Grade 12 Introduction to Calculus (45S)

Final Practice Exam

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Name: ______________________________________

Student Number: ______________________________

Attending ❑ Non-Attending ❑

Phone Number: ______________________________

Address: ____________________________________

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For Marker's Use Only

Date: ___________________________________

Final Mark: _________ /100 = _________ %

Comments:

Instructions

The final examination will be weighted as follows:

Module 1: Limits	21 marks
Module 2: Derivatives	26 marks
Module 3: Applications of Derivatives	32 marks
Module 4: Integration	21 marks
	100 marks

Time allowed: 3.0 hours

Note: You are allowed to bring the following to the exam: pencils (2 or 3 of each), blank paper, a ruler, and a scientific calculator.

Show all calculations and formulas used. Use all decimal places in your calculations and round the final answers to the correct number of decimal places. Include units where appropriate. Clearly state your final answer.

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Module 1: Limits *(21 marks)*

1. Determine $\lim\limits_{x \to 4} \dfrac{3 - \sqrt{x}}{x - 9}$. *(1 mark)*

2. Given: $\lim\limits_{x \to 9} \dfrac{3 - \sqrt{x}}{x - 9}$ *(4 marks)*

   a) Evaluate the limit algebraically.

b) Explain why evaluating the limit $\lim_{x \to 9} \frac{3 - \sqrt{x}}{x - 9}$ requires an algebraic manipulation, while $\lim_{x \to 4} \frac{3 - \sqrt{x}}{x - 9}$ does not.

3. Evaluate the following limits. *(6 marks)*

a) $\lim_{x \to 2} \left(\frac{x^2 - 3x + 2}{x - 2} \right)$

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b) $\lim_{x \to \infty} \left(\dfrac{x^2 - 7x + 1}{x^3 + 2} \right)$

c) $\lim_{x \to 2^+} \dfrac{x + 1}{(x - 2)^2}$

4. Complete the tasks below to determine if $g(x) = \begin{cases} -x + 1, \ x \le 1 \\ x^2 - 1, \ x > 1 \end{cases}$ is continuous at $x = 1$.

(4 marks)

a) Verify that $\lim\limits_{x \to 1} g(x)$ exists.

b) Is $g(x)$ continuous? Explain.

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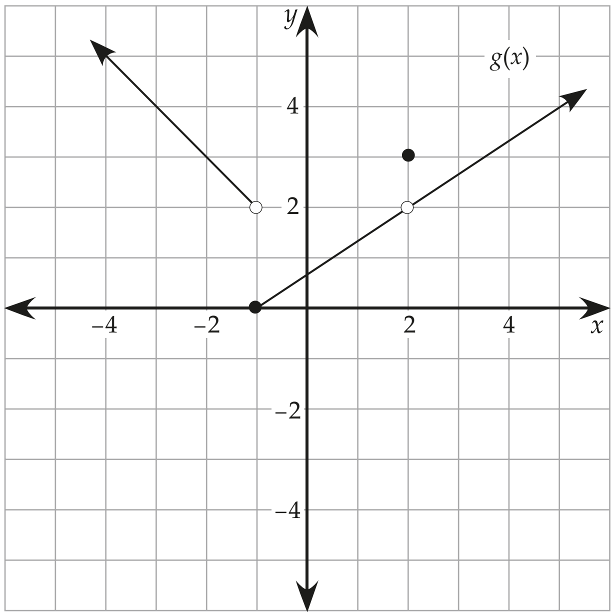
5. Use the graph of $g(x)$ below to evaluate each expression. *(6 marks)*



a) $\lim\limits_{x \to 2^-} g(x)$

b) $\lim\limits_{x \to 2^+} g(x)$

c) $g(2)$

d) $\lim\limits_{x \to -1^-} g(x)$

e) $\lim\limits_{x \to -1^+} g(x)$

f) $\lim\limits_{x \to -1} g(x)$

Module 2: Derivatives *(26 marks)*

1. Given: $g(x) = -2x^2 + 3$ *(10 marks)*

   a) Determine the slope of the secant lines PR, PS, and PT to the curve, given the coordinates P(1, 1), R(4, −29), S(3, −15), T(1.1, 0.58).

   b) Using the values from part (a) above, describe what is happening to the value of the slope of the secant line from a point (x, y) as the point approaches P.

   c) Estimate the slope of the tangent line at point P.

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d) Determine the derivative of $g(x) = -2x^2 + 3$ at $x = 1$ using the limit definition of the derivative and the difference quotient, $g'(1) = \lim_{h \to 0} \frac{g(1+h) - g(1)}{h}$.

e) Determine the equation of the tangent line to $g(x) = -2x^2 + 3$ at $x = 1$.

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2. Use derivative rules to differentiate the following and **do not** simplify your derivative. *(10 marks)*

a) $f(x) = 2x^{-4} - 5x^{\frac{2}{3}} + 7$

b) $g(x) = 6\left(2x^5 - 1\right)^3$

c) $h(x) = \dfrac{6x - 3}{2x^3 + 1}$

d) $k(x) = \left(\sqrt{x}\right)\left(5x^2 + 1\right)$

3. Determine $\frac{dy}{dx}$ in terms of x and y for the equation $x + xy^2 - y = 3$. *(4 marks)*

4. Determine y'' for $y = 3x^5 - 5x^3 - 2x^2 - 1$. *(2 marks)*

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Module 3: Applications of Derivatives *(32 marks)*

1. A ball is thrown upward so that its height above the ground after time t is $h = 20t - 5t^2$, where h is measured in metres and t is measured in seconds. *(6 marks)*

   a) Determine the equation that represents the velocity of the ball.

   b) Determine when the ball reaches its maximum height.

c) Determine the velocity of the ball when it is 15 metres high on its way down.

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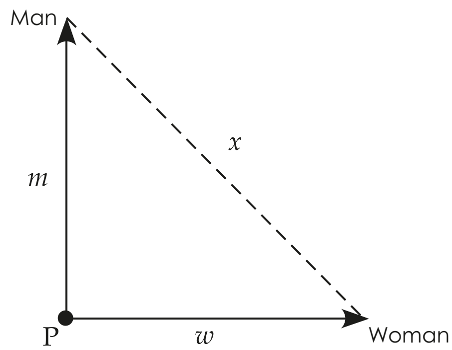
2. A man starts walking north at a speed of 1.5 m/s and a woman starts at the same point P at the same time walking east at a speed of 2 m/s. *(6 marks)*



a) How far is the man, *m*, from his starting point after one minute?

b) How far is the woman, *w*, from her starting point after one minute?

c) How far apart are the man and the woman, x, from each other after one minute?

d) At what rate is the distance between the man and the woman increasing at the instant they have been walking for one minute?

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3. Given: $g(x) = x^3 + 6x^2 + 9x + 4$ *(13 marks)*

   a) Find the intervals where the function is increasing and decreasing.

   b) Find the coordinates where the relative extreme values occur and identify each of them as a relative maximum or minimum.

c) Find the intervals of concavity and the coordinates of any points of inflection.

d) Sketch the graph of the function and label its extreme values and point(s) of inflection.

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4. The sum of two positive numbers is 12. If the product of one number cubed and the other number is a maximum, find the two numbers. *(7 marks)*

Module 4: Integration *(21 marks)*

1. If $f'(x) = 4x^5 - 2x^3 + x - 2$, and $f(0) = 3$, determine the function equation for $f(x)$. *(4 marks)*

2. Evaluate algebraically $\int_{-2}^{0} \left(5x^4 - 2x^2 - 1\right) dx$. *(3 marks)*

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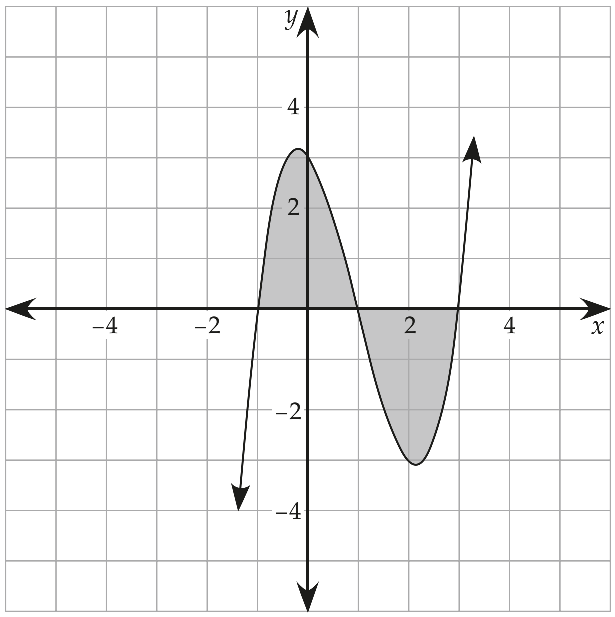
3. Write the general function equation represented by the indefinite integral $\int \left(3x^6 - 3x^{-2}\right)\, dx$. *(2 marks)*

4. Sketch and determine the area bounded by the line $y = -x + 1$ and the x-axis on the closed interval [0, 1]: *(3 marks)*

   a) geometrically, using a graph of the function

   b) algebraically, using the antiderivative

5. Determine the area bounded by the curve $y = x^3 - 3x^2 - x + 3$ and the x-axis. *(5 marks)*



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6. Find the values of each definite integral geometrically using the sketch of $f(x)$ as shown. *(4 marks)*.

a) $\int_{-2}^{4} f(x)\,dx$

b) $\int_{-4}^{0} f(x)\,dx$

c) $\int_{2}^{4} f(x)\,dx$

d) $\int_{-4}^{0} |f(x)|\,dx$