



GRADE 12 INTRODUCTION TO CALCULUS (45S)

Final Practice Exam
Answer Key

GRADE 12 INTRODUCTION TO CALCULUS

Final Practice Exam Answer Key

Name: _____

Student Number: _____

Attending ☐ Non-Attending ☐

Phone Number: _____

Address: _____

For Marker's Use Only

Date: _____

Final Mark: _____ / 100 = _____ %

Comments: _____

Instructions

The final examination will be weighted as follows:

Module 1: Limits	21 marks
Module 2: Derivatives	26 marks
Module 3: Applications of Derivatives	32 marks
Module 4: Integration	21 marks
	100 marks

Time allowed: 3.0 hours

Note: You are allowed to bring the following to the exam: pencils (2 or 3 of each), blank paper, a ruler, and a scientific calculator.

Show all calculations and formulas used. Use all decimal places in your calculations and round the final answers to the correct number of decimal places. Include units where appropriate. Clearly state your final answer.

Name: _____

Module 1: Limits (21 marks)

1. Determine $\lim_{x \rightarrow 4} \frac{3 - \sqrt{x}}{x - 9}$. (1 mark)

Answer:

(Lesson 3)

(1 mark for evaluation of limit)

$$\lim_{x \rightarrow 4} \frac{3 - \sqrt{x}}{x - 9} = \frac{3 - \sqrt{4}}{4 - 9} = \frac{3 - 2}{-5} = -\frac{1}{5}$$

The limit is $-\frac{1}{5}$.

2. Given: $\lim_{x \rightarrow 9} \frac{3 - \sqrt{x}}{x - 9}$ (4 marks)

- a) Evaluate the limit algebraically.

Answer:

(Lesson 4)

(2 marks for the algebraic manipulation of the limit)

(1 mark for evaluating the limit)

$$\lim_{x \rightarrow 9} \frac{3 - \sqrt{x}}{x - 9} = \frac{0}{0} \quad \text{I.F.}$$

$$\begin{aligned} \lim_{x \rightarrow 9} \frac{3 - \sqrt{x}}{x - 9} &= \lim_{x \rightarrow 9} \left[\frac{(3 - \sqrt{x})}{(x - 9)} \cdot \frac{(3 + \sqrt{x})}{(3 + \sqrt{x})} \right] = \lim_{x \rightarrow 9} \frac{9 - x}{(x - 9)(3 + \sqrt{x})} = \lim_{x \rightarrow 9} \frac{-1}{3 + \sqrt{x}} \\ &= \frac{-1}{3 + \sqrt{9}} = \frac{-1}{3 + 3} = \frac{-1}{6} \text{ or } -0.1\bar{6} \end{aligned}$$

The limit is $-\frac{1}{6}$.

- b) Explain why evaluating the limit $\lim_{x \rightarrow 9} \frac{3 - \sqrt{x}}{x - 9}$ requires an algebraic manipulation, while $\lim_{x \rightarrow 4} \frac{3 - \sqrt{x}}{x - 9}$ does not.

Answer:

(Lesson 4)

(1 mark for explanation)

Possible explanation could reference that the $x = 9$ is a non-permissible value that makes the numerator and denominator zero or that the resultant substitution is in the indeterminate form.

3. Evaluate the following limits. (6 marks)

a) $\lim_{x \rightarrow 2} \left(\frac{x^2 - 3x + 2}{x - 2} \right)$

Answer:

(Lesson 3)

(1 mark for simplification of limit)

(1 mark for evaluating the limit)

$$\begin{aligned} \lim_{x \rightarrow 2} \left(\frac{x^2 - 3x + 2}{x - 2} \right) &= \lim_{x \rightarrow 2} \left(\frac{(x - 2)(x - 1)}{x - 2} \right) = \lim_{x \rightarrow 2} (x - 1) \\ &= (2 - 1) = 1 \end{aligned}$$

Name: _____

b) $\lim_{x \rightarrow \infty} \left(\frac{x^2 - 7x + 1}{x^3 + 2} \right)$

Answer:

(Lesson 6)

(1 mark for dividing top and bottom by the highest power of x in the denominator)

(1 mark for simplifying both the numerator and denominator)

(1 mark for evaluating the limit)

$$\lim_{x \rightarrow \infty} \left(\frac{x^2 - 7x + 1}{x^3 + 2} \right) = \frac{\infty}{\infty} \text{ I.F.}$$

$$\begin{aligned} \lim_{x \rightarrow \infty} \left(\frac{x^2 - 7x + 1}{x^3 + 2} \right) &= \lim_{x \rightarrow \infty} \left(\frac{x^2 - 7x + 1}{x^3 + 2} \right) \cdot \left(\frac{\frac{1}{x^3}}{\frac{1}{x^3}} \right) = \lim_{x \rightarrow \infty} \left(\frac{\frac{1}{x} - \frac{7}{x^2} + \frac{1}{x^3}}{1 + \frac{2}{x^3}} \right) \\ &= \frac{0 + 0 + 0}{1 + 0} = \frac{0}{1} = 0 \end{aligned}$$

c) $\lim_{x \rightarrow 2^+} \frac{x + 1}{(x - 2)^2}$

Answer:

(Lesson 6)

(1 mark for answer)

Illustrate using $x = 2.01$.

$$\lim_{x \rightarrow 2^+} \frac{x + 1}{(x - 2)^2} \approx \frac{2.01 + 1}{(2.01 - 2)^2} = \frac{3.01}{(0.01)^2} = \frac{3.01}{0.0001} = 30100$$

$$\lim_{x \rightarrow 2^+} \frac{x + 1}{(x - 2)^2} = \frac{3}{\text{very small positive number}} \text{ and approaches } \infty$$

As x approaches 2 from the right, the right limit approaches ∞ .

$$\lim_{x \rightarrow 2^+} \left(\frac{x + 1}{(x - 2)^2} \right) = \infty$$

4. Complete the tasks below to determine if $g(x) = \begin{cases} -x + 1, & x \leq 1 \\ x^2 - 1, & x > 1 \end{cases}$ is continuous at $x = 1$.

(4 marks)

- a) Verify that $\lim_{x \rightarrow 1} g(x)$ exists.

Answer:

(Lesson 5)

(1 mark for setting up the one-side limits)

(1 mark for evaluating the limit)

$$\lim_{x \rightarrow 1^-} g(x) = \lim_{x \rightarrow 1^-} (-x + 1) = -1 + 1 = 0$$

$$\lim_{x \rightarrow 1^+} g(x) = \lim_{x \rightarrow 1^+} (x^2 - 1) = 1^2 - 1 = 1 - 1 = 0$$

$$\lim_{x \rightarrow 1^-} g(x) = \lim_{x \rightarrow 1^+} g(x)$$

$$\text{So, } \lim_{x \rightarrow 1} g(x) = 0.$$

- b) Is $g(x)$ continuous? Explain.

Answer:

(Lesson 7)

(1 mark for evaluating the function value)

(1 mark for using the definition of continuity to explain why the function is continuous)

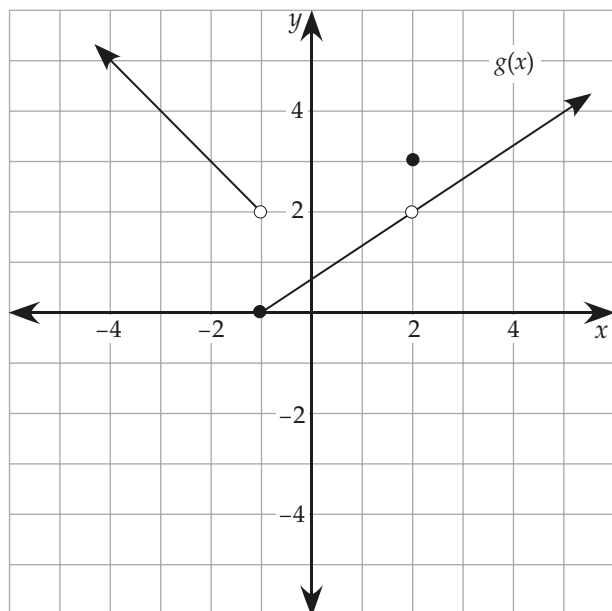
For the function to be continuous at $x = 1$, both the function and the limit values have to exist and be equal to each other at $x = 1$.

$$g(1) = -1 + 1 = 0$$

Since $g(1) = 0$ and $\lim_{x \rightarrow 1} g(x) = 0$, then the function is defined at $x = 1$ and equal to the limit value, so the function is continuous at $x = 1$.

Name: _____

5. Use the graph of $g(x)$ below to evaluate each expression. (6 marks)



(6 \times 1 mark per correct limit)

(Lesson 5)

a) $\lim_{x \rightarrow 2^-} g(x)$

Answer:

$$\lim_{x \rightarrow 2^-} g(x) = 2$$

b) $\lim_{x \rightarrow 2^+} g(x)$

Answer:

$$\lim_{x \rightarrow 2^+} g(x) = 2$$

c) $g(2)$

Answer:

$$g(2) = 3$$

d) $\lim_{x \rightarrow -1^-} g(x)$

Answer:

$$\lim_{x \rightarrow -1^-} g(x) = 2$$

e) $\lim_{x \rightarrow -1^+} g(x)$

Answer:

$$\lim_{x \rightarrow -1^+} g(x) = 0$$

f) $\lim_{x \rightarrow -1} g(x)$

Answer:

Does not exist since left-hand and right-hand limits are not equal.

Module 2: Derivatives (26 marks)

1. Given: $g(x) = -2x^2 + 3$ (10 marks)

- a) Determine the slope of the secant lines PR, PS, and PT to the curve, given the coordinates P(1, 1), R(4, -29), S(3, -15), T(1.1, 0.58).

Answer:

(Lesson 1)

(2 marks for evaluating slope properly)

$$m_{PR} = \frac{-29 - 1}{4 - 1} = \frac{-30}{3} = -10$$

$$m_{PS} = \frac{-15 - 1}{3 - 1} = \frac{-16}{2} = -8$$

$$m_{PT} = \frac{0.58 - 1}{1.1 - 1} = \frac{-0.42}{0.1} = -4.2$$

- b) Using the values from part (a) above, describe what is happening to the value of the slope of the secant line from a point (x, y) as the point approaches P.

Answer:

(Lesson 1)

(1 mark for a reasonable description)

The slopes of the secant lines are increasing since they are smaller negative numbers.

- c) Estimate the slope of the tangent line at point P.

Answer:

(Lesson 1)

(1 mark for a reasonable estimate)

The slope of the tangent line is approaching -4.2 (or is near -4.2).

Name: _____

- d) Determine the derivative of $g(x) = -2x^2 + 3$ at $x = 1$ using the limit definition of the derivative and the difference quotient, $g'(1) = \lim_{h \rightarrow 0} \frac{g(1+h) - g(1)}{h}$.

Answer:

(Lesson 2)

(1 mark for correct substitution into the limit definition of the derivative)

(1 mark for simplifying the limit)

(1 mark for evaluating the limit and finding the derivative at $x = 1$)

$$\begin{aligned} g'(1) &= \lim_{h \rightarrow 0} \frac{g(1+h) - g(1)}{h} = \lim_{h \rightarrow 0} \frac{(-2(1+h)^2 + 3) - (-2(1)^2 + 3)}{h} \\ &= \lim_{h \rightarrow 0} \frac{-2 - 4h - 2h^2 + 3 + 2 - 3}{h} \end{aligned}$$

$$\begin{aligned} g'(1) &= \lim_{h \rightarrow 0} \frac{-4h - 2h^2}{h} = \lim_{h \rightarrow 0} \frac{h(-4 - 2h)}{h} = \lim_{h \rightarrow 0} (-4 - 2h) \\ &= -4 - 2(0) = -4 - 0 = -4 \end{aligned}$$

The derivative of the function at $x = 1$ is -4 .

e) Determine the equation of the tangent line to $g(x) = -2x^2 + 3$ at $x = 1$.

Answer: (Lesson 2)

(1 mark for determining the coordinates of the point of tangency)

(1 mark for appropriate algebraic work)

(1 mark for determining the equation of the tangent line)

Use the slope and a point on the tangent line to determine the equation of the tangent line.

$$y = g(1) = -2(1)^2 + 3 = 1$$

$$m_T = -4 \text{ and } (1, 1)$$

Using slope-intercept form:

$$y = mx + b$$

$$1 = (-4)(1) + b$$

$$b = 1 + 4 = 5$$

Or, using point-slope form:

$$y - y_1 = m(x - x_1)$$

$$y - 1 = -4(x - 1)$$

The equation of the tangent line is $y = -4x + 5$.

Name: _____

2. Use derivative rules to differentiate the following and **do not** simplify your derivative.
(10 marks)

a) $f(x) = 2x^{-4} - 5x^{\frac{2}{3}} + 7$

Answer:

(Lesson 3)

(3 marks for proper use of the power rule on each of the three terms)

$$f'(x) = -8x^{-5} - \frac{10}{3}x^{-\frac{1}{3}}$$

b) $g(x) = 6(2x^5 - 1)^3$

Answer:

(Lesson 5)

(1 mark for derivative of outside function in chain rule)

(1 mark for derivative of inside function in chain rule)

$$g'(x) = 18(\text{inside})^2 \cdot \text{inside}'$$

$$g'(x) = 18(2x^5 - 1)^2 (10x^4)$$

c) $h(x) = \frac{6x - 3}{2x^3 + 1}$

Answer:

(Lesson 4)

(2 marks for correctly writing the numerator, 1 mark for each term)

(1 mark for correctly writing the denominator using the quotient rule)

$$h'(x) = \frac{(2x^3 + 1)(6) - (6x - 3)(6x^2)}{(2x^3 + 1)^2}$$

d) $k(x) = (\sqrt{x})(5x^2 + 1)$

Answer:

(Lesson 4)

(2 marks for using the product rule, one mark for each term)

$$k'(x) = \left(\frac{1}{2}x^{-\frac{1}{2}}\right)(5x^2 + 1) + (\sqrt{x})(10x)$$

3. Determine $\frac{dy}{dx}$ in terms of x and y for the equation $x + xy^2 - y = 3$. (4 marks)

(Lesson 6)

Answer:

(1 mark for power rule and constant rule)

(1 mark for product rule)

(1 mark for implicit differentiation)

(1 mark for isolating y')

$$\frac{d}{dx}(x + xy^2 - y) = \frac{d}{dx}(3)$$

$$1 + (1)y^2 + x(2yy') - y' = 0$$

$$2xyy' - y' = -1 - y^2$$

$$y'(2xy - 1) = -1 - y^2$$

$$\frac{dy}{dx} = y' = \frac{-1 - y^2}{2xy - 1}$$

4. Determine y'' for $y = 3x^5 - 5x^3 - 2x^2 - 1$. (2 marks)

Answer:

(Lesson 5)

(1 mark for the first derivative)

(1 mark for the second derivative)

$$y = 3x^5 - 5x^3 - 2x^2 - 1$$

$$y' = 15x^4 - 15x^2 - 4x$$

$$y'' = 60x^3 - 30x - 4$$

Name: _____

Module 3: Applications of Derivatives (32 marks)

1. A ball is thrown upward so that its height above the ground after time t is $h = 20t - 5t^2$, where h is measured in metres and t is measured in seconds. (6 marks)

- a) Determine the equation that represents the velocity of the ball.

Answer: (Lesson 2)

(1 mark for determining velocity)

$$h = 20t - 5t^2$$

$$v = h' = 20 - 10t$$

- b) Determine when the ball reaches its maximum height.

Answer: (Lesson 2)

(1 mark for setting velocity equal to zero)

(1 mark for determining when the ball reaches the maximum height)

$$v = 20 - 10t$$

$$0 = 20 - 10t$$

$$10t = 20$$

$$t = 2$$

The ball reaches its maximum height after 2 seconds.

- c) Determine the velocity of the ball when it is 15 metres high on its way down.

Answer:

(Lesson 2)

(1 mark for solving the quadratic equation)

(1 mark for determining the time on the way down)

(1 mark for determining the velocity)

$$15 = 20t - 5t^2$$

$$0 = -5t^2 + 20t - 15$$

$$0 = t^2 - 4t + 3$$

$$0 = (t - 3)(t - 1)$$

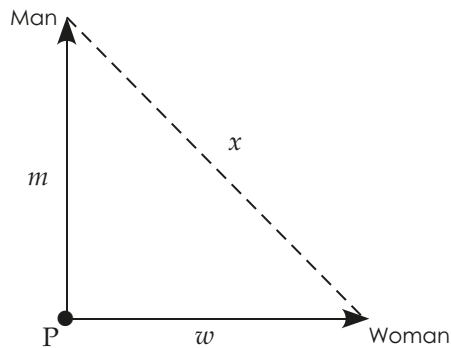
The ball is 15 metres high at $t = 1$ s or $t = 3$ s. It is on its way down when $t = 3$ s.

$$v = 20 - 10(3) = 10 - 30 = -10 \text{ m/s}$$

Since the ball is travelling downward, its velocity is negative, and the velocity of the ball on its way down at a height of 15 metres is -10 m/s.

Name: _____

2. A man starts walking north at a speed of 1.5 m/s and a woman starts at the same point P at the same time walking east at a speed of 2 m/s. (6 marks)



- a) How far is the man, m , from his starting point after one minute?

Answer: _____ (Lesson 6)

(1 mark for evaluating the man's displacement after one minute)

$$m = (1.5 \text{ m/s}) \times (60 \text{ s}) = 90 \text{ m}$$

The man is 90 metres from point P after one minute.

- b) How far is the woman, w , from her starting point after one minute?

Answer: _____ (Lesson 6)

(1 mark for evaluating the woman's displacement after one minute)

$$w = (2 \text{ m/s}) \times (60 \text{ s}) = 120 \text{ m}$$

The woman is 120 metres from point P after one minute.

- c) How far apart are the man and the woman, x , from each other after one minute?

Answer:

(Lesson 6)

(1 mark for evaluating the displacement between the man and woman after one minute)

$$w^2 + m^2 = x^2$$

$$120^2 + 90^2 = x^2$$

$$14400 + 8100 = x^2$$

$$22500 = x^2$$

Use the Pythagorean theorem:

$$x = \sqrt{22500} = 150$$

The man and woman are 150 metres apart from one another after one minute.

- d) At what rate is the distance between the man and the woman increasing at the instant they have been walking for one minute?

Answer:

(Lesson 6)

(1 mark for implicit differentiation)

(1 mark for correct substitution)

(1 mark for evaluating rate at which the distance between the man and woman changes)

Differentiate the equation that relates the variables that are all dependent on time — that is, $w^2 + m^2 = x^2$.

$$\frac{d}{dt}(w^2 + m^2) = \frac{d}{dt}(x^2)$$

Given:

$$\frac{dw}{dt} = 2 \text{ m/s}$$

$$2w \cdot \frac{dw}{dt} + 2m \cdot \frac{dm}{dt} = 2x \cdot \frac{dx}{dt}$$

$$\frac{dm}{dt} = 1.5 \text{ m/s}$$

When $w = 120$ m, $m = 90$ m, and $x = 150$ m.

$$2(120) \cdot (2) + 2(90) \cdot (1.5) = 2(150) \cdot \frac{dx}{dt}$$

$$300 \frac{dx}{dt} = 480 + 270 = 750$$

$$\frac{dx}{dt} = \frac{750}{300} = 2.5 \text{ m/s}$$

At the instant they have been walking for one minute, the distance between the man and woman is increasing at 2.5 m/s.

Name: _____

3. Given: $g(x) = x^3 + 6x^2 + 9x + 4$ (13 marks)

a) Find the intervals where the function is increasing and decreasing.

Answer:

(Lesson 3)

(1 mark for determining the first derivative)

(1 mark for determining the critical values)

$$g'(x) = 3x^2 + 12x + 9$$

Solve:

$$g'(x) = 0$$

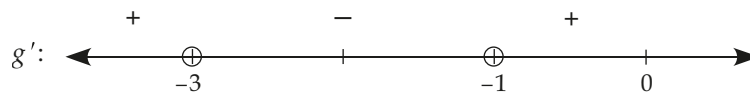
$$3x^2 + 12x + 9 = 0$$

$$x^2 + 4x + 3 = 0$$

$$(x + 1)(x + 3) = 0$$

The critical values when $g'(x) = 0$ are $-1, -3$.

(1 mark for creating sign diagram and identifying boundary values)



(1 mark for interpreting the sign diagram to determine the positive and negative intervals)

The function is increasing on $(-\infty, -3) \cup (-1, \infty)$ because the first derivative is positive; and the function is decreasing on $(-3, -1)$ because the first derivative is negative.

- b) Find the coordinates where the relative extreme values occur and identify each of them as a relative maximum or minimum.

Answer:

(Lesson 3)

Method 1

(1 mark for using the first derivative test to determine whether maximum or minimum)

(1 mark for determining the coordinates of the extreme values)

Since the function changes from increasing to decreasing at $x = -3$, then there is a local maximum there. Also, since $g(-3) = (-3)^3 + 6(-3)^2 + 9(-3) + 4$
 $= -27 + 54 - 27 + 4 = 4$, then the coordinates of the local maximum are $(-3, 4)$.

Since the function changes from decreasing to increasing at $x = -1$, then there is a local minimum there. Also, since $g(-1) = (-1)^3 + 6(-1)^2 + 9(-1) + 4$
 $= -1 + 6 - 9 + 4 = 0$, then the coordinates of the local minimum are $(-1, 0)$.

Or

Method 2

(1 mark for using the second derivative test to determine whether maximum or minimum)

(1 mark for determining the coordinates of the extreme values)

$$g'' = 6x + 12$$

Use the second derivative test at $x = -3$ and $x = -1$.

$$g''(-3) = 6(-3) + 12 = -6$$

Since the second derivative is negative at $x = -3$, then the function is concave down at that x -value and there is a local maximum there.

As in Method 1 above, the local maximum coordinates are $(-3, 4)$.

$$g''(-1) = 6(-1) + 12 = +6$$

Since the second derivative is positive at $x = -1$, then the function is concave up at that x -value and there is a local minimum there.

As in Method 1 above, the local minimum coordinates are $(-1, 0)$.

Name: _____

- c) Find the intervals of concavity and the coordinates of any points of inflection.

Answer:

(Lesson 5)

(1 mark determining the second derivative)

(1 mark for determining the critical values of $g''(x)$)

$$g''(x) = 6x + 12$$

$$0 = g''(x) = 6x + 12$$

$$6x = -12$$

$$x = -2$$

(2 × 1 mark for describing each concavity interval)

(1 mark for determining the point of inflection)

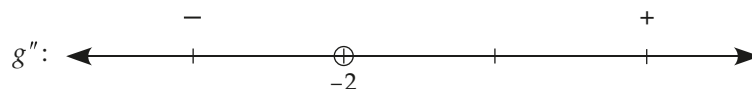
The test intervals are $(-\infty, -2)$ and $(-2, \infty)$.

Select $x = -3$ as a test point for $(-\infty, -2)$ and substitute it into the second derivative:

$$g''(-3) = 6(-3) + 12 = -18 + 12 = -6 < 0$$

Select $x = 0$ as a test point for $(-2, \infty)$ and substitute it into the second derivative:

$$g''(0) = 6(0) + 12 = 12 > 0$$



Since the second derivative is negative, then the function is concave down on $(-\infty, -2)$.

Since the second derivative is positive, then the function is concave up on $(-2, \infty)$.

Since the function changes concavity at $x = -2$ and $g(-2) = (-2)^3 + 6(-2)^2 + 9(-2) + 4 = -8 + 24 - 18 + 4 = 2$, then there is an inflection point at $(-2, 2)$.

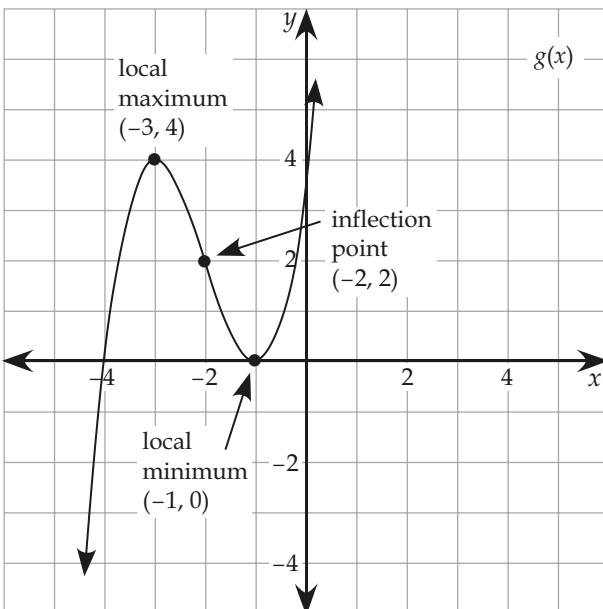
- d) Sketch the graph of the function and label its extreme values and point(s) of inflection.

Answer:

(Lesson 5)

(1 mark for plotting the extreme values and inflection point)

(1 mark for sketching the correct curve behaviour)



Name: _____

4. The sum of two positive numbers is 12. If the product of one number cubed and the other number is a maximum, find the two numbers. (7 marks)

Answer: _____ (Lesson 4)

(1 mark for defining the sum of two natural numbers)

(1 mark for creating a product equation as a function of one variable with the sum equation)

$$x + y = 12$$

$$x = 12 - y$$

You need an expression for the product to find the maximum. Substitute for x to write P in terms of y .

$$P = x \cdot y^3 \quad P = (12 - y) \cdot y^3 = 12y^3 - y^4$$

(1 mark for determining the derivative of the product equation)

$$P' = 36y^2 - 4y^3$$

(1 mark for determining the critical value)

$$0 = P' = 36y^2 - 4y^3$$

$$0 = 4y^2(9 - y)$$

$$y = 0, 9$$

Only $y = 9$ is a possible critical value because the number must be positive.

(1 mark for determining the second derivative or creating a sign diagram for P')

$$P'' = 72y - 12y^2$$

$$P''(9) = 72(9) - 12(9)^2 = -324$$

(1 mark for using the second derivative or interpreting the sign diagram for P' to determine if there is a maximum)

Since the second derivative is negative when $x = 9$, then the function is concave down and there is a maximum at that critical value.

Alternatively, you can earn the 2 marks above by creating and interpreting a sign diagram for P' . You will notice that P' is positive to the left when $y = 8$ and negative to the right when $y = 10$. There is a maximum at the critical value, $y = 9$, since P' goes from $+$ to $-$ or, said another way, P goes from increasing to decreasing.

(1 mark for determining the two natural numbers)

$$x = 12 - 9 = 3$$

The two positive numbers are 9 and 3 and the product $3(9)^3 = 2187$ is the maximum.

Module 4: Integration (21 marks)

1. If $f'(x) = 4x^5 - 2x^3 + x - 2$, and $f(0) = 3$, determine the function equation for $f(x)$. (4 marks)

Answer:

(Lesson 2)

(1 mark for determining the general antiderivative of $f'(x)$)

(1 mark for substituting initial conditions into the general antiderivative)

(1 mark for solving for constant of variation)

(1 mark for determining the specific antiderivative)

$$f(x) = \frac{4}{6}x^6 - \frac{2}{4}x^4 + \frac{1}{2}x^2 - 2x + C$$

$$f(0) = \frac{4}{6}(0)^6 - \frac{2}{4}(0)^4 + \frac{1}{2}(0)^2 - 2(0) + C = 3$$

$$C = 3$$

$$\text{Therefore, } f(x) = \frac{2}{3}x^6 - \frac{1}{2}x^4 + \frac{1}{2}x^2 - 2x + 3.$$

2. Evaluate algebraically $\int_{-2}^0 (5x^4 - 2x^2 - 1) dx$. (3 marks)

Answer:

(Lesson 3)

(1 mark for determining the antiderivative)

(1 mark for substituting the upper and lower bounds)

(1 mark for evaluating the definite integral)

$$\int_{-2}^0 (5x^4 - 2x^2 - 1) dx = \left[x^5 - \frac{2}{3}x^3 - x \right]_{-2}^0$$

$$= \left[(0)^5 - \frac{2}{3}(0)^3 - (0) \right] - \left[(-2)^5 - \frac{2}{3}(-2)^3 - (-2) \right]$$

$$= [0] - \left[(-32) - \frac{2}{3}(-8) + 2 \right] = 0 + 32 - \frac{16}{3} - 2 = \frac{90}{3} - \frac{16}{3} = \frac{74}{3} \text{ or } 24\frac{2}{3}$$

Name: _____

3. Write the general function equation represented by the indefinite integral $\int (3x^6 - 3x^{-2}) dx$. (2 marks)

Answer:

(Lesson 1)

(1 mark for determining the antiderivative)

(1 mark for stating the family of indefinite integrals with constant)

$$\int (3x^6 - 3x^{-2}) dx = \frac{3}{7}x^7 + 3x^{-1} + C = \frac{3}{7}x^7 + \frac{3}{x} + C$$

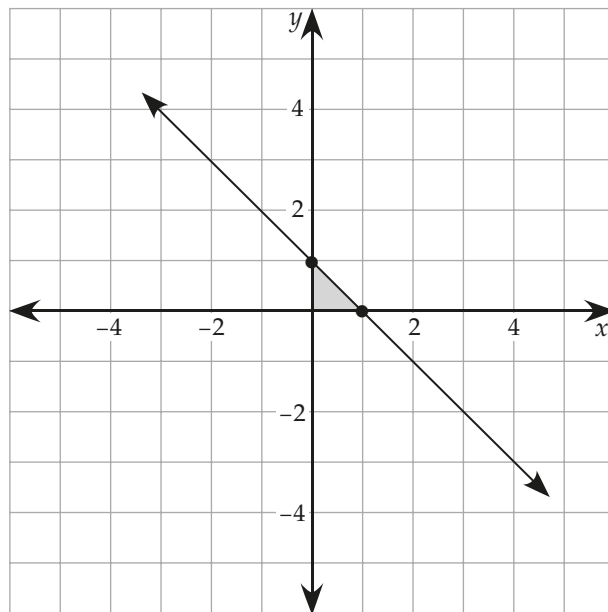
4. Sketch and determine the area bounded by the line $y = -x + 1$ and the x -axis on the closed interval $[0, 1]$: (3 marks)

- a) geometrically, using a graph of the function

Answer:

(Lesson 4)

(1 mark for sketching the area and calculating the area)



Area under the curve = area of triangle

$$= \frac{1}{2}(1)(1) = 0.5 \text{ units}^2$$

b) algebraically, using the antiderivative

Answer:

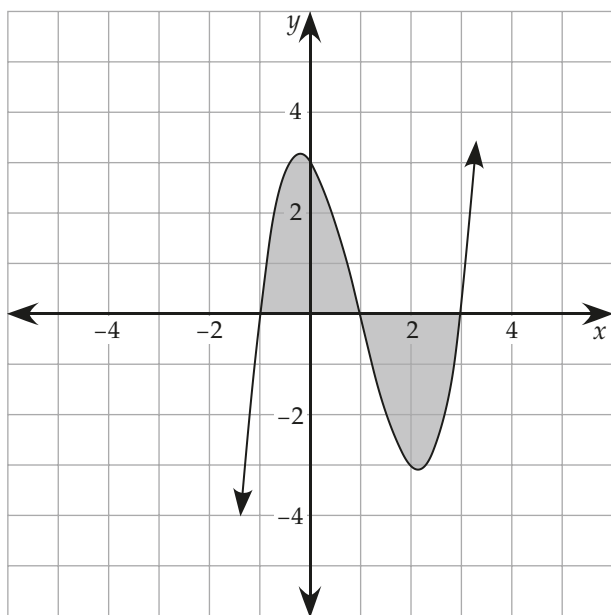
(Lesson 4)

(1 mark for setting up the definite integral)

(1 mark for evaluating the antiderivative difference at $x = 1$ and $x = 0$)

$$\begin{aligned}\text{Area} &= \int_0^1 (-x + 1) dx = \left[-\frac{1}{2}x^2 + x \right]_0^1 = \left(-\frac{1}{2}(1)^2 + 1 \right) - \left((0)^2 + 0 \right) \\ &= -\frac{1}{2} + 1 - 0 = 0.5 \text{ units}^2\end{aligned}$$

5. Determine the area bounded by the curve $y = x^3 - 3x^2 - x + 3$ and the x -axis. (5 marks)



Answer:

(Lesson 5)

(1 mark for separating the area above the axis from the area below the axis)

(1 mark for distinguishing the bounds for each area)

(1 mark for setting up the definite integral that will determine the area of the bounded region)

Total Area = Integral of Positive Interval + | Integral of Negative Interval |

$$= \int_{-1}^1 (x^3 - 3x^2 - x + 3) dx + \left| \int_1^3 (x^3 - 3x^2 - x + 3) dx \right|$$

(1 mark for determining the antiderivative)

(1 mark for evaluating the definite integral to determine the bounded area)

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$$\begin{aligned}\text{Total Area} &= \int_{-1}^1 (x^3 - 3x^2 - x + 3) dx - \int_1^3 (x^3 - 3x^2 - x + 3) dx \\&= \left[\frac{1}{4}x^4 - x^3 - \frac{1}{2}x^2 + 3x \right]_{-1}^1 - \left[\frac{1}{4}x^4 - x^3 - \frac{1}{2}x^2 + 3x \right]_1^3 \\&= \left(\frac{1}{4}(1)^4 - (1)^3 - \frac{1}{2}(1)^2 + 3(1) \right) - \left(\frac{1}{4}(-1)^4 - (-1)^3 - \frac{1}{2}(-1)^2 + 3(-1) \right) \\&\quad - \left(\frac{1}{4}(3)^4 - (3)^3 - \frac{1}{2}(3)^2 + 3(3) \right) + \left(\frac{1}{4}(1)^4 - (1)^3 - \frac{1}{2}(1)^2 + 3(1) \right) \\&= \frac{1}{4} - 1 - \frac{1}{2} + 3 - \frac{1}{4} - 1 + \frac{1}{2} + 3 - \frac{81}{4} + 27 + \frac{9}{2} - 9 + \frac{1}{4} - 1 - \frac{1}{2} + 3 \\&= 8 \text{ units}^2\end{aligned}$$

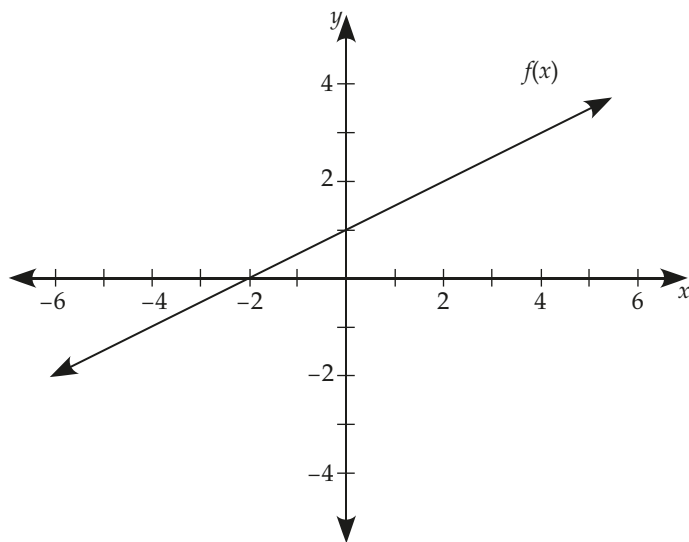
6. Find the values of each definite integral geometrically using the sketch of $f(x)$ as shown. (4 marks).

a) $\int_{-2}^4 f(x) dx$

b) $\int_{-4}^0 f(x) dx$

c) $\int_2^4 f(x) dx$

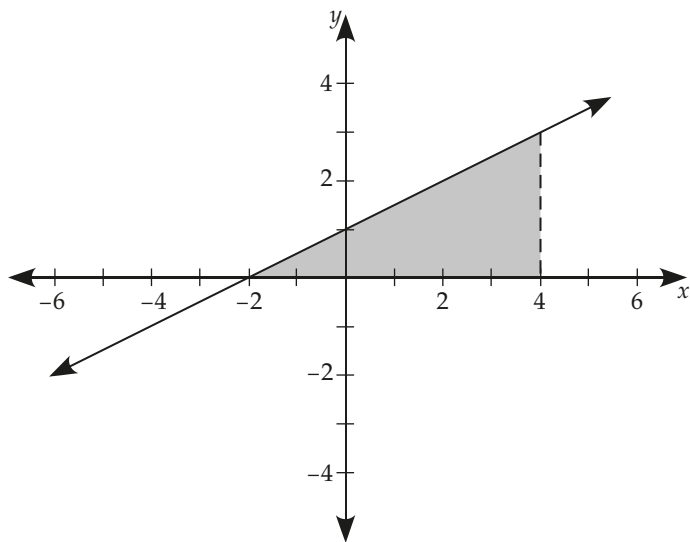
d) $\int_{-4}^0 |f(x)| dx$



Answers:

(Lesson 4)

a)

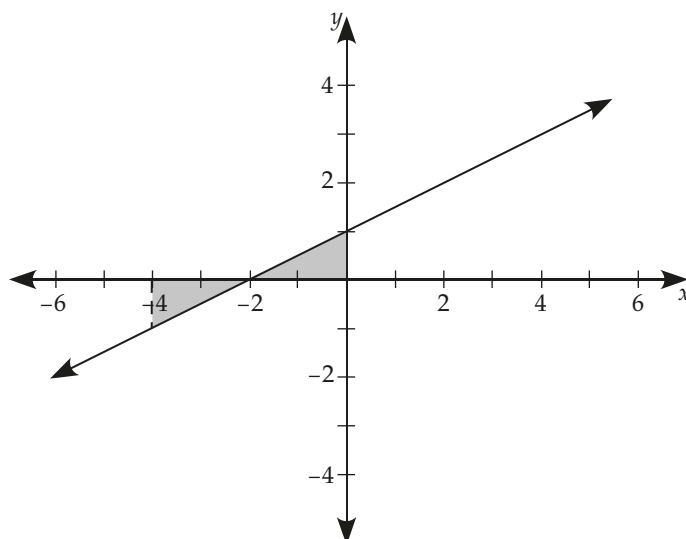


$$\text{Area} = 6 \times 3 \div 2 = 9$$

$$\text{Therefore, } \int_{-2}^4 f(x) dx = 9.$$

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b)



Area below x -axis

$$= 2 \times 1 \div 2 = 1$$

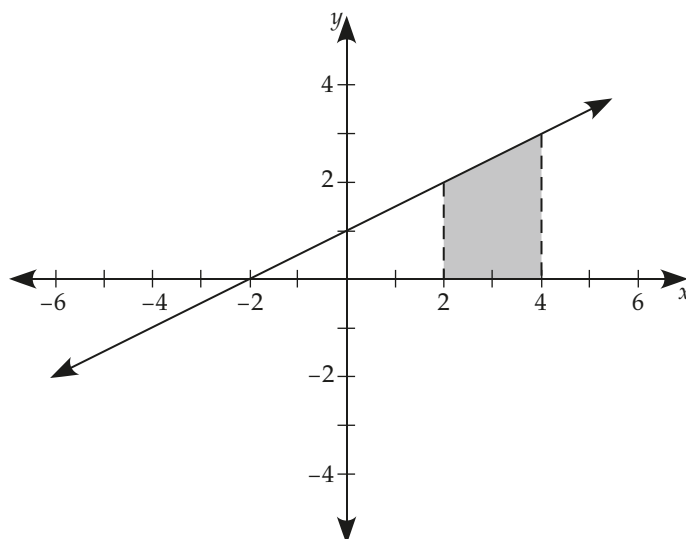
Area above x -axis

$$= 2 \times 1 \div 2 = 1$$

Area below x -axis has a negative definite integral, so

$$\int_{-4}^0 f(x) dx = 1 + (-1) = 0.$$

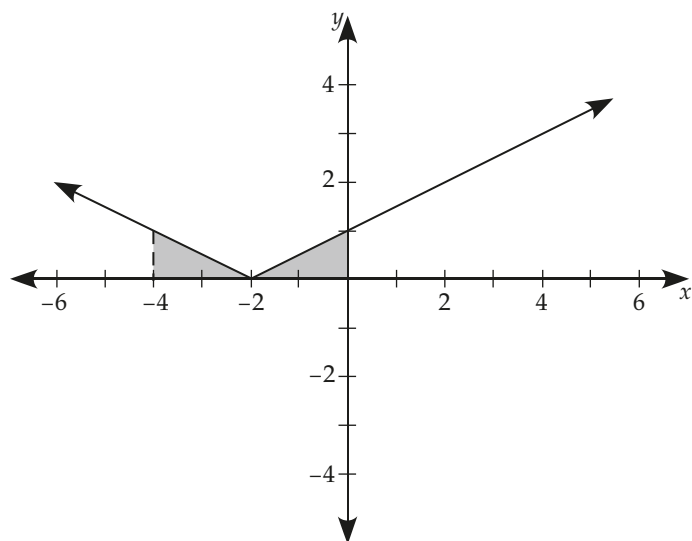
c)



$$\text{Area} = (2 \times 2) + (2 \times 1 \div 2) = 5$$

$$\text{Therefore, } \int_2^4 f(x) dx = 5.$$

d)



The absolute value of the function is entirely above the x -axis.

$$\text{Area} = (2 \times 1 \div 2) + (2 \times 1 \div 2) = 2$$

Therefore,

$$\int_{-4}^0 |f(x)| \, dx = 1 + 1 = 2.$$