CALCULUS 12 FINAL EXAM PREP.

Name ___________________________ Date _________________

*****DO NOT FORGET TO LOOK OVER MULTIPLE CHOICE QUESTIONS**

UNIT 1

All working must be shown, as applicable to obtain full marks. DON'Y SIMPLIFY PRODUCT, QUOTIENT & CHAIN RULES

1) Find the first derivative of each of the following functions:

a) $y = -6x^3 - 5$

b) $y = \pi x^4 + 8x^2 - 3x + 6$

c) $f(x) = 6\pi^2 x^3 - 4x^2 - 8\pi^3$

d) $g(x) = \frac{1}{3}x^6 - \frac{1}{2}x^8 + \frac{3}{5}x^5$

2. If $y = 8p^2 - 2kp^3 + 5m^2$ find $\frac{dy}{dp}$

3. If $f(x) = 3x^3 - 2x^2 + 7$ find $f'(x)$ at the point (-1, 2)

4. If $y = 8x^4 - 6x + 3$ find :

a) y''

b) $\left(\frac{dy}{dx}\right)^2$

c) $\frac{d^3y}{dx^3}$

5) If $y = 4mx^3 - 6p^2x^2 - 8m + 3$ find:

a) $\frac{dy}{dx}$ b) $\frac{dy}{dm}$ c) $\frac{dy}{dp}$ d) $\frac{dy}{dw}$

6) Given $y = 3x^4 - 6x + 5$ find $\frac{dy}{dx}$ at x = -1

7) Find the first derivative of each of the following functions:

a) $y = \frac{6}{x^3} - \frac{4}{x^2} + \frac{16}{x} - 8$

b) $y = 6\sqrt{x} + \frac{8}{\sqrt{x}} - \frac{15}{2\sqrt[3]{x}}$

c) $f(x) = \frac{5x^2}{1-3x^3}$

d) $g(x) = \sqrt{6x^2 - 2x + 8}$

8) Find the slope of the tangent line to $y = x^2 - 5x - 4$ at (-1, 2)

9) Use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y:

a) $8x^2 - 4y^2 = 16$

b) $x^2 + 4xy - y^2 = 10$

10) Given the position function $s(t) = 6t^3 - 8^2 t + 9$ find the velocity and acceleration as functions of time *t*.

11) How fast is the side of a square growing when the length of the side is 5 *m* and the area is increasing at 0.75 m^2/s.

12) The hypotenuse of a right triangle is of fixed length but the lengths of the other two sides *x* and *y* depend on time. How fast is *y* changing when $\frac{dx}{dt} = 8$ and $x = 24$ if the length of the hypotenuse is 25?

13) Differentiate each function:

a) $h(x) = 6\cos 4x$

b) $y = \sin\left(2x^4 + 3x\right)$

c) $y = \sin^3 4x + 4\tan 5x$

d) $f(x) = 4x\sin x$

14) Find the first derivative of each of the following functions:

a) $y = 3\ln\left(x^2 - 3\right)$

b) $k(x) = 4e^x - 3e^2 + 6e^\pi - 3e^{\pi x}$

c) $f(x) = 12^{x^2}$

d) $y = 3e^x \ln x$

15) Differentiate each of the following functions:

a) $y = Bx^3 - 3Cx^3 + D^4$

b) $f(x) = 4\cos\frac{5}{x^2}$

c) $m(x) = \frac{x^2 - 5}{x^2 + 5}$

d) $y = y^2 + xy$

UNIT 2

***All* working must be shown, as applicable to obtain full marks**

1) Find the general antiderivative of each of the following functions:

a) $5x^3 - 4x^2 + 3$

b) $x^4 + \frac{1}{x^3} + 2\pi$

c) $\frac{12}{x} + 3\cos x - 2$

d) $\cos(4x) + 4x^2$

e) $6e^x - 4e^{3x}$

f) $x^3 \cos(x^4)$

2) Find the position function $s(t)$ for an object with acceleration function $a(t) = 2t - 2$ and initial velocity $v(0) = 1$ and initial position $s(0) = 2$

3) Find each of the following indefinite integrals:

a) $\int x^3\, dx$

b) $\int \frac{2}{1-x}\, dx$

c) $\int e^{-3x}\, dx$

d) $\int \cos 3x\, dx$

4) Find the **exact** value of each of the following definite integrals:

a) $\int_{0}^{\frac{\pi}{2}} \sin x \; dx$

b) $\int_{-2}^{2} xe^{x^2} \; dx$

5) Simplify:

a) $\int_{0}^{a} e^{x} \; dx$

b) $\int_{0}^{b} 3x^{3} \; dx$

6) Simplify: $\int_{1}^{4} 3m^{2} \; dz$

7) Find the **exact** area between the curve $y = 4x$ and the x-axis over the interval $-3 \leq x \leq 4$

8) Find the **exact** area between the curve $y = 2x^2$ and the ***y*-axis** over the interval $0 \le x \le 3$

9) Find the **exact** area between $f(x) = 2x - 3$ and $g(x) = x^2 + 3$ over the interval $-2 \le x \le 5$

10) Simplify:

a) $\int_a^b 4x^2 \, dx$

b) $\int_0^b \frac{1}{1+x} \, dx$

11) Integrate:

a) $\int x^3\sqrt{x^4-3}\ dx$

$u = x^4 - 3$

b) $\int \frac{\sin x}{(\cos x)^5}\ dx$

12) Determine the area between $y = 3 - x$ *and* $x = 3y - y^2$.

UNIT 3

***All* working must be shown, as applicable to obtain full marks**

1) Evaluate each limit:

a) $\lim_{x \to -2} 3x^2 + 1$

b) $\lim_{x \to 3} \dfrac{2x^2 + 1}{3x}$

c) $\lim_{x \to 3\pi} \left(x^2 + 6\pi x - 2\pi^2\right)$

d) $\lim_{x \to a} \dfrac{(x + 2a)^2}{x^2 + a^2}$

2. By evaluating one-sided limits, find the indicated limit if it exists:

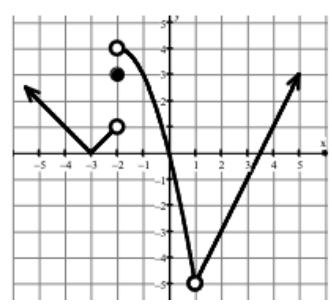
$$f(x) = \begin{cases} x^2 & \text{if } x < 2 \\ -2x + 5 & \text{if } x \geq 2 \end{cases} \qquad \text{find } \lim_{x \to 2} f(x)$$

3. Evaluate each limit.

a. $\lim_{x \to -3} f(x) =$ b. $f(1) =$ c. $\lim_{x \to 1} f(x) =$

d. $\lim_{x \to -2^+} f(x) =$ e. $f(3) =$ f. $\lim_{x \to -2^-} f(x) =$

g. $\lim_{x \to -2} f(x) =$ h. $f(-2) =$ i. $f(4) =$



4. Evaluate each of the following limits using L'Hopital's Rule:

a) $\lim_{x \to 3} \frac{9 - x^2}{x - 3}$

b) $\lim_{x \to 0} \frac{4\sin 2x}{4x}$

c) $\lim_{x \to 1} \frac{\ln x}{x - 1}$

d) $\lim_{x \to 0} \frac{1 - e^x}{\sin x}$

5) Evaluate the following limit by factoring and simplifying: $\lim_{x \to 5} \frac{x^2 - 25}{x - 5}$

6) Evaluate the following limit by rationalizing the numerator: $\lim\limits_{x\to 3} \dfrac{\sqrt{3}-\sqrt{x}}{3-x}$

7) Evaluate each of the following limits:

a) $\lim\limits_{x\to 2\pi} \left(2x^2-6\pi x\right)$

b) $\lim\limits_{x\to 4} \dfrac{16-x^2}{4-x}$

c) $\lim\limits_{x\to 0} \dfrac{2x}{e^x-1}$

d) $\lim\limits_{t\to -2} \dfrac{t^2-4t}{t-2}$

e) $\lim\limits_{x\to a} \dfrac{5x+a}{5x-a}$

f) $\lim\limits_{x\to 4} \dfrac{\ln(5-x)}{x-4}$

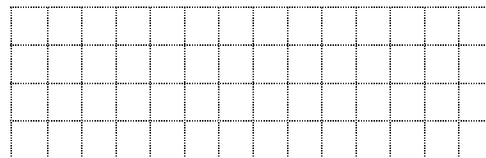
8) Use the **definition** of the derivative to find $\frac{dy}{dx}$ for $2x^2 + x$

9) Use the **definition** of the derivative to find $\frac{dy}{dx}$ for $y = \frac{1}{x+4}$

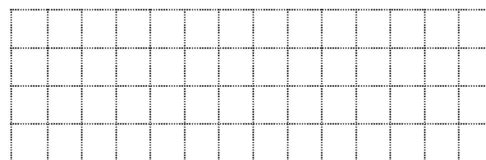
UNIT 4

<u>All</u> working must be shown, as applicable to obtain full marks

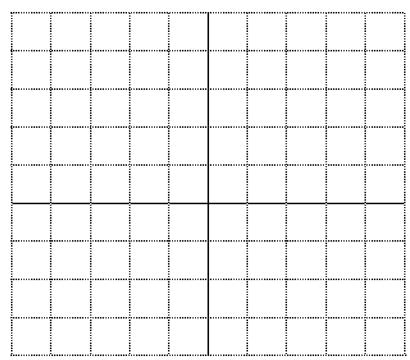
1. Find the vertical asymptotes of the following function and sketch the graph near the asymptotes: $f(x) = \dfrac{4}{(x-1)(x+3)}$



2. Find the horizontal asymptotes of the following function and sketch the graph near the asymptotes: $y = \dfrac{2x^2 - 5}{x^2 + 7}$



3. Make a rough sketch of the following function. Indicate x-intercepts, y-intercepts, vertical and horizontal asymptotes: $y = \frac{x+1}{x-2}$



4. Use Calculus and the Second Derivative Test to determine Concave up, Concave down, and Inflection Point for the following. **Sketch** function to show the above features.

$$f(x) = \frac{1}{x^2+1}$$

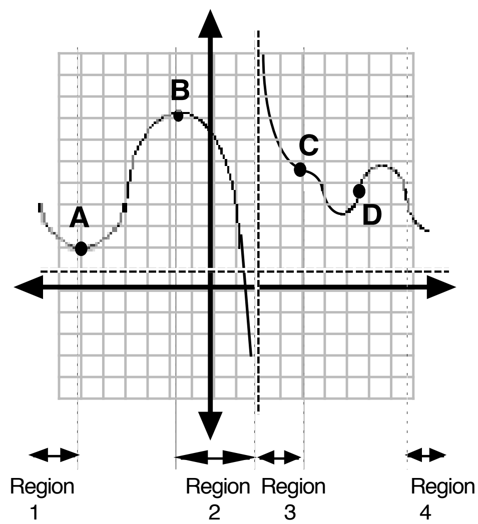
5. Use the terms increasing, decreasing, concave up, concave down to describe each region below:

Region 1

Region 2

Region 3

Region 4

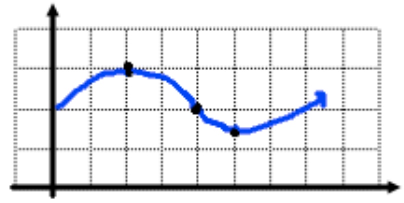


b) Use the terms local max/min, global man/min, turning point, inflection point to describe each of the following points:

A B C D

c) give the equations of **all** asymptotes:

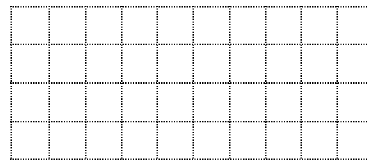
6. Given the position-time graph below:



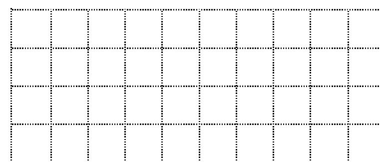
a) at what time(s) is the velocity 0?

b) at what time(s) is the acceleration 0?

v(t)

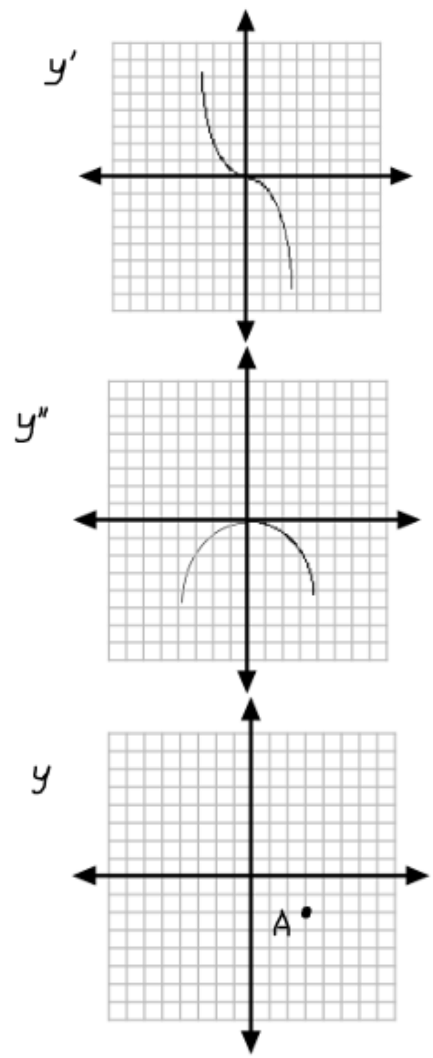


a(t)



c) when is the object slowing down?

7. Given the graphs of the first derivative y' and the second derivative y'', sketch the graph of y passing through A.



UNIT 5

***All* working must be shown, as applicable to obtain full marks**

1. a) $4 \ln x - 12 = 0$ b) $2 \ln x = \ln(2x + 8)$

2. Simplify:
   a) $\ln(e^{-4}) + \ln(e^{6})$ b) $e^{-2\ln 4}$

3. Find the equation of the tangent line to the curve y = 2 ln x at the point (e, 2).

4. Use Newton's Method to solve the following equation $x^3 - 2x^2 - 2 = 0$ using x-initial value $x = 2$. Just do two iterations $x_3 =$

5. An open field is bounded by a lake with a straight shoreline. A rectangular enclosure is to be constructed using 600 m of fencing along three sides and the lake as a natural boundary on the fourth side. What dimensions will maximize the enclosed area and what is the maximum area? Solve this problem using Calculus.

6. An open-top box is to be made from a 24 in. by 36 in. piece of cardboard by removing a square from each corner of the box and folding up the flaps on each side. What size square should be cut out of each corner to get a box with the maximum volume?

7. Evaluate each of the following **IMPROPER INTEGRALS** if possible

a) $\int_{1}^{\infty} \frac{1}{x^2}\, dx$

b) $\int_{-\infty}^{0} 2e^{x}\, dx$

8. a) Use the MVT to determine all the numbers c for the following function:
$f(x) = x^3 + 4x$, [-1, 1]

b) Use Rolle's Theorem to show that the function has a horizontal tangent line in the interval [0, 2] $f(x) = x^2 - 2x$

c) Use the IVT to show that $f(x) = x^4 - 7x^2 + 10$ has a root somewhere in the interval [0, 2]

9. Given $\boldsymbol{f}$ is a function of x and y find the partial derivative of $\boldsymbol{f}$ with respect to x and the partial derivative of $\boldsymbol{f}$ with respect to y of each of the following:

$a)\ f(x, y) = 3x^2 + x^3y$

$b)\ f(x, y) = x^2 lny - y^2$

10. Find the exact value of each of the following multiple integrals:

a) $\int_0^1 \int_2^4 x^2\, dx\, dy$

b) $\int_b^a \int_{-1}^5 \frac{x}{3}\, dx\, dy$

11. Simplify:

a) $(3 - 4i) + (1 + 9i)$

b) $(4 - 3i)(2 + 4i)$

12. Solve each of the following equations over the COMPLEX NUMBERS:

$a)\ x^2 + 25 = 0$ $b)\ 2x^2 + 3 = x^2 - 1$

13. Give the rectangular coordinates of the point whose polar coordinates are given:

$a)\ (r, \theta) = \left(3\,, \frac{\pi}{3}\right)$ $b)\ (r, \theta) = (6\,, \pi\,)$

14. Give the polar coordinates of the point whose rectangular coordinates are given:

$a)\ (x, y) = (5{,}5)$ $b)\ (x, y) = \left(\sqrt{3}, -\frac{1}{3}\right)$

15. Find a rectangular equation equivalent to the given polar equation and describe the graph:

a) $r = 2$ b) $r = 4\cos\theta$