CALCULUS 12 FINAL MOCK EXAM (2024)

MARKS: **Part 1 - MC** ____ **/ 22**
Part 2 – WS ____ **/ 58**

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80

Name _____
Date _____

PART 1: Multiple Choice Section:

1.

If $f(x)=\frac{3}{x}$ then $f'(x)=$

A. $-\frac{3}{x^2}$ B. $\frac{3}{x}$ C. $-3x$ D. $3x$ E. ***none of these***

2.

If $f(x)=\sqrt{x}$ determine the value of $f'(x)$ at $(16, 4)$

A. $-\frac{1}{4}$ B. $-\frac{1}{8}$ C. $\frac{1}{8}$ D. $\frac{1}{4}$ E. ***none of these***

3.

Give all values of x where the function $f(x)=x^3-3x+4$ is increasing

A. $x>1$ B. $x<-1$ C. $-1<x<1$ D. $x<-1$ ***or*** $x>1$

4.

If $f'(x)=-6x$ determine all values of x such that $f(x)$ is decreasing

A. $x>0$ B. $x<0$ C. $-6<x<0$ D. ***all real numbers***

5.

Determine the x-values of the critical points for the function $f(x)=x^3+3x^2-24x$

A. $x=-4,\ x=2$ B. $x=4,\ x=-2$ C. $x=0,\ x=3.62,\ x=-6.62$
D. $x=0,\ x=3.62,\ x=6.62$ E. ***none of these***

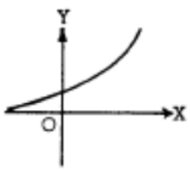
6.

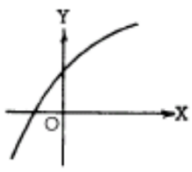
Find the slope of the tangent to $y = x^3 - 2x^2 + 6$ at $(2, 6)$

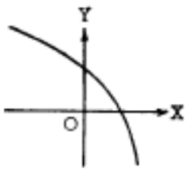
A. **4** B. **6** C. **10** D. **20** E. ***none of these***

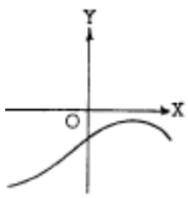
7.

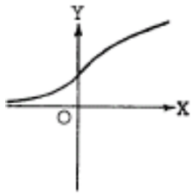
If y is a function of x such that $y' > 0$ for all x and $y'' < 0$ for all x, which of the following could be part of the graph of $f(x)$

A. 

B. 

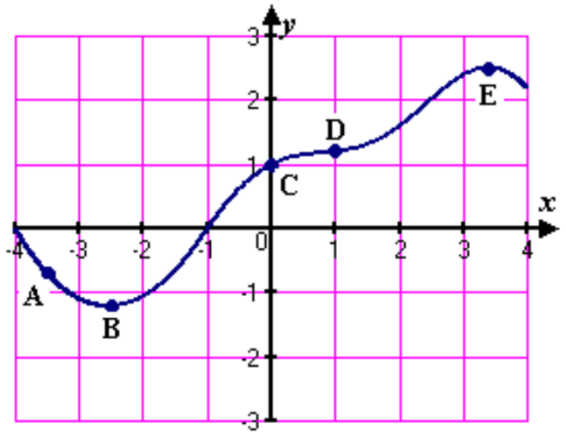
C. 

D. 

E. 

8.

At which point on the graph of $y = g(x)$ on the right is $g'(x) = 0$ and $g''(x) < 0$



A. **A** B. **B** C. **C** D. **D** E. **E**

9.

If $y = x(\ln x)^2$ then $\dfrac{dy}{dx} =$

A. $3(\ln x)^2$ B. $(\ln x)(2x + \ln x)$ C. $(\ln x)(2 + \ln x)$

D. $(\ln x)(2 + x \ln x)$ E. $(\ln x)(1 + \ln x)$

10.

Consider the curve $x + xy + 2y^2 = 6$ The slope of the line tangent to the curve at the point $(2, 1)$ is

A. $\frac{2}{3}$ B. $\frac{1}{3}$ C. $-\frac{1}{3}$ D. $-\frac{1}{5}$ E. $-\frac{3}{4}$

11.

The graph of $y = 2x^3 + 5x^2 - 6x + 7$ has a point of inflection at $x =$

A. $-\frac{5}{3}$ B. 0 C. $-\frac{5}{6}$ D. $\frac{5}{2}$ E. -2

12.

A particle moves along the x-axis so that its position at time t is $x(t) = 2t^2 - 7t + 3$ (x in cm and t in seconds). What is the velocity (in cm/sec) at time $t = 2$ seconds ?

A. -6 B. -3 C. 1 D. 4 E. ***none of these***

13.

The position function of a moving particle on the x-axis is given as $s(t) = t^3 + t^2 - 8t$ for $0 \le t \le 10$ For what values of t is the particle moving to the right ?

A. $t < -2$ B. $t > 0$ C. $t < \frac{4}{3}$ D. $0 < t < \frac{4}{3}$ E. $t > \frac{4}{3}$

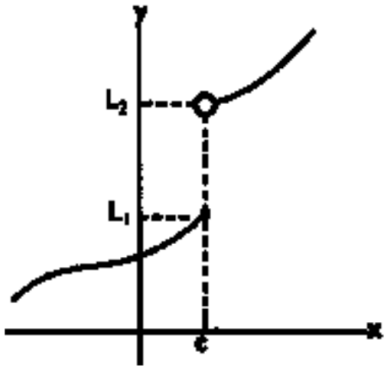
14.

A particle moves along the x-axis so that at any time t its position is given by $x(t) = te^{-2t}$ For what values of t is the particle at rest ?

A. no values B. 0 only C. $\frac{1}{2}$ only D. 1 only E. 0 and $\frac{1}{2}$

15.

Referring to the following figure showing the graph of $y = f(x)$, $\lim_{x \to c} f(x) =$



A. L_1 B. L_2 C. $\frac{L_1 + L_2}{2}$ D. $L_1 + L_2$ E. *does not exist*

16.

Given $f(x) = \begin{cases} x^2 & \text{where } x \neq 2 \\ 2 & \text{where } x = 2 \end{cases}$ then $\lim_{x \to 2^-} f(x) =$

A. 0 B. 1 C. 2 D. 4 E. *does not exist*

17.

$\lim_{x \to 0} \frac{x-1}{x^2-1} =$

A. $-\frac{1}{2}$ B. $\frac{1}{4}$ C. $\frac{1}{2}$ D. 1 E. *indeterminate*

18.

$\lim_{x \to -3} \frac{x^2+x-6}{x^2-9} =$

A. -1 B. $\frac{5}{6}$ C. 1 D. *does not exist* E. *none of these*

19.

$$\lim_{x\to\infty}\left(2x+\frac{5}{x}\right)=$$

A. 0 B. 2 C. 5 D. *does not exist* E. *none of these*

20.

$$\lim_{x\to 0}\frac{x^2-x}{x^4+x^3}=$$

A. -1 B. 0 C. $\frac{1}{2}$ D. 1 E. *does not exist*

21.

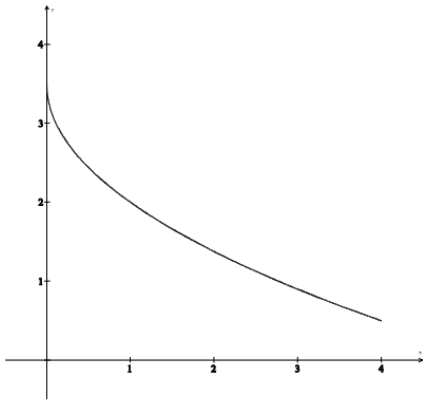
The approximate value of $y=\sqrt{4+\sin(x)}$ at $x=0.12$, obtained from the tangent to the graph at $x=0$ is

(A) 2.00
(B) 2.03
(C) 2.06
(D) 2.12
(E) 2.24

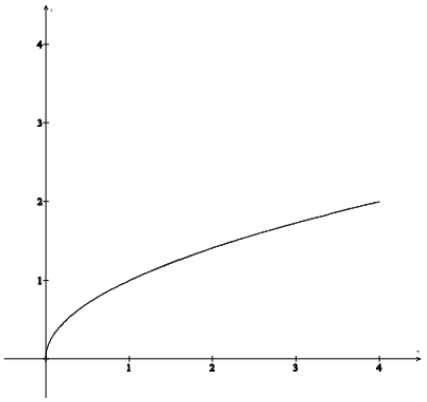
22.

If a trapezoidal sum overapproximates $\int_0^4 f(x)dx$, and a right Riemann sum underapproximates $\int_0^4 f(x)dx$, which of the following could be the graph of $y = f(x)$?

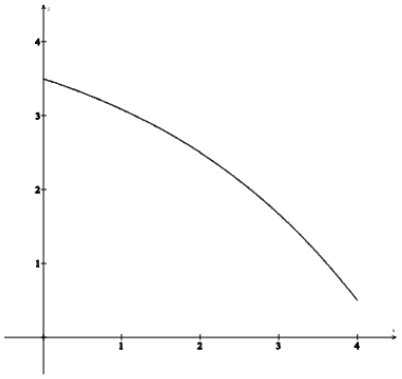
(A)



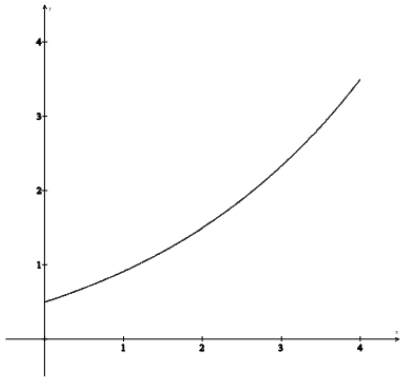
(B)



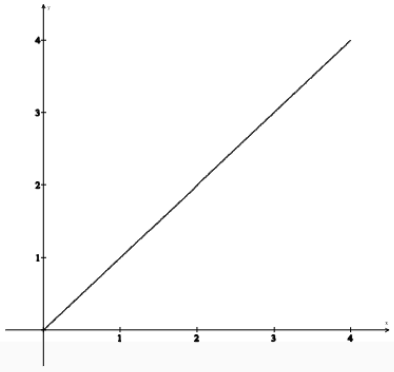
(C)



(D)



(E)



PART 2: Written Response Section:

Show all steps clearly to receive full marks. Please **CIRCLE YOUR FINAL ANSWER.**

1. Differentiate (with respect to x) each function: [1 mark each]

a) $f(x) = \frac{2}{3}x^6 - \frac{1}{2}x^4 + \frac{4}{x}$

b) $y = -2e^{x^2}$

c) $y = cos^2(x^3 - x)$

d. $g(x) = \sqrt[3]{4x^2 - x - 2}$

2. Related Rate Problems. Show all steps and diagram [3 marks]

One car leaves an intersection traveling north at 50 *mph*, and another is driving west toward the intersection at 40 *mph*. At one point, the north-bound car is 3/10 mile north of the intersection, and the west-bound car is 4/10 mile east of the intersection. At this point, how fast is the distance between the cars changing?

3. Find the antiderivative of each of the following functions: [3 marks]

a) $\int x^2 + \frac{5}{x} - 1 \; dx$

b) $\int x^3(2x^4 + 7)^3 \; dx$

4. Find the **exact** value of each of the following definite integrals: [3 mark]

a) $\int_{-1}^{a} x^3 \; dx$

b) $\int_{0}^{6} -2e^{-2x} \; dx$

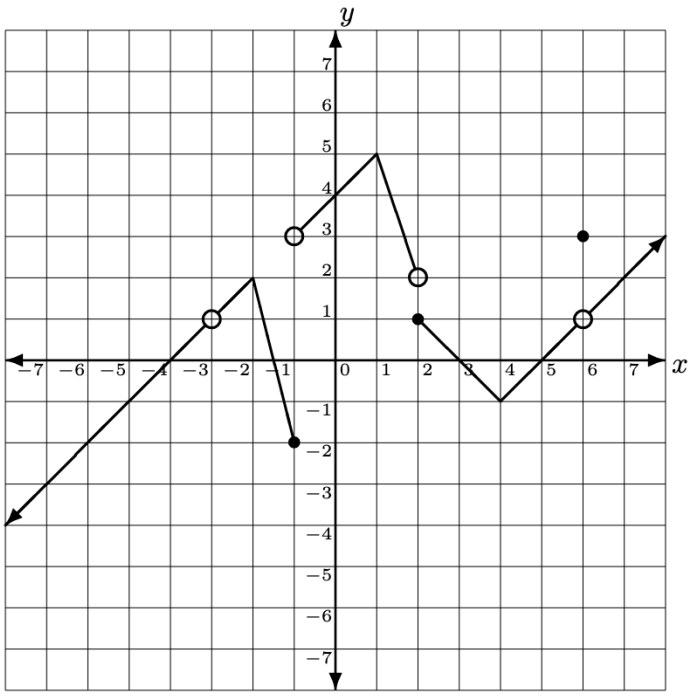
5. Find the **exact** area between the curve $y = x^2 - 5x + 4$ and the *x*-axis over the interval $0 \leq x \leq 4$ [2 marks]

6. Find the **exact** area of the region bound between two functions
$f(x) = x^3 + 4x \ \ and \ \ g(x) = 4x^2$.

[2 marks]

7. [1/2 mark each]

For the function f graphed below, find the following:



$\lim_{x \to 2^-} f(x) =$

$\lim_{x \to 2^+} f(x) =$

$\lim_{x \to 2} f(x) =$

$f(2) =$

$\lim_{x \to 6^-} f(x) =$

$\lim_{x \to 6^+} f(x) =$

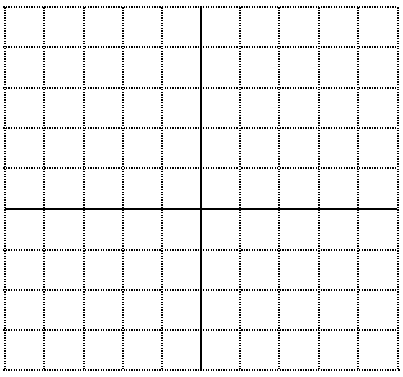
$\lim_{x \to 6} f(x) =$

$f(6) =$

8. Use the **<u>definition</u>** of the derivative to find $\frac{dy}{dx}$ for $y = \sqrt{3x - 2}$. [4 marks]

9. Make a rough sketch of the following function. Indicate x-intercepts, y-intercepts, vertical and horizontal asymptotes: [4 marks]

$$g(x) = \frac{4x^2}{2x^2 - 1}$$



10. Use the **<u>second</u> derivative test** to find and classify all local extrema of $y = x^3 - 12x - 2$

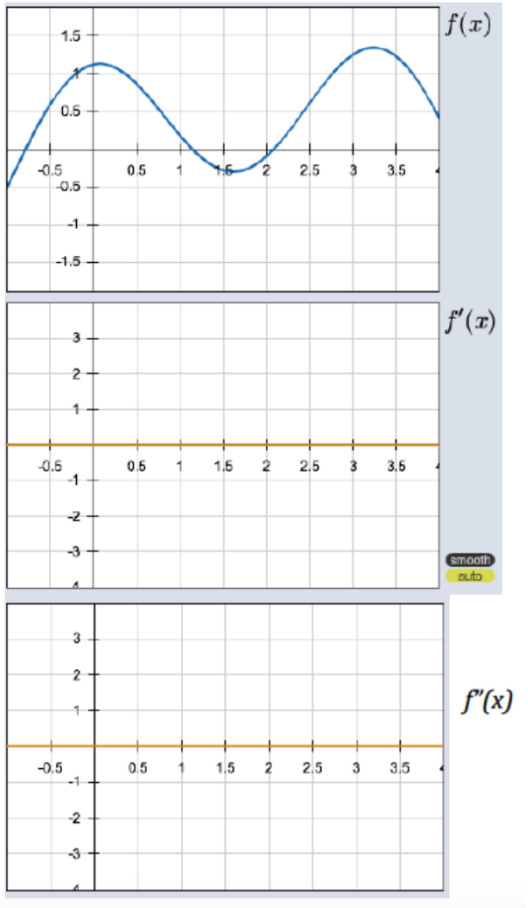
[2 marks]

11 a) Given the position-time graph below:
[2 marks]

i) at what time(s) is the velocity 0?

ii) at what time(s) is the acceleration 0?

iii) when is the object speeding up?



12. Solve the following equations **exactly** $(\ln x)^2 - 4 = 5$ [2 marks]

13. *Find the equation of the normal line to the curve* $y = \ln x^2$ *at the point* $(e, 1)$.
[2 marks]

14. If $x = \sin(xy)$, find $\frac{dy}{dx}$ at the point $\left(1, \frac{\pi}{3}\right)$. [2 marks]

15. *A open 3-dimensional box is to be constructed in such a way that its volume is 288 cm^3. It is also specified that the length of the base is 2 times the width of the base. Determine the dimensions of the box which satisfy these conditions and have the minimum possible surface area.*

[3 marks]

16. Use Newton's Method to solve the following equation $f(x) = x^3 - 4x^2 + 1 = 0$ between the interval (0,1), using x-initial value $x = 0.5$.
Just do two iterations $x_3 =$

[3 marks]

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

17. Evaluate the following **IMPROPER INTEGRALS** and state if convergent or divergent:

[2 marks]

$$\int_0^\infty e^{-x}\,dx$$

18. Determine if the Mean Value Theorem applies to the function on the given interval. If it does, find the c-value. If it doesn't, explain why not.

[2 marks]

$f(x) = \sqrt{x-4}$ $[4, 8]$

19. Find the exact value of the following multiple integrals: $\int_{1}^{3}\int_{-1}^{1} 2x + 1 \, dx \, dy$

20. Simplify: $(1+ 2i)(1 - 2i)$ [1 mark]

21. Solve the following equation over the **COMPLEX NUMBERS:** [2 marks]

$3x^2 + 5x = x - 4$

22. Give the rectangular coordinates of the point whose polar coordinates are given:

[2 marks]

$\left(6, \frac{\pi}{6}\right)$

23. Find a rectangular equation equivalent to the given polar equation and describe the graph:

$r = 4cos\theta$

[3 marks]