel to* $3x-2y=-6$

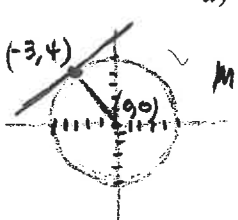
$y=\frac{3}{2}x-4$

d) $(2, -1)$ *and perpendicular to* $3x-2y=-6$

$y=\frac{-2}{3}x+\frac{1}{3}$

4. *Use algebra and geometry to find the equation in slope-intercept form of the tangent line*
to the given circle at the given point.

a) $x^2+y^2=25$ *at* $(-3, 4)$



$m=\frac{0-4}{0+3}=\frac{-4}{3}$ $\therefore$ $\perp=\frac{3}{4}$

$y=mx+b$
$4=\frac{3}{4}(-3)+b$
$4=\frac{-9}{4}+b$
$4+\frac{9}{4}=b$
$b=\frac{25}{4}$
$\therefore$ $y=\frac{3}{4}x+\frac{25}{4}$

b) $x^2+y^2=169$ *at* $(-5, -12)$

$y=\frac{-5}{12}x-\frac{169}{12}$

5. *Use the first derivative to find the slope of the tangent line to the given curve at the given point:*

a) $y = 2x^2 + 6 \quad at \ (-1, 8)$

$y' = 4x$

$y'(-1) = 4(-1)$

$\therefore mT = -4$

b) $y = -x^2 + 2x - 3 \quad at \ (2, 3)$

$mT = -2$

c) $y = 4 - 3x^3 \quad at \ (1, 1)$

$mT = -9$

d) $y = \dfrac{3x-1}{x+3} \quad at \ (-2, -7)$

$mT = 10$

6. *Find the slope of the normal line to the given curve at the given point for each of the curves in question #5.*

a) $\frac{1}{4}$ b) $\frac{1}{2}$ c) $\frac{1}{9}$ d) $-\frac{1}{10}$

7. *Find the equation in slope-intercept form of the tangent line to the given curve at the given point for each of the curves in question #5.*

a) $y = -4x + 4$

b) $y = -2x + 7$

c) $y = -9x + 10$

d) $y = 10x + 13$

8. *Find the equation in slope-intercept form of the normal line to the given curve at the given point for each of the curves in question #5.*

a) $y = \frac{1}{4}x + \frac{33}{4}$

b) $y = \frac{1}{2}x + 2$

c) $y = \frac{1}{9}x - \frac{8}{9}$

d) $y = -\frac{1}{10}x - \frac{36}{5}$

9. For each curve below find the equation of the (i) tangent line and (ii) normal line to the given curve at the given point:

a) $y=(x^3-5x+2)(3x^2-2x)$ at $(1,-2)$ b) $y=\sqrt{16x^3}$ at $(4, 32)$

1^{st} - find $y'=(x^3-5x+2)(6x-2)+(3x^2-5)(3x^2-2x)$

2^{nd} - plug in $x=1 \Rightarrow -10 \therefore m=-10$

$y=mx+b$

$-2=-10(1)+b$

$b=8$

$\therefore$

Tangent Line i) $y=-10x+8$

Normal Line ii) $m=\frac{1}{10}$

$-2=\frac{1}{10}(1)+b$

$b=\frac{-21}{10} \therefore$

$y=\frac{1}{10}x-\frac{21}{10}$

i) $y=12x-16$

ii) $y=\frac{-1}{12}x+\frac{97}{3}$

10. Tangent lines are drawn to the parabola $y=x^2$ at $(2,4)$ and $\left(\frac{-1}{8},\frac{1}{64}\right)$.

Prove that the tangents are perpendicular.

$y'=2x \quad (2,4) \Rightarrow m=4$

$y'=2x \quad \left(-\frac{1}{8},\frac{1}{64}\right) \Rightarrow m=-\frac{1}{4} \quad \therefore \perp$

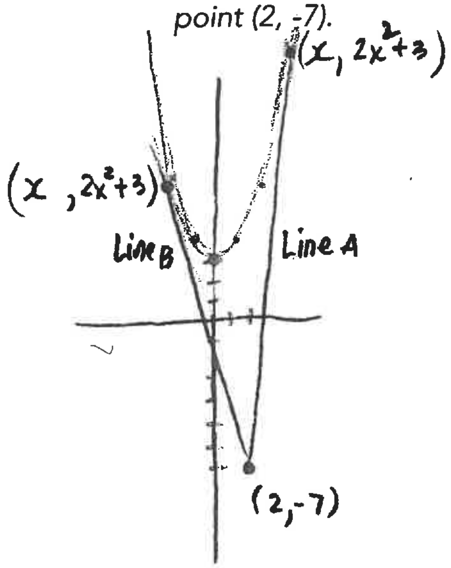
11. Find a point on the parabola $y=-x^2+3x+4$ where the slope of the tangent line is 5.

$(-1, 0)$

12. Find the equation of the normal line to the curve $y=-x^2+5x$ that has slope of -2.

$y = -2x + \frac{171}{16}$

13. Find the equations of the tangent lines to the curve $y=2x^2+3$ that pass through the point (2, -7).



LINE A $y = 20x - 47$

LINE B $y = -4x + 1$

HINT: Remember $y = 2x^2 + 3$
and use
$m = \frac{y_2 - y_1}{x_2 - x_1} = y'$

14. *Prove the curve* $y=-2x^3+x-4$ *has* <u>*no*</u> *tangent with a slope of 2.*

$$-6x^2+1=2$$

$$\frac{-6x^2}{-6}=\frac{1}{-6}$$

$$\sqrt{x^2}=\sqrt{-\frac{1}{6}}$$ Can't do

15. *At what points on the curve* $y^3-3x=5$ *is the slope of the tangent line equal to 1?*

HINT:
Implicit

$\left(-\frac{4}{3},1\right)$ and $(-2,-1)$

***BE SURE TO DO MULTIPLE CHOICE QUESTIONS ONLINE **VERSION 7**
#56 - 58, 60, 66, 69 – 71, 128, 130
https://moodle.sd79.bc.ca/pluginfile.php/1871/mod_resource/content/5/AB%20Calculus%20Version%207.pdf

4.2 - PRACTICE QUESTIONS

1. *Use Calculus to find the* ***CRITICAL POINTS*** *of each of the following functions:*

a) $y = x^2 - 6x + 5$

$y' = 2x - 6$

$2x - 6 = 0$

$x = 3$

$y = (3)^2 - 6(3) + 5$

$y = -4 \quad \therefore$

$\underline{\underline{(3, -4)}}$

b) $y = 2x^3 - 24x$

$(2, -32) \,;\, (-2, 32)$

c) $y = 2x^3 + 6x^2 + 6x$

$(-1, -2)$

2. *Use Calculus showing a* ***THUMBNAIL SKETCH*** *to find the* ***TURNING POINTS*** *of each. of the following functions*

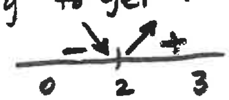
a) $y = x^2 - 4x - 3$

$y' = 2x - 4$

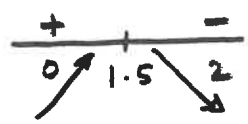
1st find critical pts (Same as above)

$(2, -7)$ * only interested in 2

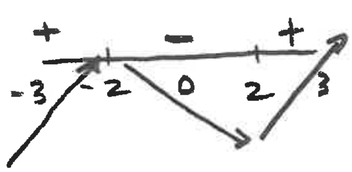
2nd pick number on each side of 2 and plug into y' to get + or −



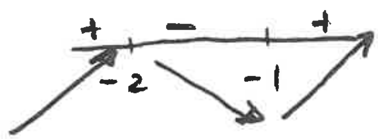
b) $y = -2x^2 + 6x + 13$



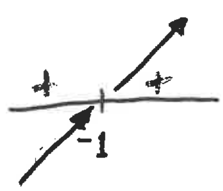
c) $y = x^3 - 12x$



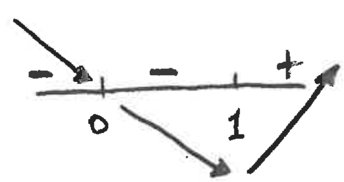
d) $y = 2x^3 + 9x^2 + 12x$



e) $y = x^3 + 3x^2 + 3x$



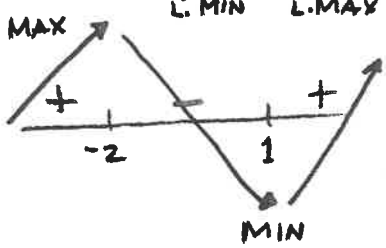
f) $y = 3x^4 - 4x^3$



3. *Find the local extrema (local maximum & minimum) using the FIRST DERIVATIVE TEST. Show a THUMBNAIL SKETCH.*

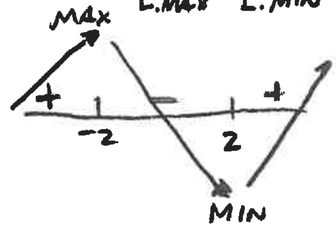
a) $y = 2x^3 + 3x^2 - 12x$

CP $(1,-7)$ & $(-2,20)$
L. MIN L.MAX



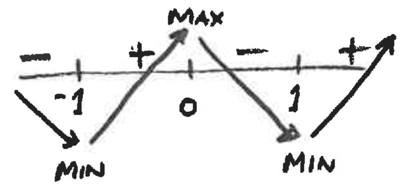
b) $y = 4x^3 - 48x + 10$

CP $(-2,74)$ & $(2,-54)$
L.MAX L.MIN



c) $y = (1 - x^2)^2 - 2$

CP $(-1,-2)$; $(0,-1)$ & $(1,-2)$
L.MIN L.MAX L.MIN



4. *Use Calculus showing a* ***THUMBNAIL SKETCH*** *to find* ***the INFLECTION POINTS*** *of each of the following functions:*

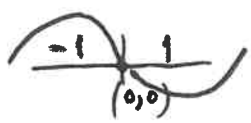
a) $y = x^3 - 12x$

1st find $y'' = 6x = 0$

I.P $(0,0)$

2nd Test for Concavity

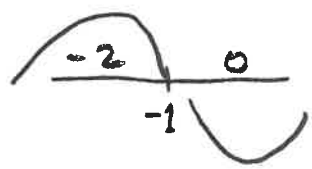
$y'' = 6(-1)$
-6 ∩
CD



$y'' = 6(1)$
$+6$ ∪
CU

Since there is a Change in Concavity
$(0,0)$ is I.P

b) $y = x^3 + 3x^2 + 3x$



I.P $(-1\ -1)$

c) $y = x^2 - 4x - 3$

$y'' = 2$ ∴

no I.P.

5. *Use the* ***SECOND DERIVATIVE TEST*** *to find and classify all local* ***extrema:***

a) $y = x^2 - 10x + 3$

$(5,-22)$ L. MIN

b) $y = x^3 - 12x + 5$

$(-2,21)$ L.MAX
$(2,-11)$ L.MIN

c) $y = x^4 - 2x^3$

$\left(\frac{3}{2}, \frac{-27}{16}\right)$ L. MIN

6. *Use Calculus to find the turning points of each of the following polynomial functions (show a thumbnail sketch). Identify each turning points as a local maximum or a local minimum and sketch the graph near each turning point.*

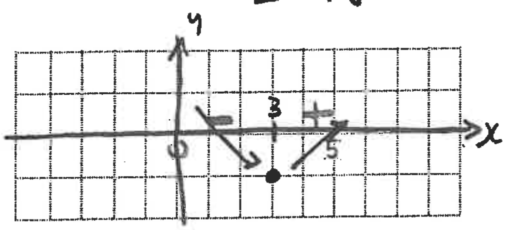
a) $y = x^2 - 6x + 8$

$y' = 2x - 6 = 0$

$x = 3 \quad y = -1$

C.P $(3, -1)$

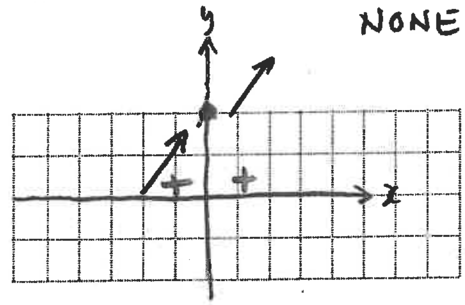
L. MIN



b) $f(x) = x^3 + 2$

CP $(0, 2)$

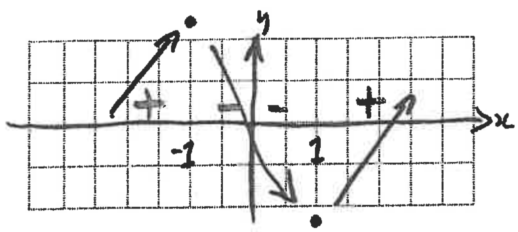
NONE



c) $g(x) = \frac{4}{3}x^3 - 4x$

CP $(1, -2\frac{2}{3})$ L. MIN

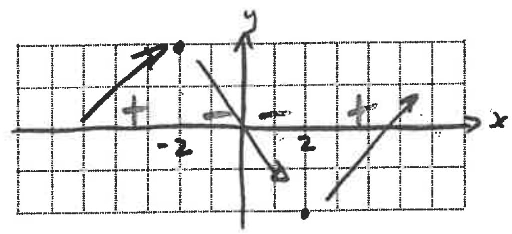
$(-1, 2\frac{2}{3})$ L. MAX



d) $y = x^3 - 12x$

CP $(2, -16)$ L. MIN

$(-2, 16)$ L. MAX

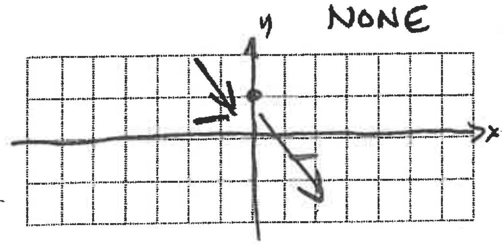


e) $y = -2x^5 + 1$

$y' = -10x^4 = 0$

$x = 0 \quad (0, 1)$

NONE

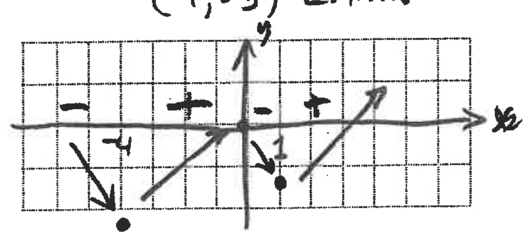


f) $h(x) = x^4 + 4x^3 - 8x^2$

CP $(-4, -128)$ L. MIN

$(0, 0)$ L. MAX

$(1, -3)$ L. MIN



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#23 - 37, 236 – 248, 272 - 292

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EXTENDED QUESTIONS

PART I:

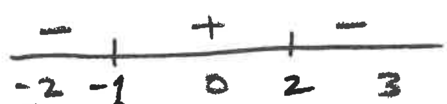
Find all intervals where the following functions are increasing or decreasing.

↳ USE FIRST DERIVATIVE TEST

1. $y = -2x^3 + 3x^2 + 12x + 2$

$y' = -6x^2 + 6x + 12 \Rightarrow -6(x-2)(x+1) = 0$

C.P.: $x = 2 \quad x = -1$



$-6(x-2)(x+1)$

$- \cdot - \cdot - \qquad - \cdot - \cdot + \qquad - \cdot + \cdot +$

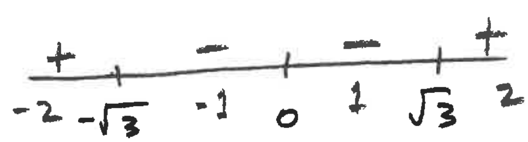
$= - \qquad = + \qquad = -$

$\boxed{\begin{array}{l} \text{DECREASING } (-\infty, -1) \text{ \& } (2, \infty) \\ \text{INCREASING } (-1, 2) \end{array}}$

2. $y = x^5 - 5x^3$

$y' = 5x^4 - 15x^2 \Rightarrow 5x^2(x^2 - 3) = 0$

C.P.: $x = 0 \quad x = \pm\sqrt{3}$ or ± 1.73



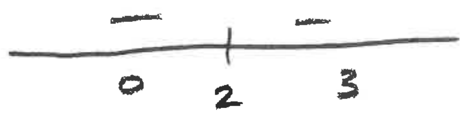
$\boxed{\begin{array}{l} \text{DECREASING } (-\sqrt{3}, \sqrt{3}) \\ \text{INCREASING } (-\infty, -\sqrt{3}) \text{ \& } (\sqrt{3}, \infty) \end{array}}$

3. $y = \dfrac{x+2}{x-2}$

$$y' = \frac{-4}{(x-2)^2} = 0$$

C.P. DNE

VA: $x = 2$ $\therefore$ DISCONTINOUS FUNCTION @ 2



$$\boxed{\text{DECREASING } (-\infty, 2) \div (2, \infty)}$$

4. $y = \dfrac{x}{x^2-1}$

$$y' = \frac{-x^2-1}{(x^2-1)^2} = 0 \Rightarrow -x^2-1 = 0$$

$x^2 = -1$ can't do

$\therefore$ C.P DNE

VA: $x = \pm 1$

$$\boxed{\text{DECREASING } (-\infty, -1) \div (-1, 1) \div (1, \infty)}$$

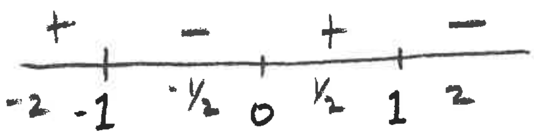
5. $y = x^2 e^{-x^2}$

$y' = x^2 \cdot e^{-x^2}(-2x) + 2x e^{-x^2} \Rightarrow$ RW $\dfrac{-2x^3}{e^{x^2}} + \dfrac{2x}{e^{x^2}} =$

$$y' = \frac{-2x^3 + 2x}{e^{x^2}}$$

$-2x^3 + 2x = 0$

$-2x(x^2 - 1) = 0$



C.P.: $x = 0$ $\quad x = \pm 1$

$\boxed{\begin{array}{l}\text{DECREASING } (-1,0) \text{ \& } (1,\infty) \\ \text{INCREASING } (-\infty,-1) \text{ \& } (0,1)\end{array}}$

6. $y = x\ln x$

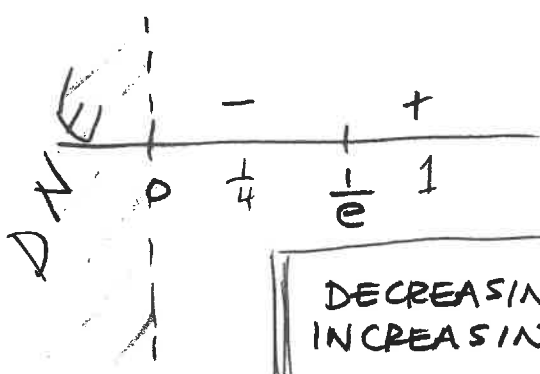
$y' = x \cdot \frac{1}{x} + 1(\ln x)$

$1 + \ln x = 0$

$e^{\ln x} = e^{-1}$

C.P. : $x = e^{-1}$ or $x = \frac{1}{e}$ or 0.37

WITH $\ln(x)$ $x > 0$



$\boxed{\begin{array}{l}\text{DECREASING } (0, \frac{1}{e}) \\ \text{INCREASING } (\frac{1}{e}, \infty)\end{array}}$

7. $y = 3x^{\frac{2}{3}} - x^2$

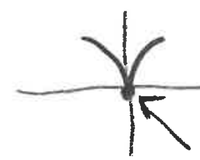
$$y' = 2x^{-1/3} - 2x \Rightarrow 2x^{-1/3}\left(1 - x^{4/3}\right) = 0$$

$\Downarrow$

C.P.: Cusp $x = 0$ $\qquad x = \pm 1$

$*$ non-differentiable @ $x=0$

Look at DESMOS



cusp

+		−		+		−

$-2 \quad -1 \quad -\tfrac{1}{2} \quad 0 \quad \tfrac{1}{2} \quad 1 \quad 2$

$$\boxed{\begin{array}{l} \text{DECREASING } (-1,0) \text{ \& } (1,\infty) \\ \text{INCREASING } (-\infty,-1) \text{ \& } (0,1) \end{array}}$$

8. $y = (2x-4)^{\frac{1}{3}}$

$$y' = \frac{1}{3}(2x-4)^{-2/3}(2) \Rightarrow \frac{2}{3(2x-4)^{2/3}} = 0$$

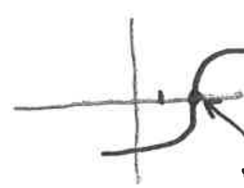
C.P: DNE

But this is non-differentiable @ $x=2$

Look AT GRAPH DESMOS

+		+

$0 \quad 2 \quad 3$



This is a Vertical Tangent

$$\boxed{\text{INCREASING } (-\infty,\infty)}$$

PART II:

Find all intervals where the following functions are concave up or concave down.

↳ USE SECOND DERIVATIVE "Concavity Test"

1. $y = -2x^3 + 3x^2 + 12x + 2$

1st STEP: find C.P $y' = 0$

$y' = -6x^2 + 6x + 12$

C.P $x = 2$ $x = -1$

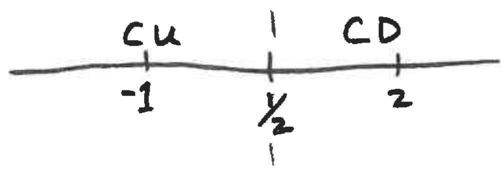
2nd STEP: find I.P $y'' = 0$

$y'' = -12x + 6 = 0$ I.P $x = \frac{1}{2}$ * Change in concavity

3rd STEP: Concavity Test $y''(C.P) = \pm$

$y''(2) = -12(2) + 6 = -$ CD

$y''(-1) = -12(-1) + 6 = +$ CU



$$\boxed{\begin{array}{l} CU\ (-\infty, \tfrac{1}{2}) \\ CD\ (\tfrac{1}{2}, \infty) \end{array}}$$

2. $y = x^5 - 5x^3$

SEE PART I for C.P. 0 $\pm\sqrt{3}$

$y'' = 20x^3 - 30x \Rightarrow 10x(2x^2 - 3) = 0$

I.P.: $x = 0$ $x = \pm\sqrt{3/2}$ or ± 1.22

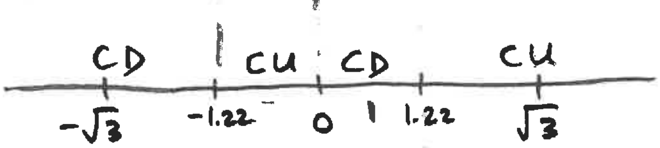
$y''(0) = 20(0)^3 - 30(0) = 0$ Test fails ∴ I.P.

$y''(\sqrt{3}) = 20(\sqrt{3})^3 - 30(\sqrt{3}) = +$ CU

$y''(-\sqrt{3}) = 20(-\sqrt{3})^3 - 30(-\sqrt{3}) = -$ CD

$y''(-1) = 10x(2x^2 - 3) = - \cdot - = +$ CU

$y''(1) = 10x(2x^2 - 3) = + \cdot - = -$ CD



$$\boxed{\begin{array}{l} CU\ (-\sqrt{3/2}, 0) \cup (\sqrt{3/2}, \infty) \\ CD\ (-\infty, -\sqrt{3/2}) \cup (0, \sqrt{3/2}) \end{array}}$$

3. $y = \dfrac{x+2}{x-2}$

$$y' = \frac{-4}{(x-2)^2} \qquad y'' = -4(x-2)^{-2}$$

$$8(x-2)^{-3} \quad \text{or}$$

$$y'' = \frac{8}{(x-2)^3} = 0$$

No I.P.

CD | CU

1 2 3

$$y''(1) = \frac{8}{(1-2)^3} = \frac{+}{-} = - \quad \text{CD}$$

$$y''(3) = \frac{8}{(3-2)^2} = \frac{+}{+} = + \quad \text{CU}$$

$$\boxed{\begin{array}{l} \text{CD } (-\infty, 2) \\ \text{CU } (2, \infty) \end{array}}$$

4. $y = \dfrac{x}{x^2-1}$

$$y' = \frac{-x^2-1}{(x^2-1)^2} \qquad y'' = \overset{F}{(-x^2-1)}\overset{S}{(x^2-1)^{-2}}$$

$$y'' = \frac{2x(x^2+3)}{(x^2-1)^3} = 0$$

$$2x = 0 \qquad \text{\&} \qquad x^2+3=0$$

$$x = 0 \qquad \qquad \sqrt{x^2} = -\sqrt{3}$$

Can't do

I.P: $x = 0$

CD | CU | CD | CU

-2 -1 -½ 0 ½ 1 2

$$y''(-2) = \frac{2x(x^2+3)}{(x^2-1)^3} = \frac{-\cdot+}{+} = - \quad \text{CD}$$

$$y''(-\tfrac{1}{2}) = \frac{2x(x^2+3)}{(x^2-1)^3} = \frac{-\cdot+}{-} = + \quad \text{CU}$$

$$y''(\tfrac{1}{2}) = \frac{2x(x^2+3)}{(x^2-1)^3} = \frac{+\cdot+}{-} = - \quad \text{CD}$$

$$y''(2) = \frac{2x(x^2+3)}{(x^2-1)^3} = \frac{+\cdot+}{+} = + \quad \text{CU}$$

$$\boxed{\begin{array}{l} \text{CD } (-\infty, -1) \text{ \& } (0, 1) \\ \text{CU } (-1, 0) \text{ \& } (1, \infty) \end{array}}$$

5. $y = x^2 e^{-x^2}$

$y' = \dfrac{-2x^3 + 2x}{e^{x^2}}$

$\dfrac{B T' - B' T}{B^2}$

$y'' = \dfrac{(\cancel{e^{x^2}})(-6x^2+2) - (2x\cancel{e^{x^2}})(-2x^3+2x)}{(e^{x^2})^2}$

$y'' = \dfrac{-6x^2 + 2 + 4x^4 - 4x^2}{e^{x^2}}$

$y'' = \dfrac{4x^4 - 10x^2 + 2}{e^{x^2}} = 0$

I.P $x = \pm 1.51$; $x = \pm 0.47$

* USED DESMOS

$y''(-2) = +$ CU

$y''(-1) = -$ CD

$y''(0) = +$ CU

$y''(1) = -$ CD

$y''(2) = +$ CU

CU	CD	CU	CD	CU
-2	-1.51, -1	-0.47, 0	0.47, 1	1.51, 2

$\boxed{\begin{array}{l} \text{CU } (-\infty, -1.51) ; (-0.47, 0.47) ; (1.51, \infty) \\ \text{CD } (-1.51, -0.47) ; (0.47, 1.51) \end{array}}$

6. $y = x\ln x$

$y' = 1 + \ln x$

$y'' = \dfrac{1}{x} = 0$

I.P DNE

CU

0 1

$y''(1) = \dfrac{1}{1} = +$ CU

$\boxed{\text{CU } (0, \infty)}$

7. $y = 3x^{\frac{2}{3}} - x^2$

$y' = 2x^{-1/3} - 2x$

$y'' = -\frac{2}{3}x^{-4/3} - 2$ or $\frac{-2}{3x^{4/3}} - 2 = 0$

I.P DNE

CD | CD

-1 0 1

$y''(-1) = -$ CD

$y''(1) = -$ CD

$\boxed{\text{CD } (-\infty, \infty)}$

8. $y = (2x-4)^{\frac{1}{3}}$

$y' = \frac{2}{3}(2x-4)^{-2/3}$

$y'' = \frac{-4}{9}(2x-4)^{-5/3}(2)$

$y'' = \frac{-8}{9(2x-4)^{5/3}} = 0$

I.P DNE

CU | CD

0 2 3

$y''(0) = +$ CU

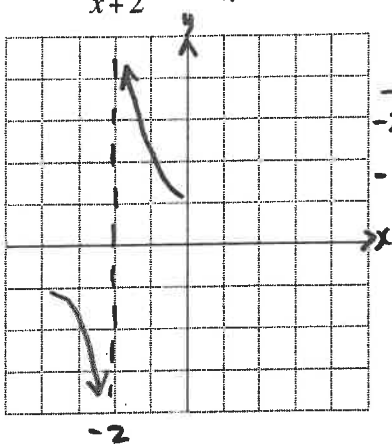
$y''(3) = -$ CD

CU $(-\infty, 2)$
CD $(2, \infty)$

4.3 - PRACTICE QUESTIONS

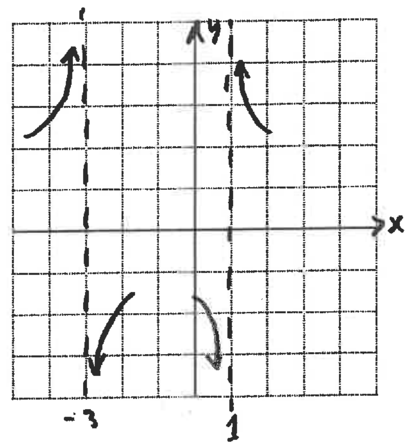
1. *Use algebra to find* **vertical asymptotes**, *if they exist, of each function and sketch the graph near the asymptotes.*

a) $y = \dfrac{1}{x+2}$ VA: $x = -2$

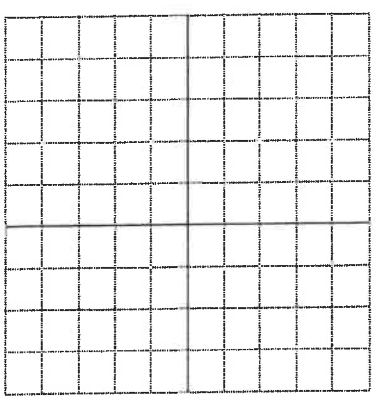


LS	RS
-2.0001	-1.9999
-10000	10000

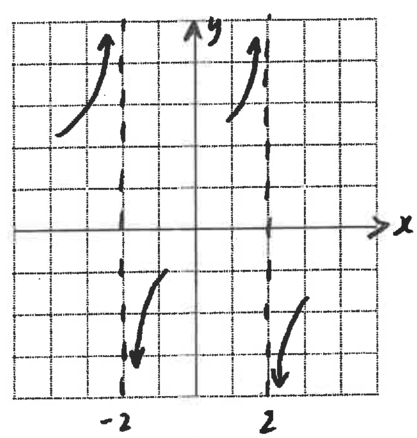
b) $f(x) = \dfrac{5}{(x-1)(x+3)}$ VA: $x = -3$ ∴ 1



c) $g(x) = 6x^3 - 5x - 3$ None, not a rational function

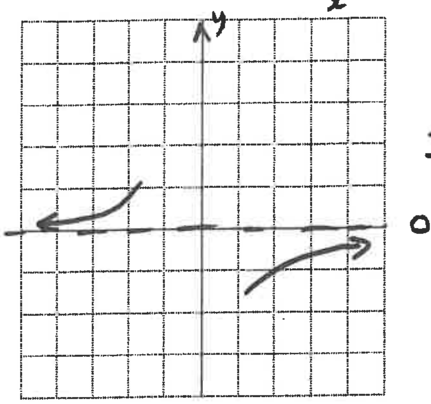


d) $h(x) = \dfrac{-6x}{x^2 - 4}$ VA: $x = \pm 2$



2. *Use Calculus to find the* **horizontal asymptotes**, *if they exist, of each function and sketch the graph near the asymptotes.*

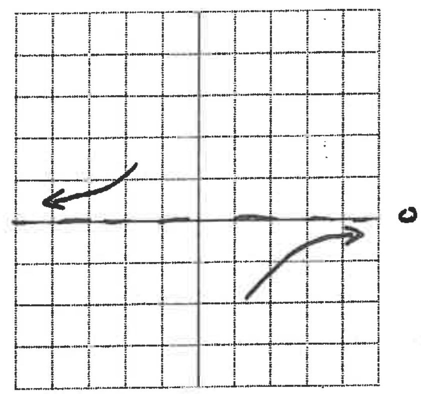
a) $y=\frac{-2}{x}$ $\Rightarrow \frac{\frac{-2}{x}}{\frac{x}{x}}$ $\lim_{x\to\infty} \frac{\frac{-2}{x}}{1} = \frac{0}{1}$



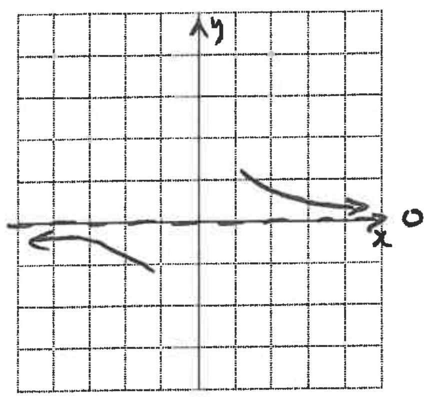
HA. 0

LS	RS
-999	999
0.002	-0.002

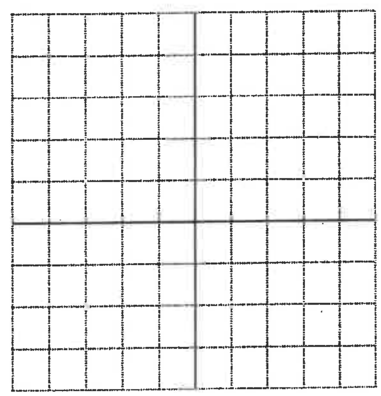
b) $f(x)=\frac{-2}{x+1}$ HA: 0



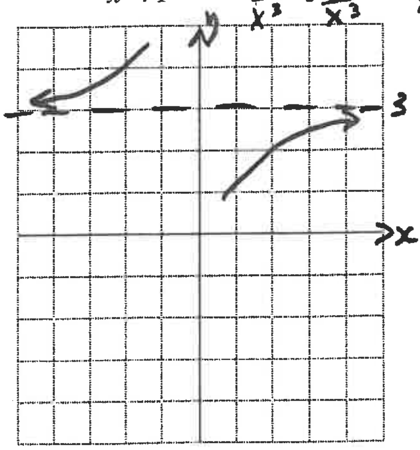
c) $g(x)=\frac{5}{x+1}$ HA: 0



d) $h(x)=4x^3-2x^2+5$ NONE

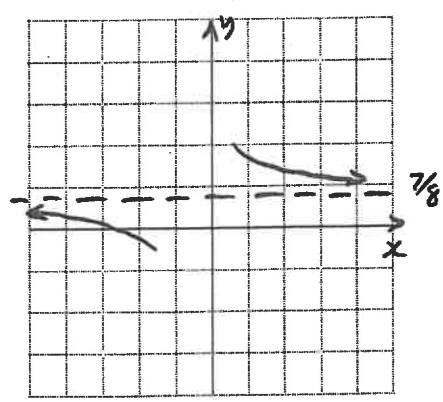


e) $y=\frac{3x^3-2}{x^3+1}$ $\Rightarrow \frac{\frac{3x^3}{x^3}-\frac{2}{x^3}}{\frac{x^3}{x^3}+\frac{1}{x^3}} \Rightarrow \lim_{x\to\infty} \frac{3-\frac{2}{x^3}}{1+\frac{1}{x^3}}$



HA 3

f) $k(x)=\frac{7x^3}{(2x-1)^3}$ HA $\frac{7}{8}$



3. *Make a rough sketch of each function below. Indicate* ***x-intercept, y-intercept, turning points, vertical*** *and* ***horizontal asymptotes.***

a) $y=\dfrac{x+4}{x+2}$

VA: $x=-2$
HA: $y=1$

xint
Let $y=0$
$0=x+4$
$x=-4$

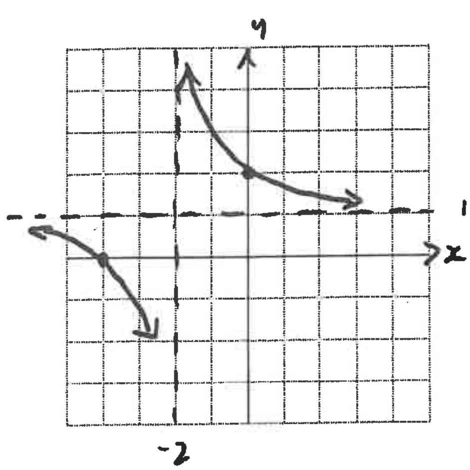
yint
Let $x=0$
$y=\dfrac{0+4}{0+2}$
$y=2$

T.P. $\dfrac{B'T-BT'}{B^2}$

$y'=\dfrac{(x+2)(1)-(1)(x+4)}{(x+2)^2}$

$y'=\dfrac{-2}{(x+2)^2}=0$

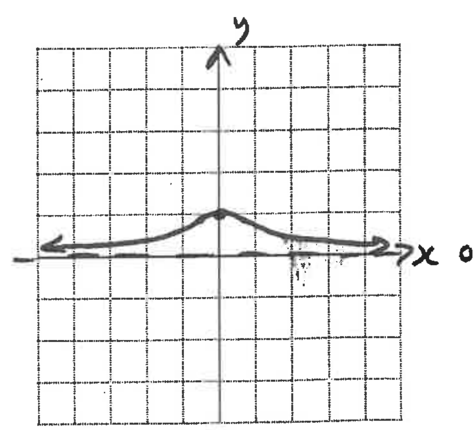
DNE



b) $f(x)=\dfrac{1}{1+x^2}$

VA : NONE
HA : $y=0$

xint DNE
yint 1
T.P (0,1)



c) $g(x) = \frac{1}{x} - \frac{1}{x^2}$ RW $\frac{x-1}{x^2}$

VA: $x = 0$
HA: $y = 0$

x-int: $x = 1$

y int: DNE

T.P. $\Rightarrow \frac{x-1}{x^2} \Rightarrow$ RW $x^{-1} - x^{-2}$

$g'(x) = -x^{-2} + 2x^{-3}$

RW $\Rightarrow \frac{2}{x^3} - \frac{1}{x^2} \Rightarrow \frac{2-x}{x^3} = 0$

T.P.: $(2, 0.25)$

d) $h(x) = 2x^3 - 2x^2 - 5x + 5$

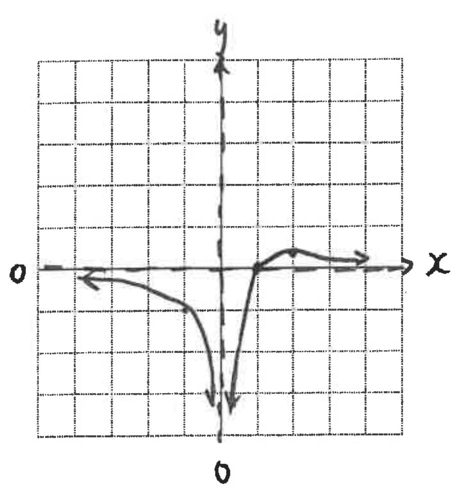
hint: Synth. Div

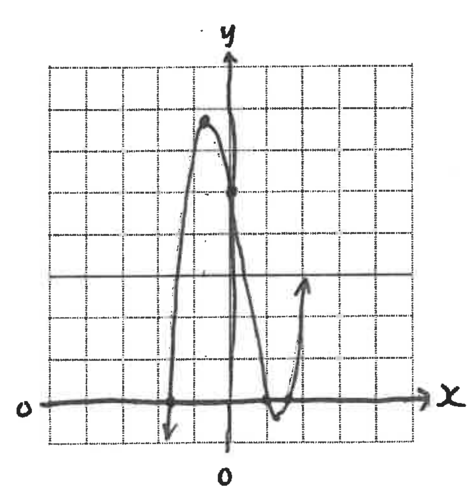
VA: DNE
HA: DNE

x-int: -1.58 ; 1 ; 1.58

y-int: 5

T.P: $(-0.64, 6.86)$ and $(1.31, -0.49)$





e) $y=\dfrac{x+1}{x}$

VA : x=0
HA : y=1

Xint	yint
-1	DNE

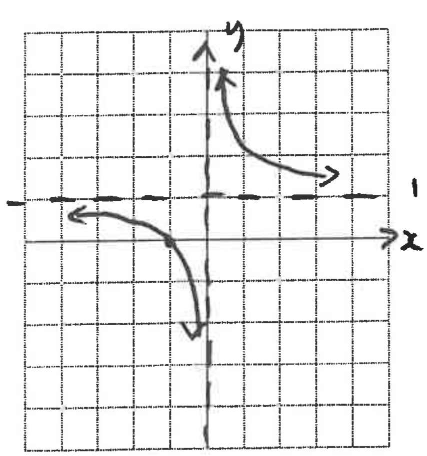
T.P
DNE

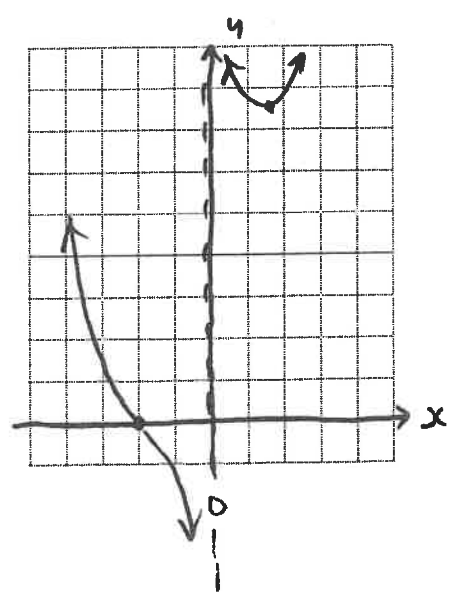
f) $y=x^2+\dfrac{8}{x}$

VA: X=0
HA DNE

Xint -2
yint DNE

T.P
(1.59, 7.56)

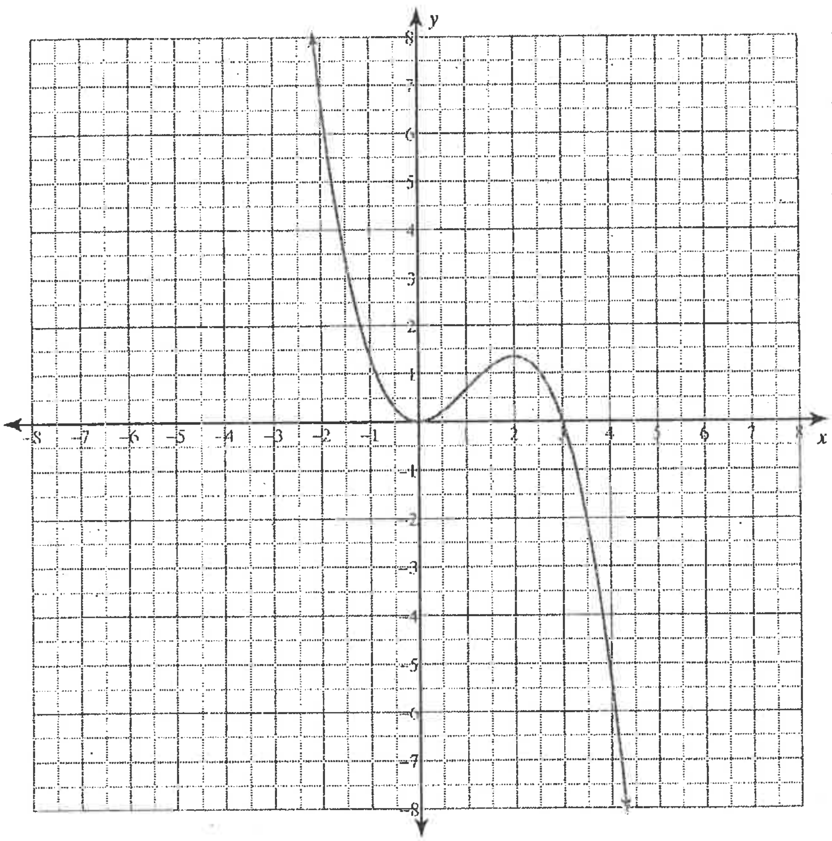




EXTENDED QUESTIONS

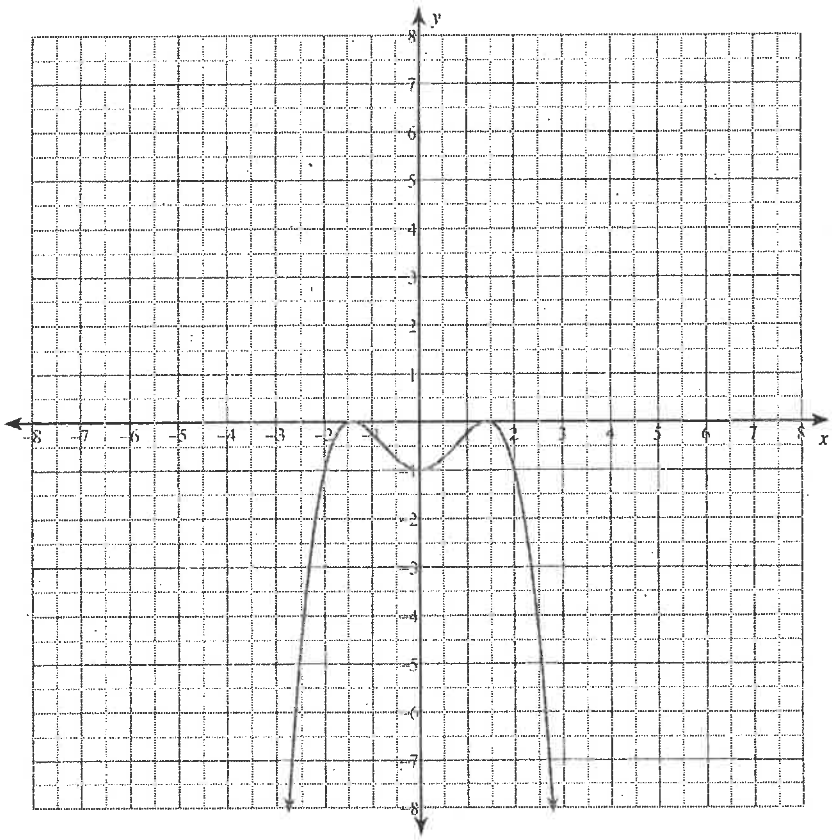
For each problem, find the: x and y intercepts, x-coordinates of the critical points, open intervals where the function is increasing and decreasing, x-coordinates of the inflection points, open intervals where the function is concave up and concave down, and relative minima and maxima. Using this information, sketch the graph of the function.

1) $y = -\dfrac{x^3}{3} + x^2$



x-intercepts at $x = 0, 3$
y-intercept at $y = 0$
Critical points at: $x = 0, 2$
Increasing: $(0, 2)$
Decreasing: $(-\infty, 0), (2, \infty)$
Inflection point at: $x = 1$
Concave up: $(-\infty, 1)$
Concave down: $(1, \infty)$
Relative minimum: $(0, 0)$
Relative maximum: $\left(2, \dfrac{4}{3}\right)$

2) $y = -\frac{x^4}{4} + x^2 - 1$



x-intercepts at $x = -\sqrt{2}, \sqrt{2}$

y-intercept at $y = -1$

Critical points at: $x = -\sqrt{2}, 0, \sqrt{2}$

Increasing: $(-\infty, -\sqrt{2}), (0, \sqrt{2})$

Decreasing: $(-\sqrt{2}, 0), (\sqrt{2}, \infty)$

Inflection points at: $x = -\frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{3}$

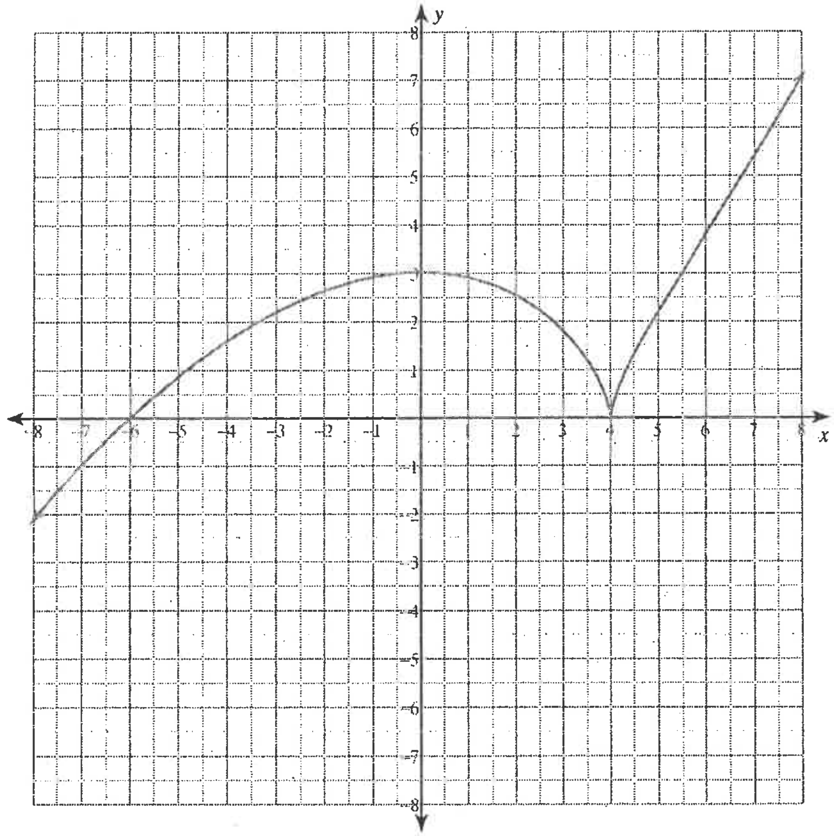
Concave up: $\left(-\frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{3}\right)$

Concave down: $\left(-\infty, -\frac{\sqrt{6}}{3}\right), \left(\frac{\sqrt{6}}{3}, \infty\right)$

Relative minimum: $(0, -1)$

Relative maxima: $(-\sqrt{2}, 0), (\sqrt{2}, 0)$

3) $y=\frac{1}{5}(x-4)^{\frac{5}{3}}+2(x-4)^{\frac{2}{3}}$



x-intercepts at $x=-6, 4$

y-intercept at $y=\frac{12\sqrt[3]{2}}{5}$

Critical points at: $x=0, 4$

Increasing: $(-\infty, 0)$, $(4, \infty)$

Decreasing: $(0, 4)$

Inflection point at: $x=6$

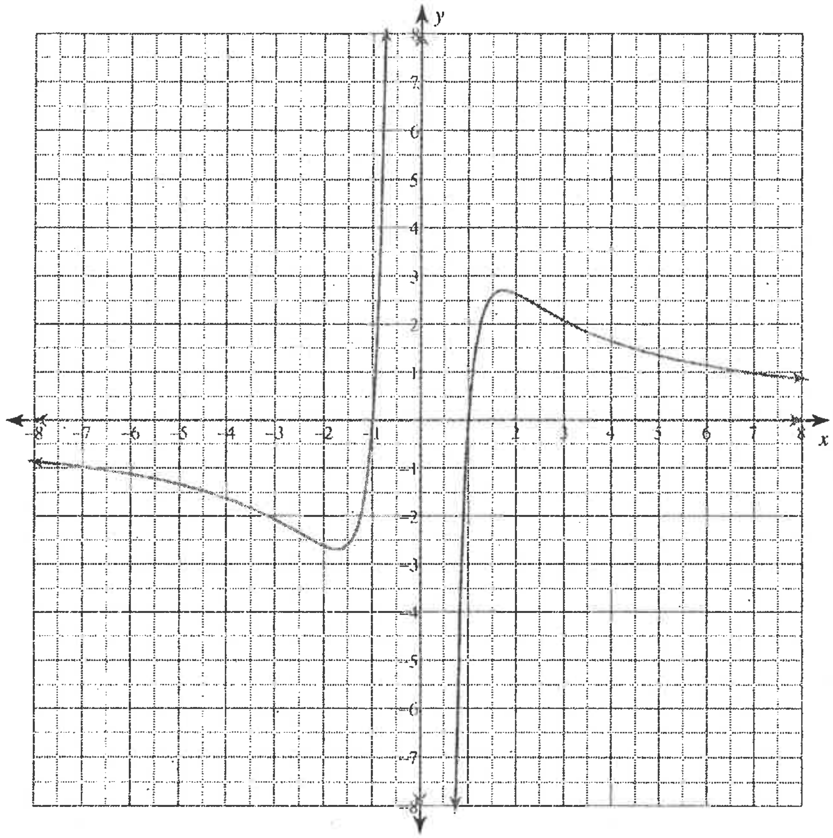
Concave up: $(6, \infty)$

Concave down: $(-\infty, 4)$, $(4, 6)$

Relative minimum: $(4, 0)$

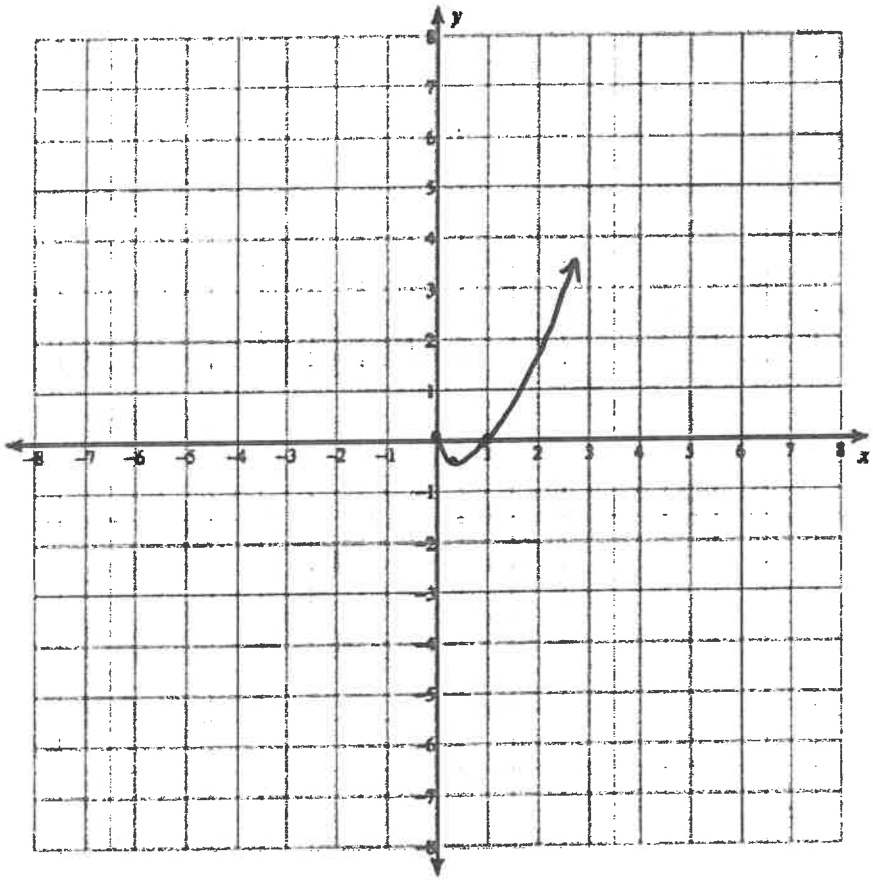
Relative maximum: $\left(0, \frac{12\sqrt[3]{2}}{5}\right)$

4) $y = \dfrac{7x^2 - 7}{x^3}$



x-intercepts at $x = -1, 1$
No y-intercepts.
Vertical asymptote at: $x = 0$
Horizontal asymptote at: $y = 0$
Critical points at: $x = -\sqrt{3}, \sqrt{3}$
Increasing: $(-\sqrt{3}, 0), (0, \sqrt{3})$
Decreasing: $(-\infty, -\sqrt{3}), (\sqrt{3}, \infty)$
Inflection points at: $x = -\sqrt{6}, \sqrt{6}$
Concave up: $(-\sqrt{6}, 0), (\sqrt{6}, \infty)$
Concave down: $(-\infty, -\sqrt{6}), (0, \sqrt{6})$
Relative minimum: $\left(-\sqrt{3}, -\dfrac{14\sqrt{3}}{9}\right)$
Relative maximum: $\left(\sqrt{3}, \dfrac{14\sqrt{3}}{9}\right)$

5) $y = x\ln(x)$



x int at $x = 1$

No y int

VA DNE

HA DNE

C.P at $x = 0.37$

Increasing $(0.37, \infty)$

Decreasing $(0, 0.37)$

Inflec Pt. DNE

CU $(0, \infty)$

CD never

Rel. Min $(0.37, -0.37)$

Rel. Max none

7

4.4 - PRACTICE QUESTIONS

1. *For the following situations, state whether the velocity is positive or negative and whether the speed is increasing or decreasing.*

	Velocity + or –	*Speed I or D*
a) braking while moving forward	+	D
b) accelerating while in reverse gear	–	I
c) accelerating while in forward gear	+	I
d) braking while moving backward	–	D

2. A particle moves in a straight line with a position function s(t), $t \geq 0$. In each case sketch the graphs of the position and velocity:

a) $s(t)=2t^3-3t^2+6t$

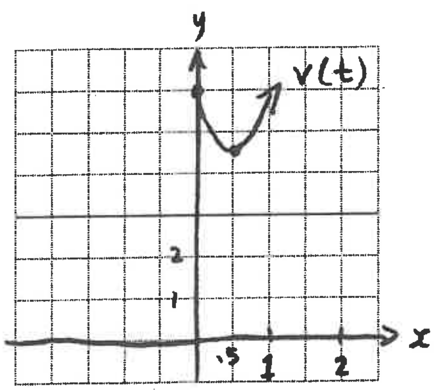
$v(t) = 6t^2 - 6t + 6$

↳ y int 6

T.P. (0.5, 4.5)

$v'(t) = a(t) = 12t - 6 = 0$

$\frac{12t}{12} = \frac{6}{12} = 0.5$



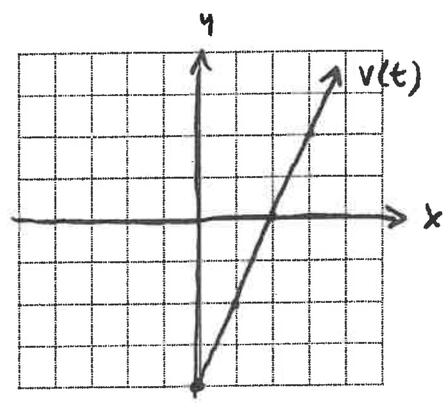
b) $s(t)=t^2-4t+4$

$v(t) = 2t - 4$

x int 2

y int −4

T.P DNE



3. For each function above complete the following chart:

	a) $s(t)=2t^3-3t^2+6t$	b) $s(t)=t^2-4t+4$
a) when is the particle at rest $v(t)=0$	never	2 sec
b) when is the particle moving in a positive direction $v(t)>0$	always	$t > 2$sec or $(2, \infty)$
c) when is the particle moving in a negative direction $v(t)<0$	never	$0<t<2$sec or $(0,2)$
d) when is the speed of the particle increasing	$t>0.5$ or $(0.5, \infty)$	$t>2$ or $(2,\infty)$
e) when is the speed of the particle decreasing	$0<t<0.5$ or $(0, 0.5)$	$0<t<2$ $(0,2)$
f) what is the total distance traveled after 6 sec of motion, starting t = 0	360	20

4.

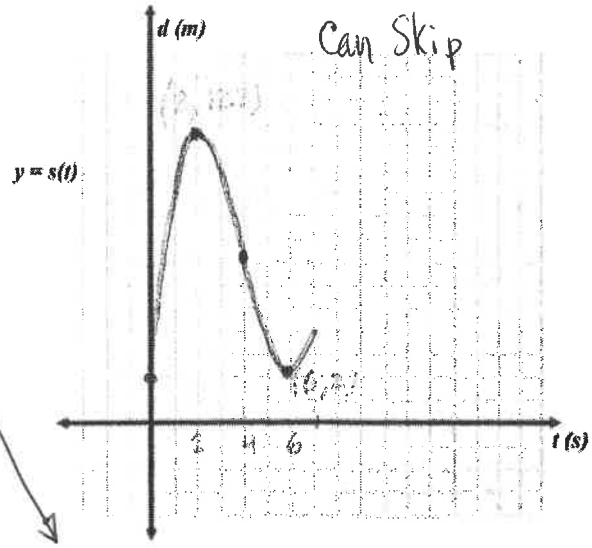
Can Skip

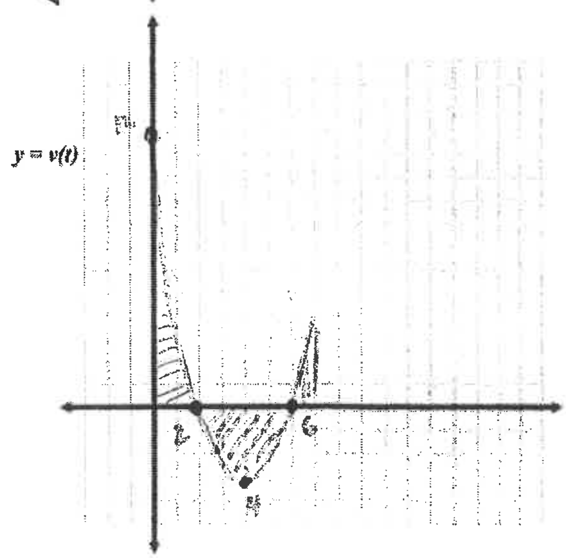
. Given the position equation:

$s(t) = \frac{1}{3}t^3 - 4t^2 + 12t + 2$; [0, 7]

$v(t) = t^2 - 8t + 12$

a) Sketch a graph of position and velocity





b) Answer the following questions:

I. When is the particle at rest?

$v(t) = 0$ 2s ; 6s

II. When is the particle moving in a positive direction?

$v(t) > 0$ $(0,2)$ $(6,7)$

III. When is the particle moving in a negative direction?

$v(t) < 0$ $(2,6)$

IV. When is the particle slowing down?

moving towards x-axis $(0,2)$ $(4,6)$

V. When is the particle speeding up?

moving away from x-axis $(2,4)$ $(6,7)$

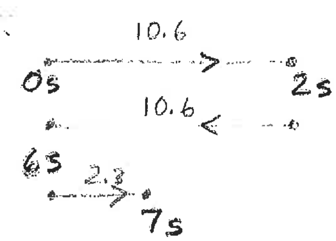
VI. What is the total distance travelled after 7 sec.?

* Break up $\int$ because neg. veloc.

$$\int_0^2 v(t) + \int_2^6 v(t) + \int_6^7 v(t)$$

$\boxed{T.D = \frac{71}{3}}$

$$\frac{32}{3} + \frac{32}{3} + \frac{7}{3} = \frac{71}{3}$$



VII. When was the particle furthest left?

0s ; 6s

VIII. When was the particle furthest right?

2s.

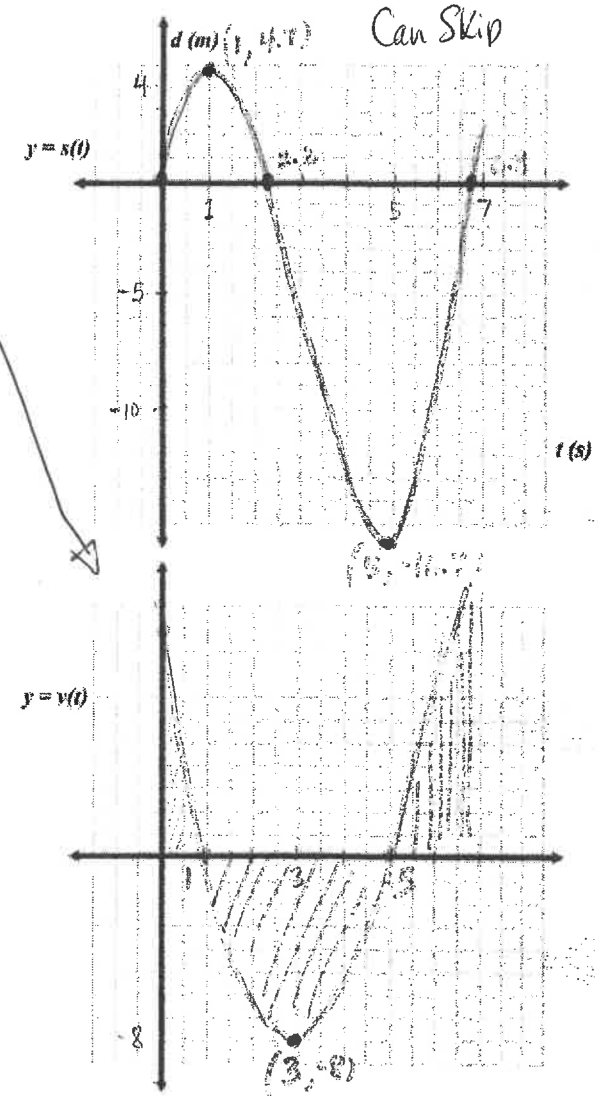
5.

Given the position equation:

$s(t) = \frac{2}{3}t^3 - 6t^2 + 10t$; [0, 7]

$v(t) = 2t^2 - 12t + 10$

a) Sketch a graph of position and velocity



b) Answer the following questions:

I. When is the particle at rest?

1 s & 5 s.

II. When is the particle moving in a positive direction?

(0, 1) (5, 7)

III. When is the particle moving in a negative direction?

(1, 5)

IV. When is the particle slowing down?

(0, 1) (3, 5)

V. When is the particle speeding up?

(1, 3) (5, 7)

VI. What is the total distance travelled after 7 sec.?

$\int_0^1 v(t) + \int_1^5 v(t) + \int_5^7 v(t)$

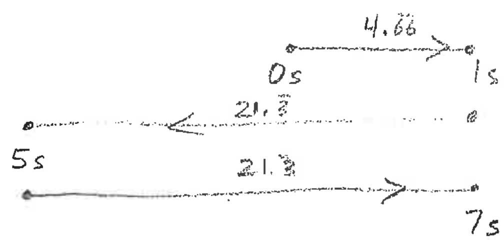
$\frac{14}{3} + \frac{64}{3} + \frac{64}{3} = \boxed{\frac{142}{3}}$

VII. When was the particle furthest left?

5 s

VIII. When was the particle furthest right?

1 s & 7 s



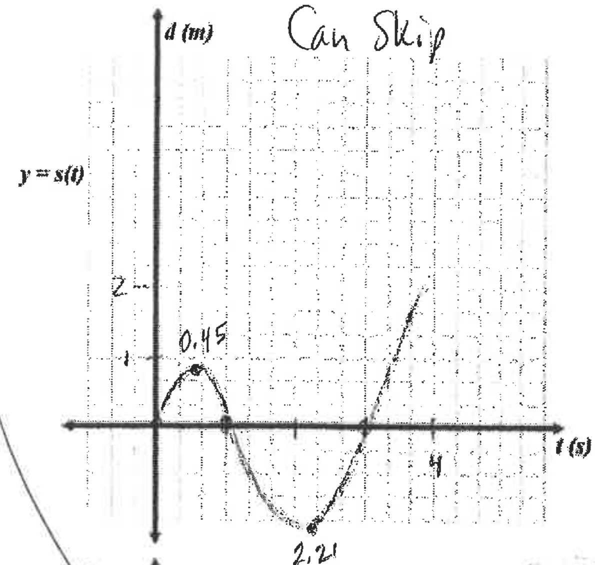
6.

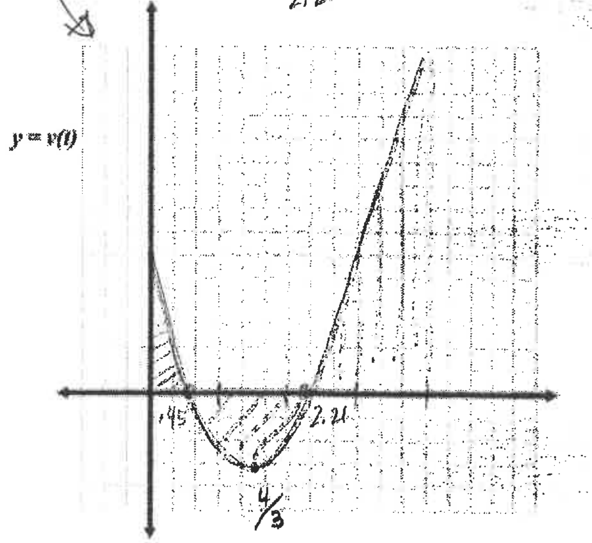
Given the position equation:

$s(t) = t^3 - 4t^2 + 3t;\ [0, 4]$ $6t - 8 = 0$

$v(t) = 3t^2 - 8t + 3$

a) Sketch a graph of position and velocity





b) Answer the following questions:

I. When is the particle at rest?
0.45 s & 2.21 s

II. When is the particle moving in a positive direction?
(0, 0.45) (2.21, 4)

III. When is the particle moving in a negative direction?
(0.45, 2.21)

IV. When is the particle slowing down?
(0, 0.45) (1.33, 2.21)

V. When is the particle speeding up?
(0.45, 1.33) (2.21, 4)

VI. What is the total distance travelled after 3 sec.?

$\boxed{5.48}$

$$\int_0^{.45} v(t) + \int_{.45}^{2.21} v(t) + \int_{2.21}^{3} v(t)$$

$$t^3 - 4t^2 + 3t \Big|_0^{.45}$$

$$(.45)^3 - 4(.45)^2 + 3(.45) \qquad (2.21)^3 - 4(2.21)^2 + 3(2.21)$$

$$[0.63 - 0] \qquad [-2.11 - 0.63]$$

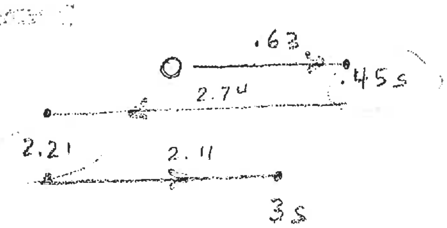
$$\underline{0.63} \qquad 2.74$$

$$(3)^3 - 4(3)^2 + 3(3)$$

$$[0 - -2.11] = 2.11$$

VII. When was the particle furthest left?
2.21 s

VIII. When was the particle furthest right?
0.45 s

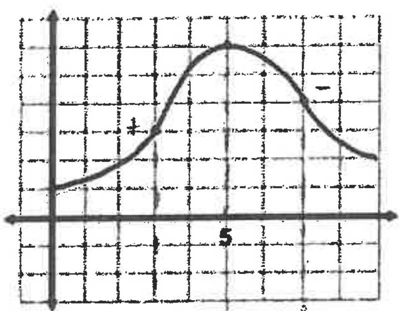


4.5 - PRACTICE QUESTIONS

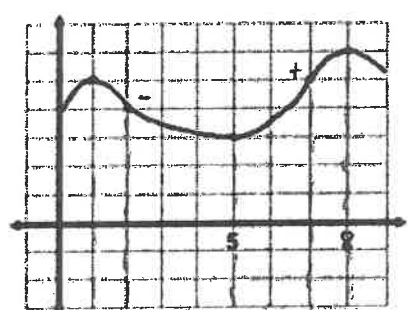
1. Answer the following questions using the graphs below:

	Graph A	Graph B
a) at what times is the velocity 0	t = 6	t = 1, 4, 8
b) at what times is the object moving in the a positive or negative direction + / -	+ v>0 (0,6) - v<0 (6,9)	(0,1) ÷ (4,8) (1,4)
c) at what times is the acceleration 0	t = 3	t = 2, 6
d) at what times is the acceleration + or -	(0,3) (3,9)	(2,6) (0,1) ÷ (6,8)
e) when is the object slowing down or speeding up	SD (3,6) SU (0,3)	(0,1) ÷ (6,8) (2,6)

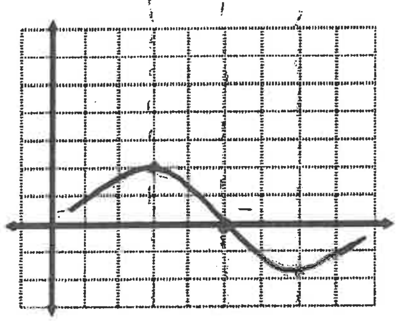
GRAPH A

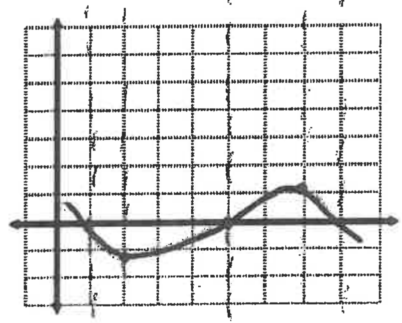


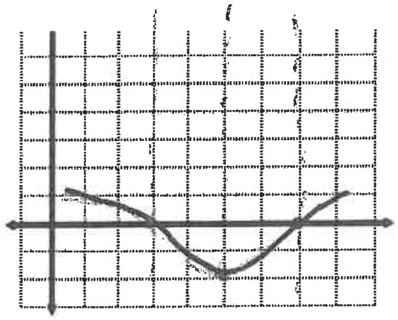
GRAPH B

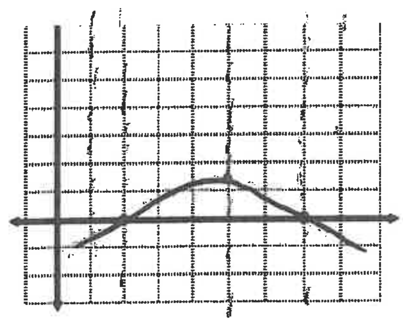


2. Make a rough sketch of the velocity and acceleration in the above:



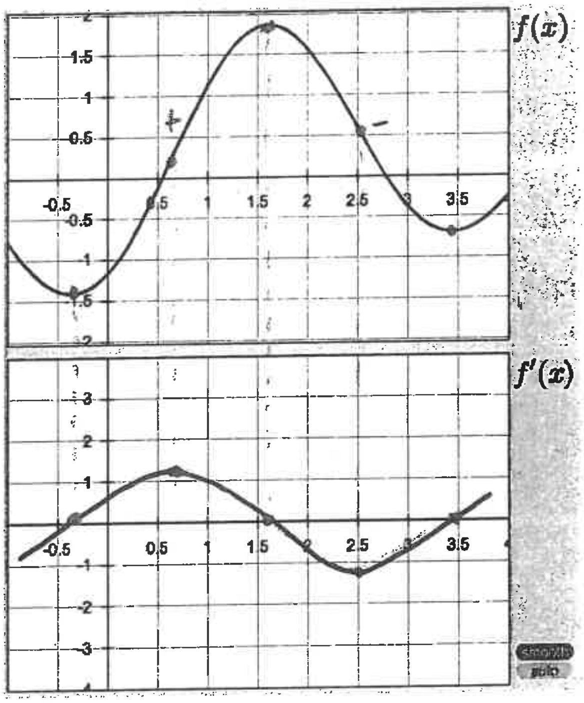




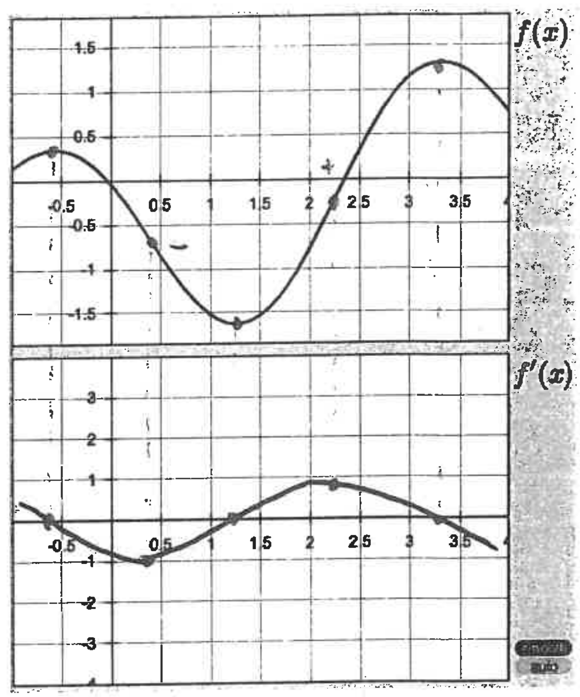


3. Given $f(x)$ Sketch $f'(x)$.

a)

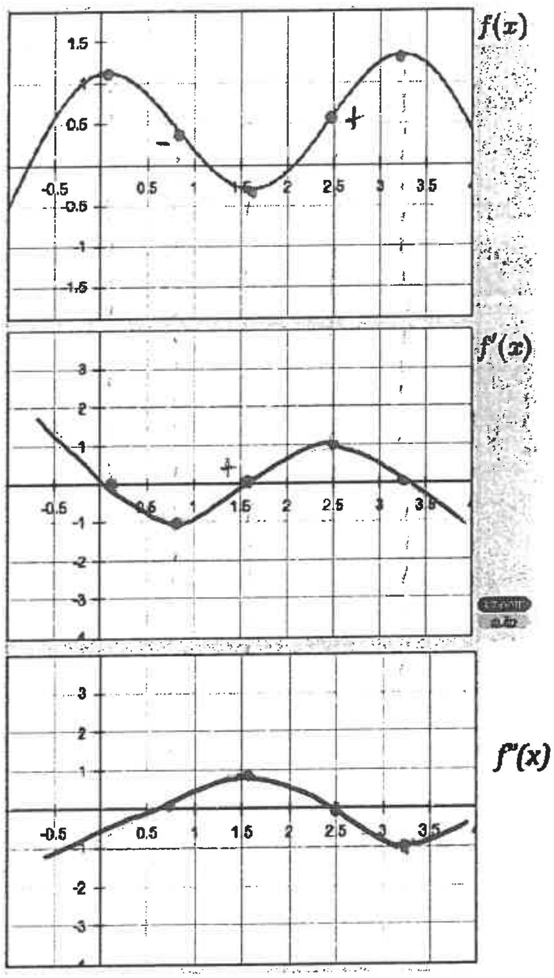


b)

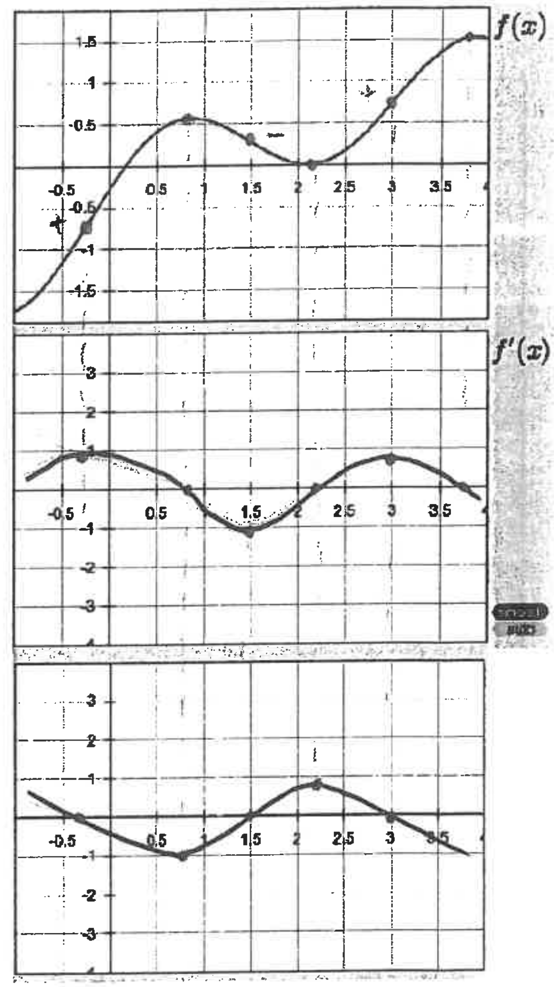


4. Given $f(x)$ Sketch $f'(x)$ and $f''(x)$

a)

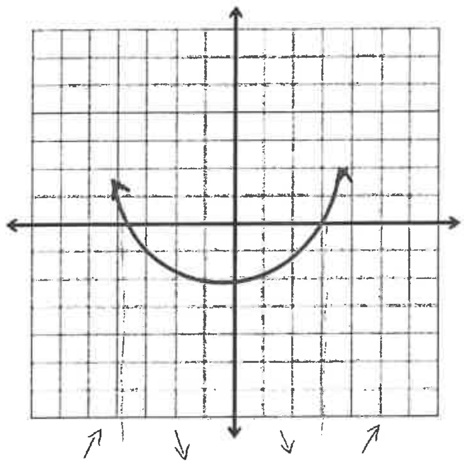


b)

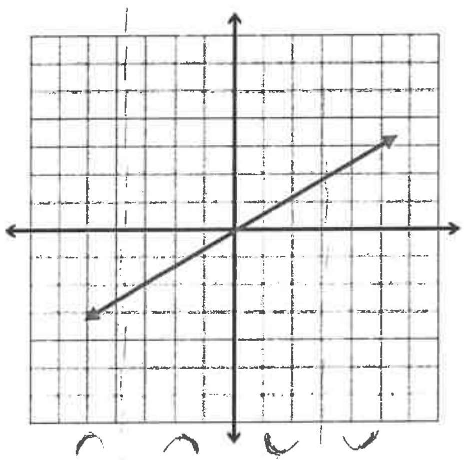


5. Sketching Position from Velocity & Acceleration Graphs that goes through Point A and/or B.

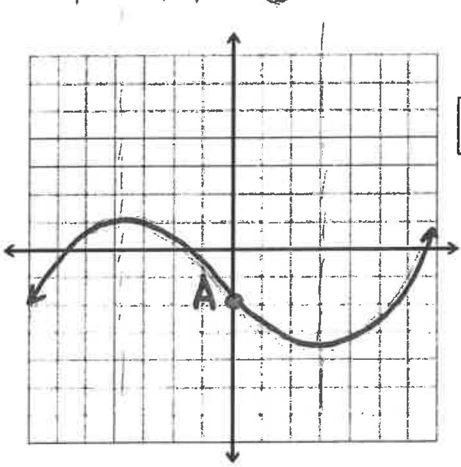
a)



y'

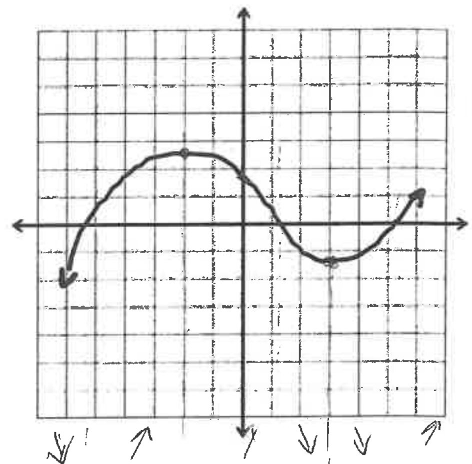


y''

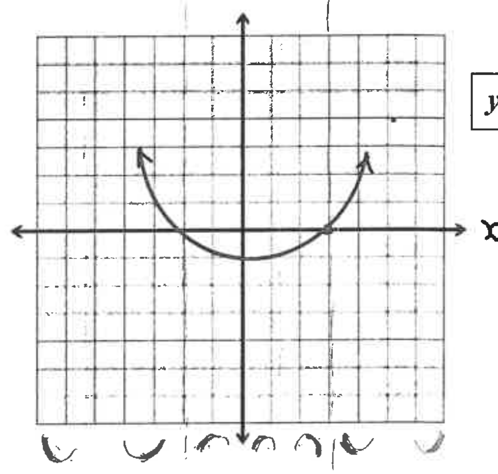


y

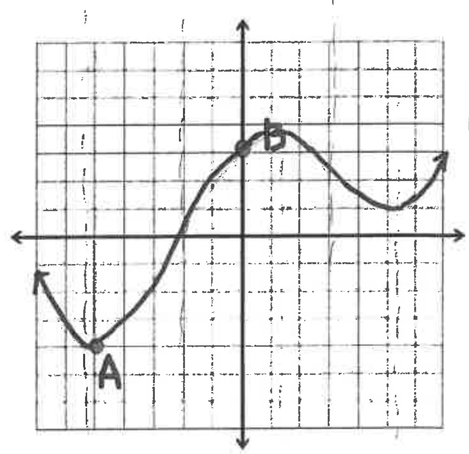
b)



y'

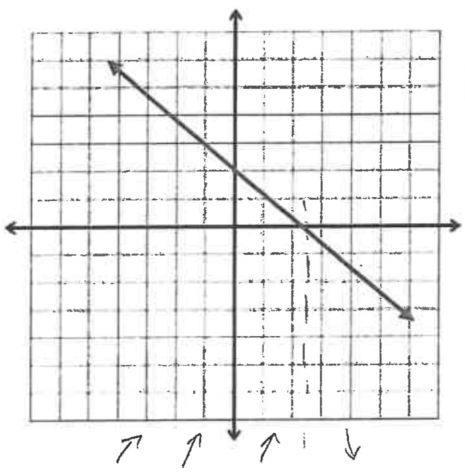


y''

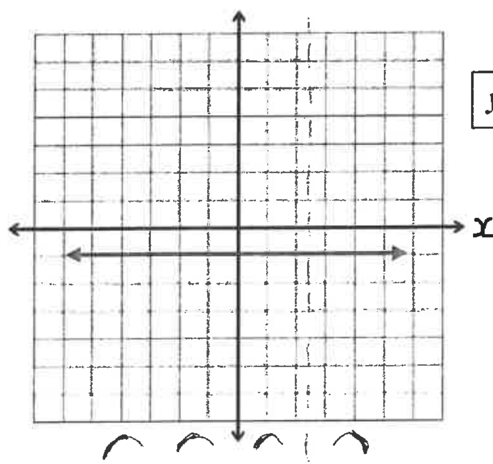


y

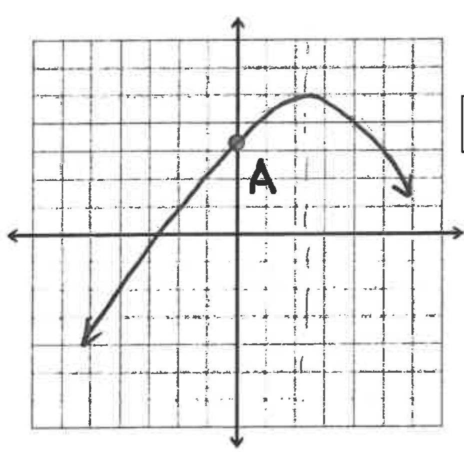
c)



y'

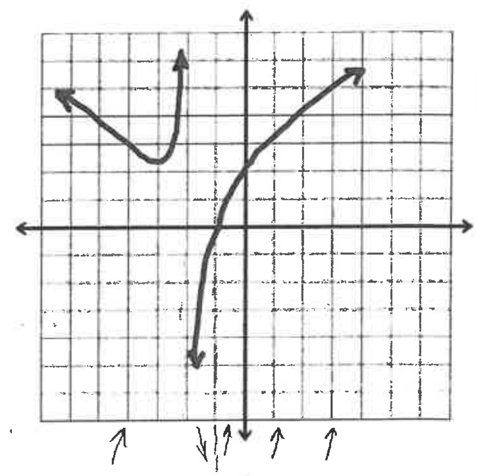


y''

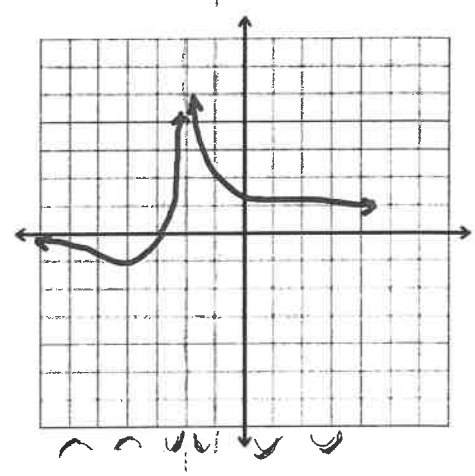


y

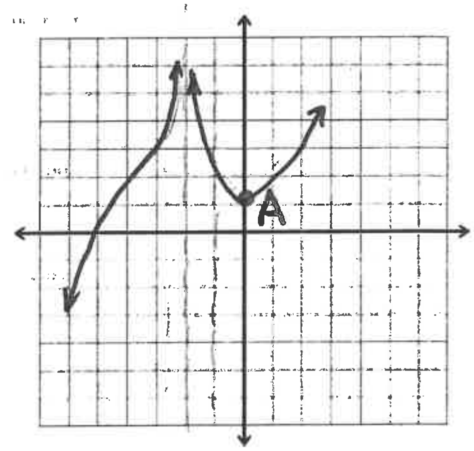
d)



y'



y''



y

4.6 - PRACTICE QUESTIONS

* Answers may vary

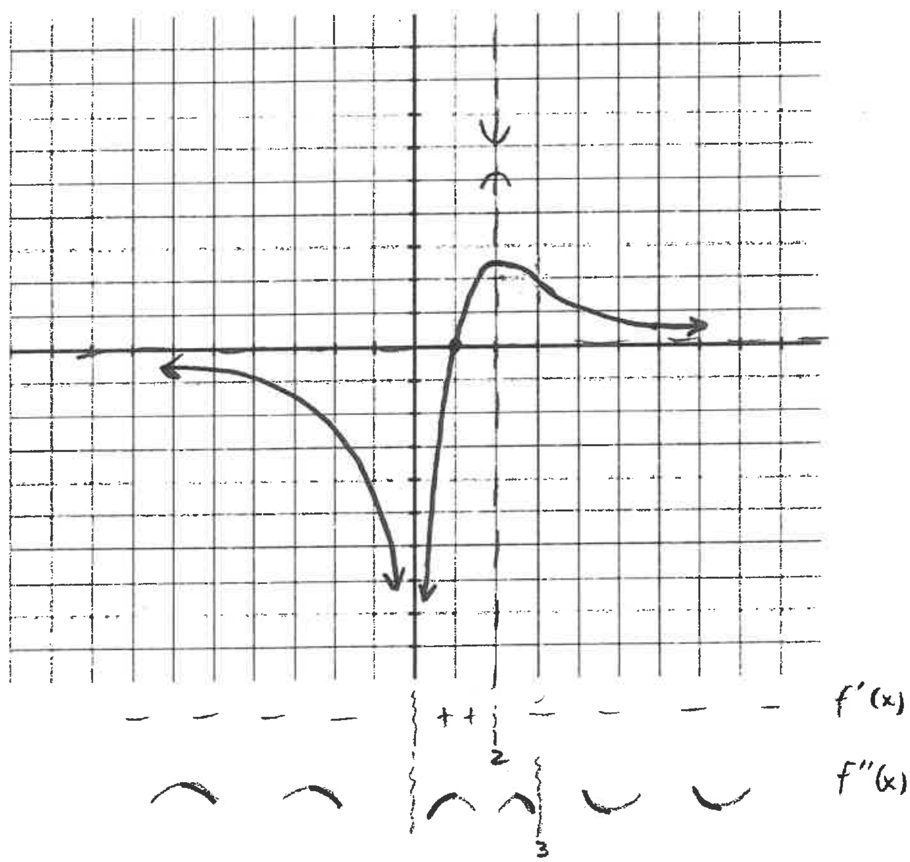
1. *Sketch a graph of a function satisfying each of the following conditions:*

a)

$f(0)$ is undefined
$f(1) = 0$
$\lim_{x \to \infty} f(x) = 0$
$\lim_{x \to -\infty} f(x) = 0$
$f'(2) = 0$
$f''(3) = 0$

Interval	$(-\infty, 0)$	$(0, 2)$	$(2, \infty)$
Sign of $f'(x)$	-	+	-

Interval	$(-\infty, 0)$	$(0, 3)$	$(3, \infty)$
Sign of $f''(x)$	-	-	+

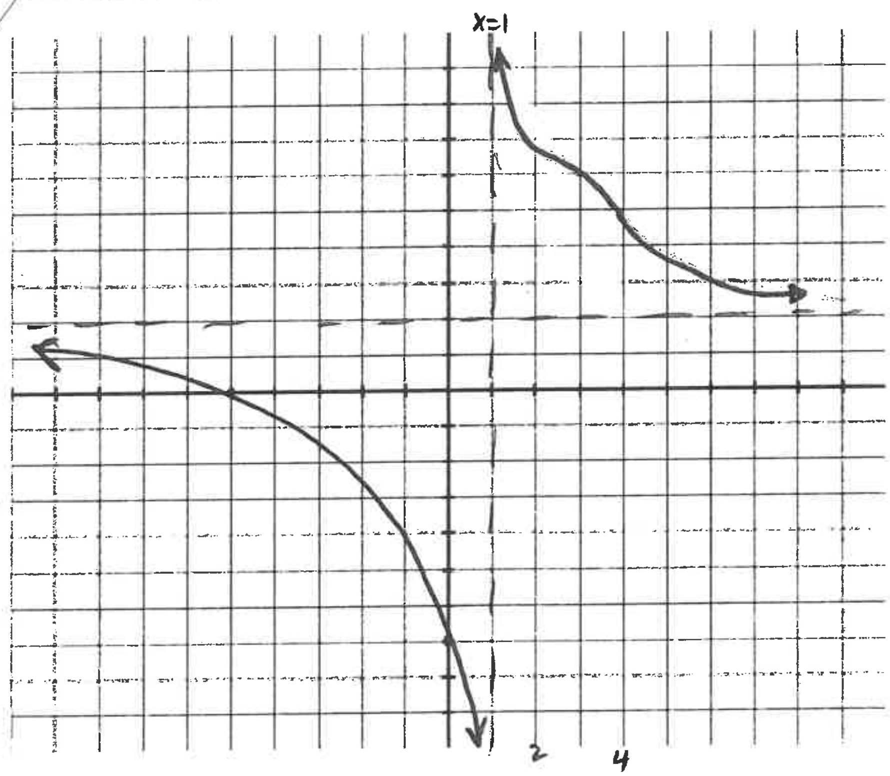


b)

$f(1)$ is undefined
$\lim_{x\to\infty} f(x) = 2$ $\lim_{x\to-\infty} f(x) = 2$
$f'(1)$ is undefined
$f''(1)$ is undefined $f''(2) = 0$ I.P. $f''(4) \neq 0$

Interval	$(-\infty,1)$	$(1,2]$	$[2,\infty)$
Sign of $f'(x)$	-	-	-

Interval	$(-\infty,1)$	$(1,2]$	$[2,4]$	$[4,\infty)$
Sign of $f''(x)$	-	+	-	+



7

c)

$f(0) = -3$
$f(-3) = 0$
$f(4) = 0$

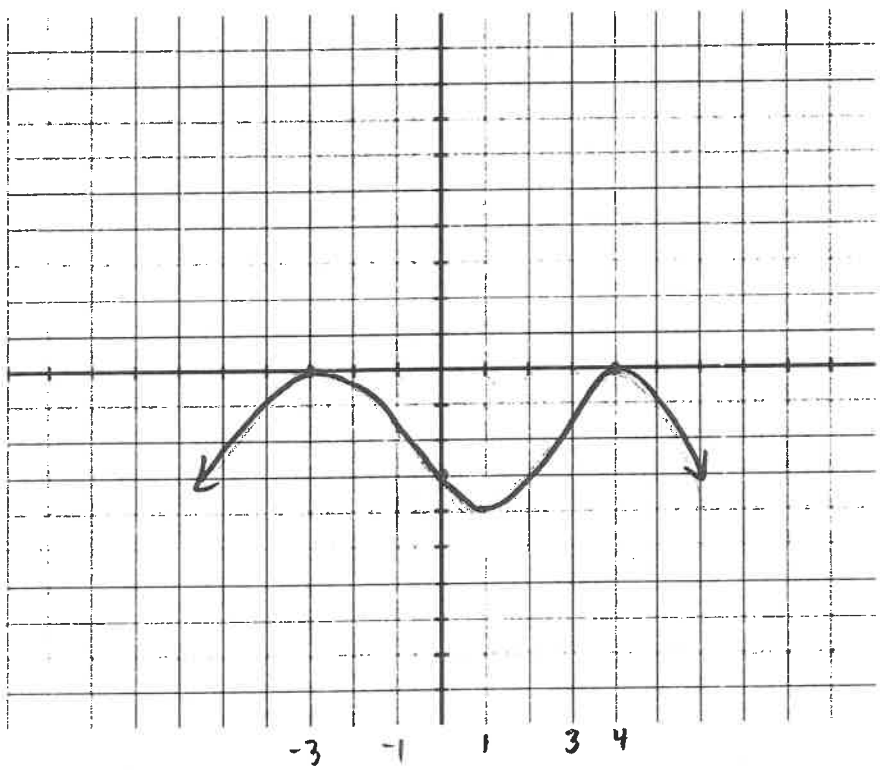
$\lim_{x \to \infty} f(x) = -\infty$
$\lim_{x \to -\infty} f(x) = -\infty$

$f'(-3) = 0$
$f'(1) = 0$
$f'(4) = 0$

$f''(-1) = 0$
$f''(3) = 0$

Interval	$(-\infty, -3]$	$[-3, 1]$	$[1, 4]$	$[4, \infty)$
Sign of $f'(x)$	+	-	+	-

Interval	$(-\infty, -1]$	$[-1, 3]$	$[3, \infty)$
Sign of $f''(x)$	-	+	-



5.1 - PRACTICE QUESTIONS

1. Solve each of the following equations *exactly*:

a) $\ln x + 3 = 0$

$\cancel{e^{\ln}} x = e^{-3}$

$x = e^{-3}$

or

$\frac{1}{e^3}$

b) $2 \ln x - 9 = 0$

$\frac{2 \ln x}{2} = \frac{9}{2}$

$\cancel{e^{\ln}} x = e^{\frac{9}{2}}$

$x = e^{9/2}$

c) $(\ln x)^2 - 4 = 0$

$x = e^2$ or e^{-2} $\left(\frac{1}{e^2}\right)$

d) $(\ln x)^2 + (\ln x) - 2 = 0$

$x = e^{-2}$ and e

or $\frac{1}{e^2}$

e) $2 \ln x = \ln 16$

Reject -4

$x = 4$

f) $2 \ln x = \ln(4x + 5)$

$x = 5$ Reject -1

7

2. Simplify:

a) $\ln(e^{-2})$

$\text{Rw } \ln e^{-2}$

$-2 \cancel{\ln e}$

$= -2$

b) $\ln(e^{-2}) + \ln(e^{3})$

$= 1$

c) $e^{(2 \ln 3)}$

$= 9$

d) $e^{(-2 \ln 3)}$

$= \frac{1}{9}$

e) e^-$\ln(\frac{1}{x})$

$= x$

f) $\ln(8e^{3})$

$= \ln 8 + 3$

3. Solve each of the following equations <u>exactly:</u>

a) $e^{2x} + e^{x} - 6 = 0$

$(e^{x}+3)(e^{x}-2)$

$\ln e^{x} = \ln 3 \quad \ln e^{x} = \ln 2$

$x = \ln(-3) \quad x = \ln(2)$

Reject

b) $e^{2x} - 2e^{x} = 8$

$x = \ln 4$ Reject $\ln(-2)$

c) $e^{x} + 4e^{-x} = 5$

$x = \ln(1)$

$= 0$

d) $e^{x} = 6e^{-x} + 1$

$x = \ln(3)$ Reject $\ln(-2)$

7

4. Differentiate each function:

a) $y = (\ln x)^2 + \ln(x^2)$

$y' = 2(\ln x)(\frac{1}{x}) + (\frac{1}{x^2})(2x)$

or $y' = \dfrac{2\ln(x) + 2}{x}$

b) $y = (x \ln x)^2$

$y' = 2(x\ln(x))(\ln(x) + 1)$

c) $y = 2e^{(x^2 - x)}$

$y' = 2e^{(x^2-x)} \cdot (2x-1)$

or $y' = (4x-2)e^{(x^2-x)}$

d) $y = 2xe^{(2x)}$

$y' = 2e^{2x} + 4x(e^{2x})$

e) $y = \ln(\pi + e^{(2x)})$

$y' = \dfrac{2e^{2x}}{\pi + e^{2x}}$

f) $y = \dfrac{e^x}{\ln x}$

$y' = \dfrac{(\ln(x))(e^x) - (\frac{1}{x})(e^x)}{(\ln(x))^2}$ or

$\dfrac{e^x(x\ln(x) - 1)}{x\ln^2(x)}$

g) $y = \ln(\sin y) + x^2$

$y' = \dfrac{1}{\sin(y)} \cdot \cos(y) y' + 2x$

$y' - \dfrac{\cos y}{\sin y} y' = 2x$

$y'\left(1 - \dfrac{\cos y}{\sin y}\right) = 2x$

$y' = \dfrac{2x}{1 - \frac{\cos y}{\sin y}}$

h) $y = e^{(2y)} + xy$

$y' = \dfrac{-y}{2e^{2y} + x - 1}$

5. Find the equation of the tangent line and of the normal line to the curve $y = \ln x$ at the point $(e, 1)$.

Tangent Line

$y = \frac{1}{e}x$

Normal Line

$y = -ex + 1 + e^2$

6. Find the equation of the tangent line and of the normal line to the curve $y = e^x$ at the point $(2, e^2)$.

Tangent Line

$y = e^2x - e^2$

Normal Line

$y = \frac{-1}{e^2}x + e^2 + \frac{2}{e^2}$

7. Find $\frac{dy}{dx}$ at the given point:

a) $\ln y - x = 0$ at $(1, e)$

$y' = e$

b) $x \ln y + xy = 2$ at $(2, 1)$

$y' = -\frac{1}{4}$

8. *Find the equation of the tangent line and of the normal line to the curve $y = \sin x$ at the point (0, 0).*

Tangent Line

$y = x$

Normal Line

$y = -x$

9. *Find the equation of the tangent line and of the normal line to the curve $y = \cos x$ at the point $\left(\frac{\pi}{3}, \frac{1}{2}\right)$.*

Tangent Line

$y = \frac{-\sqrt{3}}{2}x + \frac{3+\sqrt{3}\pi}{6}$

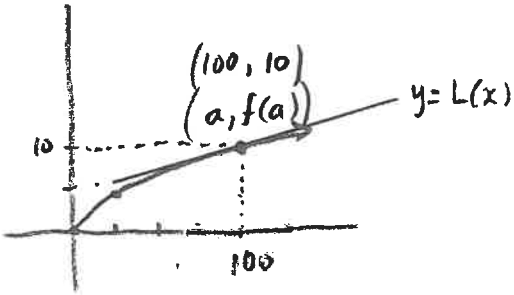
Normal Line

$y = \frac{2}{\sqrt{3}}x + \frac{1}{2} - \frac{2\pi}{3\sqrt{3}}$

10. *If $\tan(xy) = x$, find $\frac{dy}{dx}$ at the point $\left(1, \frac{\pi}{4}\right)$.*

$y' = \frac{1}{2} - \frac{\pi}{4}$

11. *Find the tangent approximation to the function* $f(x)=\sqrt{x}$ *at x = 100 and use it to approximate the square root of 102.*



$$L(x) = f(a) + f'(a)(x-a)$$

$$= 10 + \frac{1}{20}(102-100)$$

$$\boxed{= 10.1}$$

$f'(x) = \frac{1}{2}x^{-1/2}$ or

$f'(a) = \frac{1}{2\sqrt{x}} \Rightarrow f'(100) = \frac{1}{2\sqrt{100}} = \frac{1}{20}$

12. *Use Newton's Method to solve the equation* $x^2 - 2 = 0$ *using x-initial = 1*

1.42

EXTENDED QUESTIONS

Expand each logarithm.

1) $\ln\left(\frac{8^5}{7}\right)^4$

$20\ln 8 - 4\ln 7$

2) $\ln\left(c\sqrt{a\cdot b}\right)$

$\ln c + \frac{\ln a}{2} + \frac{\ln b}{2}$

3) $\ln\left(uv^6\right)^5$

$5\ln u + 30\ln v$

4) $\ln\left(x\cdot y\cdot z^6\right)$

$\ln x + \ln y + 6\ln z$

Condense each expression to a single logarithm.

5) $25\ln 5 - 5\ln 11$

$\ln \frac{5^{-25}}{11^5}$

6) $5\ln x + 6\ln y$

$\ln(y^6x^5)$

7) $\frac{\ln 5}{2} + \frac{\ln 6}{2} + \frac{\ln 7}{2}$

$\ln\sqrt{210}$

8) $20\ln a - 4\ln b$

$\ln \frac{a^{20}}{b^4}$

Solve each equation.Round your final answer to the nearest thousandth.

13) $\ln(7-p) = \ln(-5p-1)$

-2

14) $\ln -2x = \ln(3x+10)$

-2

15) $\ln 8 - \ln(x+4) = 1$

$\frac{8-4e}{e}$

16) $\ln(x+1) - \ln x = 5$

$\frac{-1}{1-e^5}$

17) $\ln(x+8) - \ln 7 = 3$

$7e^3 - 8$

18) $\ln 10 + \ln(5x-2) = 3$

$\frac{e^3+20}{50}$

19) $\ln(-4x-4) - \ln 3 = 3$

$\frac{-3e^3-4}{4}$

20) $\ln 3 + \ln(2x^2+4) = \ln 12$

0

Solve each equation. Round your answers to the nearest thousandth.

23) $-8e^{-p} = -49$

-1.812

24) $2e^{8x} = 45$

0.389

25) $e^{-4x} + 8 = 35$

-0.824

26) $e^{3r} + 4 = 59$

1.336

27) $3e^{-b} = 56$

-2.927

28) $-9.3e^{10b} = -67$

0.198

29) $4e^{r+5} = 29$

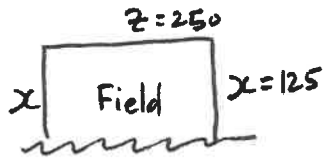
-3.019

30) $6e^{-10r} = 63$

-0.235

5.2 - PRACTICE QUESTIONS

1. An open field is bounded by a lake with a straight shoreline. A rectangular enclosure is to
be constructed by using 500 m of fencing along three sides and the lake as a natural boundary on the fourth side. What dimensions will maximize the enclosed area and what is the maximum area?



$A = L \times W$

$y = xz$

Constraint $P = 500$

$\therefore\ P = 2x + z$

$500 = 2x + z$

$z = 500 - 2x$

$y = x(500 - 2x)$

$y = -2x^2 + 500x$

$y' = -4x + 500 = 0$

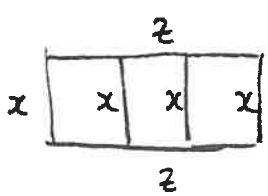
$x = 125$

$z = 500 - 2(125)$

$z = 250$

DIMENSIONS
125m × 250m
MAX. AREA
$31,250\ m^2$

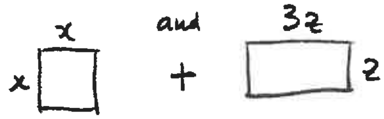
2. Two farmers have 800 m of fencing. They wish to form a rectangular enclosure and then divide it into 3 sections by running two lengths of fence parallel to one side. What should
the dimensions of the enclosure be in order to maximize the enclosed area?



$P = 4x + 2z$

DIMENSIONS
100m × 200m

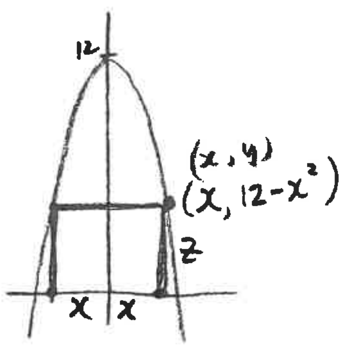
3. A piece of wire 24 in long is used to form a square and a rectangle whose length is three times its width. Determine their minimum combined area.



$P = 4x + 8z$

MIN. AREA $15.48\ in^2$

4. *Find the dimensions of the rectangle of largest area whose base in on the x-axis and the upper two vertices lie on the parabola $y = 12 - x^2$. What is the maximum area?*



Constraint

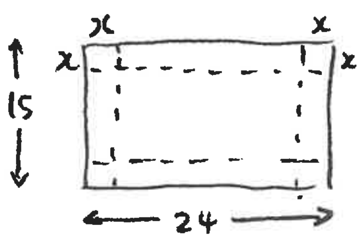
$y = 12 - x^2$

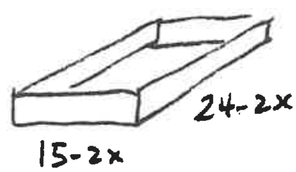
$\therefore\ z = 12 - x^2$

$y = 2x(12 - x^2)$

$\boxed{\text{MAX AREA } 32}$

5. *An open box by cutting squares of equal size from the corner of a 24 cm by 15 cm piece of sheet metal and folding up the sides. Determine the size of the cut-out that maximizes the volume of the box.*





$V = L \times W \times H$

$y = (24-2x)(15-2x)(x)$

$y = 4x^3 - 78x^2 + 360x$

$y' = 12x^2 - 156x + 360$

$12(x^2 - 13x + 30)$

$(x-3)(x-10)$

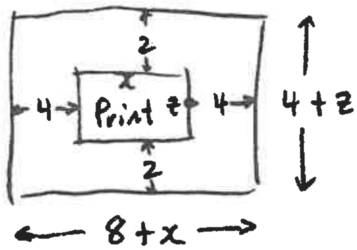
$x = 3 \quad \cancel{x = 10 \text{ Reject}}$

$\boxed{\text{CUT-OUT } 3\text{ cm}}$

6. *An open box from a 12 in by 12 in piece of cardboard by cutting away squares of equal size from the other four corners and folding up the sides. Determine the size of the cut-out that maximizes the volume of the box.*

$\boxed{\text{CUT-OUT } 2\text{ in}}$

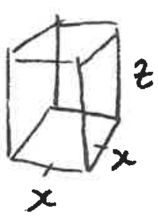
7. *A rectangular poster which is to contain 50 cm² of print, must have margins of 2 cm on each side and 4 cm on the top and bottom. What dimensions will minimize the amount of material used?*



DIMENSIONS

$\boxed{9\text{cm} \times 18\text{cm}}$

8. *Construct a closed rectangular box with square base which has a surface area of 150 cm². What is the maximum possible volume of such a box?*



Constraint

$SA = 2x^2 + 4xz$

MAX VOLUME

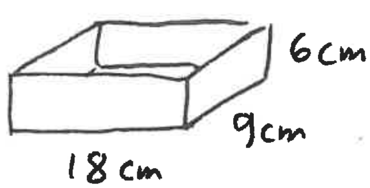
125 cm^3

9. *An open rectangular box with a base twice as long as it's wide. If its volume must be 972 cm³, what dimensions will minimize the amount of material used in its construction?*

S.A.

Constraint

Volume.



DIMENSIONS

18cm × 9cm × 6cm high

EXTENDED QUESTIONS

1. **Dimensions of a box**. *A closed 3-dimensional box is to be constructed in such a way that its volume is 4500 cm^3. It is also specified that the length of the base is 3 times the width of the base. Determine the dimensions of the box which satisfy these conditions and have the minimum possible surface area.*

DIMENSIONS:
W: 10 cm
L: 30 cm
H: 15 cm

2. **The largest garden**. *You are building a fence to completely enclose part of your backyard for a vegetable garden. You have already purchased material for a fence of a length 100 ft. What is the largest rectangular area that this fence can enclose?*

AREA: 625 ft^2

3. **Two gardens.** A fence of length 100 ft is to be enclose two gardens. One garden is to have a circular shape, and the other to be square. Determine how the fence should be cut so that the sum of the areas inside both gardens is as large as possible.

CIRCLE GARDEN: 44 ft
SQUARE GARDEN: 56 ft

4. **Dimensions of open box.** A rectangular piece of cardboard with dimension 12 cm by 24 cm is to be made into an open box (no lid) by cutting out squares from the corners and then turning up the sides. Find the size of the squares that should be cut out if the volume of the box is to be a maximum.

CUT-OUT : 2.54 cm

5. **Cost with Fixed Area.** A fence must be built in a large field to enclose a rectangular area of 25,600 m^2. One side of the area is bounded by an existing fence (no fence needed there). Material for the fence cost \$3 per meter for the ends and \$1.50 per meter for the side opposite the existing fence. Find the cost of the least expensive fencing.

COST: \$960.^{00}

6. **Cost with Fixed Area.** A fence must be built to enclose a rectangular area of 20,000 ft^2. Fencing material costs \$2.50 per foot for the two sides facing north and south and \$3.20 per foot for the other two sides. Find the cost of the least expensive fence.

COST: \$1,600.^{00}

7.

of 16,000 cm^3. The material for the top and bottom of the box costs \$3 per square centimeter, while the material for the sides costs \$1.50 per square centimeter. Find the dimension s of the box that will lead to the minimum total cost. What is the minimum total cost?

COST: \$ 7,200.^{00}

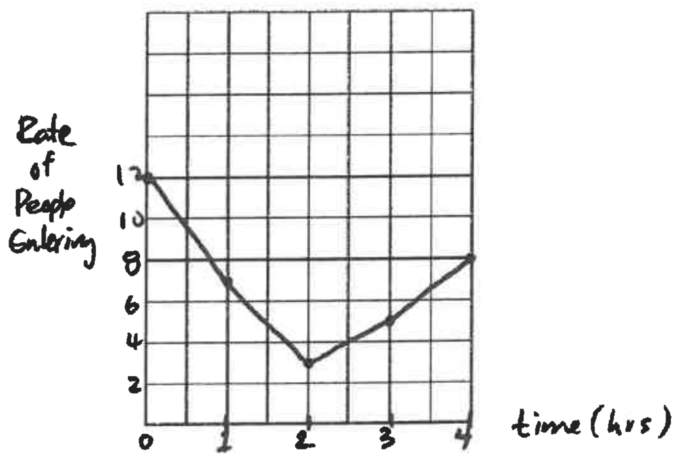
[redacted]

5.3 - PRACTICE QUESTIONS

The rate that people are entering a local office is given below in people/hour. Use the table to answer questions 1-3.

Time (hours)	0	1	2	3	4
r '(t) ppl/hr	12	7	3	5	8

Sketch and label a graph:



1. Use a left Riemann sum with 4 subintervals to approximate the total number of people entering the office over the interval $0 \le t \le 4$.

27 people

2. Use a right Riemann sum with 4 subintervals to approximate the total number of people entering the office over the interval $0 \le t \le 4$.

23 people

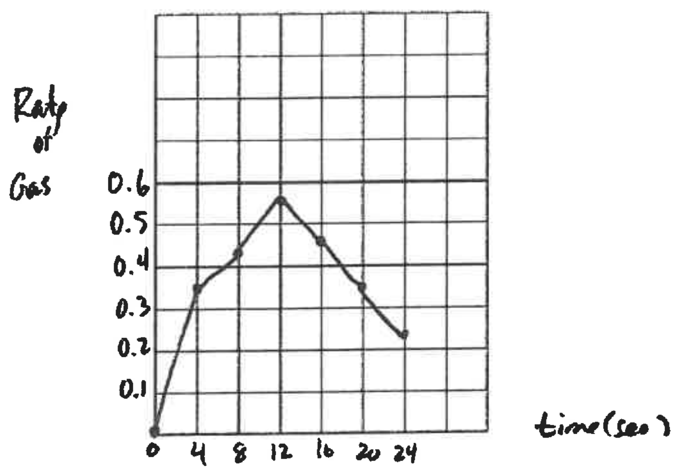
3. Use a trapezoidal approximation with 4 subintervals to approximate the total number of people entering the office over the interval $0 \le t \le 4$.

25 people

Gasoline is being pumped into a car. The rate that the gas is being pumped is given in the table below at selected times (seconds). Use the table to answer questions 4-6.

Time (sec)	0	4	8	12	16	20	24
$g'(t)$ gal/sec	0	.34	.42	.56	.45	.34	.22

Sketch and label a graph:



4. Use a left Riemann Sum with 3 subintervals to approximate the total gallons of gasoline pumped in the car over the 24 seconds.

6.96 gallons

5. Use a right Riemann Sum with 3 subintervals to approximate the total gallons of gasoline pumped in the car over the 24 seconds.

8.72 gallons

6. Use a trapezoidal Riemann Sum approximation with 3 subintervals to approximate

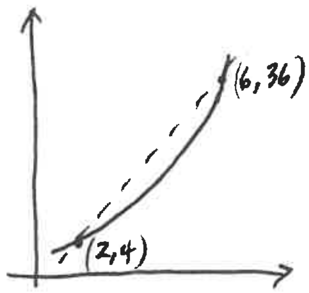
7.84 gallons

ROJECT – see your teacher to get > Creek Project

5.4 - PRACTICE QUESTIONS

1, Show how the ***MEAN VALUE THEOREM*** applies to the given function over the given interval:

a) $f(x) = x^2, \quad 2 \leq x \leq 6$



$\frac{\Delta y}{\Delta x} = \text{SLOPE} = f'(x)$

$\frac{36-4}{6-2} = 8 = 2x$

$x = 4$

$y = 16$

MVT: $(4, 16)$

b) $g(x) = x^2 + 3x + 5, \quad 1 \leq x \leq 4$

MVT: $(2.5, 18.75)$

2. Show how the ***INTERMEDIATE VALUE THEOREM*** applies to the given function over the given interval:

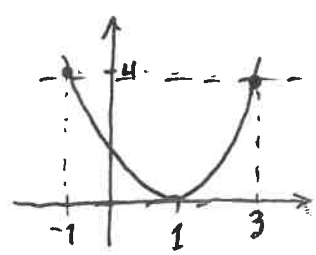
a) $f(x) = x^2, \quad 2 \leq x \leq 8$ and $f(c) = 36$

$c = 6$

3. *Use Rolle's Theorem to show that the function has a horizontal tangent in the interval [-1, 3]* $\quad f(x) = x^2 - 2x + 1$

$f(-1) = (-1)^2 - 2(-1) + 1 = 4 \therefore (-1, 4)$

$f(3) = (3)^2 - 2(3) + 1 = 4 \therefore (3, 4)$



$f'(x) = 2x - 2 = 0$

$2x = 2$

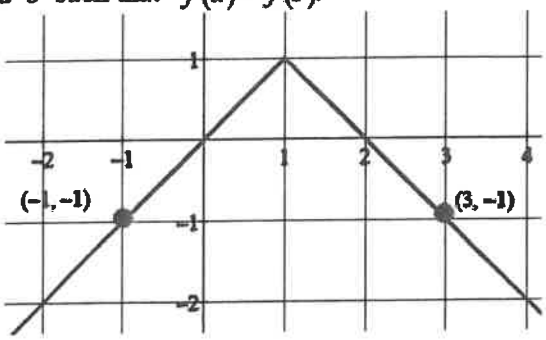
$x = 1$

$f(a) = f(b)$

EXTENDED QUESTIONS

In problems 1 and 2, state why Rolle's Theorem does not apply to the function even though there exist a and b such that $f(a)=f(b)$.

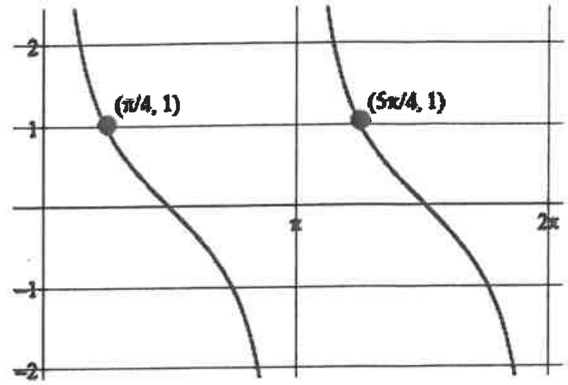
1.



$f(x)=1-|x-1|$

DOESN'T APPLY BECAUSE THE DERIVATIVE AT X=1 DNE, SO IT'S NOT DIFFERENTIABLE ON INTERVAL (-1,3)

2.



$f(x)=\cot x$

DOESN'T APPLY BECAUSE DISCONTINUOUS AT $x=\pi$

---

3. Determine whether the Mean Value Theorem (MVT) applies to the function $f(x)=3x^2-x$ on the interval $[-1, 2]$. If it applies, find all the value(s) of c guaranteed by the MVT for the indicated interval. SINCE THE f(x) IS CONTINUOUS AND DIFFERENTIABLE ON THE INTERVAL MVT CAN APPLY $c=\frac{1}{2}$

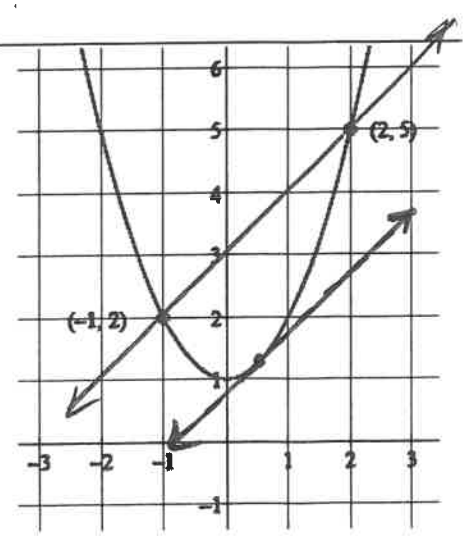
---

4. Determine whether the MVT applies to the function $f(x)=\dfrac{x+1}{x}$ on the interval $[-2, 3]$. If it applies, find all the value(s) of c guaranteed by the MVT for the indicated interval.

DOESN'T APPLY BECAUSE DISCONTINUOUS AT X=0

---

5. Consider the graph of the function $g(x)=x^2+1$ shown to the right.
   a) On the drawing provided, draw the secant line through the points $(-1, 2)$ and $(2, 5)$.
   b) Since g is both continuous and differentiable, the MVT guarantees the existence of a tangent line(s) to the graph parallel to the secant line. Sketch such line(s) on the drawing. SEE DASHED LINE
   c) Use your sketch from part (b) to visually estimate the x-coordinate at the point of tangency. $x \approx \frac{1}{2}$
   d) That x-coordinate at the point of tangency is the value of c promised by the MVT. Verify your answer to part (c) by using the conclusion of the MVT on the interval $[-1, 2]$ to find c. $c=\frac{1}{2}$



6. Given $h(x)=x^{2/3}$, explain why the hypothesis of the MVT are met on $[0, 8]$ but are not met on $[-1, 8]$.

$h'(x) = \frac{2}{3}x^{-1/3} \Rightarrow \frac{2}{3x^{1/3}}$.

Cusp at $x=0$

h(x) IS CONTINUOUS BUT NOT DIFFERENTIABLE AT $X=0$

∴ MVT DOESN'T APPLY

Some of the following questions require using the IVT and MVT "backwards." This means that a fact is stated and you need to identify what theorem was used to guarantee that fact. You might want to read again the conclusions for the IVT and MVT before attempting these problems!

13. Given $h(x)=x^3+x-1$ on the interval $[0, 2]$, will there be a value p such that $0<p<2$ and $h'(p)=5$? Justify your answer. If your answer is yes, find p.

MVT APPLIES $\Rightarrow \frac{h(2)-h(0)}{2-0} \Rightarrow \frac{9-(-1)}{2} \Rightarrow 5 = h'(p)$

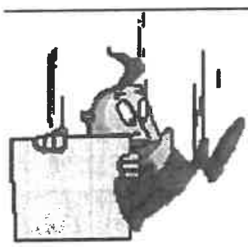
$5 = 3p^2+1$

$\Rightarrow p = \pm\frac{2}{\sqrt{3}}$ ∴ $p = \frac{2}{\sqrt{3}}$

Reject $-\frac{2}{\sqrt{3}}$ not $(0,2)$

14. Given $g(x)=x^3-x^2+x$ on the interval $[1, 3]$, will there be a value r such that $1<r<3$ and $g(r)=11$? Justify your answer. If your answer is yes, use your calculator to find r.

IVT $\quad r = 2.439$



You may use a calculator for this problem.

15. The height of an object t seconds after it is dropped from a height of 500 meters is $h(t)=-4.9t^2+500$.
   a) Find the average velocity of the object during the first three seconds. Remember: average velocity is equal to change in position divided by change in time. -14.7 ft/sec
   b) Show that at some time during the first three seconds of fall the instantaneous velocity must equal the average velocity you found in part (a). Then, find that time.

$h(t)$ is continuous and differentiable $[0,3]$ ∴ MVT

$$\frac{h(3)-h(0)}{3-0} = h'$$

$$-14.7 = -9.8t$$

$t = 1.5$ sec.

5.5 - PRACTICE QUESTIONS

1. Evaluate each of the following ***IMPROPER INTEGRALS*** *if possible:*

a) $\int_{1}^{\infty} \frac{1}{x^2}\,dx$ $\lim_{a\to\infty} \int_{1}^{a} x^{-2}\,dx$

$-\frac{1}{x}\Big|_{1}^{a} \Rightarrow \frac{-1}{a} - \frac{-1}{1}$

$\Big|_{1}^{\infty} \Rightarrow \frac{-1}{\infty} + 1$

$= 0 + 1 = 1$

b) $\int_{-\infty}^{0} 2e^{x}\,dx$

2

c) $\int_{0}^{1} \frac{1}{\sqrt{x}}\,dx$

2

d) $\int_{2}^{\infty} \frac{1}{x}\,dx$

Div ⇒ DNE

2. *Find the exact arc length of the given function between the given values of x:*

a) f(x)= 2x, from x = 4 to x = 8

$f'(x) = 2$

$\int_{a}^{b} \sqrt{1+f'(x)^2} = \int_{a}^{b} \sqrt{1+(2)^2}$

$= \int_{a}^{b} \sqrt{5}\,dx$

$x\sqrt{5}\,\Big|_{4}^{8}$

$8\sqrt{5} - 4\sqrt{5} = 4\sqrt{5}$

b) $g(x) = \frac{2}{3}x^{\frac{3}{2}}$, from x = 1 to x = 3

$\frac{16 - 2\sqrt{8}}{3}$ or

$\frac{16 - 4\sqrt{2}}{3}$

3. Given f is a function of x and y find the partial derivative of f with respect to x and the partial derivative of f with respect to y of each of the following:

a) $f(x, y) = 4x^3 - 3x^2y^2$

$\frac{df}{dx} = 12x^2 - 6xy^2$

$\frac{df}{dy} = 0 - 6x^2y$

b) $f(x, y) = x^4 \ln y + y$

$\frac{df}{dx} = 4x^3 \ln(y)$

$\frac{df}{dy} = x^4 \cdot \frac{1}{y} + 1$

4. Find the exact value of each of the following multiple integrals:

a) $\int_0^1 \int_2^4 x \, dx\, dy$

1st $\int_2^4 x\, dx \Rightarrow \frac{1}{2}x^2 \Big|_2^4 \Rightarrow \frac{1}{2}(4)^2 - \frac{1}{2}(2)^2 = 6$

$\int_0^1 6\, dy \Rightarrow 6y \Big|_0^1 \Rightarrow 6(1) - 6(0)$

$= 6$

b) $\int_b^a \int_{-1}^5 2x \, dx\, dy$

$24a - 24b$

c) $\int_1^3 \int_{-1}^1 y^2 \, dy\, dx$

$\frac{4}{3}$

d) $\int_{-1}^1 \int_{-2}^2 2x + 2y \, dy\, dx$

0

5. *Find all solutions of each* ***DIFFERENTIAL EQUATION*** *below:*

a) $\frac{dy}{dx} = 2x + 3$

$y' = 2x + 3$

$y = x^2 + 3x + C$

b) $\frac{dy}{dt} = 4t^2 + 5$

$y = \frac{4}{3}t^3 + 5t + C$

c) $\frac{dv}{dt} = 3\cos 2t - 4$

$V = \frac{3}{2}\sin(2t) - 4t + C$

6. *Find the solution of each of the* ***SEPARABLE EQUATION*** *satisfying the given initial condition:*

a) $\frac{dy}{dx} = 3x^2 + 6, \quad y(1) = 9$

$y' = 3x^2 + 6$

$y = x^3 + 6x + C$

$9 = (1)^3 + 6(1) + C$

$C = 2 \quad \therefore \quad y = x^3 + 6x + 2$

b) $\frac{dy}{dt} = (3 - 2t)^5, \quad y(2) = 1$

$y = -\frac{1}{12}(3 - 2t)^6 + \frac{13}{12}$

5.6 - PRACTICE QUESTIONS

1. Simplify:

a) $\sqrt{-9}$ b) $\sqrt{-64}$ c) $\sqrt{-20}$ d) $\sqrt{-80}$ e) $\sqrt{-2}$

$3i$ $8i$ $2i\sqrt{5}$ $4i\sqrt{5}$ $i\sqrt{2}$

f) i^5 g) i^6 h) i^{83} i) i^{202} j) i^{484}

i -1 $-i$ -1 1

2. Simplify:

a) $(2 + 6i) + (4 - 8i)$

$6-2i$

b) $(3 - 5i) - (6 - 9i)$

$-3+4i$

c) $(2 + 5i)(3 - 4i)$

$26+7i$

d) $(2 + 5i)(2 - 5i)$

29

e) $\dfrac{6}{5i}$

$-\dfrac{6i}{5}$

h) $\dfrac{4-6i}{2+i}$

$\dfrac{2-16i}{5}$

i) $\dfrac{8-2i}{8+2i}$

$\dfrac{15-8i}{17}$

3. Solve each of the following equations over the COMPLEX NUMBERS:

a) $x^2 + 9 = 0$

$\sqrt{x^2} = \sqrt{-9}$

$x = \pm 3i$

b) $3x^2 + 10 = 2x^2 + 8$

$\pm i\sqrt{2}$

c) $x^2 + 2x + 2 = 0$

$-1 \pm i$

d) $2x^2 = 2x - 3$

$\frac{1 + i\sqrt{5}}{2}$

4. Find the following integrals over the COMPLEX NUMBERS:

a) $\int_{-2i}^{4i} 2x \, dx$

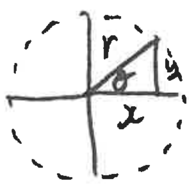
-12

b) $\int_{-2i}^{i} 4x^3 \, dx$

-15

5.7 - PRACTICE QUESTIONS

1. *Give the relationship between the POLAR COORDINATES (r, θ) and the RECTANGULAR COORDINATES (x, y):*



$r^2 = x^2 + y^2$

$r = \sqrt{x^2+y^2}$

$\sin\theta = \frac{y}{r} \Rightarrow y = r\sin\theta$

$\cos\theta = \frac{x}{r} \Rightarrow x = r\cos\theta$

$\tan\theta = \frac{y}{x}$

2. *Give the rectangular coordinates of the point whose polar coordinates are given:*

a) $\left(2, \frac{\pi}{6}\right)$

$r \quad \theta$

$x = r\cos\theta$

$= 2\cos \pi/6$

$= 2(\sqrt{3}/2)$

$x = \sqrt{3}$

$y = r\sin\theta$

$2\sin \pi/6$

$2(1/2)$

$y = 1$

$(\sqrt{3}, 1)$

b) $\left(-4, \frac{\pi}{3}\right)$

$(-2, -2\sqrt{3})$

c) (2, 0)

$(2, 0)$

3. *Give the polar coordinates of a point with the given rectangular coordinates:*

a) (1, 0)

$(1, 0)$

b) (1, 1)

$(\sqrt{2}, \pi/4)$

c) $(\sqrt{3}, 1)$

$(2, \pi/6)$

4. *Find a rectangular equation equivalent to the given polar equation and describe the graph:*

a) $r = 2$

$r^2 = 4$

$x^2 + y^2 = 4 \quad \therefore$ CIRCLE

b) $r = a$

$x^2 + y^2 = a^2 \quad \therefore$ CIRCLE

c) $\theta = \frac{\pi}{4}$

$y = x \quad \therefore$ LINE

d) $r = 2\sin\theta$

$x^2 + (y-1)^2 = 1$

$\therefore$ CIRCLE

e) $r = 4\cos\theta$

$(x-2)^2 + y^2 = 4$

$\therefore$ CIRCLE

f) $r = \tan\theta \sec\theta$

$y = x^2 \quad \therefore$ PARABOLA

5. *Change the given rectangular equation into an equivalent polar equation:*

a) $x^2 + y^2 = 16$

$r = 4$

b) $x = 3$

$r = \frac{3}{\cos\theta}$

c) $2x^2 + 2y^2 = 8$

$r = 2$

d) $y = \sqrt{3}\,x$

$\tan\theta = \sqrt{3}$

e) $x + 2y = 3$

$r = \frac{3}{\cos\theta + 2\sin\theta}$

f) $x^2 - y^2 = 1$

$r = \sqrt{\frac{1}{\cos^2\theta - \sin^2\theta}}$

PROJECT