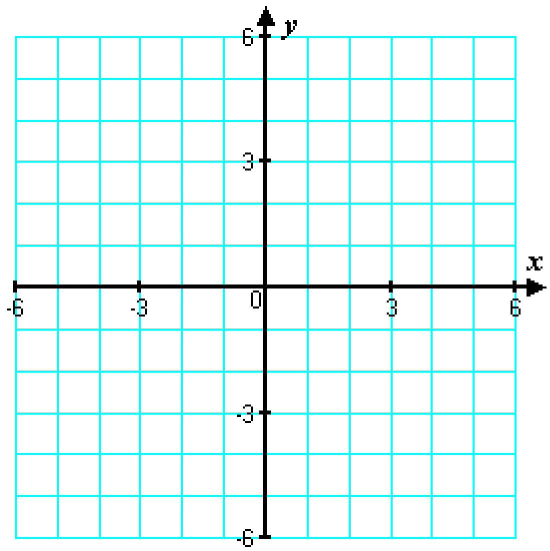
Calculus AB package #2 Feb/2009 Name: ______________________________

1.

Piecewise functions explained by example

$$f(x)=\begin{cases} x+2 & \text{for } -5\le x<-1 \\ x^2 & \text{for } -1\le x<2 \\ 5 & \text{for } 2\le x\le 5 \quad x\ne 4 \\ 1 & \text{for } x=4 \end{cases}$$

$f(-4)=$	$f'(-4)=$	$\lim_{x\to -1^+} f(x)=$
$f(1)=$	$f'(1)=$	$\lim_{x\to 2^-} f(x)=$
$f(3)=$	$f'(3)=$	$\lim_{x\to 4} f(x)=$
$f(8)=$	$f'(8)=$	



2. Find the range of the piecewise function defined by $f(x)=\begin{cases} (x-1)^2, & x<1 \\ 2x-3, & x>1 \end{cases}$

A. { all real numbers } B. $\{y>-1\}$ C. $\{y\ge -1\}$

D. $\{y\ne 1\}$ E. $\{y>0\}$

3. Given $f(x)=\begin{cases} x+3 & \text{where } x<0 \\ x-3 & \text{where } x\ge 0 \end{cases}$ then $\lim_{x\to 0^-} f(x)=$

A. -3 B. 0 C. 1 D. 3 E. ***does not exist***

4. Given $f(x)=\begin{cases} x+3 & \text{where } x<0 \\ x-3 & \text{where } x\ge 0 \end{cases}$ then $\lim_{x\to 0^+} f(x)=$

A. -3 B. 0 C. 1 D. 3 E. ***does not exist***

5. Given $f(x)=\begin{cases} x+3 & \text{where } x<0 \\ x-3 & \text{where } x\ge 0 \end{cases}$ then $\lim_{x\to 0} f(x)=$

A. -3 B. 0 C. 1 D. 3 E. ***does not exist***

6. Given $f(x)=\begin{cases} x+3 & \text{where } x<0 \\ x-3 & \text{where } x\ge 0 \end{cases}$ then $\lim_{x\to 1} f(x)=$

A. -2 B. -1 C. 0 D. 1 E. ***does not exist***

7. Given $f(x)=\begin{cases} x+3 & \text{where } x<0 \\ x-3 & \text{where } x\geq 0 \end{cases}$ then $\lim_{x\to -2} f(x)=$

A. -3 B. 0 C. 1 D. 3 E. *does not exist*

8. Given $f(x)=\begin{cases} x^2 & \text{where } x\neq 2 \\ 2 & \text{where } x=2 \end{cases}$ then $f(2)=$

A. 0 B. 1 C. 2 D. 4 E. *does not exist*

9. Given $f(x)=\begin{cases} x^2 & \text{where } x\neq 2 \\ 2 & \text{where } x=2 \end{cases}$ then $\lim_{x\to 2^-} f(x)=$

A. 0 B. 1 C. 2 D. 4 E. *does not exist*

10. Given $f(x)=\begin{cases} x^2 & \text{where } x\neq 2 \\ 2 & \text{where } x=2 \end{cases}$ then $\lim_{x\to 2^+} f(x)=$

A. 0 B. 1 C. 2 D. 4 E. *does not exist*

11. Given $f(x)=\begin{cases} x^2 & \text{where } x\neq 2 \\ 2 & \text{where } x=2 \end{cases}$ then $\lim_{x\to 2} f(x)=$

A. 0 B. 1 C. 2 D. 4 E. *does not exist*

12. Given $f(x)=\begin{cases} e^x & \text{where } x<1 \\ \ln x & \text{where } x\geq 1 \end{cases}$ then $f(1)=$

A. 0 B. 1 C. 2 D. e E. *does not exist*

13. Given $f(x)=\begin{cases} e^x & \text{where } x<1 \\ \ln x & \text{where } x\geq 1 \end{cases}$ then $\lim_{x\to 1^-} f(x)=$

A. 0 B. 1 C. 2 D. e E. *does not exist*

14. Given $f(x)=\begin{cases} e^x & \text{where } x<1 \\ \ln x & \text{where } x\geq 1 \end{cases}$ then $\lim_{x\to 1^+} f(x)=$

A. 0 B. 1 C. 2 D. e E. *does not exist*

15. Given $f(x)=\begin{cases} e^x & \text{where } x<1 \\ \ln x & \text{where } x\geq 1 \end{cases}$ then $\lim_{x\to 1} f(x)=$

A. 0 B. 1 C. 2 D. e E. *does not exist*

16. Given $f(x)=\begin{cases} x^2+1 & \text{where } x<2 \\ 4 & \text{where } x>2 \end{cases}$ then $\lim_{x\to 2^-} f(x)=$

A. 0 B. 2 C. 4 D. 5 E. *does not exist*

17. Find $f(2)$ for $f(x)=\begin{cases} x^2+4 & \text{where } x<0 \\ 3-x & \text{where } x\ge 0 \end{cases}$

A. 1 B. 3 C. 4 D. 8 E. *does not exist*

18. $f(x)=\begin{cases} 4 & \text{where } x<2 \\ x^2 & \text{where } x\ge 2 \end{cases}$ is differentiable for

A. $x<2$ B. $x\ne 2$ C. $x>2$ D. $x\ge 2$ E. all real numbers

19. Consider the function $f(x)=\begin{cases} 2x^2-x^3 & \text{for } x<2 \\ e^{2x-4} & \text{for } x\ge 2 \end{cases}$ Find $\lim\limits_{x\to 2} f(x)=$

A. 0 B. 1 C. 2 D. 8 E. *does not exist*

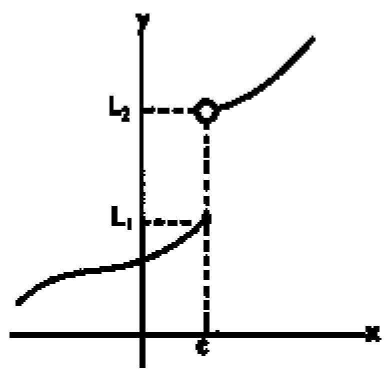
20. If $f(x)=\begin{cases} 1 & \text{if } x\le -2 \\ x^2-4 & \text{if } -2<x<2 \\ x & \text{if } x\ge 2 \end{cases}$ the range of f is

A. $y\ge -4$
B. $y\ge 1$
C. $y=1$ or $y\ge 2$
D. $-4\le y<0$ or $y=1$ or $y\ge 2$
E. all real numbers

21. The range of the piecewise function defined by $f(x)=\begin{cases} (x-1)^2 & \text{for } x<2 \\ 2x-3 & \text{for } x>2 \end{cases}$ is

A. all real numbers B. $y>1$ C. $y<1$
D. $y\ne 1$ E. $y\ge 0$

22. Referring to the following figure showing the graph of $y=f(x)$, $\lim\limits_{x\to c} f(x)=$



A. L_1 B. L_2 C. $\dfrac{L_1+L_2}{2}$ D. L_1+L_2 E. *does not exist*

23. The graph of $f(x)=\dfrac{x^2-1}{x-1}$ has a

A. hole at $x=1$ B. hole at $x=-1$ C. value $f(1)=2$
D. vertical asymptote at $x=1$ E. vertical asymptote at $x=-1$

24.

$\boxed{\text{Limits}}$ $\rightarrow$ needed to handle ***holes***, ***asymptotes***, ***sharp points***, ***endpoints***,

$\rightarrow$ **definition of derivative**

$\boxed{1}$ Direct substitution $\rightarrow$ polynomials $\lim\limits_{x\to c} f(x) = f(c)$ $\leftarrow$ too easy (difficult to see usefulness)

$\boxed{2}$ Does ***not*** exist

a) left/right $\lim\limits_{x\to 3}\dfrac{|x-3|}{x-3}$ (jump discontinuity) $\lim\limits_{x\to 3^-}\dfrac{|x-3|}{x-3} \neq \lim\limits_{x\to 3^+}\dfrac{|x-3|}{x-3}$

b) unbounded $\lim\limits_{x\to 3}\dfrac{1}{x-3} = \dfrac{1}{3-3} = \dfrac{1}{0} = \pm\infty$ $\lim\limits_{x\to 3^-}\dfrac{1}{x-3} \neq \lim\limits_{x\to 3^+}\dfrac{1}{x-3}$

c) oscillating $\lim\limits_{x\to 3}\sin\left(\dfrac{1}{x}\right)$

$\boxed{3}$ Piecewise $\rightarrow$ learn and understand Given $f(x) = \begin{cases} x+3 & \text{where } x<0 \\ x-3 & \text{where } x\geq 0\end{cases}$ then $\lim\limits_{x\to 0^-} f(x) =$

$\boxed{4a}$ Indeterminant form $\rightarrow$ $\lim\limits_{x\to 3}\dfrac{x^2-9}{x^2+x-12} = \dfrac{3^2-9}{3^2+3-12} = \boxed{\dfrac{0}{0}}$ $\leftarrow$ factor

$$\lim_{x\to 3}\frac{x^2-9}{x^2+x-12} = \lim_{x\to 3}\frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}(x+4)} = \lim_{x\to 3}\frac{x+3}{x+4} = \frac{3+3}{3+4} = \boxed{\frac{6}{7}}$$ ☑

$\boxed{4b}$ Indeterminant form $\rightarrow$ $\lim\limits_{x\to 9}\dfrac{\sqrt{x}-3}{x-9} = \dfrac{\sqrt{9}-3}{9-9} = \boxed{\dfrac{0}{0}}$ $\leftarrow$ conjugate (of numerator in this case)

$$\lim_{x\to 9}\frac{\sqrt{x}-3}{x-9} = \lim_{x\to 9}\frac{\sqrt{x}-3}{x-9}\left(\frac{\sqrt{x}+3}{\sqrt{x}+3}\right) = \lim_{x\to 9}\frac{\cancel{x-9}}{\cancel{(x-9)}\left(\sqrt{x}+3\right)} = \lim_{x\to 9}\frac{1}{\sqrt{x}+3} = \frac{1}{\sqrt{9}+3} = \boxed{\frac{1}{6}}$$

$\boxed{4c}$ Indeterminant form $\rightarrow$ $\lim\limits_{x\to 1}\dfrac{\ln x}{x^4-1} = \dfrac{\ln 1}{1^4-1} = \boxed{\dfrac{0}{0}}$ $\leftarrow$ L'hopital rule

$$\lim_{x\to 1}\frac{\frac{1}{x}}{4x^3} = \frac{\frac{1}{1}}{4(1)^3} = \boxed{\frac{1}{4}}$$

$\boxed{5}$ Infinity $\rightarrow$ horizontal asymptotes (divide by the variable with highest exponent in denominator

$$\lim_{x\to\infty}\frac{4x-5x^2}{2x^2+3x-1} = \lim_{x\to\infty}\frac{\frac{4x}{x^2}-\frac{5x^2}{x^2}}{\frac{2x^2}{x^2}+\frac{3x}{x^2}-\frac{1}{x^2}} = \lim_{x\to\infty}\frac{\frac{4}{x}-5}{2+\frac{3}{x}-\frac{1}{x^2}} = \frac{0-5}{2+0-0} = \boxed{-\frac{5}{2}}$$

$\boxed{6a}$ Trig $\rightarrow$ basic $\lim\limits_{x\to 0}\dfrac{\sin x}{x} = 1$ $\lim\limits_{x\to 0}\dfrac{\cos x - 1}{x} = 0$ $\leftarrow$ squeeze theorem proof

$\boxed{6b}$ Trig $\rightarrow$ advanced questions based on basics

$\boxed{7}$ Definition of derivative $f'(x) = \lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $\leftarrow$ secant changes into a tangent

25. $\lim\limits_{x\to 3}(x^2-2x+2) =$

A. -8 B. 4 C. 5 D. 17 E. *none of these*

26. $\lim_{x\to 3}\frac{x^2-9}{x^2+9}=$

A. -1 B. 0 C. 1 D. *does not exist* E. *none of these*

27. $\lim_{x\to 5}\frac{x-5}{x-5}=$

A. 0 B. 1 C. -1 D. $\frac{1}{2}$ E. *does not exist*

28. $\lim_{x\to 2}\frac{x^2-4}{x^2+4}=$

A. 1 B. 0 C. $\frac{1}{2}$ D. -1 E. ∞

29. $\lim_{x\to 1}\frac{x}{\ln x}=$

A. 0 B. $\frac{1}{e}$ C. 1 D. e E. *does not exist*

30. $\lim_{x\to 10}\frac{4x^2-6x+10}{50+4x^2}=$

A. -1 B. 0 C. $\frac{7}{9}$ D. 1 E. ∞

31. $\lim_{x\to 10}\frac{3x^2-7x+10}{60+3x^2}=$

A. -1 B. $\frac{2}{3}$ C. 1 D. 10 E. ∞

32. $\lim_{x\to 0}\left(\frac{\frac{1}{x-1}+1}{x}\right)=$

A. -1 B. 0 C. 1 D. 2 E. *does not exist*

33. $\lim_{x\to 1}\frac{\ln x}{x}=$

A. 1 B. 0 C. e D. $-e$ E. *does not exist*

34. $\lim_{x\to 3}\frac{6x^2-5}{4x^2+1}=$

A. -5 B. $\frac{1}{5}$ C. $\frac{49}{37}$ D. $\frac{3}{2}$ E. $\frac{13}{5}$

35. $\lim_{x\to 2}(3x^2+5)=$

A. 41 B. 17 C. 11 D. 0 E. *none of these*

36. $\lim\limits_{x\to -3}(-2x^2+1)=$

A. 37 B. 19 C. -17 D. $\pm\sqrt{2}$ E. *none of these*

37. $\lim\limits_{x\to -1}\dfrac{x^2+3x+2}{x^2+1}=$

A. 0 B. ∞ C. -1 D. *does not exist* E. *none of these*

38. $\lim\limits_{x\to -1}\dfrac{x^2+2x+3}{x^2+1}=$

A. 0 B. 1 C. ∞ D. *does not exist* E. *none of these*

39. $\lim\limits_{x\to 3}\sqrt{x^2-4}=$

A. 1 B. 5 C. -1 D. $\sqrt{5}$ E. *none of these*

40. $\lim\limits_{x\to 3}\sqrt{9-x^2}=$

A. 0 B. $\sqrt{6}$ C. $3\sqrt{2}$ D. *does not exist* E. *none of these*

41. $\lim\limits_{x\to 2^-}\sqrt{2x-3}=$

A. $1,\ -1$ B. 1 C. -1 D. $\dfrac{1}{2}$ E. *none of these*

42. $\lim\limits_{x\to 0}\dfrac{x-1}{x^2-1}=$

A. $-\dfrac{1}{2}$ B. $\dfrac{1}{4}$ C. $\dfrac{1}{2}$ D. 1 E. *indeterminate*

43. If the function f is continuous for all real numbers and if $f(x)=\dfrac{x^2-4}{x+2}$ when $x\neq -2,$ then $f(-2)=$

A. -4 B. -2 C. -1 D. 0 E. 2

44. If $f(x)=\begin{cases}\dfrac{x^2-4}{x-2} & \text{if } x\neq 2\\ k & \text{if } x=2\end{cases}$, for what value(s) of k is $f(x)$ continuous at $x=2$

A. $-2,\ 2$ B. 4 C. 8 D. 0 E. 6

45. If $f(x)=\begin{cases}\dfrac{x^2-x}{2x} & \text{for } x\neq 0\\ k & \text{for } x=0\end{cases}$ and if f is continuous at $x=0$ then $k=$

A. -1 B. $-\dfrac{1}{2}$ C. 0 D. $\dfrac{1}{2}$ E. 1

46.

Limits $\rightarrow$ needed to handle ***holes***, ***asymptotes***, ***sharp points***, ***endpoints***,

4a Indeterminant form $\rightarrow$ $\lim\limits_{x\to 3}\dfrac{x^2-9}{x^2+x-12}=\dfrac{3^2-9}{3^2+3-12}=\boxed{\dfrac{0}{0}}$ $\leftarrow$ factor

$$\lim_{x\to 3}\frac{x^2-9}{x^2+x-12}=\lim_{x\to 3}\frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}(x+4)}=\lim_{x\to 3}\frac{x+3}{x+4}=\frac{3+3}{3+4}=\boxed{\frac{6}{7}}\ \text{☑}$$

4b Indeterminant form $\rightarrow$ $\lim\limits_{x\to 9}\dfrac{\sqrt{x}-3}{x-9}=\dfrac{\sqrt{9}-3}{9-9}=\boxed{\dfrac{0}{0}}$ $\leftarrow$ conjugate (of numerator in this case)

$$\lim_{x\to 9}\frac{\sqrt{x}-3}{x-9}=\lim_{x\to 9}\frac{\sqrt{x}-3}{x-9}\left(\frac{\sqrt{x}+3}{\sqrt{x}+3}\right)=\lim_{x\to 9}\frac{\cancel{x-9}}{\cancel{(x-9)}\left(\sqrt{x}+3\right)}=\lim_{x\to 9}\frac{1}{\sqrt{x}+3}=\frac{1}{\sqrt{9}+3}=\boxed{\frac{1}{6}}$$

4c Indeterminant form $\rightarrow$ $\lim\limits_{x\to 1}\dfrac{\ln x}{x^4-1}=\dfrac{\ln 1}{1^4-1}=\boxed{\dfrac{0}{0}}$ $\leftarrow$ L'hopital rule

$$\lim_{x\to 1}\frac{\frac{1}{x}}{4x^3}=\frac{\frac{1}{1}}{4(1)^3}=\boxed{\frac{1}{4}}$$

47. $\lim\limits_{x\to 7}\dfrac{x^2-6x-7}{x^2-5x-14}=$

A. -1 B. $\dfrac{1}{2}$ C. $\dfrac{8}{9}$ D. 1 E. *none of these*

48. $\lim\limits_{x\to -3}\dfrac{x^2+x-6}{x^2-9}=$

A. -1 B. $\dfrac{5}{6}$ C. 1 D. *does not exist* E. *none of these*

49. $\lim\limits_{x\to 4}\dfrac{x^2-2x-8}{x^2-6x+8}=$

A. 0 B. 1 C. 3 D. *does not exist* E. *none of these*

50. $\lim\limits_{x\to 10}\dfrac{x^2-9x-10}{x^2-100}=$

A. $\dfrac{1}{10}$ B. $\dfrac{11}{20}$ C. 1 D. *does not exist* E. *none of these*

51. $\lim\limits_{x\to 3}\dfrac{x^2-9}{x^2+x-12}=$

A. $\dfrac{3}{4}$ B. $\dfrac{6}{7}$ C. 1 D. *does not exist* E. *none of these*

52. $\lim_{x \to 2} \frac{x^2 - x - 2}{x^2 + x - 6} =$

A. $\frac{1}{3}$ B. $\frac{3}{5}$ C. 1 D. *does not exist* E. *none of these*

53. $\lim_{x \to 1} \frac{x^2 + x - 2}{x^2 - 1} =$

A. $-\frac{1}{2}$ B. 1 C. $\frac{3}{2}$ D. *does not exist* E. *none of these*

54. $\lim_{x \to -2} \frac{x^2 - 4}{x^2 + x - 2} =$

A. 1 B. $\frac{4}{3}$ C. 2 D. *does not exist* E. *none of these*

55. $\lim_{x \to 5} \frac{x^2 - 2x - 15}{x^2 - 7x + 10} =$

A. 0 B. 1 C. $\frac{8}{3}$ D. *does not exist* E. *none of these*

56. $\lim_{x \to 2} \frac{x - 2}{x^2 - x - 2} =$

A. 0 B. $\frac{1}{3}$ C. 3 D. *does not exist* E. *none of these*

57. $\lim_{x \to 7} \frac{x^2 - 6x - 7}{x^2 - 49} =$

A. $\frac{4}{7}$ B. 1 C. $\frac{1}{7}$ D. *does not exist* E. *none of these*

58. $\lim_{x \to -1} \frac{x^2 - 5x - 6}{x^2 - 1} =$

A. 1 B. 3 C. $\frac{7}{2}$ D. 12 E. *indeterminate*

59. $\lim_{x \to 5} \frac{x - 5}{x^2 - 25} =$

A. -1 B. 0 C. $\frac{1}{10}$ D. 1 E. *does not exist*

60. $\lim_{x \to 2} \frac{2 - x}{x^2 - 4} =$

A. $-\frac{1}{2}$ B. $-\frac{1}{4}$ C. $\frac{1}{4}$ D. $\frac{1}{2}$ E. *does not exist*

61. $\lim_{x\to 1}\left(\dfrac{x^5-1}{x^2-x}\right)=$

A. 5 B. $\dfrac{5}{2}$ C. 0 D. 1 E. *does not exist*

62. $\lim_{x\to 0}\dfrac{\sqrt{x+4}-2}{x}=$

A. 0 B. $\dfrac{1}{4}$ C. ∞ D. 1 E. *none of these*

63. $\lim_{x\to 3}\dfrac{x-3}{x^2-2x-3}=$

A. 0 B. 1 C. $\dfrac{1}{4}$ D. ∞ E. *none of these*

64. $\lim_{x\to 0}\dfrac{x}{x}=$

A. 1 B. 0 C. ∞ D. -1 E. *nonexistent*

65. $\lim_{x\to 2}\dfrac{x^3-8}{x^2-4}=$

A. 4 B. 0 C. 1 D. 3 E. ∞

66. $\lim_{x\to 2}\dfrac{x^2-2}{4-x^2}=$

A. -2 B. -1 C. $-\dfrac{1}{2}$ D. 0 E. *does not exist*

67. $\lim_{h\to 0}\dfrac{\sqrt{25+h}-5}{h}=$

A. 0 B. $\dfrac{1}{10}$ C. 1 D. 10 E. *does not exist*

68. If $a\neq 0$, then $\lim_{x\to a}\dfrac{x^2-a^2}{x^4-a^4}=$

A. $\dfrac{1}{a^2}$ B. $\dfrac{1}{2a^2}$ C. $\dfrac{1}{6a^2}$ D. 0 E. *does not exist*

69. $\lim_{x\to \pi}\dfrac{\pi x-\pi^2}{2x-2\pi}=$

A. 0 B. $\dfrac{\pi}{2}$ C. π D. 2π E. ∞

70. $\lim_{x \to 0} \frac{x^5 - 16x}{x^3 - 4x} =$

A. -4 B. -2 C. 0 D. 2 E. 4

71. $\lim_{x \to 3} \frac{x^3 - 2x^2 - 3x}{x^3 - 9x} =$

A. 0 B. $\frac{2}{3}$ C. $\frac{3}{4}$ D. 1 E. ∞

72. $\lim_{x \to 1} \frac{x^3 - 1}{x^2 - 1} =$

A. 0 B. 1 C. $\frac{3}{2}$ D. 3 E. ∞

73. $\lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9} =$

A. 1 B. $\frac{1}{3}$ C. $\frac{1}{6}$ D. $-\frac{1}{3}$ E. *indeterminate*

74. $\lim_{a \to b} \left(\frac{a^2 - b^2}{a - b} + 3ab \right) =$

A. $3a^2$ B. $5b$ C. $2b + 3b^2$ D. $3a^2 + a$ E. *does not exist*

75. $\lim_{x \to a} \frac{x^2 - a^2}{a - x} =$

A. $-2a$ B. 0 C. 1 D. $2a$ E. ∞

76. $\lim_{x \to 0} \frac{1 - 2^{2x}}{1 - 2^x} =$

A. 0 B. $\frac{1}{2}$ C. 1 D. 2 E. ∞

77. $\lim_{x \to 0} \frac{x^3 + x^2 - 2x}{x^3 - x} =$

A. -1 B. 0 C. 1 D. 2 E. ∞

78. $\lim_{x \to 3} \frac{(3 - x)^2}{(x - 3)} =$

A. -2 B. -1 C. 0 D. 1 E. *does not exist*

79. $\lim_{x \to b} \frac{b - x}{\sqrt{x} - \sqrt{b}} =$

A. $-2\sqrt{b}$ B. $-\sqrt{b}$ C. $2b$ D. $\sqrt{b}$ E. $2\sqrt{b}$

80. $\lim\limits_{x\to 1}\dfrac{x^2+2x-3}{x^2-1}=$

A. -2 B. -1 C. 10 D. 1 E. 2

81. $\lim\limits_{x\to 9}\dfrac{x-9}{3-\sqrt{x}}=$

A. 6 B. -6 C. 0 D. -12 E. $+\infty$

82. If $k\neq 0$ then $\lim\limits_{x\to k}\dfrac{x^2-k^2}{x^2-kx}=$

A. 0 B. 2 C. $2k$ D. $4k$ E. ***nonexistent***

83. $\lim\limits_{x\to -2}\dfrac{x+2}{x^2-4}=$

A. $-\dfrac{1}{4}$ B. $-\dfrac{1}{2}$ C. 0 D. 1 E. ***does not exist***

84. $\lim\limits_{x\to 2}\dfrac{x^2-4}{x^3-8}=$

A. 0 B. $\dfrac{1}{3}$ C. $\dfrac{1}{2}$ D. $\dfrac{2}{3}$ E. ***does not exist***

85. $\lim\limits_{x\to 0}\dfrac{x^2-x}{x^4+x^3}=$

A. -1 B. 0 C. $\dfrac{1}{2}$ D. 1 E. ***does not exist***

86. $\lim\limits_{x\to 5}\dfrac{2x^2-50}{x^2-15x+50}=$

A. -4 B. -1 C. 0 D. 1 E. 2

87. $\lim\limits_{x\to 2}\dfrac{4x^2-16}{x-2}=$

A. -3 B. $-\dfrac{1}{4}$ C. -1 D. 0 E. 16

88. $\lim\limits_{x\to 5}\dfrac{x^2-x-20}{x-5}=$

A. 1 B. 5 C. 9 D. 10 E. ***undefined***

89. $\lim\limits_{x\to -1}\dfrac{x+x^2}{x^2-1}=$

A. $-\dfrac{1}{2}$ B. 1 C. -1 D. $\dfrac{1}{2}$ E. ***does not exist***

90. $\lim\limits_{x\to 1}\dfrac{2x-2}{x^3+2x^2-x-2}=$

A. 0 B. $\dfrac{1}{3}$ C. $\dfrac{2}{3}$ D. $+\infty$ E. $-\infty$

91. $\lim\limits_{x\to 2}\dfrac{x-2}{x^2-4}=$

A. 0 B. $\dfrac{1}{4}$ C. ∞ D. 1 E. *none of these*

92. $\lim\limits_{x\to 4}\dfrac{x^2-5x+4}{x-4}=$

A. 0 B. -1 C. 3 D. *does not exist* E. *none of these*

93. $\lim\limits_{x\to 2}\dfrac{\sqrt{3-x}-\sqrt{x-1}}{6-3x}=$

A. $\dfrac{1}{3}$ B. $\dfrac{1}{2}$ C. $\dfrac{2}{3}$ D. 1 E. $\dfrac{3}{2}$

94. $\lim\limits_{x\to 1}\dfrac{\sqrt{x}-1}{x-1}=$

A. 0 B. $\dfrac{1}{2}$ C. 1 D. $\dfrac{3}{2}$ E. *does not exist*

95. $\lim\limits_{x\to 1}\left(\dfrac{\sqrt{x+3}-2}{1-x}\right)=$

A. 0.5 B. 0.25 C. 0 D. -0.25 E. -0.5

96. $\lim\limits_{x\to -1}\dfrac{\sqrt{x^2+3}-2}{x+1}=$

A. 0 B. -2 C. $-\dfrac{1}{2}$ D. 2 E. *does not exist*

97. $\lim\limits_{h\to 0}\dfrac{e^{1+h}-e}{h}=$

A. 0 B. 1 C. 2 D. e E. e^2

98. $\lim\limits_{x\to \frac{\pi}{2}}\dfrac{2x-\pi}{x-\frac{\pi}{2}}=$

99. $\lim\limits_{x\to 1}\dfrac{\frac{1}{x+1}-\frac{1}{2}}{x-1}=$

A. $-\dfrac{1}{4}$ B. -1 C. $\dfrac{1}{4}$ D. 0 E. *does not exist*

100.

Vertical asymptotes

Let f and g be continuous on an open interval containing c If $f(c) \neq 0$, $g(c) = 0$ then the function $h(x) = \dfrac{f(x)}{g(x)}$ $\lim\limits_{x \to c} h(x) = \dfrac{k}{0} = \textit{undefined}$ (has a ***vertical*** asymptote at $x = c$)

101. $\lim\limits_{x \to -2} \dfrac{x-2}{x^2-4} =$

A. $-\dfrac{1}{4}$ B. 0 C. $\dfrac{1}{2}$ D. *does not exist* E. *none of these*

102. $\lim\limits_{x \to 2} \dfrac{x+6}{x-2} =$

A. 0 B. 1 C. 8 D. *does not exist* E. *none of these*

103. $\lim\limits_{x \to 3} \dfrac{1}{x-3} =$

A. -3 B. 0 C. 1 D. 3 E. *nonexistent*

104. $\lim\limits_{x \to -4^+} \dfrac{4x-6}{2x^2+5x-12} =$

A. 0 B. 1 C. $+\infty$ D. $-\infty$ E. *none of these*

105. Find the equation of the vertical asymptote of $y = \dfrac{5x}{x-1}$

A. $y = 1$ B. $y = 0$ C. $x = 1$ D. $x = 5$ E. $y = 5$

106. The graph of $y = \ln(x+2)$ has a vertical asymptote with equation

A. $x = -2$ B. $y = -2$ C. $x = 0$ D. $y = 0$

E. The graph has no vertical asymptote

107. Find the equation of the vertical asymptote(s) of $f(x) = \dfrac{2x}{x^2-4}$

A. $y = 0$ B. $x = 0$ C. $x = 2$ D. $x = \pm 2$ E. $y = 2$

108. The vertical asymptote of $f(x) = \dfrac{x}{x-1}$ has equation

A. $x = 0$ B. $x = 1$ C. $y = 0$ D. $y = 1$

E. no vertical asymptote

109. The vertical asymptote of $f(x)=\dfrac{4}{x+1}$ is

A. $x=-1$ B. $x=0$ C. $y=-1$ D. $y=0$

E. no vertical asymptote

110. Find the equation of the vertical asymptote of $f(x)=\dfrac{x}{4x+8}$

A. $y=\frac{1}{4}$ B. $y=-2$ C. $x=-2$ D. $y=2$ E. $x=2$

111. The graph of $f(x)=\dfrac{x^2-5x+6}{x^2-4}$ has vertical asymptotes at

A. $x=0$ only B. $x=-2$ only C. $x=2$ only

D. $x=2$ and $x=-2$ E. $x=3$ only

112. The graph of $f(x)=\dfrac{x^2-4}{x^3+3x^2-4x-12}$ has a vertical asymptote at $x=$

A. -3 only B. -2 only C. 2 only D. 3 only E. $-3,-2$ and 2

113. A function $f(x)$ has a vertical asymptote at $x=2$

The derivative of $f(x)$ is positive for all $x \neq 2$

Which of the following statements are true ?

I. $\lim\limits_{x\to 2} f(x)=+\infty$

II. $\lim\limits_{x\to 2^+} f(x)=+\infty$

III. $\lim\limits_{x\to 2^-} f(x)=+\infty$

A. **I** ***only*** B. **II** ***only*** C. **III** ***only***

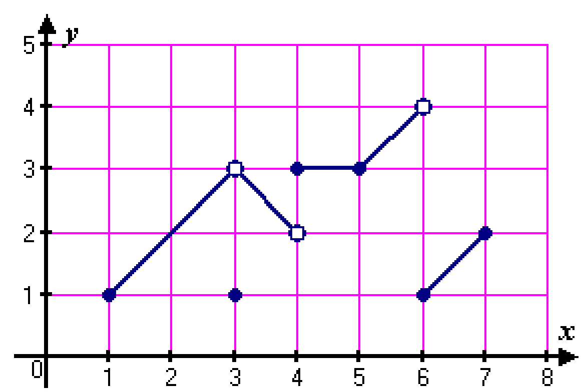
D. **I** ***and*** **II** ***only*** E. **I, II** ***and*** **III**

114. Where is the function $f(x)=\dfrac{5}{x^2-2x-15}$ discontinuous ?

A. $x=-5$ *and* $x=-3$ B. $x=-5$ *and* $x=3$ C. $x=-3$ *and* $x=5$

D. $x=3$ *and* $x=5$ E. $x=2$ *and* $x=15$

115. The graph of a function f whose domain is the closed interval $[\,1,\ 7\,]$ is shown. Which of the following statements about $f(x)$ is true ?



A. $\lim\limits_{x\to 3} f(x)=1$ B. $\lim\limits_{x\to 4} f(x)=3$

C. $f(x)$ is continuous at $x=3$ D. $f(x)$ is continuous at $x=5$

E. $\lim\limits_{x\to 6} f(x)=f(6)$

116.

$\boxed{\textbf{Definition of a derivative}}$ $\dfrac{f(x+h)-f(x)}{h}$ ← slope of a secant

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$ ← secant changes into a tangent by limit process

117. Which of the following represents the derivative of $f(x)=x^3$

A. $\lim_{h\to 0}\dfrac{(x+h)^3-x^3}{h}$ B. $\lim_{h\to 0}\dfrac{(x-h)^3+x^3}{h}$ C. $\lim_{x\to 0}\dfrac{(x+h)^3-x^3}{h}$ D. $\lim_{x\to 0}\dfrac{(x-h)^3+x^3}{h}$

118. Which expression represents the derivative of $f(x)=\dfrac{1}{x^3}$

A. $\lim_{h\to 0}\dfrac{\frac{1}{(x+h)^3}+\frac{1}{x^3}}{h}$ B. $\lim_{h\to 0}\dfrac{\frac{1}{(x+h)^3}-\frac{1}{x^3}}{h}$ C. $\lim_{x\to 0}\dfrac{\frac{1}{(x+h)^3}+\frac{1}{x^3}}{h}$ D. $\lim_{x\to 0}\dfrac{\frac{1}{(x+h)^3}-\frac{1}{x^3}}{h}$

119. Which expression represents the derivative of $f(x)=x^4$

A. $\lim_{x\to 0}\dfrac{4(x+h)^3-4x^3}{x}$ B. $\lim_{h\to 0}\dfrac{4(x+h)^3-4x^3}{h}$ C. $\lim_{x\to 0}\dfrac{(x+h)^4-x^4}{x}$

D. $\lim_{h\to 0}\dfrac{(x+h)^4-x^4}{h}$ E. *none of these*

120. Which of the following is the derivative of $f(x)$

A. $\lim_{x\to 0}\dfrac{f(x+h)-f(x)}{h}$ B. $\lim_{x\to h}\dfrac{f(x-h)+f(x)}{h}$ C. $\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

D. $\lim_{h\to 0}\dfrac{f(x-h)+f(x)}{h}$ E. *none of these*

121. Which of the following limits represents the derivative of the function $f(x)=x^2-3x+1$

A. $\lim_{h\to 0}\dfrac{2(x+h)-3}{h}$ B. $\lim_{x\to 0}\dfrac{2(x+h)-3}{h}$

C. $\lim_{h\to 0}\dfrac{(x+h)^2-3(x+h)+1-(x^2-3x+1)}{h}$ D. $\lim_{x\to 0}\dfrac{(x+h)^2-3(x+h)+1-(x^2-3x+1)}{h}$

122. Which expression represents the derivative of $f(x)=x^2+3x$

A. $\lim_{h\to 0}\dfrac{(x+h)^2+3(x+h)-(x^2+3x)}{h}$ B. $\lim_{x\to 0}\dfrac{(x+h)^2+3(x+h)-(x^2+3x)}{h}$

C. $\lim_{h\to 0}\dfrac{(x+h)^2+3(x+h)+(x^2+3x)}{h}$ D. $\lim_{x\to 0}\dfrac{(x+h)^2+3(x+h)+(x^2+3x)}{h}$

123. For the polynomial function $y = f(x)$, the expression $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ represents:

A. the minimum value of $f(x)$

B. the maximum value of $f(x)$

C. the slope of a secant line of $f(x)$

D. the slope of a tangent line of $f(x)$

124. Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two points on the graph of a polynomial function. Which expression represents the derivative at point P ?

A. $\lim\limits_{x_2\to 0}\dfrac{y_2-x_2}{y_1-x_1}$

B. $\lim\limits_{x_2\to x_1}\dfrac{y_2-x_2}{y_1-x_1}$

C. $\lim\limits_{x_2\to 0}\dfrac{y_2-y_1}{x_2-x_1}$

D. $\lim\limits_{x_2\to x_1}\dfrac{y_2-y_1}{x_2-x_1}$

E. *none of these*

125. Which of the following represents the slope of the tangent to the function $f(x)=\sqrt{x}$

A. $\lim\limits_{x\to 0}\dfrac{\sqrt{x+h}+\sqrt{x}}{h}$

B. $\lim\limits_{h\to 0}\dfrac{\sqrt{x+h}+\sqrt{x}}{h}$

C. $\lim\limits_{x\to 0}\dfrac{\sqrt{x+h}-\sqrt{x}}{h}$

D. $\lim\limits_{h\to 0}\dfrac{\sqrt{x+h}-\sqrt{x}}{h}$

126. If $f(x)=x^2$, determine the value of $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

A. x^2

B. x^2+2x

C. x^2-2x

D. $2x$

E. *none of these*

127. Which one of the following is equal to $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

A. $f'(x)$

B. $f'(x)-f(x)$

C. $f(x)$

D. 0

E. *none of these*

128. Which one of the following limits represents the derivative of the function $f(x)=x^2-2x+3$

A. $\lim\limits_{h\to 0}\dfrac{(x+h)^2-2(x+h)+3-x^2-2x+3}{h}$

B. $\lim\limits_{h\to 0}\dfrac{(x+h)^2-2(x+h)+3-x^2+2x-3}{h}$

C. $\lim\limits_{h\to 0}\dfrac{(x+h)^2-2(x+h)+3-x^2+2x+3}{h}$

D. $\lim\limits_{h\to 0}\dfrac{2(x+h)-2}{h}$

129. Given $f(x)=3x^2$, use the **definition of the derivative** to show that $f'(x)=6x$

130. Given $f(x)=x^2-3x$, use the **definition of the derivative** to show that $f'(x)=2x-3$

131. Given $f(x)=x^2+5x$, use the **definition of the derivative** to show that $f'(x)=2x+5$

132. If $f(x)=\sqrt{x-2}$ then $\dfrac{f(x+h)-f(x)}{h}=$

A. $\dfrac{\sqrt{x-2}+\sqrt{h-2}}{h}$

B. $\dfrac{\sqrt{xh-2}+\sqrt{x-2}}{h}$

C. $\dfrac{\sqrt{x-2+h}-\sqrt{x-2}}{h}$

D. $\dfrac{\sqrt{x+h}-\sqrt{2}}{h}$

E. $\dfrac{\sqrt{x+h-2-(x-2)}}{h}$

133. If $f(x)=\frac{1}{x+2}$ then $\frac{f(x+h)-f(x)}{h}=$

A. $\frac{h+4}{h(x+2)(x+h+2)}$ B. $\frac{-1}{(x+2)(x+h+2)}$ C. $\frac{-1}{(x+h)(x+h-2)}$

D. $\frac{1}{h(x+2)(x+h+2)}$ E. $\frac{1}{2h+2}$

134. $\lim_{h\to 0}\frac{3(x+h)^{37}-3x^{37}}{h}=$

A. the derivative of x^{38} B. the derivative of $3x^{37}$ C. the derivative of x^{37}

D. equal to 3 E. ***does not exist***

135. $\lim_{h\to 0}\frac{(x+h)^{\frac{1}{2}}-x^{\frac{1}{2}}}{h}=$

A. $\sqrt{x}$ B. $\frac{1}{\sqrt{x}}$ C. $\frac{\sqrt{x}}{2}$ D. $\frac{1}{2\sqrt{x}}$ E. $\frac{x^2}{2}$

136. $\lim_{h\to 0}\frac{(1+h)^6-1}{h}=$

A. 0 B. 1 C. 6 D. ∞ E. ***nonexistent***

137. $\lim_{h\to 0}\frac{\sqrt[3]{8+h}-2}{h}=$

A. 0 B. $\frac{1}{12}$ C. 1 D. 192 E. ∞

138. $\lim_{h\to 0}\frac{(2+h)^3-2^3}{h}=$

A. 0 B. 6 C. 12 D. ***does not exist*** E. ***none of these***

139. $\lim_{h\to 0}\frac{\ln(e+h)-1}{h}=$

A. 0 B. $\frac{1}{e}$ C. 1 D. e E. ***nonexistent***

140. $\lim_{h\to 0}\frac{\frac{1}{3+h}-\frac{1}{3}}{h}=$

A. $-\frac{2}{3}$ B. $-\frac{1}{9}$ C. 0 D. ***does not exist*** E. ***none of these***

141. $\lim_{h\to 0}\frac{\ln(2+h)-\ln 2}{h}=$

A. 0 B. $\ln 2$ C. $\frac{1}{2}$ D. $\frac{1}{\ln 2}$ E. ∞

142. $\lim\limits_{h\to 0} \frac{8(\frac{1}{2}+h)^8 - 8(\frac{1}{2})^8}{h} =$

A. 0 B. $\frac{1}{2}$ C. 1 D. limit does not exist

E. cannot be determined from the information given

143. $\lim\limits_{h\to 0} \frac{e^{4+h} - e^4}{h} =$

A. e^3 B. e^4 C. $4e^3$ D. $4e^4$ E. $5e^5$

144. $\lim\limits_{h\to 0} \frac{8\sqrt{16+h} - 32}{h} =$

A. -1 B. $-\frac{1}{2}$ C. $\frac{1}{2}$ D. 1 E. 4

145. $\lim\limits_{h\to 0} \frac{(2+h)^3 + (2+h) - 10}{h} =$

A. 9 B. 10 C. 11 D. 12 E. 13

146. $\lim\limits_{h\to 0} \frac{4(2+h)^3 - 2(2+h) - 28}{h} =$

A. 0 B. 26 C. 28 D. 36 E. *none of these*

147. $\lim\limits_{h\to 0} \frac{\sqrt{9+h} - 3}{h} =$

A. 0 B. $\frac{1}{6}$ C. $\frac{1}{3}$ D. 3 E. 6

148. $\lim\limits_{h\to 0} \frac{(2+h)^4 - 3(2+h) - 10}{h} =$

A. 0 B. 15 C. 26 D. 29 E. 32

149. $\lim\limits_{h\to 0} \frac{e^{x+h} - e^x}{h} =$

A. 0 B. 1 C. $+\infty$ D. $-\infty$ E. *none of these*

150. $\lim\limits_{h\to 0} \frac{(3+h)^3 + (3+h) - 30}{h} =$

A. -30 B. 0 C. 28 D. 33 E. *none of these*

151. $\lim_{h\to 0}\dfrac{\sqrt{2(6+h)-3}-\sqrt{2(6)-3}}{h}=$

A. $-\dfrac{1}{2}$ B. 0 C. $\dfrac{1}{3}$ D. 6 E. *none of these*

152. If $f(x)=\sqrt{x+2}$, then $\lim_{h\to 0}\dfrac{f(2+h)-f(2)}{h}=$

A. 4 B. 0 C. $\dfrac{1}{2}$ D. $\dfrac{1}{4}$ E. 1

153. $\lim_{h\to 0}\dfrac{2(x+h)^5-5(x+h)^3-2x^5+5x^3}{h}=$

A. 0 B. $10x^3-15x$ C. $10x^4+15x^2$ D. $10x^4-15x^2$ E. $-10x^4+15x^2$

154. $\lim_{h\to 0}\dfrac{3(\frac{1}{2}+h)^5-3(\frac{1}{2})^5}{h}=$

A. 0 B. 1 C. $\dfrac{15}{16}$ D. *does not exist* E. *cannot be determined*

155. $\lim_{h\to 0}\dfrac{e^{1+h}-e}{h}=$

A. 0 B. $\dfrac{1}{e}$ C. 1 D. e E. *does not exist*

156. $\lim_{h\to 0}\dfrac{\sqrt{1+2h}-1}{h}=$

A. 2 B. 1 C. $\dfrac{1}{2}$ D. 0 E. *does not exist*

157. $\lim_{h\to 0}\dfrac{\ln(e+h)-1}{h}=$

A. $f'(e)$ where $f(x)=\ln x$ B. $f'(e)$ where $f(x)=\dfrac{\ln x}{x}$ C. $f'(1)$ where $f(x)=\ln x$

D. $f'(0)$ where $f(x)=\ln x$ E. $f'(1)$ where $f(x)=\ln(x+e)$

158. $\lim_{h\to 0}\dfrac{1}{h}\ln\left(\dfrac{2+h}{2}\right)=$

A. e^2 B. 1 C. $\dfrac{1}{2}$ D. 0 E. *nonexistent*

159. $\lim_{h\to 0}\dfrac{(4+h)^3+(4+h)-68}{h}=$

160. If f is a differentiable function, then $f'(a)$ is given by which of the following ?

I. $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$ **II.** $\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$ **III.** $\lim_{x\to a}\frac{f(x+h)-f(x)}{h}$

A. **I *only*** B. **II *only*** C. **I *and* II *only***
D. **I *and* III *only*** E. **I, II *and* III**

161. If $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=g(x)$ then

A. $f(x)=g(x)$ B. $f'(x)=g(x)$ C. $f(x)=g'(x)$
D. $f'(x)=g'(x)$ E. $\lim_{h\to 0}\frac{g(x+h)-g(x)}{h}=f(x)$

162. If $f(x)=e^x$ which of the following is equal to $f'(e)$

A. $\lim_{h\to 0}\frac{e^{x+h}}{h}$ B. $\lim_{h\to 0}\frac{e^{x+h}-e^e}{h}$ C. $\lim_{h\to 0}\frac{e^{e+h}-e}{h}$ D. $\lim_{h\to 0}\frac{e^{x+h}-1}{h}$ E. $\lim_{h\to 0}\frac{e^{e+h}-e^e}{h}$

163. If $f(x)=6x^2+\frac{16}{x^2}$ then $\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}=$

A. 0 B. 20 C. 24 D. 32 E. ∞

164. If $\lim_{h\to 0}\frac{f(4+h)-f(4)}{h}=6$ which of the following must be true ?

A. $f(4)=6$ B. $f(h)=2$ C. $f'(4)=6$
D. $\lim_{h\to 0}\frac{f(h)}{h}=6$ E. $\lim_{h\to 0}\frac{f(4-h)+f(4)}{h}=6$

165. If the function f is continuous for all real numbers and $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}=7$ then which of the following statements must be true ?

A. $f(a)=7$ B. f is differentiable at $x=a$
C. f is differentiable for all real numbers D. f is increasing for $x>0$
E. f is increasing for all real numbers

166. Given $\lim_{h\to 0}\frac{f(6+h)-f(6)}{h}=-2$ which of the following must be true ?

I. $f'(6)$ exists **II.** $f(x)$ is continuous at $x=6$ **III.** $f(6)<0$

A. ***none*** B. **I *and* II *only*** C. **I *and* III *only***
D. **II *and* III *only*** E. **I, II *and* III**

167. Suppose $\lim\limits_{x\to 0}\dfrac{g(x)-g(0)}{x}=1$ It follows necessarily that

A. g is not defined at $x=0$

B. g is not continuous at $x=0$

C. $\lim\limits_{x\to 0} g(x)=1$

D. $g'(0)=1$

E. $g'(1)=0$

168. If f is a function such that $\lim\limits_{x\to 5}\dfrac{f(x)-f(5)}{x-5}=0$ which of the following must be true ?

A. $f(5)=0$

B. f' at $x=5$ is 0

C. f is continuous at $x=0$

D. f is not defined at $x=5$

E. the limit of $f(x)$ as x approaches 5 does not exist

169. Let f be a function defined for all real numbers. Which of the following statements about f must be true ?

A. If $\lim\limits_{x\to 2} f(x)=7$ then $f(2)=7$

B. If $\lim\limits_{x\to 5} f(x)=-3$ then -3 is in the range of f

C. If $\lim\limits_{x\to 1^-} f(x)=\lim\limits_{x\to 1^+} f(x)$ then $f(1)$ exists

D. If $\lim\limits_{x\to 3^-} f(x)\neq\lim\limits_{x\to 3^+} f(x)$ then $\lim\limits_{x\to 3} f(x)$ does not exist

E. If $\lim\limits_{x\to 4} f(x)$ does not exist, then $f(4)$ does not exist

170. If f is a function such that $\lim\limits_{x\to 2}\dfrac{f(x)-f(2)}{x-2}=0$ which of the following must be true ?

A. $f(2)=0$

B. f is not defined at $x=2$

C. The derivative of f at $x=2$ is 0

D. f is continuous at $x=0$

E. The limit of $f(x)$ as x approaches 2 does not exist

171. $\lim\limits_{a\to 4}\dfrac{2-\sqrt{a}}{4-a}=$

A. $f'(2)$, where $f(x)=\sqrt{x}$

B. $f'(2)$, where $f(x)=1-\sqrt{x}$

C. $f'(4)$, where $f(x)=\sqrt{x}$

D. $f'(2)$, where $f(x)=\sqrt{4-x}$

E. $f'(4)$, where $f(x)=\dfrac{1}{\sqrt{x}}$

172. $\lim\limits_{x\to a}\dfrac{\sqrt[3]{x}-\sqrt[3]{a}}{x-a}=$

A. 0

B. $2\sqrt[3]{a}$

C. $\dfrac{3}{2}\sqrt[3]{a^2}$

D. $\dfrac{\sqrt[3]{a}}{3a}$

E. *none of these*

173.

$\boxed{\text{Limits}}$ $\rightarrow$ needed to handle ***holes***, ***asymptotes***, ***sharp points***, ***endpoints***,

Infinity $\rightarrow$ horizontal asymptotes (divide by the variable with highest exponent in denominator

$$\lim_{x\to\infty}\frac{4x-5x^2}{2x^2+3x-1}=\lim_{x\to\infty}\frac{\frac{4x}{x^2}-\frac{5x^2}{x^2}}{\frac{2x^2}{x^2}+\frac{3x}{x^2}-\frac{1}{x^2}}=\lim_{x\to\infty}\frac{\frac{4}{x}-5}{2+\frac{3}{x}-\frac{1}{x^2}}=\frac{0-5}{2+0-0}=\boxed{-\frac{5}{2}}$$

174. $\lim_{x\to\infty}\frac{2x-7}{4x+3}=$

A. $-\frac{7}{3}$ B. $-\frac{3}{7}$ C. $\frac{1}{2}$ D. 2 E. *none of these*

175. $\lim_{n\to\infty}\frac{2n^3-1}{3n^3}=$

A. 0 B. $\frac{1}{3}$ C. $\frac{2}{3}$ D. *does not exist* E. *none of these*

176. $\lim_{n\to\infty}\frac{6n^2-4n}{3n^2+5n}=$

A. $-\frac{4}{5}$ B. 0 C. 2 D. *does not exist* E. *none of these*

177. $\lim_{x\to\infty}\frac{3x-2}{9x+8}=$

A. $\frac{1}{17}$ B. $\frac{1}{4}$ C. $\frac{1}{3}$ D. $\frac{1}{2}$ E. *none of these*

178. $\lim_{x\to\infty}\frac{4x-5x^2}{2x^2+3x-1}=$

A. $-\frac{5}{2}$ B. 0 C. 2 D. *does not exist* E. *none of these*

179. $\lim_{x\to\infty}\frac{5x^2+3x-2}{3x^2-4x+7}=$

A. $-\frac{2}{7}$ B. 1 C. $\frac{5}{3}$ D. *does not exist* E. *none of these*

180. $\lim_{x\to\infty}\frac{5x^2+4x-1}{5x^2-3x+2}=$

A. $-\frac{1}{2}$ B. $\frac{1}{2}$ C. 1 D. *does not exist* E. *none of these*

181. $\lim_{n\to\infty} \frac{3n-1}{4-2n} =$

A. $-\frac{3}{2}$ B. $-\frac{1}{4}$ C. $\frac{3}{2}$ D. *does not exist* E. *none of these*

182. $\lim_{n\to\infty} \frac{n^2+2n-3}{5-3n+6n^2} =$

A. $-\frac{3}{5}$ B. 0 C. $\frac{1}{6}$ D. *does not exist* E. *none of these*

183. $\lim_{n\to\infty} \frac{n-1}{n} =$

A. -1 B. 0 C. 1 D. *does not exist* E. *none of these*

184. $\lim_{x\to\infty} \frac{2x^4-5x^2}{3x^3+8x^2} =$

A. $-\frac{3}{11}$ B. 0 C. $\frac{2}{3}$ D. *does not exist* E. *none of these*

185. $\lim_{x\to\infty} \frac{x+5}{x+2} =$

A. 0 B. 1 C. $\frac{5}{2}$ D. *does not exist* E. *none of these*

186. $\lim_{x\to\infty} \frac{2x^2+3x+4}{2-5x^3} =$

A. $-\frac{2}{5}$ B. 0 C. 2 D. *does not exist* E. *none of these*

187. $\lim_{n\to\infty} \frac{2-n^2}{3n^3+6} =$

A. $-\frac{1}{3}$ B. 0 C. $\frac{1}{9}$ D. *does not exist* E. *none of these*

188. $\lim_{x\to\infty} \frac{x^3-1}{3-5x^3} =$

A. $-\frac{1}{3}$ B. $-\frac{1}{5}$ C. $\frac{1}{5}$ D. $\frac{1}{3}$ E. *none of these*

189. $\lim_{x\to\infty} \frac{6x^2+5x-7}{4x^2-8x+3} =$

A. $-\frac{7}{3}$ B. 0 C. $\frac{3}{2}$ D. *does not exist* E. *none of these*

190. $\lim_{x\to\infty}\left(2x+\frac{5}{x}\right)=$

A. 0 B. 2 C. 5 D. *does not exist* E. *none of these*

191. $\lim_{x\to\infty}\frac{2x^3-4x^2+3x-1}{5-2x^2+6x^3}=$

A. $-\frac{1}{5}$ B. $\frac{2}{5}$ C. $\frac{1}{3}$ D. *does not exist* E. *none of these*

192. $\lim_{x\to\infty}\frac{5x-1}{2x}=$

A. 0 B. 1 C. $\frac{5}{2}$ D. *does not exist* E. *none of these*

193. $\lim_{x\to\infty}\frac{3x^2+2x-7}{5-3x+2x^2}=$

A. $-\frac{7}{5}$ B. $\frac{3}{5}$ C. $\frac{3}{2}$ D. *does not exist* E. *none of these*

194. $\lim_{x\to\infty}\frac{1-2x+5x^4}{3+3x^2-2x^3}=$

A. $-\frac{5}{2}$ B. $\frac{1}{3}$ C. 1 D. *does not exist* E. *none of these*

195. $\lim_{x\to\infty}\frac{2x-7}{4x+3}=$

A. $-\frac{7}{3}$ B. $-\frac{3}{7}$ C. $\frac{1}{2}$ D. 2 E. *none of these*

196. $\lim_{x\to\infty}\frac{1}{x-1}=$

A. -1 B. 0 C. 1 D. *does not exist* E. *none of these*

197. $\lim_{x\to\infty}\frac{2x}{x+2}=$

A. 0 B. $\frac{1}{2}$ C. 1 D. 2 E. ∞

198. $\lim_{x\to\infty}\frac{x^2-5}{2x^2+1}=$

A. -5 B. $\frac{1}{2}$ C. 1 D. 2 E. ∞

199. $\lim_{x\to\infty}\frac{x^2-5}{2x+1}=$

A. -5 B. $\frac{1}{2}$ C. 1 D. 2 E. ∞

200. $\lim_{x\to-\infty} \frac{2^{-x}}{2^x} =$

A. -1 B. 1 C. 0 D. ∞ E. *none of these*

201. $\lim_{x\to\infty} \frac{x-5}{2x^2+1} =$

A. -5 B. $-\frac{1}{2}$ C. 0 D. $\frac{1}{2}$ E. ∞

202. $\lim_{x\to\infty} \frac{100x}{x^2-1} =$

A. -1 B. 0 C. 1 D. 100 E. *does not exist*

203. $\lim_{x\to\infty} \frac{x^2-4x+4}{4x^2-1} =$

A. $\frac{1}{4}$ B. 1 C. 4 D. 8 E. *does not exist*

204. $\lim_{x\to\infty} \frac{1-x}{1+x} =$

A. -1 B. 0 C. ∞ D. 1 E. $\frac{1}{2}$

205. $\lim_{x\to\infty} \frac{4-x^2}{x^2-1} =$

A. 1 B. 0 C. -4 D. -1 E. ∞

206. $\lim_{x\to\infty} \frac{4-x^2}{4x^2-x-2} =$

A. -2 B. $-\frac{1}{4}$ C. 1 D. 2 E. *nonexistent*

207. $\lim_{x\to-\infty} \frac{5x^3+27}{20x^2+10x+9} =$

A. $-\infty$ B. -1 C. 0 D. 3 E. ∞

208. $\lim_{x\to\infty} \frac{3x^2+27}{x^3-27} =$

A. 3 B. ∞ C. 1 D. -1 E. 0

209. $\lim_{x\to\infty} \frac{2^{-x}}{2^x} =$

A. -1 B. 1 C. 0 D. ∞ E. *none of these*

210. $\lim_{x\to\infty} \frac{2x^2+1}{(2-x)(2+x)} =$

A. -4 B. -2 C. 1 D. 2 E. *nonexistent*

211. $\lim_{x\to\infty} \frac{3x^2-4}{2-7x-x^2} =$

A. 3 B. 1 C. -3 D. ∞ E. 0

212. $\lim_{x\to\infty} \frac{20x^2-13x+5}{5-4x^3} =$

A. -5 B. ∞ C. 0 D. 5 E. 1

213. $\lim_{x\to\infty} \frac{\sqrt{x}-4}{4-3\sqrt{x}} =$

A. $-\frac{1}{3}$ B. -1 C. ∞ D. 0 E. $\frac{1}{3}$

214. $\lim_{x\to\infty} \frac{x^2-4}{2+x-4x^2} =$

A. -2 B. $-\frac{1}{4}$ C. $\frac{1}{2}$ D. 1 E. *limit does not exist*

215. $\lim_{x\to\infty} \frac{x^3-2x^2+3x-4}{4x^3-3x^2+2x-1} =$

A. 4 B. 1 C. $\frac{1}{4}$ D. 0 E. -1

216. $\lim_{n\to\infty} \frac{4n^2}{n^2+10000n} =$

A. 0 B. $\frac{1}{2500}$ C. 1 D. 4 E. *does not exist*

217. $\lim_{n\to\infty} \frac{3n^3-5n}{n^3-2n^2+1} =$

A. -5 B. -2 C. 1 D. 3 E. *does not exist*

218. $\lim_{x\to\infty} \frac{x^2-4}{2+x-4x^2} =$

A. -2 B. $-\frac{1}{4}$ C. $\frac{1}{2}$ D. 1 E. *does not exist*

219. $\lim_{x\to-\infty} \frac{10-2^x}{10+2^{-x}} =$

A. -1 B. 0 C. 1 D. 10 E. ∞

220. $\lim_{x\to\infty}\frac{3x^5-4}{x-2x^5}=$

A. $-\frac{3}{2}$ B. -1 C. 0 D. 1 E. $\frac{3}{2}$

221. $\lim_{x\to\infty}\frac{x^2-1}{1-2x^2}=$

A. -1 B. $-\frac{1}{2}$ C. $\frac{1}{2}$ D. 1 E. *does not exist*

222. $\lim_{x\to\infty}\frac{6\sqrt[3]{x}+3\sqrt[6]{x}+4}{\sqrt[3]{8x}-\sqrt[6]{x}-10}=$

A. $-\frac{2}{5}$ B. 3 C. $-\frac{13}{10}$ D. 0 E. ∞

223. $\lim_{x\to\infty}\frac{10^8x^5+10^6x^4+10^4x^2}{10^9x^6+10^7x^5+10^5x^3}=$

A. 0 B. 1 C. -1 D. $\frac{1}{10}$ E. $-\frac{1}{10}$

224. $\lim_{x\to+\infty}\frac{x-\frac{1}{2x}}{2x+\frac{1}{6x}}=$

A. -3 B. $-\frac{1}{2}$ C. $-\frac{1}{3}$ D. $\frac{1}{2}$ E. 2

225. $\lim_{x\to\infty}\frac{x^2-6}{2+x-3x^2}=$

A. -3 B. $-\frac{1}{3}$ C. $\frac{1}{3}$ D. 2 E. *does not exist*

226. $\lim_{x\to\infty}\frac{3x^2+1}{(3-x)(3+x)}=$

A. -9 B. -3 C. 1 D. 3 E. *does not exist*

227. $\lim_{x\to+\infty}\frac{x-\frac{1}{2x}}{2x-\frac{1}{6x}}=$

A. -3 B. $-\frac{1}{2}$ C. $-\frac{1}{3}$ D. $\frac{1}{2}$ E. 2

228. $\lim_{x\to\infty}\left[x^2\left(\frac{1}{x-2}-\frac{1}{x-3}\right)\right]=$

A. 0 B. 1 C. -1 D. ∞ E. $-\infty$

229. $\lim_{x\to\infty}\frac{x^3-4x+1}{2x^3-5}=$

A. $-\frac{1}{5}$ B. $\frac{1}{2}$ C. $\frac{2}{3}$ D. 1 E. *does not exist*

230. $\lim_{x\to-\infty}\frac{3x}{\sqrt{3x^2-4}}=$

A. $\sqrt{3}$ B. 1 C. 0 D. $-\sqrt{3}$ E. *does not exist*

231. $\lim_{x\to\infty}\frac{1}{\sqrt{x}\left(\sqrt{x+1}-\sqrt{x-1}\right)}=$

A. 0 B. $\frac{1}{2}$ C. 1 D. 2 E. *does not exist*

232. $\lim_{x\to-\infty}\frac{2x+3}{\sqrt{x^2+x+1}}=$

A. -2 B. -1 C. 0 D. 2 E. *nonexistent*

233. $\lim_{x\to-\infty}\frac{2x-1}{1+2x}=$

A. -1 B. 0 C. 1 D. 2 E. *nonexistent*

234. $\lim_{x\to-\infty}\frac{x^2+4x-5}{x^3-1}=$

A. 0 B. $\frac{1}{3}$ C. 5 D. $-\infty$ E. ∞

235. $\lim_{x\to-\infty}\frac{4x^2+x-7}{x^2-5x-3}=$

A. 0 B. $\frac{7}{3}$ C. 4 D. 1 E. *nonexistent*

236. $\lim_{x\to\infty}\frac{3x^2+2x+2}{4x^2+x+5}=$

A. 0 B. $-\frac{3}{4}$ C. $\frac{3}{4}$ D. $\frac{2}{5}$ E. ∞

237. $\lim_{x\to\infty}\frac{3x^2-5x+4}{6x^2+7x-1}=$

A. -4 B. $-\frac{5}{7}$ C. 0 D. $\frac{1}{6}$ E. $\frac{1}{2}$

238. $\lim\limits_{x\to\infty} \dfrac{\sqrt{3x^2+2x+1}}{x+1} =$

A. 3 B. ∞ C. $\sqrt{3}$ D. 0 E. $\dfrac{\sqrt{3}}{2}$

239. $\lim\limits_{x\to\infty} \dfrac{5x^2-3x+1}{4x^2+2x+5} =$

A. 0 B. $\dfrac{4}{5}$ C. $\dfrac{3}{11}$ D. $\dfrac{5}{4}$ E. *does not exist*

240. $\lim\limits_{x\to+\infty} \dfrac{2x^2+3x-5}{5x-3x^2-2} =$

A. $-\dfrac{2}{3}$ B. 0 C. $\dfrac{2}{5}$ D. 1 E. *nonexistent*

241. $\lim\limits_{x\to-\infty} \dfrac{x^3-111x^2+3x-2}{1-2x+22x^2+3x^3} =$

A. -2 B. $-\dfrac{2}{3}$ C. $-\dfrac{1}{3}$ D. $\dfrac{1}{3}$ E. 1

242. $\lim\limits_{x\to 0} \dfrac{\frac{3}{x^2}}{\frac{2}{x^2}+\frac{105}{x}} =$

A. 0 B. 1 C. $\dfrac{3}{2}$ D. $\dfrac{3}{107}$ E. *none of these*

243. If $\lim\limits_{n\to\infty} \dfrac{6n^2}{200-4n+kn^2} = \dfrac{1}{2}$, then $k =$

A. 3 B. 6 C. 12 D. 8 E. 2

244. $\lim\limits_{x\to\infty} \sqrt[3]{\dfrac{8+x^2}{x(x+1)}} =$

A. 0 B. 2 C. $\sqrt[3]{9}$ D. 1 E. *does not exist*

245. $\lim\limits_{x\to+\infty} \left(\dfrac{1}{x} - \dfrac{x}{x-1}\right) =$

A. -1 B. 0 C. 1 D. 2 E. *none of these*

246. $\lim\limits_{x\to\infty} \left(1+\dfrac{1}{x}\right)^x =$

A. 1 B. 0 C. ∞ D. 2 E. e

247. $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n+2} =$

A. e^2 B. $e+2$ C. $2e$ D. e E. $e+e^2$

248. $\lim_{x\to\infty}\frac{\sqrt{9x^2+2}}{4x+3} =$

A. $\frac{3}{2}$ B. $\frac{3}{4}$ C. $\frac{\sqrt{2}}{3}$ D. 1 E. *does not exist*

249. $\lim_{x\to\infty}\frac{\sqrt{x-2}}{x-2} =$

A. -2 B. 0 C. 1 D. 2 E. *does not exist*

250. Which of the following are asymptotes of $y+xy-2x=0$

I. $x=-1$ II. $x=1$ III. $y=2$

A. I *only* B. II *only* C. III *only*
D. I *and* III *only* E. II *and* III *only*

251. The horizontal asymptotes of $f(x)=\frac{1-|x|}{x}$ are given by

A. $y=1$ B. $y=-1$ C. $x=0,\ x=1,\ x=-1$
D. $y=0$ E. $y=1,\ y=-1$

252. The vertical asymptote and horizontal asymptote for $f(x)=\frac{\sqrt{x}}{x+4}$ are

A. $x=-4,\ y=0$ B. no vertical asymptote, $y=0$
C. no vertical or horizontal asymptote D. $x=-4$, no horizontal asymptote
E. $x=-4,\ y=1$

253. For $x\geq 0$ the horizontal line $y=2$ is an asymptote for the graph of the function f Which of the following statements must be true ?

A. $f(0)=2$ B. $f(x)\neq 2$ for all $x\geq 0$ C. $f(2)$ is undefined
D. $\lim_{x\to 2} f(x)=\infty$ E. $\lim_{x\to\infty} f(x)=2$

254. Find the equation of the horizontal asymptote of $y=\frac{5x}{x-1}$

A. $y=1$ B. $y=0$ C. $x=1$ D. $x=5$ E. $y=5$

255. Find the equation of the horizontal asymptote of $f(x)=\frac{2x-1}{4x+1}$

A. $y=2$ B. $x=2$ C. $y=\frac{1}{2}$ D. $x=\frac{1}{2}$ E. $y=-1$

256. The horizontal asymptote of $f(x)=\dfrac{x}{x-1}$ is

A. $x=0$ B. $x=1$ C. $y=0$ D. $y=1$

E. no horizontal asymptote

257. The horizontal asymptote of $f(x)=\dfrac{4}{x+1}$ is

A. $x=-1$ B. $x=0$ C. $y=-1$ D. $y=0$

E. no horizontal asymptote

258. Find the equation of the horizontal asymptote of $y=\dfrac{x^2}{2x^2-2}$

A. $y=0$ B. $x=\frac{1}{2}$ C. $y=\frac{1}{2}$ D. $x=1$ E. $x=\pm 1$

259. $f(x)=\dfrac{(x-1)^2}{x^2-1}$ has

A. a hole at $x=-1$ B. holes at $x=-1$ and $x=1$

C. vertical asymptotes at $x=-1$ and $x=1$ D. a horizontal asymptote at $y=-1$

E. a hole at $x=1$ and a vertical asymptote at $x=-1$

260. How many vertical and horizontal asymptotes are there to the graph of $y=\dfrac{x^2}{x^2-1}$

A. **2** vertical and **1** horizontal B. **1** vertical and **1** horizontal C. **1** vertical and no horizontal

D. **2** vertical and no horizontal E. **1** vertical and **2** horizontal

261. The graph of $y=e^{-x}+1$ has a horizontal asymptote with equation

A. $x=0$ B. $y=0$ C. $x=1$ D. $y=1$

E. The graph has no horizontal asymptote

262. An asymptote for $y=\dfrac{(x+2)(x-7)}{x-5}$ is

A. $x=0$ B. $x=-2$ C. $x=5$ D. $x=-5$ E. $y=-2$

263. The graph of $f(x)=\dfrac{4}{x^2-1}$ has

A. one vertical asymptote, at $x=1$

B. the y-axis as vertical asymptote

C. the x-axis as horizontal asymptote and $x=\pm 1$ as vertical asymptotes

D. two vertical asymptotes, at $x=\pm 1,$ but no horizontal asymptote

E. no asymptote

264. Which statement below is true about the curve $y=\dfrac{2x^2+4}{2+7x-4x^2}$

A. the line $x=-\dfrac{1}{4}$ is a vertical asymptote

B. the line $x=1$ is a vertical asymptote

C. the line $y=-\dfrac{1}{4}$ is a horizontal asymptote

D. the line $y=2$ is a horizontal asymptote

E. the graph has no vertical or horizontal asymptotes

265. The graph of which of the following equations has $y=1$ as an asymptote ?

A. $y=\ln x$ B. $y=\sin x$ C. $y=\dfrac{x}{x+1}$ D. $y=\dfrac{x^2}{x-1}$ E. $y=e^{-x}$

266. The graph of $y=\dfrac{2x^2+2x+3}{4x^2-4x}$ has

A. a horizontal asymptote at $y=\dfrac{1}{2}$ but no vertical asymptotes

B. no horizontal asymptotes but two vertical asymptotes, at $x=0$ and $x=1$

C. a horizontal asymptote at $y=\dfrac{1}{2}$ and two vertical asymptotes, at $x=0$ and $x=1$

D. a horizontal asymptote at $x=2$ but no vertical asymptotes

E. a horizontal asymptote at $y=\dfrac{1}{2}$ and two vertical asymptotes, at $x=\pm 1$

267. Which statement below is true about the curve $y=\dfrac{2x^2+4}{2+7x-4x^2}$

A. the line $x=-\dfrac{1}{4}$ is a vertical asymptote

B. the line $x=1$ is a vertical asymptote

C. the line $y=-\dfrac{1}{4}$ is a horizontal asymptote

D. the line $x=2$ is a horizontal asymptote

E. the graph has no vertical or horizontal asymptotes

268. The graph of the function $f(x)=\dfrac{|x^3|}{x^3-8}$ has the following asymptotes:

A. one vertical and one horizontal

B. two vertical and one horizontal

C. one vertical and two horizontal

D. three vertical and one horizontal

E. two vertical and two horizontal

269. If the graph of $y=\dfrac{ax+b}{x+c}$ has a horizontal asymptote $y=2$ and a vertical asymptote $x=-3$ then $a+c=$

A. -5 B. -1 C. 0 D. 1 E. 5

270. If $f(x)=e^x$ which of the following lines is an asymptote to the graph of f

A. $y=0$ B. $x=0$ C. $y=x$ D. $y=-x$ E. $y=1$

271. The equation for the horizontal asymptote for the function $f(x)=\dfrac{(2x-5)(3x+6)(x+1)^4}{(x-9)^6}$ is

A. $y=0$ B. $y=1$ C. $y=4$ D. $y=6$ E. $y=10$

272. The graph of $f(x)=\dfrac{x^2-x-2}{2x^2-x-1}$ has a horizontal asymptote which it

A. crosses at $x=\dfrac{1}{2}$ B. crosses at $x=-\dfrac{1}{2}$ C. crosses at $x=3$

D. crosses at $x=-3$ E. never intercepts

273. The number of horizontal asymptotes for the graph of the curve $y^2=\dfrac{2x}{x-1}$ is

A. 0 B. 1 C. 2 D. 3 E. 4

274. The graph of $y=\dfrac{3x}{1+|x|}$ has the following number of vertical and horizontal asymptotes:

A. no vertical and one horizontal B. no vertical and two horizontal

C. one vertical and one horizontal D. two vertical and no horizontal

E. two vertical and two horizontal

275. The graph of $y=\dfrac{2x^2}{4-x^2}$ has

A. no horizontal or vertical asymptotes

B. one vertical asymptote and no horizontal asymptotes

C. no vertical asymptotes and one horizontal asymptote

D. two vertical asymptotes and one horizontal asymptote

E. two horizontal asymptotes and one vertical asymptote

276. The equation of the horizontal asymptote for the graph of $f(x)=\dfrac{2x^3-7x^2+8x-1}{(x-2)(4x-3)(x+1)}$ is

A. $y=0$ B. $y=\dfrac{1}{2}$ C. $y=1$ D. $x=2$ E. *none of these*

277. If $y=7$ is a horizontal asymptote of a rational function f, then which of the following must be true ?

A. $\lim\limits_{x\to 7} f(x)=\infty$ B. $\lim\limits_{x\to -\infty} f(x)=-7$ C. $\lim\limits_{x\to 0} f(x)=7$

D. $\lim\limits_{x\to 7} f(x)=0$ E. $\lim\limits_{x\to \infty} f(x)=7$

278. The graph of $f(x)=\dfrac{x}{x+1}$ has

A. no asymptotes and no inflection points
B. no asymptotes and one inflection point
C. one horizontal asymptote and no vertical asymptotes
D. one vertical asymptote and no horizontal asymptotes
E. one horizontal asymptote and one vertical asymptote

279. The equation of the horizontal asymptote for the graph of $y=\dfrac{2-e^{\frac{1}{x}}}{2+e^{\frac{1}{x}}}$ is

A. $y=-1$ B. $y=-\dfrac{1}{2}$ C. $y=\dfrac{1}{3}$ D. $y=\dfrac{1}{2}$ E. $y=1$

280. The graph of $y=\dfrac{\sin x}{x}$ has

I. a vertical asymptote at $x=0$
II. a horizontal asymptote at $y=0$
III. an infinite number of zeros

A. **I *only*** B. **II *only*** C. **III *only***
D. **I *and* III *only*** E. **II *and* III *only***

281. The graph of $f(x)=\dfrac{\sin x}{|x|}$ has

A. no horizontal asymptotes and no vertical asymptotes
B. one horizontal asymptote and no vertical asymptotes
C. one horizontal asymptote and one vertical asymptote
D. one horizontal asymptote and two vertical asymptotes
E. two horizontal asymptotes and one vertical asymptote

282. Which of the following best describes the behavior of the function $f(x)=\dfrac{x^2-2x}{x^2-4}$ at the values not in its domain ?

A. One vertical asymptote, no removable discontinuities
B. Two vertical asymptotes
C. Two removable discontinuities
D. One removable discontinuity, one vertical asymptote, $x=2$
E. One removable discontinuity, one vertical asymptote, $x=-2$

283. The graph of which of the following equations has $y=1$ as an asymptote ?

A. $y=\cos x$ B. $y=e^x$ C. $y=\dfrac{x^3}{x^2+1}$ D. $y=\dfrac{x^2}{x^2-5}$ E. $y=-\ln x$

284. If $f(x)=e^x+2$ which of the following lines is an asymptote to the graph of f

A. $y=-2$ B. $x=0$ C. $y=2$ D. $x=2$ E. $y=0$

285. If the graph of $y=\dfrac{ax+b}{x+c}$ has a horizontal asymptote $y=-2$, a vertical asymptote $x=4$, and an x-intercept of 1.5, then $a-b+c=$

A. -3 B. 1 C. 5 D. -9 E. -1

286. The function $f(x)=\dfrac{4x^2-3}{2x^2+1}$ is

I. unbounded
II. bounded below by $y=-3$
III. bounded above by $y=2$
IV. bounded below by $y=2$

A. **I *only*** B. **II *only*** C. **III *only***
D. **IV *only*** E. **II *and* III *only***

287. Let $f(x)=\dfrac{2}{x^2+4}$

Which of the following are true ?

I. $f(x)$ has an absolute maximum at $x=0$
II. $f(x)$ has a vertical asymptote at $x=-2$
III. $f(x)$ has a horizontal asymptote at $y=\dfrac{1}{2}$

A. **I *only*** B. **II *only*** C. **I *and* II *only***
D. **II *and* III *only*** E. **I, II *and* III**

288. The equation of the horizontal asymptote of $f(x)=\dfrac{|x|}{|x|+x}$ is:

A. $y=\dfrac{1}{2}$ B. $y=0$ C. $y=1$ D. $y=-1$ E. $x=0$

289. If $y=\dfrac{2(x-1)^2}{x^2}$, then which of the following must be true ?

I. the range is $y\geq 0$ **II**. the y-intercept is 1 **III**. the horizontal asymptote is $y=2$

A. **I *only*** B. **II *only*** C. **III *only***
D. **I *and* II *only*** E. **I *and* III *only***

290. Which of the following is true about the function f if $f(x)=\dfrac{(x-1)^2}{2x^2-5x+3}$

I. f is continuous at $x=1$
II. The graph of f has a vertical asymptote at $x=1$
III. The graph of f has a horizontal asymptote at $y=\dfrac{1}{2}$

A. **I *only*** B. **II *only*** C. **III *only***
D. **II *and* III *only*** E. **I, II *and* III**

291.

$\boxed{\text{Limits}}$ $\rightarrow$ needed to handle ***holes***, ***asymptotes***, ***sharp points***, ***endpoints***,

$\rightarrow$ **definition of derivative**

$\boxed{6a}$ Trig $\rightarrow$ basic $\quad \lim_{x\to 0}\frac{\sin x}{x}=1 \quad \lim_{x\to 0}\frac{1-\cos x}{x}=0$ $\leftarrow$ squeeze theorem proof

$\boxed{6b}$ Trig $\rightarrow$ advanced questions based on basics

292. $\lim_{t\to 0}\frac{\sin t}{t}=$

A. 0 B. 1 C. $\frac{\pi}{2}$ D. -1 E. ***does not exist***

293. $\lim_{x\to 0}\frac{\sin 7x}{7x}=$

A. 0 B. $\frac{1}{7}$ C. 1 D. 7 E. ***indeterminate***

294. $\lim_{x\to 0}\frac{\sin 7x}{x}=$

A. 0 B. $\frac{1}{7}$ C. 1 D. 7 E. ***indeterminate***

295. $\lim_{x\to 0}\frac{\sin x}{7x}=$

A. 0 B. $\frac{1}{7}$ C. 1 D. 7 E. ***indeterminate***

296. $\lim_{x\to \pi}\frac{\sin x}{x}=$

A. $-\pi$ B. -1 C. 0 D. 1 E. π

297. $\lim_{x\to \frac{\pi}{2}}\frac{\sin x}{x}=$

A. 0 B. 1 C. $\frac{2}{\pi}$ D. $\frac{\pi}{2}$ E. ***does not exist***

298. $\lim_{x\to 0}\frac{\sin 2x}{x\cos x}=$

A. -1 B. 0 C. 1 D. 2 E. ***does not exist***

299. $\lim_{x\to \frac{\pi}{3}}\frac{1-\cos x}{x}=$

A. $\frac{\pi}{3}$ B. $\frac{3}{\pi}$ C. $\frac{3}{2\pi}$ D. $\frac{3(1-\sqrt{3})}{2\pi}$ E. 0

300. $\lim\limits_{x\to 0}\dfrac{\sin 3x}{7x}=$

A. $\dfrac{3}{7}$ B. $\dfrac{7}{3}$ C. 3 D. 7 E. *does not exist*

301. $\lim\limits_{\theta\to 0}\dfrac{\sin\theta}{\sec\theta}=$

A. 1 B. 0 C. $\dfrac{\pi}{2}$ D. -1 E. ∞

302. $\lim\limits_{x\to\frac{\pi}{4}}(\sin^2 x-1)=$

A. 0 B. 1 C. -1 D. $\dfrac{\pi}{4}$ E. $-\dfrac{1}{2}$

303. $\lim\limits_{x\to 0}\sin\dfrac{1}{x}=$

A. ∞ B. 1 C. *nonexistent* D. -1 E. *none of these*

304. $\lim\limits_{x\to\infty}x\sin\dfrac{1}{x}=$

A. 0 B. ∞ C. *nonexistent* D. -1 E. 1

305. $\lim\limits_{x\to\pi}\dfrac{\sin(\pi-x)}{\pi-x}=$

A. 1 B. 0 C. ∞ D. *nonexistent* E. *none of these*

306. $\lim\limits_{x\to 0}\dfrac{\cos x-1}{x}=$

A. -1 B. 0 C. 1 D. ∞ E. *none of these*

307. $\lim\limits_{x\to 0}\dfrac{\sin 2x}{x}=$

A. 1 B. 2 C. $\dfrac{1}{2}$ D. 0 E. ∞

308. $\lim\limits_{x\to 0}\dfrac{\sin 3x}{\sin 4x}=$

A. 1 B. $\dfrac{4}{3}$ C. $\dfrac{3}{4}$ D. 0 E. *nonexistent*

309. $\lim\limits_{x\to 0}\dfrac{1-\cos x}{x}=$

A. *nonexistent* B. 1 C. 2 D. ∞ E. *none of these*

310. $\lim_{x\to 0} \frac{\tan \pi x}{x} =$

A. $\frac{1}{\pi}$ B. 0 C. 1 D. π E. ∞

311. $\lim_{x\to 0} \frac{\sec x - \cos x}{x^2} =$

A. 0 B. $\frac{1}{2}$ C. 1 D. 2 E. *none of these*

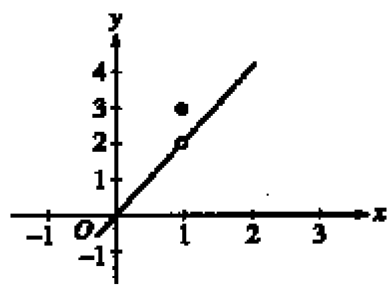
312. $\lim_{h\to 0} \frac{\cos(\frac{\pi}{2}+h)}{h} =$

A. 1 B. -1 C. 0 D. *does not exist* E. *none of these*

313. $\lim_{h\to 0} \frac{\sin(\frac{\pi}{2}+h)-1}{h} =$

A. 1 B. -1 C. 0 D. ∞ E. *none of these*

314. The graph of the function f is show in the figure.
The value of $\lim_{x\to 1} \sin(f(x)) =$



Graph of f

A. 0.909 B. 0.841 C. 0.141 D. -0.416 E. *nonexistent*

315. $\lim_{x\to 0} \frac{1-\cos^2(2x)}{x^2} =$

A. -2 B. 0 C. 1 D. 2 E. 4

316. $\lim_{x\to 0} x \csc x =$

A. $-\infty$ B. -1 C. 0 D. 1 E. ∞

317. $\lim_{x\to \frac{\pi}{4}} \frac{\sin(x-\frac{\pi}{4})}{x-\frac{\pi}{4}} =$

A. 0 B. $\frac{1}{\sqrt{2}}$ C. $\frac{\pi}{4}$ D. 1 E. *nonexistent*

318. $\lim_{\theta\to 0} \frac{1-\cos\theta}{2\sin^2\theta} =$

A. 0 B. $\frac{1}{8}$ C. $\frac{1}{4}$ D. 1 E. *nonexistent*

319. $\lim_{x\to 0}\frac{\tan x}{x}=$

A. -1 B. $-\frac{1}{2}$ C. 0 D. $\frac{1}{2}$ E. 1

320. $\lim_{x\to 0}\frac{\cos x-1}{\sin^2 x}=$

A. -1 B. $-\frac{1}{2}$ C. 0 D. $\frac{1}{2}$ E. 1

321. $\lim_{x\to 0}\frac{\sin 3x}{\tan 2x}=$

A. 0 B. $\frac{2}{3}$ C. 1 D. $\frac{3}{2}$ E. 3

322. $\lim_{x\to 0}\frac{-\cos x+1-\sin x}{x}=$

A. -1 B. 0 C. 1 D. ∞ E. *undefined*

323. $\lim_{x\to 0}\frac{2\sin x\cos x}{2x}=$

A. -2 B. -1 C. $\frac{1}{2}$ D. 1 E. *undefined*

324. $\lim_{x\to 0}\frac{\sin^2 3x}{x^2}=$

A. 0 B. 1 C. 3 D. 9 E. *undefined*

325. $\lim_{x\to 0}4\frac{\sin x\cos x-\sin x}{x^2}=$

A. 2 B. $\frac{40}{3}$ C. ∞ D. 0 E. *undefined*

326. $\lim_{x\to 0}\frac{\cos^2 x-1}{2x\sin x}=$

A. -1 B. $-\frac{1}{2}$ C. 1 D. $\frac{1}{2}$ E. 0

327. $\lim_{x\to 0}\frac{\sin 2x}{x\cos x}=$

A. 0 B. 1 C. $\frac{1}{2}$ D. 2 E. *does not exist*

328. $\lim_{x\to 0}\frac{1-\cos x}{x^2}=$

A. 0 B. $\frac{1}{2}$ C. 1 D. 2 E. *does not exist*

329. $\lim_{x\to 0}\frac{\sin 2x-2x}{x^3}=$

A. ∞ B. 1 C. $-\frac{4}{3}$ D. $-\infty$ E. *does not exist*

330. $\lim_{x\to\frac{\pi}{4}}\frac{\tan x-1}{\sin x-\cos x}=$

A. 1 B. $\sqrt{2}$ C. 2 D. $2\sqrt{2}$ E. *does not exist*

331. $\lim_{x\to\frac{\pi}{2}}(\sec x-\tan x)=$

A. 1 B. 0 C. $\ln 3$ D. $\frac{\pi}{2}$ E. *does not exist*

332. $\lim_{x\to\pi/2}\frac{\sin x}{x}=$

A. 0 B. $\frac{2}{\pi}$ C. $-\frac{\pi}{2}$ D. $\frac{2\sqrt{2}}{\pi}$ E. *none of these*

333. $\lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h}=$

A. 0 B. 1 C. $\sin x$ D. $\cos x$ E. *nonexistent*

334. $\lim_{h\to 0}\frac{\tan 3(x+h)-\tan 3x}{h}=$

A. 0 B. $3\sec^2(3x)$ C. $\sec^2(3x)$ D. $3\cot(3x)$ E. *nonexistent*

335. $\lim_{x\to 0}\frac{\tan(x+h)-\tan x}{h}=$

A. $\sec x$ B. $-\sec x$ C. $\sec^2 x$ D. $-\sec^2 x$ E. *does not exist*

336. $\lim_{h\to 0}\frac{\cos(x+h)-\cos x}{h}=$

A. $\sin x$ B. $-\sin x$ C. $\cos x$ D. $-\cos x$ E. *does not exist*

337. $\lim_{h\to 0}\frac{\tan(2(x+h))-\tan(2x)}{h}=$

A. 0 B. $2\cot(2x)$ C. $\sec^2(2x)$ D. $2\sec^2(2x)$ E. *does not exist*

338. $\lim_{h\to 0}\dfrac{\tan(\frac{\pi}{6}+h)-\tan(\frac{\pi}{6})}{h}=$

A. $\dfrac{\sqrt{3}}{3}$ B. $\dfrac{4}{3}$ C. $\sqrt{3}$ D. 0 E. $\dfrac{3}{4}$

339. $\lim_{h\to 0}\dfrac{\sec(\pi+h)-\sec\pi}{h}=$

A. -1 B. 0 C. $\dfrac{1}{\sqrt{2}}$ D. 1 E. $\sqrt{2}$

340. $\lim_{h\to 0}\dfrac{\sin(\pi+h)-\sin\pi}{h}=$

A. 1 B. 0 C. -1 D. $+\infty$ E. $-\infty$

341. $\lim_{h\to 0}\dfrac{\cos(\frac{3\pi}{2}+h)-\cos(\frac{3\pi}{2})}{h}=$

A. 1 B. $\dfrac{\sqrt{2}}{2}$ C. 0 D. -1 E. *does not exist*

342. $\lim_{\Delta x\to 0}\dfrac{\sin(\frac{\pi}{3}+\Delta x)-\sin(\frac{\pi}{3})}{\Delta x}=$

A. $-\dfrac{1}{2}$ B. 0 C. $\dfrac{1}{2}$ D. $\dfrac{\sqrt{3}}{2}$ E. *nonexistent*

343. $\lim_{h\to 0}\dfrac{\csc(\frac{\pi}{4}+h)-\csc(\frac{\pi}{4})}{h}=$

A. $\sqrt{2}$ B. $-\sqrt{2}$ C. 0 D. $-\dfrac{\sqrt{2}}{2}$ E. *undefined*

344. $\lim_{h\to 0}\dfrac{\cos(\frac{\pi}{2}+h)-\cos\frac{\pi}{2}}{h}=$

A. $-\infty$ B. -1 C. 0 D. 1 E. ∞

345. $\lim_{h\to 0}\dfrac{\sec(\frac{\pi}{3}+h)-\sec(\frac{\pi}{3})}{h}=$

A. $\dfrac{1}{2}$ B. $\sqrt{3}$ C. 2 D. $2\sqrt{3}$ E. *undefined*

346. $\lim_{h\to 0}\dfrac{\cot(\frac{5\pi}{6}+h)-\cot\frac{5\pi}{6}}{h}=$

A. -4 B. $-\sqrt{3}$ C. $\sqrt{3}$ D. 4 E. *cannot be determined*

347. $\lim_{k\to 0}\frac{\tan(\frac{\pi}{4}+k)-\tan(\frac{\pi}{4})}{k}=$

A. $\frac{1}{\sqrt{3}}$ B. 1 C. $\sqrt{2}$ D. 2 E. 3

348. $\lim_{h\to 0}\frac{\tan(\frac{\pi}{4}+h)-\tan(\frac{\pi}{4})}{h}=$

A. -1 B. 0 C. 1 D. 2 E. *does not exist*

349. $\lim_{h\to 0}\frac{\cos(\frac{\pi}{4}+h)-\cos\frac{\pi}{4}}{h}=$

A. $-\frac{\sqrt{3}}{2}$ B. $-\frac{\sqrt{2}}{2}$ C. 0 D. $\frac{\sqrt{2}}{2}$ E. 1

350. $\lim_{h\to 0}\frac{\sin(\frac{\pi}{2}+h)-\sin\frac{\pi}{2}}{h}=$

A. 0 B. 1 C. -1 D. $\frac{\pi}{2}$ E. *does not exist*

351. $\lim_{h\to 0}\frac{\sin(\frac{\pi}{2}+h)-1}{h}=$

A. $f'\left(\frac{\pi}{2}\right)$, where $f(x)=\cos x$

B. $f'(1)$, where $f(x)=\sin x$

C. $f'(1)$, where $f(x)=\cos x$

D. $f'\left(\frac{\pi}{2}\right)$, where $f(x)=\sin 2x$

E. $f'\left(\frac{\pi}{2}\right)$, where $f(x)=\sin x$

352. If $g(x)=x+\cos x$ then $\lim_{h\to 0}\frac{g(x+h)-g(x)}{h}=$

A. $\sin x+\cos x$ B. $\sin x-\cos x$ C. $1-\sin x$ D. $1-\cos x$ E. $x^2-\sin x$

353. $\lim_{x\to\infty}\frac{3\sqrt{x^4-3x}}{2x^2+\cos x}=$

A. 0 B. $\frac{3}{2}$ C. 6 D. ∞ E. *none of these*

354. $\lim_{x\to 0}\frac{\sin x}{|x|}=$

A. -1 B. 0 C. 1 D. $\frac{1}{2}$ E. *none of these*

355.

$\boxed{\textbf{Continuity}} \rightarrow$ A function $f(x)$ is said to be continuous at $x = c$ if

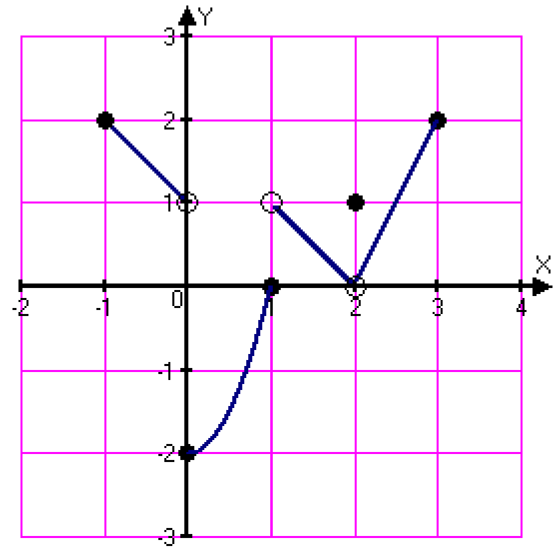
$\boxed{1}$ $f(c)$ is defined $\boxed{2}$ $\lim\limits_{x \to c} f(x)$ exists $\boxed{3}$ $\lim\limits_{x \to c} f(x) = f(c)$

If any of these conditions fails to be satisfied, the function is ***discontinuous*** at $x = c$

356.

This question and the ***next four*** are based on the function f shown in the graph and defined below:

$$f(x) = \begin{cases} 1 - x & -1 \le x < 0 \\ 2x^2 - 2 & 0 \le x \le 1 \\ -x + 2 & 1 < x < 2 \\ 1 & x = 2 \\ 2x - 4 & 2 < x \le 3 \end{cases}$$



$\boxed{\lim\limits_{x \to 2} f(x) =}$

A. ***equals 0*** B. ***equals 1*** C. ***equals 2*** D. ***nonexistent*** E. ***none of these***

357. The function f is defined on $[-1, 3]$

A. ***if*** $x \ne 0$ B. ***if*** $x \ne 1$ C. ***if*** $x \ne 2$ D. ***if*** $x \ne 3$ E. at each x in $[-1, 3]$

358. The function has a removable discontinuity at

A. $x = 0$ B. $x = 1$ C. $x = 2$ D. $x = 3$ E. ***none of these***

359. On which of the following intervals is f continuous ?

A. $-1 \le x \le 0$ B. $0 < x < 1$ C. $1 \le x \le 2$ D. $2 \le x \le 3$ E. ***none of these***

360. The function has a jump discontinuity at

A. $x = -1$ B. $x = 1$ C. $x = 2$ D. $x = 3$ E. ***none of these***

361. For what value(s) of x does $f(x) = \dfrac{x-1}{x^2-1}$ have a removable discontinuity ?

A. $x = 0$ B. $x = 1$ C. $x = -1$

D. $x = 1$ ***and*** $x = -1$ E. all real numbers

362. $f(x)=\dfrac{1}{x^2}$ is continuous for all real numbers EXCEPT

A. $x=0$ B. $x=1$ *only* C. $x=1$ *and* $x=-1$

D. $x=-1$ *only* E. $x=2$

363. $f(x)$ is a continuous function on $[\text{ a, b }]$ Which of the following statements are true ?

I. $f(x)$ is differentiable on (a, b)

II. There is a number c in (a, b) such that $f'(c)=\dfrac{f(b)-f(a)}{b-a}$

III. $f(x)$ has a maximum value on $[\text{ a, b }]$

A. **I** *only* B. **II** *only* C. **III** *only* D. **I** *and* **III** E. **I**, **II** *and* **III**

364. If f is continuous on $[-4,\ 4]$ such that $f(-4)=11$ and $f(4)=-11$, then

A. $f(0)=0$ B. $\lim\limits_{x\to 2} f(x)=8$

C. it is possible that f is not defined at $x=0$ D. $\lim\limits_{x\to 3} f(x)=\lim\limits_{x\to -3} f(x)$

E. there is at least one $c\in[-4,\ 4]$ such that $f(c)=8$

365. If f is continuous on $[\ 4,\ 7\ ]$, ***how many*** of the following statements must be true ?

I. f has a maximum value on $[\ 4,\ 7\ ]$ **III**. $f(7)>f(4)$

II. f has a minimum value on $[\ 4,\ 7\ ]$ **IV**. $\lim\limits_{x\to 6} f(x)=f(6)$

A. **0** B. **1** C. **2** D. **3** E. **4**

366. If $f(x)=\begin{cases} 2x^2+3 & \text{if } x\geq 1 \\ g(x) & \text{if } x<1 \end{cases}$, then f will be continuous at $x=1$ if $g(x)=$

A. x B. $\cos(x+4)$ C. $6-x$ D. x^2+2 E. $2x^2-3$

367. Given that $f(x)=|x-3|+2$, which one of the following statements is false ?

A. f is continuous at $x=3$ B. f is differentiable at $x=3$ C. $f'(5)=1$

D. $f'(0)=-1$ E. $f(2)=f(4)$

368. Given that f is a function, ***how many*** of the following statements are true ?

I. if f is continuous at $x=c$, then $f'(c)$ exists

II. if $f'(c)$ exists, then f is continuous at $x=c$

III. $\lim\limits_{x\to c} f(x)=f(c)$

IV. if f is continuous on (a, b), then f is continuous on $[a, b]$

A. **0** B. **1** C. **2** D. **3** E. **4**

369. Which of the following is a point of discontinuity for $f(x)=\dfrac{x^2-4}{x^2+2x-3}$

A. -3 B. 2 C. 0 D. -1 E. -2

370. ***How many*** of the following functions are **NOT** continuous over the set of real numbers ?

I $y=\dfrac{x}{x^4+1}$ **II** $y=|x+1|$ **III** $y=\dfrac{14}{x^{16}-9}$ **IV** $y=x^{\frac{3}{5}}$

A. 0 B. 1 C. 2 D. 3 E. 4

371. The graph of $y=\dfrac{x^2-9}{3x-9}$ has

A. a vertical asymptote at $x=3$

B. a horizontal asymptote at $y=\dfrac{1}{3}$

C. a removable discontinuity at $x=3$

D. an infinite discontinuity at $x=3$

E. ***none of these***

372. The function $f(x)=\begin{cases}\dfrac{x^2}{x} & \text{if } x\neq 0\\ 0 & \text{if } x=0\end{cases}$

A. is continuous everywhere

B. is continuous except at $x=0$

C. has a removable discontinuity at $x=0$

D. has an infinite discontinuity at $x=0$

E. has $x=0$ as a vertical asymptote

373. Suppose $f(x)=\begin{cases}\dfrac{3x(x-1)}{x^2-3x+2} & \text{for } x\neq 1,2\\ -3 & \text{for } x=1\\ 4 & \text{for } x=2\end{cases}$ then $f(x)$ is continuous

A. ***except at*** $x=1$

B. ***except at*** $x=2$

C. ***except at*** $x=1$ ***or*** 2

D. ***except at*** $x=0,\ 1,$ ***or*** 2

E. ***at each real number***

374. Suppose $\lim\limits_{x\to -3^-} f(x)=-1$; $\lim\limits_{x\to -3^+} f(x)=-1$; $f(-3)$ is not defined. Which, if any, of the following statements may be false ?

A. $\lim\limits_{x\to -3} f(x)=-1$

B. f has a removable discontinuity at $x=-3$

C. if we redefine $f(-3)$ to be equal to -1, then the new function will be continuous at $x=-3$

D. f is continuous everywhere except at $x=-3$

E. all of the preceding statements are true

375. Let $f(x)=\begin{cases}\dfrac{x^2+x}{x} & \text{if } x\neq 0\\ 1 & \text{if } x=0\end{cases}$ Which of the following statements, **I**, **II**, and **III**, are true ?

I. $f(0)$ exists **II.** $\lim\limits_{x\to 0} f(x)$ exists **III.** f is continuous at $x=0$

A. **I** ***only*** B. **II** ***only*** C. **I** ***and*** **II** ***only*** D. ***all of them*** E. ***none of them***

376. A function $f(x)$ equals $\dfrac{x^2-x}{x-1}$ for all x except $x=1$. In order that the function be continuous at $x=1$, the value of $f(1)$ must be

A. 0 B. 1 C. 2 D. ∞ E. ***none of these***

377. If $f(x)=\begin{cases} x^2 & \text{for } x\le 1 \\ 2x-1 & \text{for } x>1 \end{cases}$, then

A. $f(x)$ is not continuous at $x=1$
B. $\lim\limits_{x\to 1} f(x)$ does not exist
C. $f'(1)$ exists and equals 1
D. $f'(1)=2$
E. $f(x)$ is continuous at $x=1$ but $f'(1)$ does not exist

378. Suppose $f(x)=\begin{cases} x^2 & \text{if } x<-2 \\ 4 & \text{if } -2<x\le 1 \\ 6-x & \text{if } x>1 \end{cases}$ Which statement is true ?

A. f is discontinuous only at $x=-2$
B. f is discontinuous only at $x=1$
C. f is discontinuous at $x=-2$ and at $x=1$
D. f is continuous everywhere
E. if $f(-2)$ is defined to be 4, then f will be continuous everywhere

379. Which statement is true ?

A. If $f(x)$ is continuous at $x=c$, then $f'(c)$ exists
B. If $f'(c)=0$, then f has a local maximum or minimum at $(c, f(c))$
C. If $f''(c)=0$, then f has an inflection point at $(c, f(c))$
D. If f is differentiable at $x=c$, then f is continuous at $x=c$
E. If f is continuous on (a, b), then f attains a maximum value on (a, b)

380. Determine a value of k such that $f(x)$ is continuous, where $f(x)=\begin{cases} 3kx-5 & \text{for } x>2 \\ 4x-5k & \text{for } x\le 2 \end{cases}$

A. 1 B. $\dfrac{13}{11}$ C. $\dfrac{3}{11}$ D. $-\dfrac{3}{11}$ E. -3

381. Find the value of k such that $f(x)=\begin{cases} kx-1 & \text{for } x<2 \\ kx^2 & \text{for } x\ge 2 \end{cases}$ is continuous for all real numbers.

A. 1 B. $\dfrac{1}{2}$ C. $-\dfrac{1}{6}$ D. $-\dfrac{1}{2}$ E. ***none of these***

382. If $f'(a)$ does **NOT** exist, which of the following **MUST** be true ?

A. $f(x)$ is discontinuous at $x=a$
B. $\lim\limits_{x\to a} f(x)$ does not exist
C. f has a vertical tangent at $x=a$
D. f has a "hole" for $x=a$
E. none of these is necessarily true

383. The function $f(x)=\frac{x^2+5x+6}{x^2-4}$ has

A. only a removable discontinuity at $x=-2$

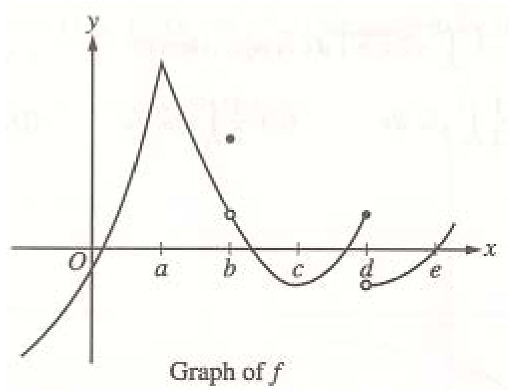
B. only a removable discontinuity at $x=2$

C. a removable discontinuity at $x=-2$ and a nonremovable discontinuity at $x=2$

D. removable discontinuities at $x=-2$ and $x=-3$

E. nonremovable discontinuities at $x=2$ and $x=-3$

384. The graph of a function f is shown. At which value of x is f continuous, but not differentiable ?

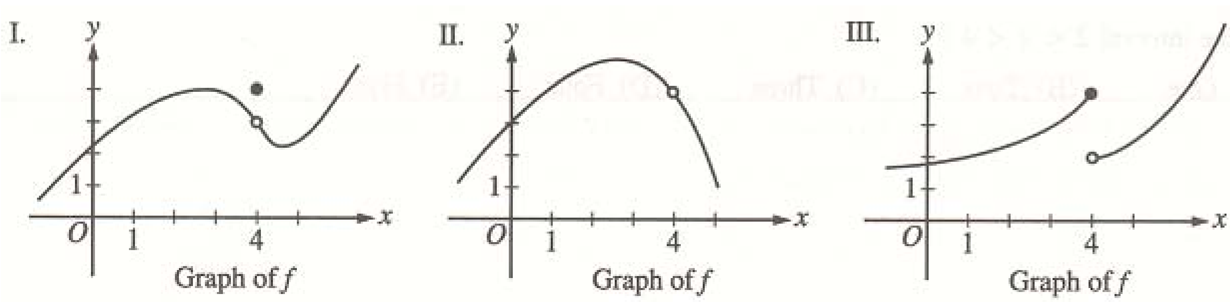


A. a B. b C. c D. d E. e

385. If $f(x)=\begin{cases} \dfrac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}, & \text{for } x\neq 2 \\ k, & \text{for } x=2 \end{cases}$ and if f is continuous at $x=2$, then $k=$

A. 0 B. $\frac{1}{6}$ C. $\frac{1}{3}$ D. 1 E. $\frac{7}{5}$

386. For which of the following does $\lim_{x\to 4} f(x)$ exist ?



A. **I** *only* B. **II** *only* C. **III** *only*

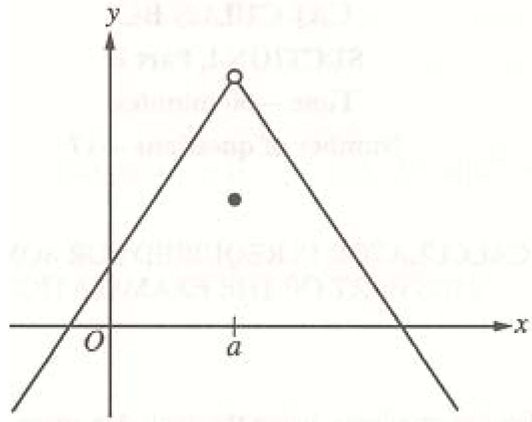
D. **I** *and* **II** *only* E. **I** *and* **III** *only*

387. Which of the following functions shows that the statement " If a function is continuous at $x=0$, then it is differentiable at $x=0$ " is false ?

A. $f(x)=x^{-\frac{4}{3}}$ B. $f(x)=x^{-\frac{1}{3}}$ C. $f(x)=x^{\frac{1}{3}}$ D. $f(x)=x^{\frac{4}{3}}$ E. $f(x)=x^{3}$

388.

The graph of the function f is shown in the diagram. Which of the following statements must be false ?



Graph of f

A. $f(a)$ exists

B. $f(x)$ is defined for $0 < x < a$

C. f is not continuous at $x = a$

D. $\lim_{x \to a} f(x)$ exists

E. $\lim_{x \to a} f'(x)$ exists

389. Let g be a continuous function on the closed interval $[0, 1]$. Let $g(0) = 1$ and $g(1) = 0$
Which of the following is **NOT** necessarily true ?

A. There exists a number h in $[0, 1]$ such that $g(h) \geq g(x)$ for all x in $[0, 1]$

B. For all a and b in $[0, 1]$, if $a = b$, then $g(a) = g(b)$

C. There exists a number h in $[0, 1]$ such that $g(h) = \frac{1}{2}$

D. There exists a number h in $[0, 1]$ such that $g(h) = \frac{3}{2}$

E. For all h in the open interval $(0, 1)$, $\lim_{x \to h} g(x) = g(h)$

390. At $x = 3$ the function given by $f(x) = \begin{cases} x^2, & \text{for } x < 3 \\ 6x - 9, & \text{for } x \geq 3 \end{cases}$ is

A. undefined

B. continuous but not differentiable

C. differentiable but not continuous

D. neither continuous nor differentiable

E. both continuous and differentiable

391. Let f be the function defined by the following $f(x) = \begin{cases} \sin x & \text{for } x < 0 \\ x^2 & \text{for } 0 \leq x < 1 \\ 2 - x & \text{for } 1 \leq x < 2 \\ x - 3 & \text{for } x \geq 2 \end{cases}$

For what values of x is f **NOT** continuous ?

A. 0 only B. 1 only C. 2 only D. 0 and 2 only E. 0, 1 and 2

392. Let f be a function such that $\lim\limits_{h\to 0}\dfrac{f(2+h)-f(2)}{h}=5$ Which of the following must be true ?

I. f is continuous at $x=2$ **II.** f is differentiable at $x=2$

III. The derivative of f is continuous at $x=2$

A. **I** ***only*** B. **II** ***only*** C. **I** ***and*** **II** ***only***

D. **I** ***and*** **III** ***only*** E. **II** ***and*** **III** ***only***

393. If f is continuous for $a \le x \le b$ and differentiable for $a < x < b$, which of the following could be false ?

A. $\int_a^b f(x)\,dx$ exists

B. $f'(c)=0$ for some c such that $a<c<b$

C. f has a minimum value on $a \le x \le b$

D. f has a maximum value on $a \le x \le b$

E. $f'(c)=\dfrac{f(b)-f(a)}{b-a}$ for some c such that $a<c<b$

394. If $f(x)=\begin{cases} \ln x & \text{for } 0<x\le 2 \\ x^2 \ln 2 & \text{for } 2<x\le 4 \end{cases}$, then $\lim\limits_{x\to 2} f(x)=$

A. $\ln 2$ B. $\ln 8$ C. $\ln 16$ D. 4 E. ***nonexistent***

395. Let f be the function given by $f(x)=|x|$. Which of the following statements about f are true ?

I. f is continuous at $x=0$

II. f is differentiable at $x=0$

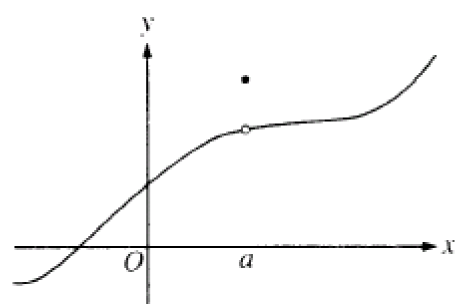
III. f has an absolute minimum at $x=0$

A. **I** ***only*** B. **II** ***only*** C. **III** ***only***

D. **I** ***and*** **III** ***only*** E. **II** ***and*** **III** ***only***

396.

The graph of a function f is shown in the diagram. Which of the following statements about f is false ?



A. f is continuous at $x=a$

B. f has a relative maximum at $x=a$

C. $x=a$ is in the domain of f

D. $\lim\limits_{x\to a^+} f(x)$ is equal to $\lim\limits_{x\to a^-} f(x)$

E. $\lim\limits_{x\to a} f(x)$ exists

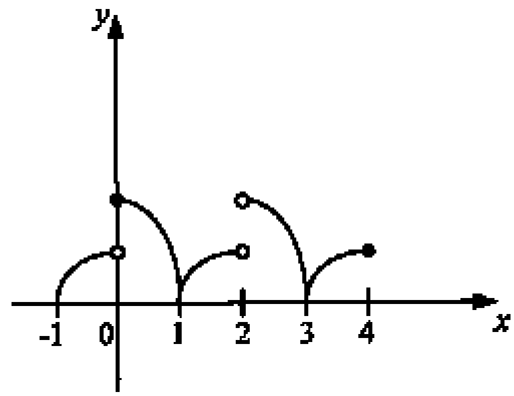
397. Let f be defined as follows, where $a \neq 0$. $f(x)=\begin{cases} \dfrac{x^2-a^2}{x-a} & \text{for } a \neq 0 \\ 0 & \text{for } x = a \end{cases}$

Which of the following are true about f

I. $\lim_{x\to a} f(x)$ exists **II.** $f(a)$ exists **III.** $f(x)$ is continuous at $x = a$

A. ***none***
B. **I** ***only***
C. **II** ***only***
D. **I** ***and*** **II** ***only***
E. **I, II** ***and*** **III**

398. The function shown is defined on the closed interval $-1 \leq x \leq 4$ for



A. all x
B. all x except $x = 0$
C. all x except $x = 1$
D. all x except $x = 2$
E. all x except $x = 0$ and $x = 2$

399. Let $f(x)=\begin{cases} \dfrac{x^3+8}{x+2}, & \text{if } x \neq -2 \\ 4, & \text{if } x = -2 \end{cases}$

Which of the following four statements are true ?

I. $f(x)$ is defined at $x = -2$
II. $f(x)$ is continuous at $x = -2$
III. $\lim_{x\to -2} f(x)$ exists
IV. $f(x)$ is differentiable at $x = -2$

A. **I** ***only***
B. **I** ***and*** **II** ***only***
C. **I** ***and*** **III** ***only***
D. **II** ***and*** **III** ***only***
E. **I, II, III** ***and*** **IV**

400. For what value of k is the function $y=\begin{cases} x+k & \text{if } x<2 \\ x^2+4 & \text{if } x \geq 2 \end{cases}$ continuous at $x = 2$

A. **2**
B. **4**
C. **6**
D. **8**
E. **10**

401. The function $f(x)=\dfrac{2x-2}{\ln x}$ is defined for all $x > 0$ except $x = 1$. The value that must be assigned to $f(1)$ to make $f(x)$ continuous at $x = 1$ is

A. -2
B. -1
C. 0
D. 1
E. 2

402. If the function defined by $f(x)=\begin{cases} x^3 & \text{if } x \leq 2 \\ 2x+k & \text{if } x > 2 \end{cases}$ is continuous at $x = 2$, then $k =$

A. **0**
B. **2**
C. **4**
D. **6**
E. **8**

403. The function $f(x)=\dfrac{e^{-x+1}-1}{\ln x}$ is defined for all $x>0$ except $x=1$ The value that must be assigned to $f(1)$ to make $f(x)$ continuous at $x=1$ is

A. -1 B. $-\dfrac{1}{e}$ C. 0 D. $\dfrac{1}{e}$ E. 1

404. The function $f(x)=\dfrac{x^3-8}{x-2}$ is not defined at $x=2$

Which of the following expressions for $f(2)$ can be used to make $f(x)$ continuous at $x=2$

I. $f(2)=12$

II. $f(2)=\lim\limits_{x\to 2} f(x)$

III. $f(2)=\lim\limits_{x\to 2^-} f(x)$

A. $\mathbf{I}$ *only* B. $\mathbf{II}$ *only* C. $\mathbf{I}$ *and* $\mathbf{II}$ D. $\mathbf{II}$ *and* $\mathbf{III}$ E. $\mathbf{I},\mathbf{II}$ *and* $\mathbf{III}$

405. What value should be assigned to $f(x)=\dfrac{x}{e^x-1}$ at $x=0$ to make $f(x)$ continuous at $x=0$

A. -1 B. 0 C. $\dfrac{1}{2}$ D. 1 E. *none of these*

406. If $f(x)=\begin{cases} e^{-x}+2 & \text{for } x<0 \\ ax+b & \text{for } x\ge 0 \end{cases}$ is differentiable at $x=0$ then $a+b=$

A. 0 B. 1 C. 2 D. 3 E. 4

407. Suppose that f is a continuous function defined for all real numbers x with $f(-5)=3$ and $f(-1)=-2$ If $f(x)=0$ for one and only one value of x, then which of the following could be x

A. -7 B. -2 C. 0 D. 1 E. 2

408. If $f(x)=\begin{cases} x^2+2 & \text{for } x\le 1 \\ 2x+1 & \text{for } x>1 \end{cases}$ then $f'(1)=$

A. $\dfrac{1}{2}$ B. 1 C. 2 D. 3 E. *does not exist*

409. If f is a continuous function defined by $f(x)=\begin{cases} x^2+bx & \text{for } x\le 5 \\ 5\sin(\frac{\pi}{2}x) & \text{for } x>5 \end{cases}$ then $b=$

A. -6 B. -5 C. -4 D. 4 E. 5

410. Consider the function $f(x)=\begin{cases} \frac{\sin x}{x} & \text{for } x\ne 0 \\ k & \text{for } x=0 \end{cases}$ In order for $f(x)$ to be continuous at $x=0$, the value of k must be

A. 0 B. 1 C. -1 D. π

E. a number greater than 1

411. Which function is **NOT** continuous everywhere ?

A. $y=|x|$ B. $y=x^{\frac{2}{3}}$ C. $y=\sqrt{x^2+1}$ D. $y=\dfrac{x}{x^2+1}$ E. $y=\dfrac{4x}{(x+1)^2}$

412. Consider the function f defined on $\dfrac{\pi}{2}\le x\le\dfrac{3\pi}{2}$ by $f(x)=\dfrac{\tan x}{\sin x}$ for all $x\ne\pi$ If f is continuous at $x=\pi$ then $f(\pi)=$

A. 2 B. 1 C. 0 D. -1 E. -2

413. If the function f is continuous for all positive real numbers and if $f(x)=\dfrac{\ln x^2-x\ln x}{x-2}$ when $x\ne 2$ then $f(2)=$

A. -1 B. -2 C. $-e$ D. $-\ln 2$ E. *undefined*

414. Which of the following functions is both continuous and differentiable at all x in the interval $-2\le x\le 2$

A. $f(x)=|x^2-1|$ B. $f(x)=\sqrt{x^2-1}$ C. $f(x)=\sqrt{x^2+1}$

D. $f(x)=\dfrac{1}{x^2-1}$ E. *none of these*

415. Let f be defined by $f(x)=\begin{cases}\frac{x^2-2x+1}{x-1} & \text{for } x\ne 1\\ k & \text{for } x=1\end{cases}$ Determine the value of k for which f is continuous for all real x

A. 0 B. 1 C. 2 D. 3 E. *none of these*

416. Which of the following is continuous at $x=0$

I. $f(x)=|x|$ **II.** $f(x)=e^x$ **III.** $f(x)=\ln(e^x-1)$

A. **I** *only* B. **II** *only* C. **I** *and* **II** *only*

D. **II** *and* **III** *only* E. *none*

417. If f is continuous at $x=2$, and if $f(x)=\begin{cases}\dfrac{\sqrt{x+2}-\sqrt{2x}}{x-2} & \text{for } x\ne 2\\ k & \text{for } x=2\end{cases}$ then $k=$

A. $-\dfrac{1}{2}$ B. $-\dfrac{1}{4}$ C. 0 D. $\dfrac{1}{4}$ E. $\dfrac{1}{2}$

418. Which of the following values for k makes the function $f(x)=\begin{cases}\ln(x+k) & \text{for } 0<x<3\\ \cos(kx) & \text{for } x\le 0\end{cases}$ continuous at $x=0$

A. 0 B. 1 C. $\dfrac{\pi}{2}$ D. e E. π

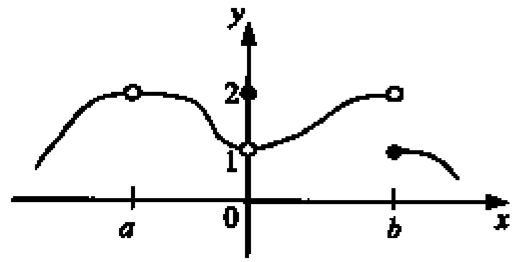
419. Which of the following functions are continuous but not differentiable at $x = 0$

$\mathbf{I.}\ f(x) = x^{\frac{1}{3}}$ $\quad$ $\mathbf{II.}\ g(x) = |x|$ $\quad$ $\mathbf{III.}\ h(x) = x|x|$

A. **I *only*** $\quad$ B. **II *only*** $\quad$ C. **I *and* II *only***

D. **II *and* III *only*** $\quad$ E. **I, II *and* III**

420. The graph of the function f is shown in the diagram. Which of the following statements about f is true ?



A. $f(a)$ exists $\quad$ B. $\lim_{x\to a} f(x) = 2$ $\quad$ C. $\lim_{x\to b} f(x) = 1$

D. $\lim_{x\to b^-} f(x) = \lim_{x\to b^+} f(x)$ $\quad$ E. f is continuous at $x = 0$

421. If $f(x) = \begin{cases} \dfrac{|x|-2}{x-2} & \text{for } x \neq 2 \\ k & \text{for } x = 2 \end{cases}$ then the value of k for which $f(x)$ is continuous for all real values of x is $k =$

A. -2 $\quad$ B. -1 $\quad$ C. 0 $\quad$ D. 1 $\quad$ E. 2

422. What is the $\lim_{x\to \ln 2} g(x)$ given $g(x) = \begin{cases} e^x & \text{if } x > \ln 2 \\ 4 - e^x & \text{if } x \leq \ln 2 \end{cases}$

A. -2 $\quad$ B. $\ln 2$ $\quad$ C. e^2 $\quad$ D. 2 $\quad$ E. ***nonexistent***

423. A function f is continuous on $[\,1,\ 5\,]$ and some of the values of f are shown in the table. If f has only one root r on the closed interval $[\,1,\ 5\,]$, and $r \neq 3$ then a possible value of b is

x	1	3	5
$f(x)$	-2	b	-1

A. -1 $\quad$ B. 0 $\quad$ C. 1 $\quad$ D. 3 $\quad$ E. 5

424. If $\lim_{x\to a} f(x) = \mathrm{L}$ where **L** is a real number, which of the following must be true ?

$\mathbf{I.}\ f(a) = \mathrm{L}$ $\quad$ $\mathbf{II.}\ \lim_{x\to a^-} f(x) = \mathrm{L}$ $\quad$ $\mathbf{III.}\ \lim_{x\to a^+} f(x) = \mathrm{L}$

A. **I *only*** $\quad$ B. **I *and* II *only*** $\quad$ C. **I *and* III *only***

D. **II *and* III *only*** $\quad$ E. **I, II *and* III**

425. Which of the following functions are continuous for all real numbers x

$\mathbf{I.}\ f(x) = |x|$ $\quad$ $\mathbf{II.}\ f(x) = \tan x$ $\quad$ $\mathbf{III.}\ f(x) = 3x^2 + x - 7$

A. **I *only*** $\quad$ B. **II *only*** $\quad$ C. **III *only***

D. **I *and* II *only*** $\quad$ E. **I *and* III *only***

426. Let $f(x)=\dfrac{x^3-2x^2-29x-42}{x^2-9}$ Which of the following statements is true ?

A. $f(x)$ has a removable discontinuity at $x=-3$

B. $f(x)$ has a jump discontinuity at $x=3$

C. If $f(3)=-\frac{5}{3}$ then $f(x)$ is continuous at $x=3$

D. $f(x)$ has nonremovable discontinuities at $x=-3$ and $x=3$

E. $\lim_{x\to -3} f(x)=\infty$

427. The function f is continuous on the closed interval $[-2,\ 1]$. Some values of f are shown in the table. The equation $f(x)=\frac{3}{2}$ must have at least two solutions in the interval $[-1,\ 1]$ if $k=$

x	-2	-1	0	1
$f(x)$	-3	7	k	3

A. 1 B. $\dfrac{3}{2}$ C. 2 D. $\dfrac{5}{2}$ E. 3

428. Which of the following statements are always true ?

I. A function that is continuous at $x=c$ must be differentiable at $x=c$

II. A function that is differentiable at $x=c$ must be continuous at $x=c$

III. A function that is ***not*** continuous at $x=c$ must ***not*** be differentiable at $x=c$

IV. A function that is ***not*** differentiable at $x=c$ must ***not*** be continuous at $x=c$

A. ***none of them*** B. **I *and* III *only*** C. **II *and* III *only***
D. **II *and* IV *only*** E. **I, II, III *and* IV**

429. A function $f(x)$ is equal to $\dfrac{x^2-6x+9}{x-3}$ for all $x>0$ except $x=3$. In order for the function to be continuous at $x=3$, what must the value of $f(3)$ be ?

A. 5 B. 4 C. 2 D. 1 E. 0

430. If the function f is continuous for all real numbers and if $f(x)=\dfrac{2x^2+x-15}{x+3}$ when $x\neq -3$, then $f(-3)=$

A. -15 B. -11 C. -3 D. 0 E. $\dfrac{5}{2}$

431. A function $f(x)$ is equal to $\dfrac{x^2-4}{x-2}$ for all $x>0$ except $x=2$. In order for the function to be continuous at $x=2$, what must the value of $f(2)=$

A. -4 B. -2 C. 0 D. 2 E. 4

432. A function $f(x)$ is equal to $\dfrac{\sin 2x}{x}$ for all x except $x=0$. In order for the function to be continuous at $x=0$, what must the value of $f(0)$ be ?

A. 2 B. 1 C. 0 D. -1 E. -2

433. If the function f is continuous for all real numbers and if $f(x)=\dfrac{x^2-7x+12}{x-4}$ when $x \neq 4$, then $f(4)=$

A. 1 B. $\dfrac{8}{7}$ C. -1 D. 0 E. *undefined*

434. If $f(x)=\begin{cases} x^2+5 & \text{if } x<2 \\ 7x-5 & \text{if } x\geq 2 \end{cases}$, for all real numbers x, which of the following must be true ?

I. $f(x)$ is continuous everywhere
II. $f(x)$ is differentiable everywhere
III. $f(x)$ has a local minimum at $x=2$

A. **I** ***only*** B. **I** ***and*** **II** ***only*** C. **II** ***and*** **III** ***only*** D. **I** ***and*** **III** ***only*** E. **I, II,** ***and*** **III**

435. Let $f(x)=\begin{cases} x+2a, & \text{if } x<1 \\ ax^2+7x-4, & \text{if } x\geq 1 \end{cases}$ If a is such that $f(x)$ is continuous at $x=1$, is $f(x)$ also differentiable at $x=1$ Justify your answer.

436. $f(x)=\begin{cases} bx^3+7, & \text{if } x<3 \\ ax^2+3, & \text{if } x\geq 3 \end{cases}$ Find the values of a and b such that $f(x)$ is differentiable at $x=3$

437. If $f(x)=\begin{cases} 3x^2+5 & \text{if} & x<1 \\ x^3+2x+5 & \text{if} & 1\leq x\leq 4 \\ x+c & \text{if} & x>4 \end{cases}$ for what value of c is $f(x)$ continuous at $x=4$

438. For what value of c is the function defined by $f(x)=\begin{cases} -x & \text{if} & x<-1 \\ -x^2+x+c & \text{if} & -1\leq x\leq 2 \\ 2x-3 & \text{if} & x>2 \end{cases}$ continuous at $x=-1$

439. $f(x)=\begin{cases} cx^2+5 & \text{if } x<1 \\ ax+b & \text{if } x\geq 1 \end{cases}$ The slope of the line tangent to the graph of $f(x)$ is -3 at $x=-1$ Find the values of a, b and c such that $f'(x)$ is defined for all real numbers.

440. Given $f(x)=\begin{cases} ax^2 & \text{if } x\leq 2 \\ bx-6 & \text{if } x>2 \end{cases}$ find the value of a that will make $f(x)$ differentiable at $x=2$

441. The function $f(x)=\dfrac{3x^2-3x}{x^2-1}$ is not defined at $x=\pm 1$ What value should be assigned to $f(1)$ to make $f(x)$ continuous at that point ?

442. Given $f(x)=\begin{cases} ax^2+bx+1 & \text{if } x<\frac{1}{2} \\ x^3+2 & \text{if } x\geq \frac{1}{2} \end{cases}$ find the values of a and b that will make $f(x)$ differentiable at $x=\frac{1}{2}$

443. The function $f(x)=\begin{cases} ax^3 & \text{if } x\leq 2 \\ 2x+k & \text{if } x>2 \end{cases}$ is differentiable at $x=2$. Find a and k

444. The position s of a moving particle at time t is given by $s(t)=\begin{cases} ct^2 & \text{if } t\leq 4 \\ 3t+d & \text{if } t>4 \end{cases}$ Find the constants c and d if the path of motion and the velocity are both continuous at $t=4$

445. $f(x)=\begin{cases} cx^2+\frac{9}{2} & \text{if } x<3 \\ ax+b & \text{if } x\geq 3 \end{cases}$ The slope of the line tangent to the graph of $f(x)$ at $x=-1$ is 1. Find the values of a, b and c such that $f'(x)$ is defined over the domain of f

446. Let f be defined by $f(x)=\begin{cases} ax+b & \text{if } x<2 \\ x^2+x+1 & \text{if } x\geq 2 \end{cases}$ Find the values of a and b such that f is differentiable at $x=2$

447. What value must be assigned to $f\left(\dfrac{1}{2}\right)$ if $f(x)=\dfrac{2x^2+5x-3}{2x^2-9x+4}$ is to be continuous at $x=\dfrac{1}{2}$

448. A function is defined for $-2<x<2$ by $f(x)=\begin{cases} 5x^2+ax+b & \text{if } -2<x\leq 0 \\ (2x+4)^{\frac{5}{2}} & \text{if } 0<x<2 \end{cases}$ Find the values of a and b if $f(x)$ is differentiable at $x=0$

449. $f(x)=\dfrac{1-e^{2x}}{1-e^{x}}$ What value must be assigned to $f(x)$ at $x=0$ to make $f(x)$ continuous at $x=0$

450. Given $f(x)=\begin{cases} ax+b & \text{if } x<\frac{1}{2} \\ 3x^2 & \text{if } x\geq \frac{1}{2} \end{cases}$ find the values of a and b for which this function is differentiable at $x=\dfrac{1}{2}$

451. If $f(x)=\begin{cases}\dfrac{2x-6}{x-3} & x\neq 3\\ 5 & x=3\end{cases}$, then $\lim\limits_{x\to 3} f(x)=$

A. 5 B. 1 C. 2 D. 6 E. 0

452. If $f(x)=\begin{cases}x+1 & x\leq 1\\ 3+ax^2 & x>1\end{cases}$, then $f(x)$ is continuous for all x if $a=$

A. 1 B. -1 C. $\frac{1}{2}$ D. 0 E. -2

453. If $f(x)=\dfrac{1}{10-\sqrt{x^2+64}}$ is not continuous at c, then $c=$

A. 6 B. -6 C. ± 6 D. ± 4 E. 5

454. Which of the following satements is/are true ?

I If f is continuous everywhere, then f is differentiable everywhere

II If f is differentiable everywhere, then f is continuous everywhere

III If f is continuous and $f(x)\geq 2$ for every x in $[3,\ 7]$, then $\int_3^7 f(x)\,dx>8$

A. **I *only*** B. **II *only*** C. **III *only*** D. **I *and* III *only*** E. **II *and* III *only***

455. A particle moves along a straight line. Its velocity is $V(t)=\begin{cases}t^2 & \text{for } 0\leq t\leq 2\\ t+2 & \text{for } t\geq 2\end{cases}$

The distance travelled by the particle in the interval $1\leq t\leq 3$ is

A. 3 B. 7.1 C. 4.3 D. 5 E. 6.8

456. For what value of c is $f(x)=\begin{cases}-cx+5, & x<-1\\ 3x^2+2, & x\geq -1\end{cases}$ continuous ?

A. 3 B. -3 C. 6 D. ***none*** E. 0

457. Use $f(x)=\begin{cases}2-x^2 & \text{for } x\geq 0\\ 2+x & \text{for } x<0\end{cases}$ and find $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

A. 0 B. 1 C. -1 D. 2 E. ***none of these***

458. If f is the function given by $f(x)=\begin{cases}0, & \text{if } x\leq 0\\ x, & \text{if } x\geq 0\end{cases}$ which of the following statements are true ?

I. $\lim\limits_{x\to 0} f(x)=0$ **II.** f is continuous at $x=0$ **III.** f is differentiable at $x=0$

A. **I *only*** B. **II *only*** C. **I *and* II *only*** D. **I *and* III *only*** E. **I, II, *and* III**

459. Which of the following functions shows that the statement " If a function is continuous at $x=0$, then it is differentiable at $x=0$ " is **FALSE** ?

A. $f(x)=x^2$ B. $f(x)=x^{\frac{5}{2}}$ C. $f(x)=x^{-\frac{4}{5}}$ D. $f(x)=x^{-\frac{5}{4}}$ E. $f(x)=x^{\frac{1}{2}}$

460. If f is a function such that $\lim_{x\to 0}\frac{f(x)-f(2)}{x-2}=0$ then which of the following must be true ?

I. f is continuous at $x=2$ **II.** f is differentiable at $x=2$ **III.** $f'(2)=0$

A. **I *only*** B. **II *only*** C. **I *and* II *only*** D. **II *and* III *only*** E. **I, II, *and* III**

461. Which of the following must be true if f is a continuous function on $[\,a,\ b\,]$

I. f is differentiable on $(\,a,\ b)$

II. $f(c)=0$, for some c in $(\,a,\ b)$

III. There exists c and d in $[\,a,\ b]$ for which $f(c)\le f(x)\le f(d)$ for all x in $[\,a,\ b\,]$

A. **I *only*** B. **II *only*** C. **III *only*** D. **I *and* II *only*** E. **II *and* III *only***

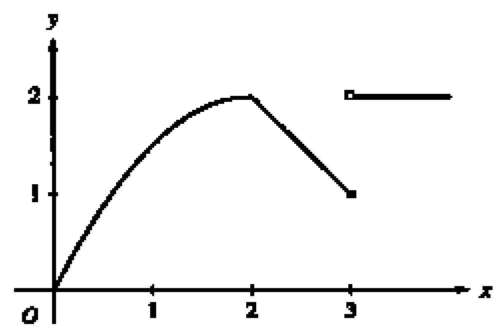
462. Let $f(x)=\begin{cases}\dfrac{x^2-1}{x-1} & \text{if } x\ne 1\\ 4 & \text{if } x=1\end{cases}$ Which of the following statements, **I**, **II** and **III** are true ?

I. $\lim_{x\to 1} f(x)$ exists **II.** $f(1)$ exists **III.** f is continuous at $x=1$

A. ***I only*** B. ***II only*** C. ***I and II only***

D. ***all of them*** E. ***none of them***

463. The graph of the function f is shown in the diagram. Which of the following statements about f is false ?



A. $f(x)$ is continuous at $x=2$

B. $\lim_{x\to 2} f(x)=\lim_{x\to 3^+} f(x)$

C. $f(3)$ is not defined

D. $f(x)$ is not differentiable at $x=2$

E. $f'(3)$ does not exist

464. Let $f(x)=\begin{cases}\dfrac{e^x-1}{x}, & x\ne 0\\ 1, & x=0\end{cases}$ Which of the following statements is true ?

A. f is continuous at $x=0$ B. $\lim_{x\to 0^+} f(x)=0$ C. $\lim_{x\to 0^+} f(x)>1$

D. $\lim_{x\to 0^-} f(x)<1$ E. $\lim_{x\to 0^-} f(x)$ does not exist

465. The function f is given by $f(x)=\begin{cases}\ln 2x, & \text{for } 0<x<2\\ 2\ln x, & \text{for } x\ge 2\end{cases}$ The limit $\lim_{x\to 2} f(x)=$

A. 0 B. $\frac{1}{2}$ C. 1 D. $2\ln 2$ E. ***nonexistent***

466. The function f is given by

$$f(x)=\begin{cases}\dfrac{x^3}{|x|}, & x\neq 0\\ 0, & x=0\end{cases}$$

I. f is continuous at the point $x=0$

II. f is differentiable at the point $x=0$

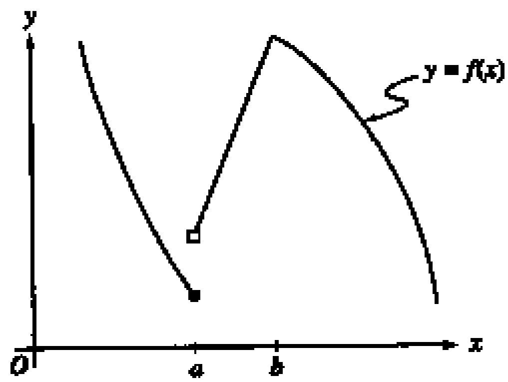
III. $x=0$ is a point of inflection for a graph of f

Which of the following statements are true ?

A. **I *only***
B. **III *only***
C. **I *and* II *only***
D. **I *and* III *only***
E. **I, II *and* III**

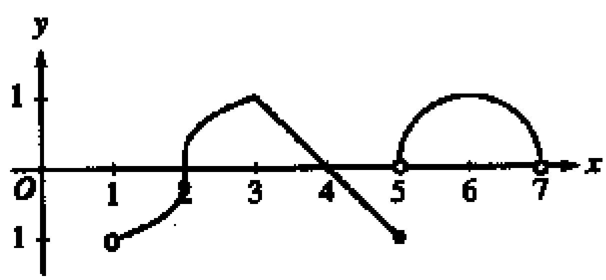
467.

The graph of f is shown in the diagram. Which of the following statements about f is false ?



A. $\lim\limits_{x\to b} f(x)$ exists
B. $\lim\limits_{x\to a^+} f(x)$ exists
C. f has a relative minimum at $x=a$
D. f has a relative maximum at $x=b$
E. f' is continuous at $x=b$

468. The function f, whose graph is shown in the diagram, is defined on the open interval $(1,\ 7)$. It has a vertical tangent when $x=2$ and a horizontal tangent when $x=6$ For what values of x, $1<x<7$, is f not differentiable ?



A. **2** only
B. **3** and **5** only
C. **5** and **6** only
D. **2**, **3** and **5** only
E. **2**, **3**, **5** and **6** only

469. If f is the function given by $f(x)=\begin{cases}e^2x, & x<0\\ \cos x+1, & x\geq 0\end{cases}$ then $\lim\limits_{x\to 0} f(x)=$

A. **0**
B. **1**
C. **2**
D. **3**
E. ***nonexistent***

470. Determine c so that $f(x)$ is continuous everywhere when $f(x)=\begin{cases}x-2 & \text{for } x\leq 5\\ cx-3 & \text{for } x>5\end{cases}$

A. 0
B. $\dfrac{6}{5}$
C. 1
D. $\dfrac{5}{6}$
E. ***none of these***

471. Let f be the function given by $f(x)=\begin{cases} x+2 & \text{if } x\le 3 \\ 4x-7 & \text{if } x>3 \end{cases}$ Which of the following statements are true about f

I. $\lim\limits_{x\to 3} f(x)$ exists **II.** f is continuous at $x=3$ **III.** f is differentiable at $x=3$

A. *none* B. **I** *only* C. **II** *only*

D. **I** *and* **II** *only* E. **I, II** *and* **III**

472. Let m and b be real numbers and let the function f be defined by

$$f(x)=\begin{cases} 1+3bx+2x^2 & \text{for } x\le 1 \\ mx+b & \text{for } x>1 \end{cases}$$

If f is both continuous and differentiable at $x=1$ then

A. $m=1,\ b=1$ B. $m=1,\ b=-1$ C. $m=-1,\ b=1$ D. $m=-1, b=-1$ E. *none of these*

473. The function f is continuous at $x=1$

If $f(x)=\begin{cases} \dfrac{\sqrt{x+3}-\sqrt{3x+1}}{x-1} & \text{for } x\ne 1 \\ k & \text{for } x=1 \end{cases}$ then $k=$

A. 0 B. 1 C. $\dfrac{1}{2}$ D. $-\dfrac{1}{2}$ E. *none of these*

474. The function f is defined on all the reals such that $f(x)=\begin{cases} x^2+kx-3 & \text{for } x\le 1 \\ 3x+b & \text{for } x>1 \end{cases}$

For which of the following values of k and b will the function be both continuous and differentiable on its entire domain ?

A. $k=-1,\ b=-3$ B. $k=1,\ b=3$ C. $k=1,\ b=4$

D. $k=1,\ b=-4$ E. $k=-1,\ b=6$

475. The function $f(x)=\begin{cases} 4-x^2 & \text{for } x\le 1 \\ mx+b & \text{for } x>1 \end{cases}$ is continuous and differentiable for all real numbers.

The values of m and b are

A. $m=2,\ b=1$ B. $m=2,\ b=5$ C. $m=-2,\ b=1$ D. $m=-2,\ b=5$ E. *none of these*

476. The function f is defined for all real numbers by $f(x)=\begin{cases} e^{-x}+3 & \text{for } x>0 \\ ax+b & \text{for } x\le 0 \end{cases}$

If f is differentiable at $x=0$, then $a+b=$

A. 0 B. 1 C. 2 D. 3 E. 4

477. Find the values of a and b that assure that $f(x)=\begin{cases} \ln(3-x) & \text{if } x<2 \\ a-bx & \text{if } x\ge 2 \end{cases}$ is differentiable at $x=2$

A. $a=3,\ b=1$ B. $a=1,\ b=2$ C. $a=2,\ b=1$ D. $a=1,\ b=3$ E. $a=-2,\ b=-1$

478.

$\boxed{\text{Trig derivatives}} \rightarrow$

Trig limits $\rightarrow \lim_{x\to 0} \frac{\sin x}{x} = 1$

$\rightarrow \lim_{x\to 0} \frac{1-\cos x}{x} = 0$

(squeeze theorem proofs pg 80)

y	y'
$\sin x$	$\cos x$
$\sec x$	$\sec x \tan x$
$\tan x$	$\sec^2 x$
$\cos x$	$-\sin x$
$\csc x$	$-\csc x \cot x$
$\cot x$	$-\csc^2 x$

479. Find the derivative of $y = \sin(x^2)$

A. $\cos(x^2)$ B. $2\sin x \cos x$ C. $2x\sin(x^2)$ D. $2x\sin x$ E. $2x\cos(x^2)$

480. If $y = \ln(\tan x)$ then $y' =$

A. $\frac{2}{\sin 2x}$ B. $\sec^2 x$ C. $\frac{1}{x\tan x}$ D. $\cot x$ E. $\sec^2 x \tan x$

481. Find the derivative of $y = e^x \sin x$

A. $e^x \cos x$ B. $e^x + \cos x$ C. $e\cos x$ D. $\ln(\sin x)$ E. $e^x(\sin x + \cos x)$

482. The derivative of $y = \tan(x^2)$ is

A. $\sec^2(x^2)$ B. $2x\sec^2(x^2)$ C. $2x\sec(x^2)$

D. $\sec(x^2)$ E. $2x\sec^2(x^2)\tan(x^2)$

483. If $y = \ln e^{\tan^2 x}$ then $y'\left(\frac{\pi}{4}\right) =$

A. -2 B. 1 C. 2 D. $2\sqrt{2}$ E. 4

484. Given $f(x) = x\cos x$ the second derivative of $f(x) =$

A. $-x\sin x$ B. $-\cos x$ C. $-x\cos x$

D. $-x\cos x - 2\sin x$ E. $x\sin x$

485. Find the derivative of $\cos^2 x$

A. $\sin^2 x$ B. $1-\sin^2 x$ C. $2\cos x$ D. $-2\sin x\cos x$ E. $-2\sin(2x)$

486. Find the value of the derivative of $e^x \sin x$ at $x = \pi$

A. 0 B. 1 C. e D. e^π E. $-e^\pi$

487. The y-intercept of the line tangent to $y = x\sin x$ at $x = \pi$ is

A. $-\pi$ B. π C. $-\pi^2$ D. π^2 E. 1

488. If $f(x)=\ln(\sin x)$ then $f'\left(\frac{\pi}{4}\right)=$

A. $-\frac{1}{2}\ln 2$ B. $\frac{\sqrt{2}}{2}$ C. 0 D. 1 E. *undefined*

489. If $y=\sin 2x-x$ then $y'\left(\frac{\pi}{2}\right)=$

A. -3 B. -1 C. 0 D. 1 E. *undefined*

490. The number of critical points of the function $f(x)=-x\sin x$ on $[-6,\ 6]$ is

A. 2 B. 3 C. 4 D. 5 E. *infinite number*

491. If $y=3\tan^2\left(\frac{x}{3}\right)$ then $y'(\pi)=$

A. 1 B. $\frac{8\sqrt{3}}{3}$ C. 9 D. $8\sqrt{3}$ E. 27

492. If $f(x)=2\sin^2 5x$ then $f'(x)=$

A. $10\sin 10x$ B. $20\sin 5x$ C. $10\sin 5x$ D. $4\cos 5x$ E. $20\cos 5x$

493. If $y=e^{\sin x}$, then $\frac{dy}{dx}=$

A. $e^{\sin x}$ B. $e^{\cos x}$ C. $e^x\sin x$ D. $e^{\sin x}\cos x$ E. $e^{1-\sin x}$

494. If $g(x)=x^2\cos x$ then $g'(x)=$

A. $2x\cos x$ B. $-x^2\sin x$ C. $x^2\sin x+2x$
D. $2x\cos x-x^2\sin x$ E. $2x\cos x+x^2\sin x$

495. If $f(x)=\cos 3x$, find all values in the interval $(0,\ \pi)$ for which $f^{(3)}(x)=0$

A. $\frac{\pi}{6}$ B. $\frac{\pi}{4}$ C. $\frac{\pi}{3},\ \frac{2\pi}{3}$ D. $\frac{\pi}{6},\ \frac{5\pi}{6}$ E. $\frac{\pi}{2}$

496. The normal line to $y=2\sin x$ at $x=\frac{\pi}{3}$ intersects the x-axis at $(x_1,\ 0)$ What is the value of x_1

A. $\frac{1}{2}$ B. $\frac{\pi+6}{3}$ C. $\frac{2\pi}{3}$ D. $\frac{\pi+3\sqrt{3}}{3}$ E. $\frac{\pi}{6}$

497. If $f(x)=\sin x+\cos x+e^x$, then $f'(x)=$

A. $-\cos x+\sin x+1$ B. $\cos x-\sin x+e^x$ C. $-\cos x-\sin x+e^x$
D. $x\sin x+x\cos x+e^x$ E. $\cos x-\sin x+1$

498. If $y = \sin u$, $u = 3w$, and $w = e^{2x}$, then $\frac{dy}{dx} =$

A. $6e^{2x}\cos(3e^{2x})$
B. $3\cos e^{2x}$
C. $e^{2\cos(3e^{2x})}$
D. $-6\sin(6e^{2x})$
E. $6x\cos e^{2x}$

499. In the interval $[\,0,\ \pi\,]$, where are the inflection points for the function $y = \sin^2 x - \cos^2 x$

A. $x = 0$ *and* $x = \pi$
B. $x = \frac{\pi}{2}$
C. $x = \frac{\pi}{4}$ *and* $x = \frac{3\pi}{4}$
D. $x = \frac{\pi}{3}$ *and* $x = \frac{2\pi}{3}$
E. ***no*** points of inflection in the indicated interval

500. Find y', if $y = \ln(\sec x + \tan x)$

A. $\sec x$
B. $\frac{1}{\sec x}$
C. $\tan x + \frac{\sec^2 x}{\tan x}$
D. $\frac{1}{\sec x + \tan x}$
E. $-\frac{1}{\sec x + \tan x}$

501. Find $\frac{dy}{dx}$ given $y = x^2 \sin\frac{1}{x}$ $(x \neq 0)$

A. $2x\sin\frac{1}{x} - x^2\cos\frac{1}{x}$
B. $-\frac{2}{x}\cos\frac{1}{x}$
C. $2x\cos\frac{1}{x}$
D. $2x\sin\frac{1}{x} - \cos\frac{1}{x}$
E. $-\cos\frac{1}{x}$

502. Find $\frac{dy}{dx}$ if $y = \frac{1}{2\sin 2x}$

A. $-\csc 2x\cot 2x$
B. $\frac{1}{4\cos 2x}$
C. $-4\csc 2x\cot 2x$
D. $\frac{\cos 2x}{2\sqrt{\sin 2x}}$
E. $-\csc^2 2x$

503. Find $\frac{dy}{dx}$, if $y = e^{-x}\cos 2x$

A. $-e^{-x}(\cos 2x + 2\sin 2x)$
B. $e^{-x}(\sin 2x - \cos 2x)$
C. $2e^{-x}\sin 2x$
D. $-e^{-x}(\cos 2x + \sin 2x)$
E. $-e^{-x}\sin 2x$

504. Find $\frac{dy}{dx}$, if $y = \sec^2\sqrt{x}$

A. $\frac{\sec\sqrt{x}\tan\sqrt{x}}{\sqrt{x}}$
B. $\frac{\tan\sqrt{x}}{\sqrt{x}}$
C. $2\sec\sqrt{x}\tan^2\sqrt{x}$
D. $\frac{\sec^2\sqrt{x}\tan\sqrt{x}}{\sqrt{x}}$
E. $2\sec^2\sqrt{x}\tan\sqrt{x}$

505. If $y = a\sin ct + b\cos ct$, where a, b, and c are constants, then $\dfrac{d^2y}{dt^2}$ is

A. $ac^2(\sin t + \cos t)$ B. $-c^2y$ C. $-ay$

D. $-y$ E. $a^2c^2\sin ct - b^2c^2\cos ct$

506. The equation of the tangent to the curve $y = x\sin x$ at the point $\left(\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$ is

A. $y = x - \pi$ B. $y = \dfrac{\pi}{2}$ C. $y = \pi - x$ D. $y = x + \dfrac{\pi}{2}$ E. $y = x$

507. If $x \neq 0$, then the slope of $x\sin\dfrac{1}{x}$ equals zero whenever

A. $\tan\dfrac{1}{x} = x$ B. $\tan\dfrac{1}{x} = -x$ C. $\cos\dfrac{1}{x} = 0$ D. $\sin\dfrac{1}{x} = 0$ E. $\tan\dfrac{1}{x} = \dfrac{1}{x}$

508. If $f(x) = \cos x \sin 3x$ then $f'\left(\dfrac{\pi}{6}\right) =$

A. $\dfrac{1}{2}$ B. $-\dfrac{\sqrt{3}}{2}$ C. 0 D. 1 E. $-\dfrac{1}{2}$

509. If $y = \sin^3(1-2x)$ then $\dfrac{dy}{dx} =$

A. $3\sin^2(1-2x)$ B. $-2\cos^3(1-2x)$ C. $-6\sin^2(1-2x)$

D. $-6\sin^2(1-2x)\cos(1-2x)$ E. $-6\cos^2(1-2x)$

510. $\dfrac{d}{dx}\left(\sin(\cos x)\right) =$

A. $\cos(\cos x)$ B. $\sin(-\sin x)$ C. $\left(\sin(-\sin x)\right)\cos x$

D. $-\left(\cos(\cos x)\right)\sin x$ E. $-\left(\sin(\cos x)\right)\sin x$

511. If $y = x^2\sin 2x$ then $\dfrac{dy}{dx} =$

A. $2x\cos 2x$ B. $4x\cos 2x$ C. $2x(\sin 2x + \cos 2x)$

D. $2x(\sin 2x - x\cos 2x)$ E. $2x(\sin 2x + x\cos 2x)$

512. A particle moves along the x-axis so that at any time $t \geq 0$, its velocity is given by $v(t) = 3 + 4.1\cos(0.9t)$ What is the acceleration of the particle at time $t = 4$

A. -2.016 B. -0.677 C. 1.633 D. 1.814 E. 2.978

513. Let f be the function with derivative given by $f'(x) = \sin(x^2+1)$ How many relative extrema does f have on the interval $2 < x < 4$

A. *one* B. *two* C. *three* D. *four* E. *five*

514. If $y = \sin(3x)$ then $\dfrac{dy}{dx} =$

A. $-3\cos(3x)$ B. $-\cos(3x)$ C. $-\dfrac{1}{3}\cos(3x)$ D. $\cos(3x)$ E. $3\cos(3x)$

515. If $f(x) = x + \sin x$ then $f'(x) =$

A. $1 + \cos x$ B. $1 - \cos x$ C. $\cos x$
D. $\sin x - x\cos x$ E. $\sin x + x\cos x$

516. If $y = \cos^2 3x$ then $\dfrac{dy}{dx} =$

A. $-6\sin 3x\cos 3x$ B. $-2\cos 3x$ C. $2\cos 3x$
D. $6\cos 3x$ E. $2\sin 3x\cos 3x$

517. If $y = \cos^2 x - \sin^2 x$ then $y' =$

A. -1 B. 0 C. $-2\sin(2x)$
D. $-2(\cos x + \sin x)$ E. $2(\cos x - \sin x)$

518. If $f(x) = \sin x$, then $f'\left(\dfrac{\pi}{3}\right) =$

A. $-\dfrac{1}{2}$ B. $\dfrac{1}{2}$ C. $\dfrac{\sqrt{2}}{2}$ D. $\dfrac{\sqrt{3}}{2}$ E. $\sqrt{3}$

519. If $y = 2\cos\left(\dfrac{x}{2}\right)$, then $\dfrac{d^2y}{dx^2} =$

A. $-8\cos\left(\dfrac{x}{2}\right)$ B. $-2\cos\left(\dfrac{x}{2}\right)$ C. $-\sin\left(\dfrac{x}{2}\right)$ D. $-\cos\left(\dfrac{x}{2}\right)$ E. $-\dfrac{1}{2}\cos\left(\dfrac{x}{2}\right)$

520. If $y = \tan x - \cot x$, then $\dfrac{dy}{dx} =$

A. $\sec x\csc x$ B. $\sec x - \csc x$ C. $\sec x + \csc x$
D. $\sec^2 x - \csc^2 x$ E. $\sec^2 x + \csc^2 x$

521. If $f(x) = (x-1)^2\sin x$ then $f'(0) =$

A. -2 B. -1 C. 0 D. 1 E. 2

522. An equation of the line tangent to the graph of $y = x + \cos x$ at the point $(0, 1)$ is

A. $y = 2x + 1$ B. $y = x + 1$ C. $y = x$ D. $y = x - 1$ E. $y = 0$

523. If $f(x) = \tan 2x$, then $f'\left(\dfrac{\pi}{6}\right) =$

A. $\sqrt{3}$ B. $2\sqrt{3}$ C. 4 D. $4\sqrt{3}$ E. 8

524. If $f(x)=\sin^2(3-x)$ then $f'(0)=$

A. $-2\cos 3$ B. $-2\sin 3\cos 3$ C. $6\cos 3$ D. $2\sin 3\cos 3$ E. $6\sin 3\cos 3$

525. If $y=3\sin x+4\cos x$ then $y''-y=$

A. $-6\sin x-8\cos x$ B. $-6\sin x+8\cos x$ C. $6\sin x-8\cos x$
D. $6\sin x+8\cos x$ E. 0

526. If $y=\sin x+e^{-x}$ then $y+y''=$

A. 0 B. $2\sin x$ C. $2e^{-x}$ D. $2\sin x+2e^{-x}$ E. $2\sin x-2e^{-x}$

527. If $f(x)=\sqrt{4\sin x+2}$ then $f'(0)=$

A. -2 B. 0 C. 1 D. $\frac{\sqrt{2}}{2}$ E. $\sqrt{2}$

528. The equation of the tangent line to the graph of $y=\cos x+\tan(2x)$ at the point $(0, 1)$ is

A. $y=0$ B. $y=2x+1$ C. $y=2x$ D. $y=2x-1$ E. $y=x+1$

529. If $f(x)=(x-1)^2\cos x$ then $f'(0)=$

A. -2 B. -1 C. 0 D. 1 E. 2

530. If $f(x)=\dfrac{\sin^2 x}{1-\cos x}$ then $f'(x)=$

A. $\cos x$ B. $\sin x$ C. $-\sin x$ D. $-\cos x$ E. $2\cos x$

531. For $x\neq 0$ the slope of the tangent to $y=x\cos x$ equals zero whenever

A. $\tan x=-x$ B. $\tan x=\frac{1}{x}$ C. $\tan x=x$ D. $\sin x=x$ E. $\cos x=x$

532. If $y=\cos^2 x-\sin^2 x$ then $y'=$

A. -1 B. 0 C. $-2(\cos x+\sin x)$
D. $2(\cos x+\sin x)$ E. $-4(\cos x)(\sin x)$

533. If $y=\cos^2(2x)$ then $\dfrac{dy}{dx}=$

A. $2\cos 2x\sin 2x$ B. $4\cos 2x$ C. $2\cos 2x$ D. $-2\cos 2x$ E. $-4\sin 2x\cos 2x$

534. A particle moves along the x-axis and its position for time $t\geq 0$ is $x(t)=\cos(2t)+\sec t$
When $t=\pi$ the acceleration of the particle is

A. -6 B. -5 C. -4 D. -3 E. *none of these*

535. If $g(x)=\tan^2(e^x)$ then $g'(x)=$

A. $2e^x\tan(e^x)\sec^2(e^x)$ B. $2\tan(e^x)\sec^2(e^x)$ C. $2\tan^2(e^x)\sec(e^x)$
D. $e^x\sec^2(e^x)$ E. $2e^x\tan(e^x)$

536.

$\boxed{\textbf{Mean Value Theorem}} \rightarrow$ for derivatives

If $f(x)$ is a function that is continuous on $[a,b]$ and differentiable on (a,b) then there is at least one number $c \in (a,b)$ such that $\underbrace{\boxed{\dfrac{f(b)-f(a)}{b-a}}}_{\text{average rate of change}} = \underbrace{\boxed{f'(c)}}_{\text{instantaneous rate of change}}$

537. How many values of c satisfy the conclusion of the Mean Value Theorem for $f(x)=x^3+1$ on the interval $[-1,\ 1\ ]$

A. 0 B. 1 C. 2 D. 3 E. 4

538. The value of c guaranteed by the Mean Value Theorem for $f(x)=\dfrac{2}{x-1}$ on the interval $[\ 3,\ 5]$

A. $1+2\sqrt{2}$ B. $2\sqrt{2}$ C. $1+\sqrt{2}$ D. 2 E. $1-2\sqrt{2}$

539. Given that $f(x)=x^4-3$, find $c \in (\ 0,\ 2)$ such that $\dfrac{f(2)-f(0)}{2-0}=f'(c)$

A. 1 B. $\sqrt{2}$ C. $\sqrt{3}$ D. $\sqrt[3]{2}$ E. $\sqrt[4]{3}$

540. Given that $f(x)=x^3$, find c such that $f(3)-f(1)=(3-1)f'(c)$

A. 6 B. 13 C. $\sqrt{13}$ D. $\dfrac{27}{2}$ E. $\sqrt{\dfrac{13}{3}}$

541. If $f(x)=2x^3-6x$, at what point on the interval $0 \le x \le \sqrt{3}$, if any, is the tangent to the curve parallel to the secant line ?

A. 1 B. -1 C. $\sqrt{2}$ D. 0 E. ***nowhere***

542. Let f be the function given by $f(x)=x^3-3x^2$ What are all values of c that satisfy the conclusion of the Mean Value Theorem of differential calculus on the closed interval $[\ 0,\ 3\ ]$

A. 0 only B. 2 only C. 3 only D. 0 and 3 E. 2 and 3

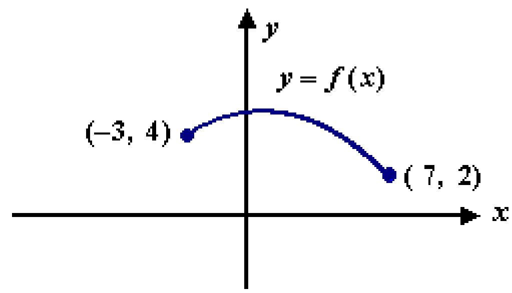
543. Find the point on the graph of $y=\sqrt{x}$ between $(\ 1,\ 1)$ and $(\ 9,\ 3)$ at which the tangent to the graph has the same slope as the line through $(\ 1,\ 1)$ and $(\ 9,\ 3)$

A. $(\ 1,\ 1)$ B. $\left(\ 2,\ \sqrt{2}\right)$ C. $\left(\ 3,\ \sqrt{3}\right)$ D. $(\ 4,\ 2)$ E. ***none of these***

544. If c is the number that satisfies the conclusion of the Mean Value Theorem for $f(x)=x^3-2x^2$ on the interval $0 \le x \le 2$ then $c=$

A. 0 B. $\dfrac{1}{2}$ C. 1 D. $\dfrac{4}{3}$ E. 2

545. The graph of $y = f(x)$ on the closed interval $[-3,\ 7]$ is shown. If f is continuous on $[-3,\ 7]$ and differentiable on $(-3,\ 7)$, then there exist a c, $-3 < c < 7$ such that



A. $f(c) = 0$ B. $f'(c) = -5$ C. $f'(c) = \frac{1}{5}$ D. $f'(c) = -\frac{1}{5}$ E. $f'(c) = \text{undefined}$

546. Let f be the function given by $f(x) = x^3$ What are all values of c that satisfy the conclusion of the **M**ean **V**alue **T**heorem on the closed interval $[\ -1,\ 2\ ]$

A. 0 only B. 1 only C. $\sqrt{3}$ only
D. -1 and 1 E. $-\sqrt{3}$ and $\sqrt{3}$

547. At time $t \ge 0$ the position of a particle moving along the x-axis is given by $x(t) = \frac{t^3}{3} + 2t + 2$ For what value of t in the interval $[\ 0,\ 3\ ]$ will the instantaneous velocity of the particle equal the average velocity of the particle from time $t = 0$ to time $t = 3$

A. 1 B. $\sqrt{3}$ C. $\sqrt{7}$ D. 3 E. 5

548. Let $f(x)$ be a function with a continuous derivative on the interval $(\ 1,\ 3)$ such that $f(1) = 2$ and $f(3) = -4$ Which of the following must be true for some a in $(\ 1,\ 3)$

A. $f'(a) = 6$ B. $f'(a) = 3$ C. $f'(a) = 0$ D. $f'(a) = -3$ E. $f'(a) = -6$

549. There is a point between $\mathbf{P}(1,0)$ and $\mathbf{Q}(e,1)$ on the graph of $y = \ln x$ such that the tangent to the graph at that point is parallel to the line through points **P** and **Q**. The x-coordinate of this point is

A. $e-1$ B. e C. -1 D. $\frac{1}{e-1}$ E. $\frac{1}{e+1}$

550. Let f be a continuous function on the interval $[-1,\ 3\ ]$ If $f(-1) = 9$ and $f(3) = 1$, then the Mean Value Theorem guarantees that

A. $f'(0) = 0$ B. $f'(c) = -2$ for some c between -1 and 3
C. $f'(c) = 2$ for some c between -1 and 3 D. $f(c) = 5$ for some c between -1 and 3
E. $f(c) < 0$ for all c between -1 and 3

551. If $f'(x)$ exists for all x and $f(1) = 10$ and $f(8) = -4$ then, for at least one value of c in the open interval $(\ 1,\ 8)$, which of the following must be true ?

A. $f(c) = 12$ B. $f(c) = -12$ C. $f'(c) = 2$ D. $f'(c) = -2$ E. $f(c^2) = 16$

552. $f(x)$ is a differentiable function with $f(1) = -3$ and $f(5) = 4$ Which of the following must be true ?

A. $f(0) = k$ for some k in $(1, 5)$

B. $f(x)$ is increasing on $(1, 5)$

C. $f'(x) = \frac{7}{4}$ for all x in $(1, 5)$

D. $f'(k) = 0$ for some k in $(1, 5)$

E. $f'(k) = \frac{7}{4}$ for some k in $(1, 5)$

553. Let f be a continuous function on the interval $[-2, 4]$ If $f(-2) = 3$ and $f(4) = -3$, then the Mean Value Theorem guarantees that

A. $f(0) = 0$

B. $f'(c) = -1$ for some c between -2 and 4

C. $f'(c) = 1$ for some c between -3 and 3

D. $f(c) = 1$ for some c between -2 and 4

E. $f(c) = 1$ for some c between -3 and 3

554. There is a point between $\mathbf{P}(-1, 1)$ and $\mathbf{Q}(7, 3)$ on the graph of $y = \sqrt{x+2}$ such that the line tangent to the graph at that point is parallel to the line through **P** and **Q** The coordinates of this point are

A. $\left(1, \sqrt{3}\right)$

B. $(2, 2)$

C. $\left(3, \sqrt{5}\right)$

D. $\left(4, \sqrt{6}\right)$

E. ***none of these***

555. If f is a differentiable function where $f(0) = -1$ and $f(4) = 3$ then which of the following must be true ?

I. there exists a c in $[0, 4]$ where $f(c) = 0$

II. there exists a c in $[0, 4]$ where $f'(c) = 0$

III. there exists a c in $[0, 4]$ where $f'(c) = 1$

A. **I *only***

B. **II *only***

C. **I *and* II *only***

D. **I *and* III *only***

E. **I, II *and* III**

556. Let f be a continuous function on the interval $[-1, 9]$ If $f(-1) = 2$ and $f(9) = 7$, then which of the following are necessarily true ?

I. $f'(c) = \frac{1}{2}$ for some c between -1 and 9

II. $f(c) > 0$ for all c between -1 and 9

III. $f(c) = 5$ for some c between -1 and 9

A. **I *only***

B. **II *only***

C. **I *and* II *only***

D. **I *and* III *only***

E. **I, II, *and* III**

557. The point $(c, f(c))$ on the curve $f(x) = \sqrt{x}$ between $x = a = 0$ and $x = b = 4$ that satisfies $f'(x) = \dfrac{f(b) - f(a)}{b - a}$ is

A. $\left(\dfrac{1}{2}, \dfrac{\sqrt{2}}{2}\right)$

B. $(1, 1)$

C. $\left(2, \sqrt{2}\right)$

D. $\left(3, \sqrt{3}\right)$

E. ***none of these***

558. Let $f(x)$ be a differentiable function defined only on the interval $-2 \le x \le 10$ The table gives the value of $f(x)$ and its derivative $f'(x)$ at several points of the domain. The line tangent to the graph of $f(x)$ and parallel to the segment between the endpoints intersects the y-axis at the point

x	-2	0	2	4	6	8	10
$f(x)$	26	27	26	23	18	11	2
$f'(x)$	1	0	-1	-2	-3	-4	-5

A. $(0,\ 27)$ B. $(0,\ 28)$ C. $(0,\ 31)$ D. $(0,\ 36)$ E. $(0,\ 43)$

559. A function f is continuous for $0 \le x \le 5$ and differentiable for $0 < x < 5$. Given that $f(0) = -2$ and $f(5) = 3$, which of the following statements must be true ?

I. $f'(c) = 1$ for some c such that $0 < c < 5$

II. $f(c) = 0$ for some c such that $0 < c < 5$

III. $f(c) = -1$ for some c such that $0 < c < 5$

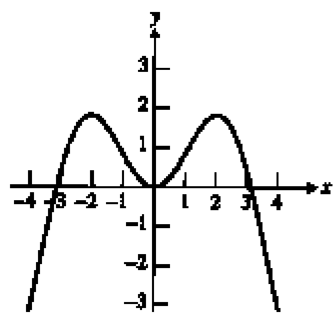
A. **I *only*** B. **II *only*** C. **I *and* II *only***
D. **II *and* III *only*** E. **I, II *and* III**

560. Consider the function $f(x) = \sqrt{x-2}$ On what intervals are the hypotheses of the Mean Value Theorem satisfied ?

A. $[\,0,\ 2\,]$ B. $[\,1,\ 5\,]$ C. $[\,2,\ 7\,]$ D. ***none of these***

561.

Consider the following graph of $f(x) = x\sin x$ on the domain $[-4,\ 4\,]$ How many values of c in $(-4,\ 4)$ appear to satisfy the Mean Value Theorem equation ?



A. none B. one C. two D. three E. four or more

562. The function $f(x)$ is continuous on the closed interval $[-3,\ 5\,]$ and differentiable on the open interval $(-3,\ 5)$. If $f'(x) > 0$ over the interval and if $f(-3) = -4$ and $f(5) = 12$, then $f(-1)$ cannot equal

A. -6 B. -1 C. 4 D. 5 E. 10

563. The function f is continuous and differentiable on the closed interval $[\,0,\ 4\,]$ The table gives selected values of f on this interval. Which of the following statements must be true ?

x	0	1	2	3	4
$f(x)$	2	3	4	3	2

A. The minimum value of f on $[\,0,\ 4\,]$ is 2 B. The maximum value of f on $[\,0,\ 4\,]$ is 4

C. $f(x) > 0$ for $0 < x < 4$ D. $f'(x) < 0$ for $2 < x < 4$

E. There exists c with $0 < c < 4$ for which $f'(c) = 0$

564.

$\boxed{\textbf{Mean Value Theorem}}$ $\rightarrow$ for integrals (text page 283)

If $f(x)$ is a function that is continuous on $[a,b]$, there exists a number $c \in [a,b]$

such that $\int_a^b f(x)dx = (b-a)f(c)$ ***or*** $\dfrac{1}{(b-a)}\int_a^b f(x)dx = \underbrace{\boxed{f(c)}}_{\substack{\text{mean value} \\ \text{of } f(x) \text{ on} \\ [a,b]}}$

565. Find the average value of $f(x) = 2x - x^2$ on the interval $[\ 0,\ 2\ ]$

A. 0 B. $\frac{1}{2}$ C. 1 D. $\frac{2}{3}$ E. $\frac{4}{3}$

566. Find the average value of $f(x) = \sqrt{x}$ on the interval $[\ 1,\ 4\ ]$

A. $\frac{1}{3}$ B. $\frac{7}{9}$ C. $\frac{14}{9}$ D. $\frac{7}{2}$ E. $\frac{14}{3}$

567. What is the average value of $y = 3t^3 - t^2$ over the interval $-1 \le t \le 2$

A. $\frac{11}{4}$ B. $\frac{7}{2}$ C. 8 D. $\frac{33}{4}$ E. 16

568. The average value of $\sqrt{3x}$ on the closed interval $[\ 0,\ 9\ ]$ is

A. $\frac{2\sqrt{3}}{3}$ B. $2\sqrt{3}$ C. 6 D. $6\sqrt{3}$ E. $18\sqrt{3}$

569. What is the average (mean) value of $3t^3 - t^2$ over the interval $-1 \le t \le 2$

A. $\frac{11}{4}$ B. $\frac{7}{2}$ C. 8 D. $\frac{33}{4}$ E. 16

570. The average value of $\sqrt{x}$ over the interval $0 \le x \le 2$ is

A. $\frac{1}{3}\sqrt{2}$ B. $\frac{1}{2}\sqrt{2}$ C. $\frac{2}{3}\sqrt{2}$ D. 1 E. $\frac{4}{3}\sqrt{2}$

571. The average value of $f(x) = x^2\sqrt{x^3+1}$ on the closed interval $[\ 0,\ 2\ ]$ is

A. $\frac{26}{9}$ B. $\frac{13}{3}$ C. $\frac{26}{3}$ D. 13 E. 26

572. What is the average value of y for the part of the curve $y = 3x - x^2$ which is in the first quadrant ?

A. -6 B. -2 C. $\frac{3}{2}$ D. $\frac{9}{4}$ E. $\frac{9}{2}$

573. The average value of the function $y=\sqrt{2x+1}$ from $x=4$ to $x=12$ is

A. $\frac{49}{24}$ B. $\frac{49}{12}$ C. $\frac{97}{23}$ D. $\frac{97}{12}$ E. $\frac{49}{6}$

574. The average value of the function $y=3x^2$ over the interval $1\le x\le 3$ is

A. 2 B. 12 C. 13 D. 24 E. 26

575. The average value of the function $f(x)=3x^2-4$ from $x=2$ to $x=4$ is

A. 6 B. 12 C. 18 D. 24 E. 48

576. The average value of the function $f(x)=4x^3-2x$ over the interval $2\le x\le 3$ is

A. 30 B. 37 C. 60 D. 74 E. 90

577. What is the average (mean) value of $2t^3-3t^2+4$ over the interval $-1\le t\le 1$

A. 0 B. $\frac{7}{4}$ C. 3 D. 4 E. 6

578. The average (mean) value of $\frac{1}{x}$ over the interval $1\le x\le e$ is

A. 1 B. $\frac{1}{e}$ C. $\frac{1}{e^2}-1$ D. $\frac{1+e}{2}$ E. $\frac{1}{e-1}$

579. The average value of $f(x)=e^{2x}+1$ on the intervall $0\le x\le \frac{1}{2}$ is

A. e B. $\frac{e}{2}$ C. $\frac{e}{4}$ D. $2e-1$ E. $\frac{e^{2x}+1}{2}$

580. If $f(x)=\sqrt{x-1}$ then the average value of f over the interval $1\le x\le 5$ is

A. $\frac{1}{4}$ B. $\frac{1}{2}$ C. $\frac{4}{3}$ D. $\frac{8}{3}$ E. $\frac{16}{3}$

581. The average value of $(3x+1)^2$ on the interval -1 to 1 is

A. $-\frac{1}{9}$ B. $\frac{1}{9}$ C. $\frac{2}{9}$ D. 4 E. 8

582. What is the average value of $f(x)=e^{2x}$ over the interval $[\,1,\ 4\,]$

A. e^8-e^2 B. e^6 C. e^4 D. $\frac{e^8-e^2}{6}$ E. $\frac{2(e^8-e^2)}{3}$

583. The average value of $y=(2x+5)^3$ over the interval $[\,1,\ 4\,]$ is

A. 360 B. 618 C. 927 D. 1090 E. 6540

584. The average value of the function $f(x)=(x-1)^2$ on the interval from $x=1$ to $x=5$ is

A. $-\frac{16}{3}$ B. $\frac{16}{3}$ C. $\frac{64}{3}$ D. $\frac{66}{3}$ E. $\frac{256}{3}$

585. The average value of e^{3x} on the interval $[\ 0,\ 4\ ]$ is

A. $\dfrac{e^{12}-1}{12}$ B. $\dfrac{e^{12}}{12}$ C. $\dfrac{e^{12}-1}{4}$ D. $\dfrac{e^{12}}{4}$ E. $\dfrac{e^{12}-1}{3}$

586. The average value of $\dfrac{1}{x}$ on the closed interval $[\ 1,\ 3\ ]$ is

A. $\dfrac{1}{2}$ B. $\dfrac{2}{3}$ C. $\dfrac{\ln 2}{2}$ D. $\dfrac{\ln 3}{2}$ E. $\ln 3$

587. If $\int_a^b f(x)\,dx = 8$, $a = 2$, f is continuous, and the average value of f on $[\ a,\ b]$ is 4, then $b =$

A. 0 B. 2 C. 4 D. 3 E. 5

588. The average value of a continuous function $f(x)$ on the closed interval $[\ 3,\ 7\ ]$ is 12 What is the value of $\int_3^7 f(x)dx$

A. 3 B. 4 C. 12 D. 36 E. 48

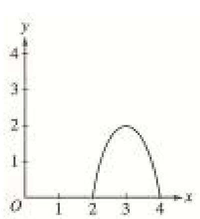
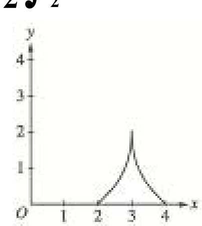
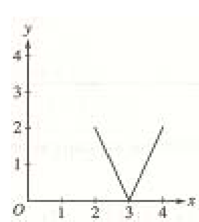
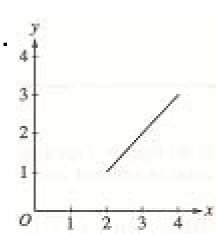
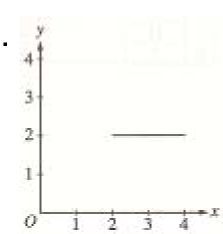
589. The average value of $f(x) = x^3$ over the interval $a \le x \le b$ is

A. $3b + 3a$ B. $b^2 + ab + a^2$ C. $\dfrac{b^3 + a^3}{2}$ D. $\dfrac{b^3 - a^3}{2}$ E. $\dfrac{b^4 - a^4}{4(b-a)}$

590. If the average value of $y = x^2$ over the interval $[\ 1,\ b\ ]$ is $\dfrac{13}{3}$ then the value of b could be

A. $\dfrac{7}{3}$ B. 3 C. $\dfrac{11}{3}$ D. 4 E. $\dfrac{13}{3}$

591. On the closed interval $[\ 2,\ 4\ ]$ which of the following could be the graph of a function f with the property that $\dfrac{1}{4-2}\int_2^4 f(t)\,dt = 1$

A.  B.  C.  D.  E. 

592. Find the average value of $f(x) = x^2 - 2$ on the interval $[\ 0,\ 2\ ]$

A. $-\dfrac{4}{3}$ B. $-\dfrac{2}{3}$ C. 0 D. $\dfrac{2}{3}$ E. $\dfrac{4}{3}$

593. The average value of the function $f(x) = e^{-x^2}$ on the closed interval $[\ -1,\ 1\ ]$ is

A. 0 B. 0.368 C. 0.747 D. 1 E. 1.494

594. The average value of the function $f(x)=\ln^2 x$ on the interval $[\ 2,\ 4\ ]$ is

A. -1.204 B. 1.204 C. 2.159 D. 2.408 E. 8.636

595. The average value of the function $y=e^x$ on the interval from $x=-2$ to $x=2$ is

A. 1.637 B. 1.813 C. 1.881 D. 1.924 E. 2.114

596. The average value of $f(x)=x\ln x$ on the interval $1\le x\le e$ is

A. 0.772 B. 1.221 C. 1.359 D. 1.790 E. 2.097

597. Find the average value of $f(x)=\sqrt[3]{x+3}$ on the interval $[-3,-2]$

A. 0.681 B. 0.750 C. 0.909 D. 1.282 E. 2.280

598. What is the average value of the function $f(x)=\dfrac{x}{x^2+1}$ on the interval $[\ 0,\ 2\ ]$

A. 0.38 B. 0.40 C. 0.42 D. 0.44 E. 0.50

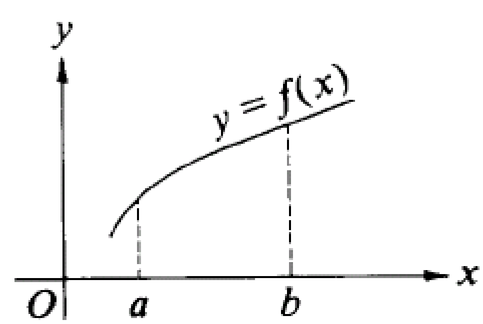
599. If $f(x)=2+|x|$ find the average value of the function f on the interval $-1\le x\le 3$

A. $\dfrac{7}{4}$ B. $\dfrac{9}{4}$ C. $\dfrac{11}{4}$ D. $\dfrac{13}{4}$ E. $\dfrac{15}{4}$

600. If the position of a particle on the x-axis at time t is $-5t^2$, then the average velocity of the particle for $0\le t\le 3$ is

A. -45 B. -30 C. -15 D. -10 E. -5

601. If f is the continuous, strictly increasing function on the interval $a\le x\le b$ as shown, which of the following must be true ?



I. $\int_a^b f(x)dx < f(b)(b-a)$

II. $\int_a^b f(x)dx > f(a)(b-a)$

III. $\int_a^b f(x)dx = f(c)(b-a)$ for some number c such that $a<c<b$

A. I *only* B. II *only* C. III *only*
D. I *and* III *only* E. I, II *and* III

602. If $f(x)$ is a continuous and even function and $\int_0^4 f(x)dx=-5$ and $\int_4^6 f(x)dx=2$ then the average value of $f(x)$ over the interval from $x=-6$ to $x=4$ is

A. -0.2 B. -0.8 C. 0.2 D. 1.2 E. 2

603. $\int e^{\sin x} \cos x \, dx =$

A. $e^{\cos x} + \sin x + C$ B. $e^{\cos x} + C$ C. $e^{\sin x} + C$

D. $e^{\cos x} \sin x + C$ E. *none of these*

604. $\int (\sec^2 x + \sec x \tan x) \, dx =$

A. $\tan x + \csc x + C$ B. $\tan x + \sec x + C$ C. $\frac{\sec^3 x}{3} + \tan^2 x + C$

D. $\ln\left|\sec x + \tan x\right| + C$ E. *none of these*

605. $\int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \tan x \sin x \cot x \csc x \, dx =$

A. $\frac{\pi}{6}$ B. 1 C. $-\frac{\pi}{6}$ D. $\frac{\pi}{12}$ E. $-\frac{\pi}{12}$

606. $\int \sin 7x \, dx =$

A. $\cos 7x + C$ B. $7\sin 7x + C$ C. $-\frac{1}{7}\cos 7x + C$ D. $-7\cos 7x + C$ E. $7\cos\frac{1}{7}x + C$

607. $\int_{0}^{\frac{\pi}{4}} \cos^2 x \sin x \sec x \, dx =$

A. $-\frac{1}{2}$ B. 1 C. $\frac{3}{2}$ D. 2 E. $\frac{1}{4}$

608. $\int \cos(4x+7) \, dx =$

A. $4\sin 4x + C$ B. $4\sin(4x+7) + C$ C. $\frac{1}{4}\sin(4x+7) + C$

D. $-4\sin(4x+7) + C$ E. $-\frac{1}{4}\sin(4x+7) + C$

609. $\int \cos 3x \, dx =$

A. $3\sin 3x + C$ B. $-\sin 3x + C$ C. $-\frac{1}{3}\sin 3x + C$ D. $\frac{1}{3}\sin 3x + C$ E. $\frac{1}{2}\cos^2 3x + C$

610. $\int t\cos(2t)^2 dt =$

A. $\frac{1}{8}\sin(4t^2) + C$ B. $\frac{1}{2}\cos^2(2t) + C$ C. $-\frac{1}{8}\sin(4t^2) + C$

D. $\frac{1}{4}\sin(2t)^2 + C$ E. *none of these*

611. $\int \sin 2\theta \, d\theta =$

A. $\frac{1}{2}\cos 2\theta + C$ B. $-2\cos 2\theta + C$ C. $-\sin^2\theta + C$ D. $\cos^2\theta + C$ E. $-\frac{1}{2}\cos 2\theta + C$

612. $\int \frac{du}{\cos^2 3u} =$

A. $-\frac{\sec 3u}{3} + C$ B. $\tan 3u + C$ C. $u + \frac{\sec 3u}{3} + C$

D. $\frac{1}{3}\tan 3u + C$ E. $\frac{1}{3\cos 3u} + C$

613. $\int \tan\theta \, d\theta =$

A. $-\ln|\sec\theta| + C$ B. $\sec^2\theta + C$ C. $\ln|\sin\theta| + C$ D. $\sec\theta + C$ E. $-\ln|\cos\theta| + C$

614. $\int \frac{dx}{\sin^2 2x} =$

A. $\frac{1}{2}\csc 2x \cot 2x + C$ B. $-\frac{2}{\sin 2x} + C$ C. $-\frac{1}{2}\cot 2x + C$

D. $-\cot x + C$ E. $-\csc 2x + C$

615. $\int \cot 2u \, du =$

A. $\ln|\sin u| + C$ B. $\frac{1}{2}\ln|\sin 2u| + C$ C. $-\frac{1}{2}\csc^2 2u + C$

D. $-\sec 2u + C$ E. $2\ln|\sin 2u| + C$

616. $\int \cos\theta \, e^{\sin\theta} \, d\theta =$

A. $e^{\sin\theta+1} + C$ B. $e^{\sin\theta} + C$ C. $-e^{\sin\theta} + C$

D. $e^{\cos\theta} + C$ E. $e^{\sin\theta}(\cos\theta - \sin\theta) + C$

617. $\int e^{2\theta} \sin e^{2\theta} \, d\theta =$

A. $\cos e^{2\theta} + C$ B. $2e^{4\theta}(\cos e^{2\theta} + \sin e^{2\theta}) + C$ C. $-\frac{1}{2}\cos e^{2\theta} + C$

D. $-2\cos e^{2\theta} + C$ E. *none of these*

618. $\int_0^{\frac{\pi}{4}} \sin 2\theta \, d\theta =$

A. 2
B. $\frac{1}{2}$
C. -1
D. $-\frac{1}{2}$
E. -2

619. $\int_0^{\pi} \cos^2\theta \sin\theta \, d\theta =$

A. $-\frac{2}{3}$
B. $\frac{1}{3}$
C. 1
D. $\frac{2}{3}$
E. 0

620. $\int_0^{\frac{\pi}{6}} \frac{\cos\theta}{1+2\sin\theta} d\theta =$

A. $\ln 2$
B. $\frac{3}{8}$
C. $-\frac{1}{2}\ln 2$
D. $\frac{3}{2}$
E. $\ln\sqrt{2}$

621. $\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} \frac{\cos 2x}{\sin^2 2x} dx =$

A. $-\frac{1}{4}$
B. 1
C. $\frac{1}{2}$
D. $-\frac{1}{2}$
E. -1

622. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^3\theta \cos\theta \, d\theta =$

A. $\frac{3}{16}$
B. $\frac{1}{8}$
C. $-\frac{1}{8}$
D. $-\frac{3}{16}$
E. $\frac{3}{4}$

623. $\int x \cos x^2 dx =$

A. $\sin x^2 + C$
B. $2\sin x^2 + C$
C. $-\frac{1}{2}\sin x^2 + C$
D. $\frac{1}{4}\cos^2 x^2 + C$
E. $\frac{1}{2}\sin x^2 + C$

624. $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cot x \, dx =$

A. $\ln\frac{1}{2}$
B. $\ln 2$
C. $-\ln\left(2-\sqrt{3}\right)$
D. $\ln\left(\sqrt{3}-1\right)$
E. *none of these*

625. $\int \cos^2 x \sin x \, dx =$

A. $-\frac{\cos^3 x}{3} + C$
B. $-\frac{\cos^3 x \sin^2 x}{6} + C$
C. $\frac{\sin^2 x}{2} + C$
D. $\frac{\cos^3 x}{3} + C$
E. $\frac{\cos^3 x \sin^2 x}{6} + C$

626. $\int_0^{\frac{\pi}{4}} \sin x\, dx =$

A. $-\frac{\sqrt{2}}{2}$ B. $\frac{\sqrt{2}}{2}$ C. $-\frac{\sqrt{2}}{2}-1$ D. $-\frac{\sqrt{2}}{2}+1$ E. $\frac{\sqrt{2}}{2}-1$

627. $\int x^2 \cos(x^3)\, dx =$

A. $-\frac{1}{3}\sin(x^3)+C$ B. $\frac{1}{3}\sin(x^3)+C$ C. $-\frac{x^3}{3}\sin(x^3)+C$

D. $\frac{x^3}{3}\sin(x^3)+C$ E. $\frac{x^3}{3}\sin\left(\frac{x^4}{4}\right)+C$

628. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos x}{\sin x}\, dx =$

A. $\ln\sqrt{2}$ B. $\ln\frac{\pi}{4}$ C. $\ln\sqrt{3}$ D. $\ln\frac{\sqrt{3}}{2}$ E. $\ln e$

629. $\int \sin(2x+3)\, dx =$

A. $\frac{1}{2}\cos(2x+3)+C$ B. $\cos(2x+3)+C$ C. $-\cos(2x+3)+C$

D. $-\frac{1}{2}\cos(2x+3)+C$ E. $-\frac{1}{5}\cos(2x+3)+C$

630. $\int \tan(2x)\, dx =$

A. $-2\ln|\cos(2x)|+C$ B. $-\frac{1}{2}\ln|\cos(2x)|+C$ C. $\frac{1}{2}\ln|\cos(2x)|+C$

D. $2\ln|\cos(2x)|+C$ E. $\frac{1}{2}\sec(2x)\tan(2x)+C$

631. $\int_0^{\frac{\pi}{3}} \sin(3x)\, dx =$

A. -2 B. $-\frac{2}{3}$ C. 0 D. $\frac{2}{3}$ E. 2

632. $\int \sin(2x+3)\, dx =$

A. $-2\cos(2x+3)+C$ B. $-\cos(2x+3)+C$ C. $-\frac{1}{2}\cos(2x+3)+C$

D. $\frac{1}{2}\cos(2x+3)+C$ E. $\cos(2x+3)+C$

633.

Volume Disc $\underbrace{V = \pi\int_a^b [R(x)]^2\, dx}_{\textit{horizontal}\text{ axis of revolution}}$ $\underbrace{V = \pi\int_c^d [R(y)]^2\, dy}_{\textit{vertical}\text{ axis of revolution}}$

Volume Washer $V = \pi\int_a^b \underbrace{[R(x)]^2}_{outer} - \underbrace{[r(x)]^2}_{inner}\, dx = \pi\int_a^b \underbrace{[R(x)]^2}_{disc} dx - \underbrace{\pi\int_a^b [r(x)]^2}_{hole} dx$

Volume Cross Section $V = \int_a^b [\textit{triangle}, \textit{square}, \textit{semicircle}]\, dx$

634. An integral for the volume obtained by revolving, around the x-axis, the region bounded by $y = 2x - x^2$ and the x-axis is

A. $4\pi\int_0^1 x(2x - x^2)dx$ B. $2\pi\int_0^2 x(2x - x^2)dx$ C. $\pi\int_0^2 (2x - x^2)^2 dx$

D. $\pi\int_0^2 \left[(2x)^2 - (x^2)^2\right]dx$ E. *none of these*

635. The region enclosed by the x-axis, the line $x = 3$, and the curve $y = \sqrt{x}$ is rotated about the x-axis. What is the volume of the solid generated ?

A. 3π B. $2\sqrt{3}\pi$ C. $\frac{9}{2}\pi$ D. 9π E. $\frac{36\sqrt{3}}{5}\pi$

636. What is the volume of the solid generated by revolving the area bounded by $y = e^x$, $x = 0$ and $x = 1$ about the x-axis.

A. $\pi(e-1)$ B. $\frac{\pi}{2}e^2$ C. $\frac{\pi}{2}(e^2 - 1)$ D. $\pi(e^2 - 1)$ E. *none of these*

637. If the region enclosed by the graphs of $y = \sqrt{x-1}$, $x = 4$ and the x-axis is revolved about the x-axis, the volume of the solid generated is

A. $2\pi\sqrt{3}$ B. $\frac{7\pi}{2}$ C. 4π D. $\frac{9\pi}{2}$ E. 12π

638. What is the volume of the solid obtained when the region bounded by $x = 4$, $x = 9$, $y = 0$, and $y = \sqrt{x}$ is rotated about the x-axis ?

A. 1 B. 2 C. $\frac{65\pi}{2}$ D. 65 E. 65π

639. Find the volume of the solid formed by rotating the graph of $x^2 + 4y^2 = 4$ about the x-axis.

A. $\frac{8}{3}$ B. $\frac{8}{3}\pi$ C. $\frac{16}{3}$ D. $\frac{32}{3}$ E. $\frac{32}{3}\pi$

640. Which definite integral represents the volume of a sphere with radius $\mathbf{5}$

A. $\pi\int_{-5}^{5}(x^2+25)\,dx$ B. $\pi\int_{0}^{5}(25-x^2)\,dx$ C. $\pi\int_{-5}^{5}(5-x^2)\,dx$

D. $2\pi\int_{0}^{5}(25-x^2)\,dx$ E. $\pi\int_{-5}^{5}(x^2-25)\,dx$

641. What is the volume of the solid obtined by rotating the region bounded by $y=1-x^2$ and $y=0$ about the x-axis ?

A. $2\pi\ units^3$ B. $\frac{16\pi}{15}\ units^3$ C. $\frac{5\pi}{3}\ units^3$ D. $\frac{12\pi}{7}\ units^3$ E. $\pi\ units^3$

642. Which of the following integrals represents the volume of the solid obtained by rotating the region bounded by the graph of $y=-\sqrt{x}$, the x-axis and the line $x=4$ about the x-axis ?

A. $\pi\int_{0}^{4}y^2dy$ B. $\pi\int_{0}^{2}y^2dy$ C. $\pi\int_{0}^{4}\left(\sqrt{x}\right)^2dx$

D. $\pi\int_{0}^{2}\left(-\sqrt{x}\right)^2dx$ E. $\pi\int_{0}^{4}(-x)\,dx$

643. The region bounded by $y=x^{\frac{1}{3}}$, $x=0$, $x=1$, and the x-axis is revolved about the x-axis. In terms of cubic units, what is the volume of the solid generated ?

A. $\frac{3\pi}{5}$ B. $\frac{3\pi}{4}$ C. π D. $\frac{\pi}{3}$ E. 2π

644. The region in the first quadrant bounded by the graph of $y=\sec x$, $x=\frac{\pi}{4}$ and the axes is rotated about the x-axis. What is the volume of the solid generated ?

A. $\frac{\pi^2}{4}$ B. $\pi-1$ C. π D. 2π E. $\frac{8\pi}{3}$

645. The volume of the solid obtained by revolving the region enclosed by the ellipse $x^2+9y^2=9$ about the x-axis is

A. 2π B. 4π C. 6π D. 9π E. 12π

646. The area bounded by $y=e^x$, $x=-1$, $x=1$ and the x-axis is revolved about the x-axis. The volume thus generated is

A. $\pi\left(e^2-\frac{1}{e^2}\right)$ B. $\frac{\pi}{2}\left(e^2+\frac{1}{e^2}\right)$ C. $\frac{\pi}{2}\left(e^2-\frac{1}{e^2}\right)$ D. $\pi\left(e^2+\frac{1}{e^2}\right)$ E. $\frac{\pi}{4}\left(e^2+\frac{1}{e^2}\right)$

647. What is the volume of the solid generated by rotating about the x-axis the region enclosed by the curve $y=\sec x$ and the lines $x=0$, $y=0$ and $x=\frac{\pi}{3}$

A. $\frac{\pi}{\sqrt{3}}$ B. π C. $\pi\sqrt{3}$ D. $\frac{8\pi}{3}$ E. $\pi\ln\left(\frac{1}{2}+\sqrt{3}\right)$

648. Which definite integral represents the volume of a sphere with radius **2**

A. $\pi\int_{-2}^{2}(x^2-4)dx$ B. $\pi\int_{-2}^{2}(x^2+4)dx$ C. $2\pi\int_{0}^{2}(4-x^2)dx$

D. $2\pi\int_{-2}^{2}(4-x^2)dx$ E. $\pi\int_{0}^{2}(4-x^2)dx$

649. The volume of a solid generated by revolving the area bounded by $x=-1$, $x=1$ and $y=e^{-x}$ about the x-axis is

A. $\frac{1}{2}\pi e^2\left(1-\frac{1}{e^4}\right)$ B. $2\pi e^2\left(1-\frac{1}{e^2}\right)$ C. $\frac{1}{2}\pi e^2\left(1-\frac{1}{e^2}\right)$ D. $2\pi e^2\left(1-\frac{1}{e^4}\right)$ E. *none of these*

650. The volume of the solid formed by revolving the region bounded by the graph of $y=(x-3)^2$ and the coordinate axes about the x-axis is given by which of the following integrals ?

A. $\pi\int_{0}^{3}(x-3)^2dx$ B. $\pi\int_{0}^{3}(x-3)^4dx$ C. $2\pi\int_{0}^{3}(x-3)^2dx$

D. $2\pi\int_{0}^{3}x(x-3)^2dx$ E. $2\pi\int_{0}^{3}x(x-3)^4dx$

651. What is the volume of the solid generated by revolving the region bounded by the x-axis and the graph of $y=4x-x^2$ about the x-axis ?

A. $\frac{32\pi}{15}$ B. $\frac{32\pi}{3}$ C. $\frac{256\pi}{15}$ D. $\frac{512\pi}{15}$ E. $\frac{2048\pi}{15}$

652. Let **R** be the region in the first quadrant bounded by the x-axis and the curve $y=2x-x^2$
The volume produced when **R** is revolved about the x-axis is

A. $\frac{16\pi}{15}$ B. $\frac{8\pi}{3}$ C. $\frac{4\pi}{3}$ D. 16π E. 8π

653. The region in the first quadrant between the x-axis from $x=0$ to $x=3$, and the graph $y=x$, is rotated about the x-axis. The volume of the resulting solid of revolution is given by

A. $\int_{0}^{3}\pi x^2dx$ B. $\int_{0}^{3}2\pi x^2dx$ C. $\int_{0}^{3}\pi x\,dx$ D. $\int_{0}^{6}2\pi y^3dy$ E. $\int_{0}^{6}\pi(2+\sqrt{6+y})^2dy$

654. Find the volume of the solid formed by revolving the region bounded by $y=x^3$, $y=1$, and $x=2$ about the x-axis.

A. $\frac{127\pi}{7}$ B. $\frac{120\pi}{7}$ C. $\frac{240\pi}{7}$ D. $\frac{1013\pi}{10}$ E. *none of these*

655. Find the volume of the solid formed by revolving the region bounded by the graphs of $y=-x^2+4$ and $y=0$ about the x-axis.

A. $\frac{1472\pi}{15}$ B. $\frac{736\pi}{15}$ C. $\frac{2944\pi}{15}$ D. $\frac{32\pi}{3}$ E. *none of these*

656. The ellipse $\frac{x^2}{2}+\frac{y^2}{9}=1$ is revolved around the y-axis. The number of cubic units in the resulting solid is

A. 4π B. $\frac{4\pi}{3}$ C. 8π D. $\frac{8\pi}{3}$ E. 6π

657. The region **R** in the first quadrant is enclosed by the lines $x=0$ and $y=5$ and the graph of $y=x^2+1$ The volume of the solid generated when **R** is revolved about the y-axis is

A. 6π B. 8π C. $\frac{34\pi}{3}$ D. 16π E. $\frac{544\pi}{15}$

658. If the region enclosed by the y-axis, the line $y=2$ and the curve $y=\sqrt{x}$ is revolved about the y-axis, the volume of the solid generated is

A. $\frac{32\pi}{5}$ B. $\frac{16\pi}{3}$ C. $\frac{16\pi}{5}$ D. $\frac{8\pi}{3}$ E. π

659. The volume of the solid generated by revolving about the y-axis the region bounded by the graph of $y=x^3$, the line $y=1$ and the y-axis is

A. $\frac{\pi}{4}$ B. $\frac{2\pi}{5}$ C. $\frac{3\pi}{5}$ D. $\frac{2\pi}{3}$ E. $\frac{3\pi}{4}$

660. What is the volume of the solid obtained by rotating the region under the graph $y=\sqrt{x^3+1}$ between $x=1$ and $x=2$, around the x-axis ?

A. $\pi\int_1^2 x^3+1\,dx$ B. $\pi\int_1^2 2\sqrt{x^3+1}\,dx$ C. $\pi\int_0^{\ln 2} x^3+1\,dx$

D. $\pi\int_0^{\ln 2} 2\sqrt{x^3+1}\,dx$ E. $\pi\int_1^{\ln 2} \sqrt{x^3+1}\,dx$

661. A solid is generated when the region in the first quadrant enclosed by the graph of $y=(x^2+1)^3$, the line $x=1$, the x-axis, and the y-axis is revolved about the x-axis. Its volume is found by evaluating which of the following integrals ?

A. $\pi\int_1^8 (x^2+1)^3dx$ B. $\pi\int_1^8 (x^2+1)^6dx$ C. $\pi\int_0^1 (x^2+1)^3dx$

D. $\pi\int_0^1 (x^2+1)^6dx$ E. $2\pi\int_0^1 (x^2+1)^6dx$

662. Let **R** be the region in the first quadrant that is enclosed by the graph of $y=\tan x$, the x-axis, and the line $x=\frac{\pi}{3}$

***a*)** Find the area of **R**

***b*)** Find the volume of the solid formed by revolving **R** about the x-axis.

663. The volume generated by revolving $y = x^3$ around the y-axis is $(-1 \le x \le 1)$

A. π B. 2π C. $\frac{6\pi}{5}$ D. $\frac{2\pi}{5}$ E. $\frac{4\pi}{5}$

664. Let $\mathbf{R}$ be the region bounded by the x-axis, the graph of $y = \sqrt{x}$, and the line $x = 4$

$a)$ Find the area of the region $\mathbf{R}$

$b)$ Find the value of h such that the vertical line $x = h$ divides the region $\mathbf{R}$ into two regions of equal area.

$c)$ Find the volume of the solid generated when $\mathbf{R}$ is revolved about the x-axis.

$d)$ The vertical line $x = k$ divides the region $\mathbf{R}$ into two regions such that when these two regions are revolved about the x-axis, they generate solids with equal volumes. Find the value of k

665. Let $\mathbf{R}$ be the region in the first quadrant bounded by the graph of $y = 8 - x^{\frac{3}{2}}$ the x-axis and and y-axis.

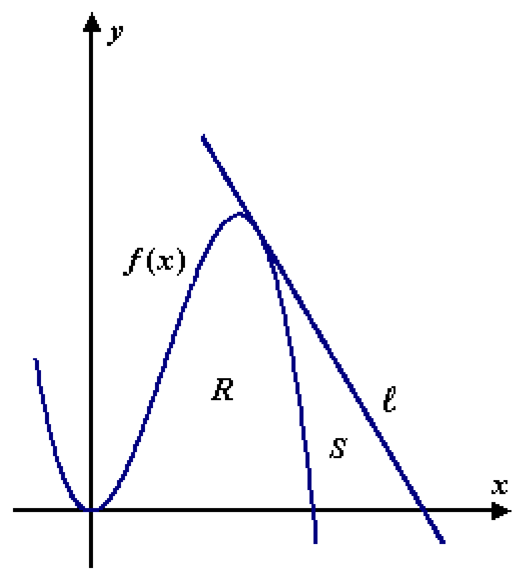
$a)$ Find the area of the region $\mathbf{R}$

$b)$ Find the volume of the solid generated when $\mathbf{R}$ is revolved about the x-axis.

$c)$ The vertical line $x = k$ divides the region $\mathbf{R}$ into two regions such that when these two regions are revolved about the x-axis, they generate solids with equal volumes. Find the value of k

666.

Let f be the function given by $f(x) = 4x^2 - x^3$, and let ℓ be the line $y = 18 - 3x$, where ℓ is tangent to the graph of f. Let $\mathbf{R}$ be the region bounded by the graph of f and the x-axis, and let $\mathbf{S}$ be the region bounded by the graph of f, the line ℓ, and the x-axis, as shown on the right.



$a)$ Show that ℓ is tangent to the graph of $y = f(x)$ at the point $x = 3$

$b)$ Find the area of $\mathbf{S}$

$c)$ Find the volume of the solid generated when R is revolved about the x-axis

667. Find the volume of the solid formed by rotating about the x-axis the region enclosed by the graph of $y = \sqrt{x} + 1$, the x-axis, the y-axis and the line $x = 4$

A. **7.667** B. **9.333** C. **22.667** D. **37.699** E. **71.209**

668. Find the volume of the solid generated when the region bounded by the y-axis, $y = e^x$, and $y = 2$ is rotated around the y-axis.

A. 0.296 B. 0.592 C. 2.427 D. 3.998 E. 27.577

669. Find the volume of the solid formed by rotating the region bounded by the graph of $y = \sqrt{x} + 1$, the y-axis and the line $y = 3$ about the y-axis.

A. 6.40 B. 8.378 C. 20.106 D. 100.531 E. 145.77

670. The area bounded by the curve $y = e^{-x}$ and the lines $y = 0$, $x = 0$ and $x = 10$ is rotated about the x-axis. Which of the following is the best approximation for the volume of the solid of revolution so generated ?

A. 0.78 B. 1.57 C. 2.71 D. 3.15 E. 6.28

671. The region in the first quadrant bounded above by the graph of $y = \sqrt{x}$ and below by the x-axis on the interval $\left[\ 0,\ 4\ \right]$ is revolved about the x-axis. If a plane perpendicular to the x-axis at the point where $x = k$ divides the solid into parts of equal volume, then $k =$

A. 2.77 B. 2.80 C. 2.83 D. 2.86 E. 2.89

672. The region bounded by $y = e^x$, $y = 1$, and $x = 2$ is rotated about the x-axis. The volume of the solid generated is given by the integral:

A. $\pi \int_0^2 e^{2x} dx$ B. $2\pi \int_0^{e^2} (2 - \ln y)(y - 1)\, dy$ C. $\pi \int_0^2 (e^{2x} - 1)\, dx$

D. $2\pi \int_0^{e^2} y(2 - \ln y)\, dy$ E. $\pi \int_0^2 (e^x - 1)^2\, dx$

673. Let $\mathbf{R}$ be the region in the first quadrant enclosed by the lines $x = 0$ and $y = 2$ and the graph of $y = e^x$ The volume of the solid generated when $\mathbf{R}$ is revolved about the x-axis is given by

A. $\pi \int_0^2 (4 - e^{2x}) dx$ B. $\pi \int_0^{\ln 2} (2 - e^x)^2\, dx$ C. $2\pi \int_0^{\ln 2} x(2 - e^x) dx$

D. $\pi \int_0^{\ln 2} (4 - e^{2x}) dx$ E. $2\pi \int_0^2 x(2 - e^x) dx$

674. The volume of the solid generated by rotating about the x-axis the region enclosed between the curve $y = 3x^2$ and the line $y = 6x$ is given by

A. $\pi \int_0^3 (6x - 3x^2)^2 dx$ B. $\pi \int_0^2 (6x - 3x^2)^2 dx$ C. $\pi \int_0^2 (9x^4 - 36x^2) dx$

D. $\pi \int_0^2 (36x^2 - 9x^4) dx$ E. $\pi \int_0^2 (6x - 3x^2) dx$

675. If the region bounded between $y = x^2$ and the horizontal line $y = 1$ is rotated about the x-axis, the volume of the resulting solid of revolution is

A. $\frac{2\pi}{3}$ B. $\frac{4\pi}{5}$ C. $\frac{16\pi}{15}$ D. $\frac{4\pi}{3}$ E. $\frac{8\pi}{5}$

676. The volume of revolution formed by rotating the region bounded by $y = x^3$, $y = x$, $x = 0$, $x = 1$ about the x-axis is represented by

A. $\pi\int_0^1 (x^3 - x)^2\,dx$ B. $\pi\int_0^1 (x^6 - x^2)\,dx$ C. $2\pi\int_0^1 (x^2 - x^6)\,dx$

D. $\pi\int_0^1 (x^2 - x^6)\,dx$ E. $2\pi\int_0^1 (x^6 - x^2)\,dx$

677. Let $\mathbf{R}$ be the region between the graphs of $y = 1$ and $y = \sin x$ from $x = 0$ to $x = \frac{\pi}{2}$. The volume of the solid obtained by revolving $\mathbf{R}$ about the x-axis is given by

A. $2\pi\int_0^{\frac{\pi}{2}} x\sin x\,dx$ B. $2\pi\int_0^{\frac{\pi}{2}} x\cos x\,dx$ C. $\pi\int_0^{\frac{\pi}{2}} (1 - \sin x)^2\,dx$

D. $\pi\int_0^{\frac{\pi}{2}} \sin^2 x\,dx$ E. $\pi\int_0^{\frac{\pi}{2}} (1 - \sin^2 x)\,dx$

678. Find the volume of the solid generated by rotating the area bounded by $y = x^2$ and $x = y^2$ about the x-axis.

679. Let $\mathbf{R}$ be the region in the first quadrant enclosed by the lines $x = \ln 3$ and $y = 1$ and the graph of $y = e^{\frac{x}{2}}$ The volume of the solid generated when $\mathbf{R}$ is revolved about the line $y = -1$ is

A. **5.128** B. **7.717** C. **12.845** D. **15.482** E. **17.973**

680. Let $\mathbf{R}$ be the region in the first quadrant that is enclosed by the graph of $f(x) = \ln(x+1)$, the x-axis and the line $x = e$ What is the volume of the solid generated when $\mathbf{R}$ is rotated about the line $y = -1$

A. **5.037** B. **6.545** C. **10.073** D. **20.146** E. **28.686**

681. If the region bounded between $y = \frac{1}{x}$ and the x-axis between the vertical lines $x = 1$ and $x = e$ is rotated about the line $y = -2$, the volume of the resulting solid of revolution is represented by

A. $\pi\int_1^e \left(\frac{1}{x} + 2\right)^2 dx$ B. $\pi\int_1^e \left(\frac{1}{x^2} + 2\right) dx$ C. $\pi\int_1^e \left(\frac{1}{x} + 2\right) dx$

D. $\pi\int_1^e \left[\left(\frac{1}{x} + 2\right)^2 - 4\right] dx$ E. $\pi\int_1^e \left[\left(\frac{1}{x} + 2\right)^2 + 4\right] dx$

682. What is the approximate volume of the solid obtained by revolving about the x-axis the region in the first quadrant enclosed by the curves $y = x^3$ and $y = \sin x$

A. **0.061** B. **0.438** C. **0.215** D. **0.225** E. **0.278**

683. What is the volume of the solid obtained by rotating the region between $y = \dfrac{6}{x+1}$ and $y = 4 - x$ around the x-axis ?

A. $\pi \int_1^3 (4-x)^2 - \left(\dfrac{6}{x+1}\right)^2 dx$ B. $\pi \int_1^3 \left(\dfrac{6}{x+1}\right)^2 - (4-x)^2 dx$ C. $\pi \int_1^2 (4-x)^2 - \left(\dfrac{6}{x+1}\right)^2 dx$

D. $2\pi \int_1^2 (4-x) - \left(\dfrac{6}{x+1}\right) dx$ E. $2\pi \int_1^3 (4-x) - \left(\dfrac{6}{x+1}\right) dx$

684. Let $\mathbf{R}$ be the region between the curves $y = \sqrt{x^3+1}$ and $y = x+1$, for which x is positive. What is the volume of the solid obtained by rotating $\mathbf{R}$ around the x-axis ?

A. $\pi \int_0^3 (x^3 - x^2 - 2x - 2)\, dx$ B. $\pi \int_0^2 (-x^3 + x^2 + 2x)\, dx$ C. $\pi \int_0^2 (x^3 - x^2 - 2x - 2)\, dx$

D. $\pi \int_1^3 (x^3 - x^2 - 2x + 2)\, dx$ E. $\pi \int_1^2 (-x^3 + x^2 + 2x + 2)\, dx$

685. The region enclosed by the graphs of $y = e^{x-1}$ and $y = -x$ and the vertical lines $x = 0$ and $x = 2$ is rotated about the line $y = -3$. Which of the following gives the volume of the generated solid ?

A. $\pi \int_0^2 \left((e^{x-1} - 3)^2 - (-x-3)^2\right) dx$ B. $\pi \int_0^2 \left((e^{x-1} + 3)^2 - (-x+3)^2\right) dx$

C. $\pi \int_0^2 \left((e^{x-1})^2 - (-x)^2 - 3^2\right) dx$ D. $\pi \int_{-2}^{e} \left((\ln y - 2)^2 - (-y-3)^2\right) dy$

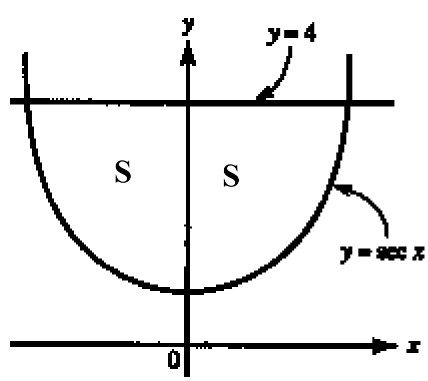
E. $\pi \int_{-2}^{e} \left((\ln y + 4)^2 - (-y+3)^2\right) dy$

686. Find the volume of the solid formed by rotating the region bounded by the graph of $y = \sqrt{x} + 1$, the y-axis and the line $y = 3$ about the line $y = 5$

A. **13.333** B. **17.657** C. **41.888** D. **92.153** E. **242.95**

687.

The region $\mathbf{S}$ in the diagram is bounded by $y = \sec x$ and $y = 4$ What is the volume of the solid formed when $\mathbf{S}$ is rotated about the x-axis ?



A. **0.304** B. **39.867** C. **53.126** D. **54.088** E. **108.177**

688. The region enclosed by the line $x+y=1$ and the coordinate axes is rotated about the line $y=-1$ What is the volume of the solid generated ?

A. $\frac{17\pi}{2}$ B. $\frac{12\pi}{4}$ C. $\frac{2\pi}{3}$ D. $\frac{3\pi}{4}$ E. $\frac{4\pi}{3}$

689. The region in the first quadrant enclosed by the graphs of $y=x$ and $y=2\sin x$ is revolved about the x-axis. The volume of the solid generated is

A. **1.895** B. **2.126** C. **5.811** D. **6.678** E. **13.355**

690. The volume of the solid generated by revolving about the y-axis the region bounded by the graphs of $y=\sqrt{x}$ and $y=x$ is

A. $\frac{2\pi}{15}$ B. $\frac{\pi}{6}$ C. $\frac{2\pi}{3}$ D. $\frac{16\pi}{15}$ E. $\frac{56\pi}{15}$

691. What is the volume of the solid obtained by revolving about the y-axis the region enclosed by the graphs of $x=y^2$ and $x=9$

A. 36π B. $\frac{81\pi}{2}$ C. $\frac{486\pi}{5}$ D. $\frac{1994}{5}$ E. $\frac{1944\pi}{5}$

692. Identify the definite integral that computes the volume of the solid generated by revolving the region bounded by the graph of $y=x^3$ and the line $y=x$ between $x=0$ and $x=1$, about the y-axis.

A. $\pi\int_0^1 (x^2-x^4)dx$ B. $\pi\int_0^1 (y^{\frac{1}{3}}-y)^2\,dy$ C. $\pi\int_0^1 (x^4-x^2)dx$

D. $\pi\int_0^1 (y^{\frac{2}{3}}-y^2)dy$ E. *none of these*

693. The region enclosed by the graph of $y=x^2$, the line $x=2$, and the x-axis is revolved about the y-axis. The volume of the solid generated is

A. 8π B. $\frac{32}{5}\pi$ C. $\frac{16}{3}\pi$ D. 4π E. $\frac{8}{3}\pi$

694. When the region enclosed by graphs of $y=x$ and $y=4x-x^2$ is revolved about the y-axis, the volume of the solid generated is given by

A. $\pi\int_0^3 (x^3-3x^2)\,dx$ B. $\pi\int_0^3 (x^2-(4x-x^2)^2)\,dx$ C. $\pi\int_0^3 (3x-x^2)^2\,dx$

D. $2\pi\int_0^3 (x^3-3x^2)\,dx$ E. $2\pi\int_0^3 (3x^2-x^3)\,dx$

695. The region in the first quadrant enclosed by the y-axis and the graph of $y=\cos x$ and $y=x$ is rotated about the x-axis. The volume of the solid generated is

A. **0.484** B. **0.877** C. **1.520** D. **1.831** E. **3.040**

696. Let **R** denote the region enclosed between the graph of $y = x^2$ and the graph of $y = 2x$

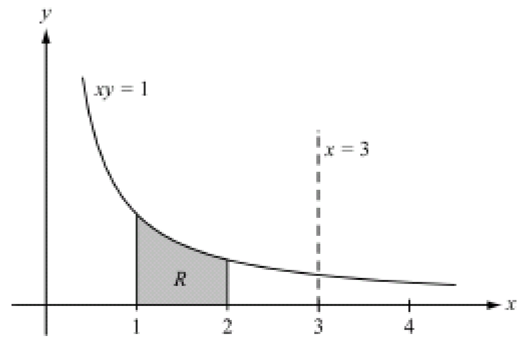
a) Find the area of region **R**

b) Find the volume of the solid obtained by revolving the region **R** about the y-axis.

697. In the figure, the shaded region **R** is bounded by the graphs of $xy = 1$, $x = 1$, $x = 2$ and $y = 0$

a) Find the volume of the solid generated by revolving the region **R** about the x-axis.

b) Find the volume of the solid generated by revolving the region **R** about the line $x = 3$



698. Let **R** be the region bounded by the curves $f(x) = \dfrac{4}{x}$ and $g(x) = (x-3)^2$

a) Find the area of **R**

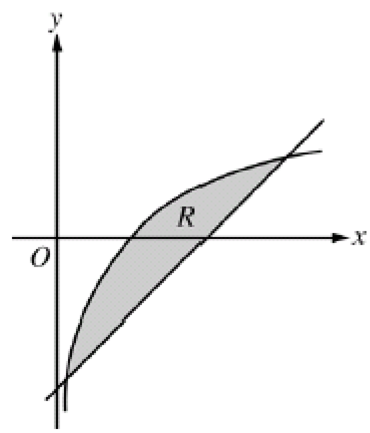
b) Find the volume of the solid generated by revolving **R** about the x-axis.

699. Let **R** be the shaded region bounded by the graph $y = \ln x$ and the line $y = x - 2$, as shown on the right

a) Find the area of **R**

b) Find the volume of the solid generated when **R** is rotated about the horizontal line $y = -3$

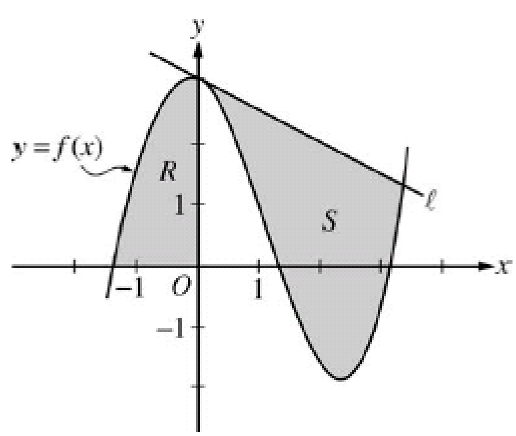
c) Write, but do not evaluate, an integral expression that can be used to find the volume of the solid generated when **R** is rotated about the y-axis



700. Let f be the function given by

$$f(x) = \frac{x^3}{4} - \frac{x^2}{3} - \frac{x}{2} + 3\cos x$$

Let **R** be the shaded region in the second quadrant bounded by the graph of f, and let **S** be the shaded region bounded by the graph of f and the line ℓ, the line tangent to the graph of f at $x = 0$, as shown.



a) Find the area of **R**

b) Find the volume of the solid generated when **R** is rotated about the horizontal line $y = -2$

c) Write, but do not evaluate, an integral expression that can be used to find the area of **S**

701. Consider the closed curve in the xy-plane given by $x^2+2x+y^4+4y=5$

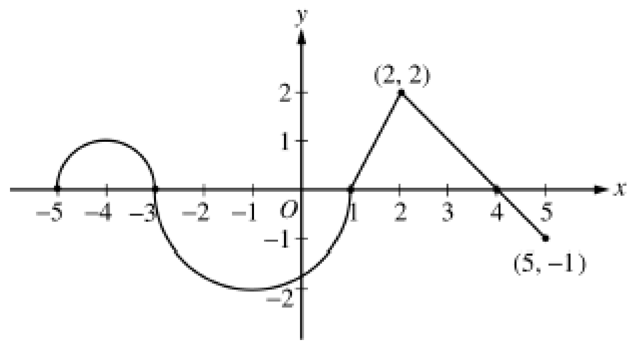
a) Show that $\dfrac{dy}{dx}=\dfrac{-(x+1)}{2(y^3+1)}$

b) Write an equation for the line tangent to the curve at the point $(-2,\ 1)$

c) Find the coordinates of the two points on the curve where the line tangent to the curve is vertical.

d) Is it possible for this curve to have a horizontal tangent at points where it intersects the x-axis ? Explain your reasoning.

702.



Graph of f'

Let f be a function defined on the closed interval $-5 \le x \le 5$ with $f(1)=3$. The graph of f', the derivative of f, consists of two semicircles and two line segments, as shown above.

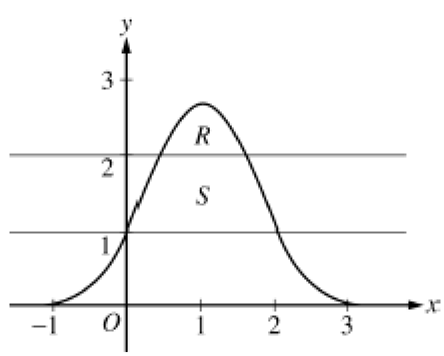
(a) For $-5 < x < 5$, find all values x at which f has a relative maximum. Justify your answer.

(b) For $-5 < x < 5$, find all values x at which the graph of f has a point of inflection. Justify your answer.

(c) Find all intervals on which the graph of f is concave up and also has positive slope. Explain your reasoning.

(d) Find the absolute minimum value of $f(x)$ over the closed interval $-5 \le x \le 5$. Explain your reasoning.

703.



Let R be the region bounded by the graph of $y=e^{2x-x^2}$ and the horizontal line $y=2$, and let S be the region bounded by the graph of $y=e^{2x-x^2}$ and the horizontal lines $y=1$ and $y=2$, as shown above.

(a) Find the area of R.

(b) Find the area of S.

(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y=1$.

704. A particle moves along the x-axis so that its velocity v at time t for $0 \le t \le 5$ is given by $v(t) = \ln(t^2 - 3t + 3)$ The particle is at position $x = 8$ at time $t = 0$

a) Find the acceleration of the particle at time $t = 4$

b) Find all times t in the open interval $0 < t < 5$ at which the particle changes direction. During which time intervals, for $0 \le t \le 5$, does the particle travel to the left ?

c) Find the position of the particle at time $t = 2$

d) Find the average speed of the particle over the interval $0 \le t \le 2$

705. The base of a solid is the region enclosed by $y = \sin x$ and the x-axis on the interval $[\ 0,\ \pi\ ]$ Cross sections perpendicular to the x-axis are semicircles with diameter in the plane of the base. Write an integral that represents the volume of the solid.

A. $\dfrac{\pi}{8}\displaystyle\int_0^{\pi} (\sin x)^2\,dx$ B. $\dfrac{\pi}{8}\displaystyle\int_0^{1} (\sin x)^2\,dx$ C. $\dfrac{\pi}{4}\displaystyle\int_0^{\pi} (\sin x)^2\,dx$

D. $\dfrac{\pi}{8}\displaystyle\int_0^{\pi} \sin x\,dx$ E. $\dfrac{\pi}{2}\displaystyle\int_0^{\pi} (\sin x)^2\,dx$

706. The base of a solid is the region enclosed by the graph of $x = 1 - y^2$ and the y-axis. If all plane cross-sections perpendicular to the x-axis are semicircles with diameters parallel to the y-axis, then the volume is:

A. $\dfrac{\pi}{8}$ B. $\dfrac{\pi}{4}$ C. $\dfrac{\pi}{2}$ D. $\dfrac{3\pi}{4}$ E. $\dfrac{3\pi}{2}$

707. Let the first quadrant region enclosed by the graph of $y = \dfrac{1}{x}$ and the lines $x = 1$ and $x = 4$ be the base of a solid. If cross sections perpendicular to the x-axis are semicircles, the volume of the solid is

A. $\dfrac{3\pi}{64}$ B. $\dfrac{3\pi}{32}$ C. $\dfrac{3\pi}{16}$ D. $\dfrac{3\pi}{8}$ E. $\dfrac{3\pi}{4}$

708. The base of a solid is the region in the first quadrant bounded by the line $x + 2y = 4$ and the coordinate axes. What is the volume of the solid if every cross section perpendicular to the x-axis is a semicircle ?

A. $\dfrac{2\pi}{3}$ B. $\dfrac{4\pi}{3}$ C. $\dfrac{8\pi}{3}$ D. $\dfrac{32\pi}{3}$ E. $\dfrac{64\pi}{3}$

709. The base of a solid is the region enclosed by the ellipse $4x^2 + y^2 = 1$ If all plane cross sections perpendicular to the x-axis are semicircles, then its volume is

A. $\dfrac{\pi}{6}$ B. $\dfrac{\pi}{4}$ C. $\dfrac{\pi}{3}$ D. $\dfrac{\pi}{2}$ E. $\dfrac{2\pi}{3}$

710. The base of a solid is a region enclosed by the circle $x^2 + y^2 = 4$ What is the approximate volume of the solid if the cross sections of the solid perpendicular to the x-axis are semicircles ?

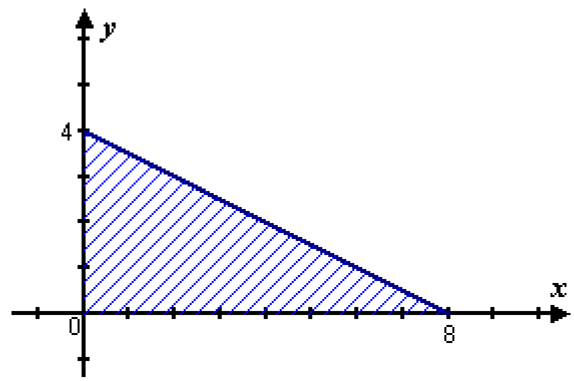
A. 8π B. $\frac{16\pi}{3}$ C. $\frac{32\pi}{3}$ D. $\frac{64\pi}{3}$ E. $\frac{512\pi}{15}$

711. The base of a solid is the region in the first quadrant bounded by the curve $y = \sqrt{\sin x}$ for $0 \le x \le \pi$ If each cross section of the solid perpendicular to the x-axis is a semicircle, the volume of the solid is

A. π B. $\frac{\pi}{2}$ C. $\frac{\pi}{4}$ D. $\frac{\pi}{8}$ E. $\frac{\pi}{12}$

712.

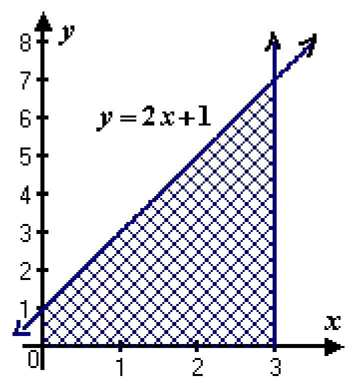
The base of a solid is a region in the first quadrant bounded by the x-axis, the y-axis, and the line $x + 2y = 8$, as shown in the figure on the right. If cross sections of the solid perpendicular to the x-axis are semicircles, what is the volume of the solid ?



A. **12.566** B. **14.661** C. **16.755** D. **67.021** E. **134.041**

713.

The base of a solid is the region in the first quadrant bounded by the x-axis, y-axis, and the lines $y = 2x + 1$ and $x = 3$, as shown in the diagram. If cross-sections of the solid perpendicular to the x-axis are semicircles, what is the volume of the solid ?

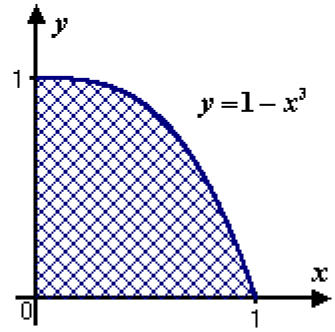


A. **14.137** B. **22.384** C. **28.274** D. **44.768** E. **89.535**

714. The base of a solid is the region enclosed by $y = e^x$, the x-axis, the y-axis and the line $x = \ln 3$ Cross sections perpendicular to the x-axis are squares. Write an integral that represents the volume of the solid.

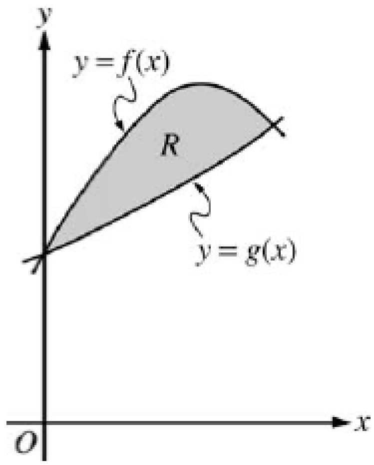
A. $\int_0^{\ln 3} e^x dx$ B. $\int_0^{(\ln 3)^2} e^{2x} dx$ C. $\int_0^{\ln 3} e^{2x} dx$ D. $\pi \int_0^{\ln 3} e^{2x} dx$ E. $\pi \int_0^{3} e^{2x} dx$

715. The base of a solid is a region in the first quadrant bounded by the x-axis, the y-axis, and the graph of $y = 1 - x^3$, as shown in the diagram. If cross-sections of the solid perpendicular to the x-axis are semicircles, what is the volume of the solid ?



A. **0.252** B. **0.505** C. **1.010** D. **2.020** E. **2.356**

716. Let f and g be the functions given by $f(x) = 1 + \sin(2x)$ and $g(x) = e^{\frac{x}{2}}$ Let **R** be the shaded region in the first quadrant enclosed by the graphs of f and g as shown in the diagram.



$a)$ Find the area of **R**

$b)$ Find the volume of the solid generated when **R** is revolved about the x-axis.

$c)$ The region **R** is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles with diameters extending from $y = f(x)$ to $y = g(x)$. Find the volume of this solid.

717. Let **R** be the region in the first and second quadrants bounded above by the graph of $y = \dfrac{20}{1+x^2}$ and below by the horizontal line $y = 2$

$a)$ Find the area of **R**

$b)$ Find the volume of the solid generated when **R** is rotated about the x-axis

$c)$ The region **R** is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles. Find the volume of this solid.

718. The base of a solid is the region enclosed by $y = e^x$ and the lines $y = 1$ and $x = \ln 3$ Cross sections perpendicular to the y-axis are squares. Write an integral that represents the volume of the solid.

A. $\pi \int_1^3 (\ln 3 - \ln y)^2 dy$ B. $\int_1^3 (\ln 3 - \ln y)^2 dy$ C. $\int_0^{\ln 3} (\ln 3 - \ln y)^2 dy$

D. $\int_1^3 ((\ln 3)^2 - (\ln y)^2) dy$ E. $\int_0^1 (e^{2x} - 1) dx$

719. A solid has as its base the region bounded by $y = \sqrt{x}$, the x-axis, and the vertical line $x = 4$. Each cross-section of the solid perpendicular to the y-axis is a square. Which one of the following expressions represents the volume of the solid ?

A. $\int_0^4 \sqrt{x}\,dx$ B. $\int_0^4 x\,dx$ C. $\int_0^2 (4-y^2)\,dy$ D. $\int_0^2 (4-y)^2\,dy$ E. $\int_0^2 (4-y^2)^2\,dy$

720. The base of a solid is the region in the first quadrant enclosed by the graph of $y = 2 - x^2$ and the coordinate axes. If every cross section of the solid perpendicular to the y-axis is a square, the volume of the solid is given by

A. $\pi\int_0^2 (2-y)^2\,dy$ B. $\int_0^2 (2-y)\,dy$ C. $\pi\int_0^{\sqrt{2}} (2-x^2)^2\,dx$

D. $\int_0^{\sqrt{2}} (2-x^2)^2\,dx$ E. $\int_0^{\sqrt{2}} (2-x^2)\,dx$

721. The base of a solid is a region in the first quadrant bounded by the x-axis, the y-axis, the graph of $y = x^2 + 1$, and the vertical line $x = 2$ If cross sections perpendicular to the x-axis are squares, what is the volume of the solid ?

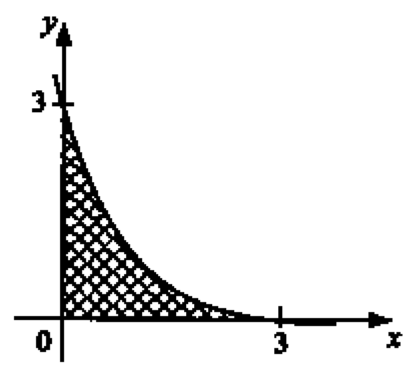
A. 6.400 B. 8.667 C. 10.786 D. 13.733 E. 17.333

722. The base of a solid is the region enclosed by the graph of $y = 3(x-2)^2$ and the coordinate axes. If every cross section perpendicular to the x-axis is a square, then the volume of the solid is

A. 8.0 B. 19.2 C. 24.0 D. 25.6 E. 57.6

723.

The base of a solid is the region in the first quadrant bounded by the x-axis, the y-axis, and the graph of $y = (3-x)e^{-x}$ as shown in the diagram. If cross sections of the solid perpendicular to the x-axis are squares, what is the volume of the solid ?



A. 2.050 B. 3.081 C. 3.249 D. 10.208 E. 12.998

724. The base of a solid is the region enclosed by the graph of $x^2 + 4y^2 = 4$ Cross sections of the solid perpendicular to the x-axis are squares. Find the volume of the solid.

A. $\frac{8}{3}$ B. $\frac{8}{3}\pi$ C. $\frac{16}{3}$ D. $\frac{32}{3}$ E. $\frac{32}{3}\pi$

725. The base of a solid is the region bounded by the parabola $y^2 = 4x$ and the line $x = 2$. Each plane section perpendicular to the x-axis is a square. The volume of the solid is

A. 6 B. 8 C. 10 D. 16 E. 32

726. Let **R** be the region in the first quadrant under the graph of $y = \dfrac{1}{\sqrt{x}}$ for $4 \le x \le 9$

a) Find the area of **R**

b) If the line $x = k$ divides the region **R** into two regions of equal area, what is the value of k

c) Find the volume of the solid whose base is the region **R** and whose cross sections cut by planes perpendicular to the x-axis are squares.

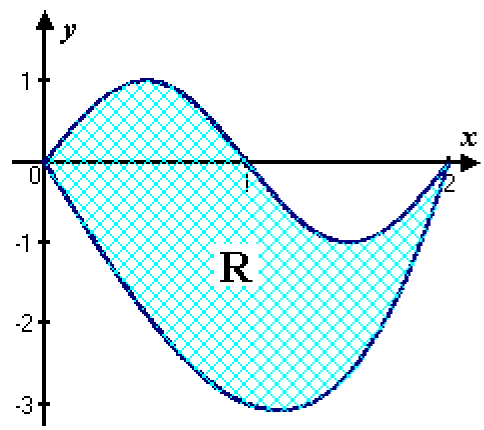
727. The base of a solid is the region enclosed by the graph of $y = e^{-x}$, the coordinate axes, and the line $x = 3$. If all plane cross sections perpendicular to the x-axis are squares, then its volume is

A. $\dfrac{1-e^{-6}}{2}$ B. $\dfrac{1}{2}e^{-6}$ C. e^{-6} D. e^{-3} E. $1-e^{-3}$

728. The base of a solid is the region in the first quadrant enclosed by the parabola $y = 4x^2$, the line $x = 1$, and the x-axis. Each plane section of the solid perpendicular to the x-axis is a square. The volume of the solid is

A. $\dfrac{4\pi}{3}$ B. $\dfrac{16\pi}{5}$ C. $\dfrac{4}{3}$ D. $\dfrac{16}{5}$ E. $\dfrac{64}{5}$

729. Let **R** be the region bounded by the graphs of $y = \sin(\pi x)$ and $y = x^3 - 4x$ as shown in the diagram.



a) Find the area of **R**

b) The horizontal line $y = -2$ splits the region **R** into two parts. Write, but do not evaluate, an integral expression for the area of the part of **R** that is below this horizontal line.

c) The region **R** is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

d) The region **R** models the surface of a small pond. At all points in **R** at a distance x from the y-axis, the depth of the water is given by $h(x) = 3 - x$. Find the volume of water in the pond.

730. Let **R** be the region in the first quadrant bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{3}$

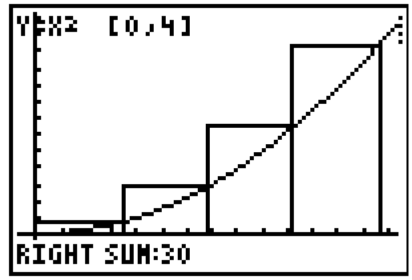
a) Find the area of **R**

b) Find the volume of the solid generated when **R** is rotated about the vertical line $x = -1$

c) The region **R** is the base of a solid. For this solid, the cross sections perpendicular to the y-axis are squares. Find the volume of this solid.

731. Definite Integrals (exact)

$$\int_0^4 x^2 dx = \left[\frac{x^3}{3}\right]_0^4 = \frac{64}{3} = \boxed{21\tfrac{1}{3}} \leftarrow \textit{exact}\,!!!$$

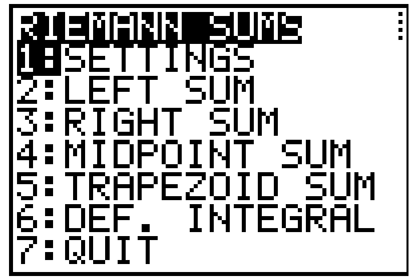


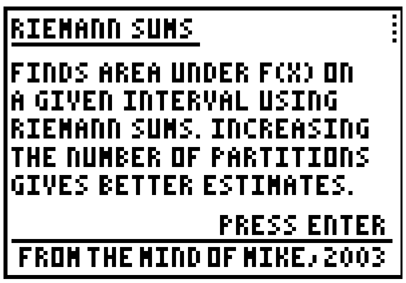
Riemann Sums (approximations)

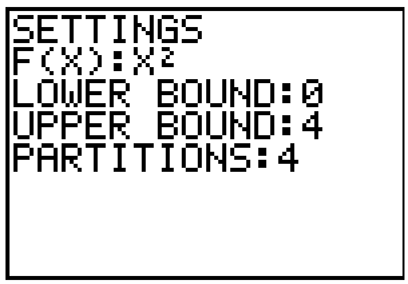
$\rightarrow$ increasing/decreasing functions

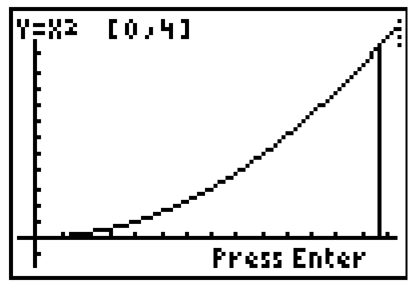
$\rightarrow$ concavity/number of partitions

Partitions	4	10	100
Left(under)	14	18.24	21.0144
Right(over)	30	24.64	21.6544
Midpoint	21	21.28	21.3328
Trapezoid	22	21.44	21.3344









732. The lower sum of $f(x) = \sqrt{x}$ on the interval $[\,0,\ 1\,]$ with four equal subintervals is

A. **0.25** B. **0.518** C. **0.667** D. **0.768** E. **3.073**

733. The left-hand sum for $f(x) = x^5$ on the interval $[-1,\ 1\,]$ using four equal subintervals is

A. -1 B. $-\frac{1}{2}$ C. 0 D. $\frac{1}{2}$ E. 1

734. Use a Riemann sum and four inscribed rectangles to approximate $\int_0^4 x^2 + 1\,dx =$

A. **18** B. **21** C. **24** D. **25** E. **26**

735. If $\int_1^2 (4 + \ln x)\,dx$ is approximated by a midpoint Riemann sum with four subintervals of equal length, then the value is

A. **4.297** B. **4.388** C. **4.470** D. **4.514** E. **4.669**

736. When $\int_{-1}^5 \sqrt{x^3 - x + 1}\,dx$ is approximated by using the mid-points of **3** rectangles of equal width, then the approximation is nearest to

A. **22.6** B. **22.9** C. **23.2** D. **23.5** E. **23.8**

737. If $\int_1^7 \ln x\,dx$ is approximated by **3** circumscribed rectangles of equal width on the x-axis, then the approximation is

A. $\frac{1}{2}(\ln 3+\ln 5+\ln 7)$ B. $\frac{1}{2}(\ln 1+\ln 3+\ln 5)$ C. $2(\ln 3+\ln 5+\ln 7)$
D. $2(\ln 3+\ln 5)$ E. $\ln 1+2\ln 3+2\ln 5+\ln 7$

738. An approximation for $\int_{-1}^{2} e^{\sin(1.5x-1)}dx$ using a right-hand Riemann sum with three equal subdivisions is nearest to

A. **2.5** B. **3.5** C. **4.5** D. **5.5** E. **6.5**

739. The midpoint-sum approximation for $\int_{-4}^{2} x^2\,dx$ using three subintervals of equal length is

A. **11** B. **14** C. **22** D. **24** E. **28**

740. If the definite integral $\int_1^3 (x^2+1)\,dx$ is approximated by using the Trapezoid Rule with $n=4$, the error is

A. 0 B. $\frac{7}{3}$ C. $\frac{1}{12}$ D. $\frac{65}{6}$ E. $\frac{97}{3}$

741. Use the trapezoid rule with $n=4$ to approximate the area between the curve $y=x^3-x^2$ and the x-axis from, $x=3$ to $x=4$

A. **35.266** B. **27.766** C. **63.031** D. **31.516** E. **25.125**

742. If the trapezoidal rule is applied to $\int_2^3 x^3\,dx$ with $\Delta x=\frac{1}{2}$, the approximate value for the integral is

A. $22\frac{1}{3}$ B. $20\frac{1}{4}$ C. $19\frac{3}{4}$ D. $17\frac{1}{8}$ E. $16\frac{9}{16}$

743. If the Trapezoidal Rule is used with $n=5$, then $\int_0^1 \frac{dx}{1+x^2}$ is equal, to three decimal places, to

A. **0.784** B. **1.567** C. **1.959** D. **3.142** E. **7.837**

744. If $\mathbf{M(4)}$ is used to approximate $\int_0^1 \sqrt{1+x^3}\,dx$, then the definite integral is equal, to two decimal places, to

A. **1.00** B. **1.11** C. **1.20** D. **2.22** E. **3.33**

745. Use a right-hand Riemann sum with **4** equal subdivisions to approximate the integral $\int_{-1}^{3} |2x-3|\,dx$

A. **13** B. **10** C. **8.5** D. **8** E. **6**

746. Let $\boldsymbol{R}$ be the region in the first quadrant enclosed by the $\boldsymbol{x}$-axis and the graph of $\boldsymbol{y = \ln x}$ from $\boldsymbol{x = 1}$ to $\boldsymbol{x = 4}$ If the Trapezoid Rule with $\mathbf{3}$ subdivisions is used to approximate the area of $\boldsymbol{R}$, the approximation is

A. $\mathbf{4.970}$ B. $\mathbf{2.510}$ C. $\mathbf{2.497}$ D. $\mathbf{2.485}$ E. $\mathbf{2.473}$

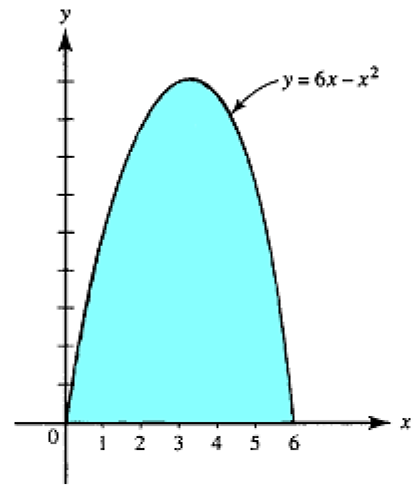
747. $\mathbf{L}$ and $\mathbf{R}$ are the left-hand and right-hand Riemann sums, respectively, of $\boldsymbol{f(x) = 3x - x^2}$ on $\boldsymbol{[\,1,\ 3\,]}$, divided into $\mathbf{4}$ subintervals of equal length. Which of the following statements is true ?

A. $\mathbf{R = 0}$ B. $\mathbf{L < R}$ C. $\mathbf{L > R}$ D. $\mathbf{L = R}$

E. cannot determine whether $\mathbf{L}$ is greater than $\mathbf{R}$ or less than $\mathbf{R}$ from the given information

748.

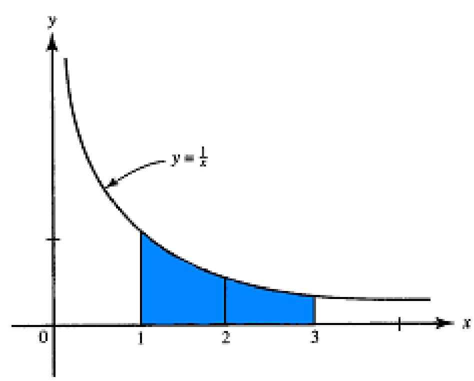
If we approximate the area of the shaded region by $\boldsymbol{M(20)}$ (that is, the midpoint sum with $\mathbf{20}$ subintervals), then the difference $\boldsymbol{M(20) - \int_0^6 f(x)\,dx}$ is equal to



A. $\mathbf{0.004}$ B. $\mathbf{0.008}$ C. $\mathbf{0.010}$ D. $\mathbf{0.045}$ E. ***none of these***

749.

The area of the following shaded region is equal exactly to $\mathbf{\ln\ 3}$. If we approximate $\mathbf{\ln\ 3}$ using $\boldsymbol{L}\mathbf{(2)}$ and $\boldsymbol{R}\mathbf{(2)}$, which of the inequalities follows ?



A. $\boldsymbol{\frac{1}{2} < \int_1^2 \frac{1}{x}dx < 1}$ B. $\boldsymbol{\frac{1}{3} < \int_1^3 \frac{1}{x}dx < 2}$ C. $\boldsymbol{\frac{1}{2} < \int_0^2 \frac{1}{x}dx < 2}$

D. $\boldsymbol{\frac{1}{3} < \int_2^3 \frac{1}{x}dx < \frac{1}{2}}$ E. $\boldsymbol{\frac{5}{6} < \int_1^3 \frac{1}{x}dx < \frac{3}{2}}$

750. If $\boldsymbol{\int_0^6 (x^2 - 2x + 2)\,dx}$ is approximated by three inscribed rectangles of equal width on the $\boldsymbol{x}$-axis, then the approximation is

A. $\mathbf{24}$ B. $\mathbf{26}$ C. $\mathbf{28}$ D. $\mathbf{48}$ E. $\mathbf{76}$

751. Which of the following is true for $f(x) = \cos x$ on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ using four equal subintervals ?

A. left-hand sum < right-hand sum
B. right-hand sum < left-hand sum
C. midpoint sum < left-hand sum
D. midpoint sum = left-hand sum
E. left-hand sum = right-hand sum

752. The function f is continuous in the closed interval $[\,1, 5\,]$ and has values that are given in the table. If two subintervals of equal length are used, what is the midpoint Riemann sum approximation of $\int_1^5 f(x)\,dx =$

x	1	2	3	4	5
$f(x)$	15	10	9	6	5

A. -3 B. 9 C. 14 D. 32 E. 35

753. The function f is continuous on the closed interval $[\,2, 14\,]$ and has values as shown in the table. Using the subintervals $[\,2, 5\,]$, $[\,5, 10\,]$ and $[\,10, 14\,]$ what is the approximation of $\int_2^{14} f(x)\,dx$ found by using a right Riemann sum ?

x	2	5	10	14
$f(x)$	12	28	34	30

A. 296 B. 312 C. 343 D. 374 E. 390

754. A table of values for a continuous function f is shown. If four equal subintervals of $[\,0, 2\,]$ are used, which of the following is the trapezoidal approximation of $\int_0^2 f(x)\,dx$

x	0.0	0.5	1.0	1.5	2.0
$f(x)$	3	3	5	8	13

A. 8 B. 12 C. 16 D. 24 E. 32

755. The function f is continuous on the closed interval $[\,2, 8\,]$ and has values that are given in the table. Using the subintervals $[\,2, 5\,]$, $[\,5, 7\,]$ and $[\,7, 8\,]$ what is the trapezoidal approximation of $\int_2^8 f(x)\,dx$

x	2	5	7	8
$f(x)$	10	30	40	20

A. 110 B. 130 C. 160 D. 190 E. 210

756. For the function whose values are given in the table, $\int_0^6 f(x)\,dx$ is approximated by a Riemann Sum using the value at the midpoint of each of three intervals of width **2**
The approximation is

x	0	1	2	3	4	5	6
$f(x)$	0	0.25	0.48	0.68	0.84	0.95	1

A. 2.64 B. 3.64 C. 3.72 D. 3.76 E. 4.64

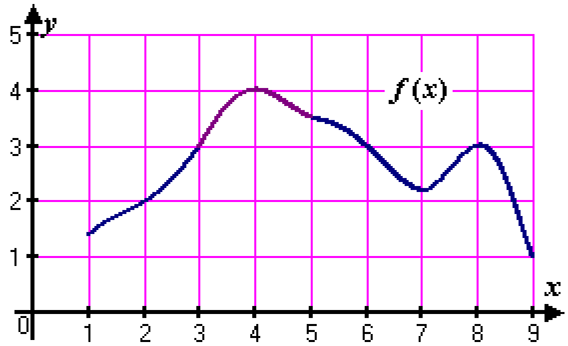
757. The function f is continuous on the closed interval $[5, 12]$ and differentiable on the open interval $(5, 12)$ and f has the values given the the table. Using the subintervals $[5, 6]$, $[6, 9]$, $[9, 11]$, and $[11, 12]$, what is the right-hand Riemann sum approximation to $\int_5^{12} f(x)dx$

x	5	6	9	11	12
$f(x)$	10	7	11	12	8

A. **64** B. **65** C. **66** D. **68.5** E. **72**

758.

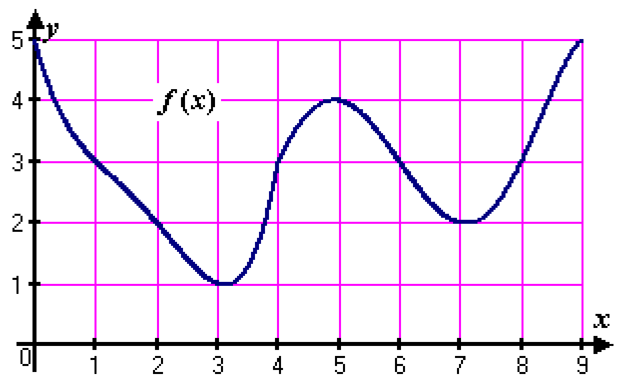
The graph of f over the interval $[1, 9]$ is shown in the figure. Using the data in the figure, find a midpoint approximation with **4** equal subdivisions for $\int_1^9 f(x)dx$



A. **20** B. **21** C. **22** D. **23** E. **24**

759.

Consider the function f whose graph is shown at the right. Use the Trapezoid Rule with $n = 4$ to estimate the value of $\int_1^9 f(x)dx$



A. **21** B. **22** C. **23** D. **24** E. **25**

760. The following table lists the known values of a function f If the Trapezoid Rule is used to approximate $\int_1^5 f(x)dx$ the result is

x	1	2	3	4	5
$f(x)$	0	1.1	1.4	1.2	1.5

A. **4.1** B. **4.3** C. **4.5** D. **4.7** E. **4.9**

761. The table contains values of a continuous function f at several values of x.

x	1	2	3	4	5	6
$f(x)$	0.14	0.21	0.28	0.36	0.44	0.54

Estimate $\int_2^5 f(x)dx$ using a trapezoidal approximation with three equal subintervals.

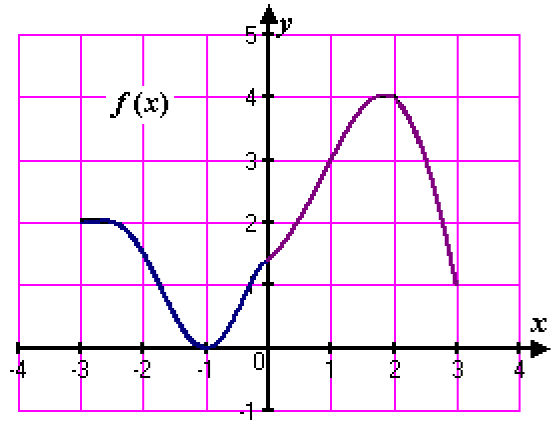
A. **0.85** B. **0.965** C. **1.08** D. **1.29** E. **1.93**

762. Use the Trapezoid Rule with $n = 3$ to approximate the area under $y = x^2$ from $x = 1$ to $x = 4$

A. $\frac{45}{3}$ B. $\frac{43}{3}$ C. $\frac{43}{2}$ D. **43** E. **21**

763. The graph of f is shown at the right.

Approximate $\int_{-3}^{3} f(x)\,dx$ using the Trapezoid Rule with **3** equal subdivisions.



A. $2\frac{1}{4}$ B. $4\frac{1}{2}$ C. **9** D. **18** E. **36**

764. The table shows the velocity readings of a car taken every **30** seconds of a five-minute interval.

Time (sec)	**0**	**30**	**60**	**90**	**120**	**150**	**180**	**210**	**240**	**270**	**300**
Velocity (mph)	**60**	**55**	**50**	**45**	**40**	**45**	**50**	**60**	**30**	**40**	**45**

What is the approximate distance (in miles) traveled by the car during this five-minute interval, using a midpoint Riemann sum with **60**-second subintervals ?

A. **3.583** B. **3.708** C. **3.750** D. **3.833** E. **4.083**

765. Water drains continuously from a tank. The rate (in gallons per second) at which the water drains out is measured at the times (in seconds) given in the table. What is the trapezoidal approximation, based on all of the data in the table, for the total amount of water that has drained from the tank in the first ten seconds ?

Time (sec)	**0**	**3**	**8**	**10**
Rate (gal/sec)	**16**	**10**	**6**	**5**

A. **37** gallons B. **70** gallons C. **79.5** gallons D. **90** gallons E. **110** gallons

766. Let f be a continuous function on $[\,0,\ 6\,]$ and have the selected values as shown in the table. If you use the subintervals $[\,0,\ 2\,]$, $[\,2,\ 4\,]$ and $[\,4,\ 6\,]$, what is the trapezoidal approximation of $\int_{0}^{6} f(x)\,dx$

x	**0**	**2**	**4**	**6**
$f(x)$	**0**	**1**	**2.25**	**6.25**

A. **9.5** B. **12.75** C. **19** D. **25.5** E. **38.25**

767. Let f be a continuous function on $[\,4,\ 10\,]$ and has selected values as shown in the table. Using three ***right endpoint rectangles*** of equal length, what is the approximate value of $\int_{4}^{10} f(x)\,dx =$

x	**4**	**6**	**8**	**10**
$f(x)$	**2**	**2.4**	**2.8**	**3.2**

A. **8.4** B. **9.6** C. **14.4** D. **16.8** E. **20.8**

768. The function f is continuous on the closed interval $[\,4,\ 6\,]$ and has values that are given in the table. Using four equal subintervals, what is the trapezoidal approximation to $\int_4^6 f(x)\,dx$

x	4	4.5	5	5.5	6
$f(x)$	6	4	8	6	10

A. **12** B. **13** C. **14** D. **15** E. **17**

769. The function f is continuous on the closed interval $[\,0,\ 10\,]$ and has the values given in the table. Using the subintervals $[\,0,\ 1\,]$, $[\,1,\ 3\,]$, $[\,3,\ 7\,]$, $[\,7,\ 10\,]$ what is the left Riemann sum estimate for $\int_0^{10} f(x)dx =$

x	0	1	3	7	10
$f(x)$	1	−1	4	2	3

A. **15** B. **17.5** C. **20** D. **21** E. **22.5**

770. The function f is continuous on the closed interval $[\,0,\ 3\,]$ and has values that are given in the table. Using the subintervals $[\,0,\ 1\,]$, $[\,1,\ 2\,]$ and $[\,2,\ 3\,]$, what is the trapezoidal approximation to $\int_0^3 f(x)\,dx$

x	0	1	2	3
$f(x)$	2	5	4	3

A. **11** B. **11.5** C. **12** D. **12.5** E. **13**

771. The table gives values for the velocity of a particle at certain times t between $t = 0$ and $t = 60$ The approximation of the total distance traveled by the particle during the time period $0 \le t \le 60$, computed using a right-hand Riemann sum with four equal subintervals, is

time (seconds)	0	15	30	45	60
velocity (feet per second)	6	10	8	7	4

A. **29** feet B. **435** feet C. **450** feet D. **465** feet E. **525** feet

772. The function f is continuous on the interval $[\,0,\ 8\,]$ and has values that are given in the table. Using the subintervals $[\,0,\ 3\,]$, $[\,3,\ 4\,]$, $[\,4,\ 8\,]$, what is the trapezoidal approximation of $\int_0^8 f(x)\,dx$

x	0	3	4	8
$f(x)$	4	6	2	12

A. **26** B. $\frac{128}{3}$ C. **47** D. **48** E. **68**

773. Use the trapezoidal method with **4** divisions to approximate the area of the region bounded by the graph of $y = \frac{1}{2x}$, the lines $x = 1$ and $x = 3$, and the x-axis

A. $\frac{67}{60}$ B. $\frac{67}{120}$ C. $\frac{91}{240}$ D. $\frac{91}{120}$ E. $\frac{67}{30}$

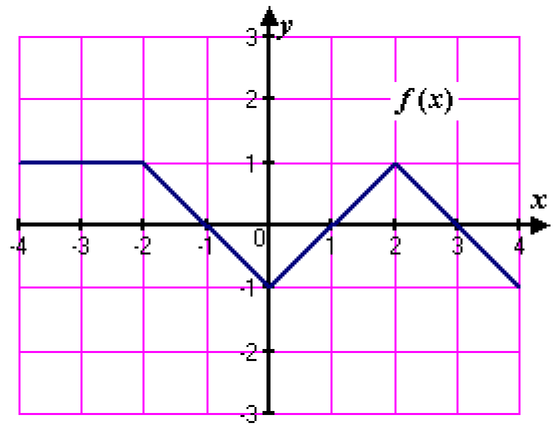
774. What is the approximation of the area under the graph of $f(x)=\sqrt{1+x^3}$ using the trapezoidal sum with all the points in the partition $\{\ 1,\ \frac{5}{4},\ 2,\ 3\ \}$

A. 4.642 B. 6.307 C. 7.971 D. 8.071 E. 12.614

775. If the definite integral $\int_1^7 \ln x\,dx$ is approximated by 3 circumscribed rectangles of equal width on the x-axis, then the approximation is

A. $\frac{1}{2}(\ln 3+\ln 5+\ln 7)$ B. $\frac{1}{2}(\ln 1+\ln 3+\ln 5)$ C. $2(\ln 3+\ln 5+\ln 7)$
D. $2(\ln 3+\ln 5)$ E. $\ln 1+2\ln 3+2\ln 5+\ln 7$

776. The graph of f is shown at the right. Which of the following statements must be true ?



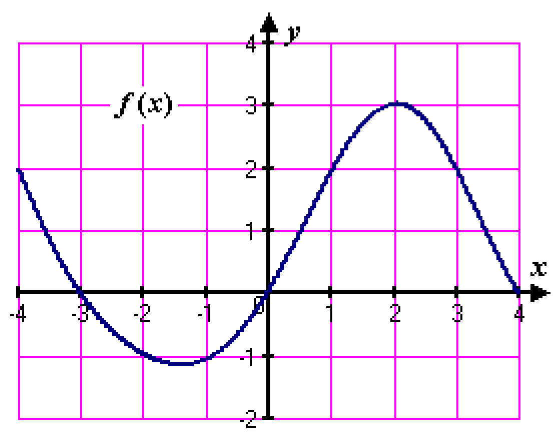
I. $f'(3)>f'(1)$

II. $\int_0^2 f(x)dx>f'(3.5)$

III. $\int_1^0 f(x)dx=\int_2^3 f(x)dx$

A. **I** *only* B. **II** *only* C. **I** *and* **II** *only*
D. **II** *and* **III** *only* E. **I, II** *and* **III**

777. The graph of a function f whose domain is the interval $[-4,\ 4]$ is shown in the figure. Which of the following statements are true ?



I. The average rate of change of f over the interval from $x=-2$ to $x=3$ is $\frac{1}{5}$

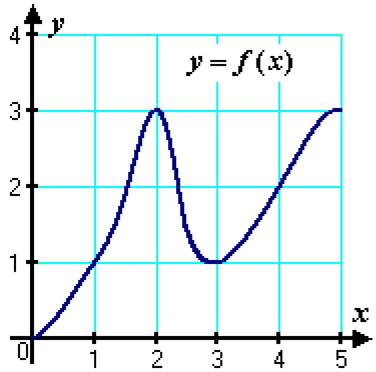
II. The slope of the tangent line at the point where $x=2$ is 0

III. The left-sum approximation of $\int_{-1}^{3} f(t)dt$ with 4 equal subdivisions is 4

A. **I** *only* B. **I** *and* **II** *only* C. **II** *and* **III** *only*
D. **I** *and* **III** *only* E. **I, II** *and* **III**

778.

Use the Trapezoid Rule with $n = 4$ to approximate the integral $\int_1^5 f(x)\,dx$ for the function f whose graph is shown on the right.



A. **7** B. **8** C. **9** D. **10** E. **11**

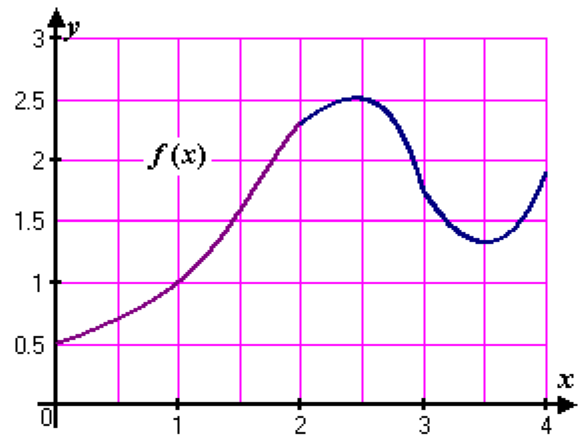
779.

A graph of the function is shown on the right. Which of the following statements are true ?

I. $f(1) > f'(3)$

II. $\int_1^2 f(x)\,dx > f'(3.5)$

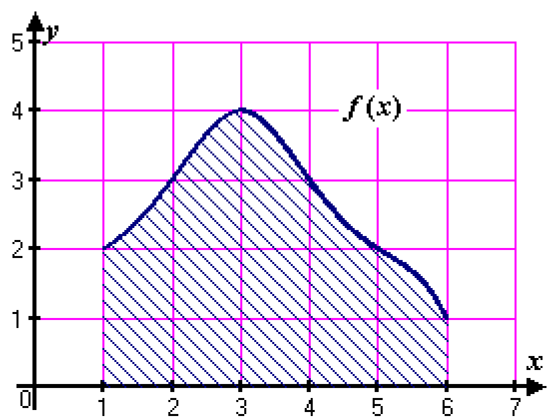
III. $\lim_{h\to 0} \frac{f(2+h) - f(2)}{h} > \frac{f(2.5) - f(2)}{2.5 - 2}$



A. **I *only*** B. **II *only*** C. **I *and* II *only***
D. **II *and* III *only*** E. **I, II *and* III**

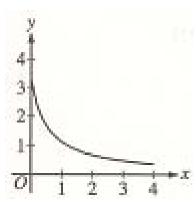
780.

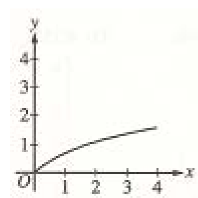
The region shaded in the figure on the right is rotated about the x-axis. Using the Trapezoid Rule with **5** equal subdivisions, the approximate volume of the resulting solid is

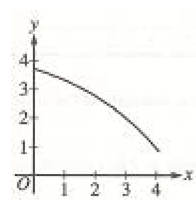


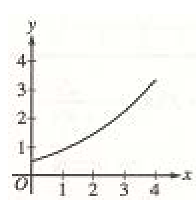
A. **23** B. **47** C. **127** D. **254** E. **400**

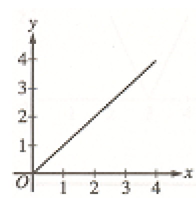
781. If a trapezoidal sum overapproximates $\int_0^4 f(x)\,dx$ and a right Riemann sum underapproximates $\int_0^4 f(x)\,dx$, which of the following could be the graph of $y = f(x)$

A.


B.


C.


D.


E.


782. If the definite integral $\int_0^2 e^{x^2}\,dx$ is first approximated by using two inscribed rectangles of equal width and then approximated by using the trapezoidal rule with $n = 2$, the difference between the two approximations is

A. **53.60** B. **30.51** C. **27.80** D. **26.80** E. **12.78**

783.

Distance from the river's edge (feet)	0	8	14	22	24
Depth of the water (feet)	0	7	8	2	0

A scientist measures the depth of the Doe River at Picnic Point. The river is **24** feet wide at this location. The measurements are taken in a straight line perpendicular to the edge of the river. The data are shown in the table above. The velocity of the water at Picnic Point, in feet per minute, is modeled by $v(t) = 16 + 2\sin\left(\sqrt{t+10}\right)$ for $0 \le t \le 120$ minutes.

a) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the area of the cross section of the river at Picnic Point, in square feet. Show the computations that lead to your answer.

b) The volumetric flow at a location along the river is the product of the cross-sectional area and the velocity of the water at that location. Use you approximation from part (*a*) to estimate the average value of the volumetric flow at Picnic Point, in cubic feet per minute, from $t = 0$ to $t = 120$ minutes.

c) The scientist proposes the function f, given by $f(x) = 8\sin\left(\frac{\pi x}{24}\right)$, as a model for the depth of the water, in feet, at Picnic Point x feet from the river's edge. Find the area of the cross section of the river at Picnic Point based on this model.

d) Recall that the volumetric flow is the product of the cross-sectional area and the velocity of the water at a location. To prevent flooding, water must be diverted if the average value of the volumetric flow at Picnic Point exceeds **2100** cubic feet per minute for a **20**-minute period. Using your answer from part (*c*), find the average value of the volumetric flow during the time interval $40 \le t \le 60$ minutes. Does this value indicate that the water mnust be diverted ?

784.

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

Concert tickets went on sale at noon ($t = 0$) and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0 \le t \le 9$ Values of $L(t)$ at various times t are shown in the table

a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at $5:30$ P.M. ($t = 5.5$) Show the computations that lead to your answer. Indicate units of measure.

b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.

c) For $0 \le t \le 9$ what is the fewest number of times at which $L'(t)$ must equal 0 Give a reason for your answer.

d) The rate at which tickets were sold for $0 \le t \le 9$ is modeled by $r(t) = 550te^{-\frac{t}{2}}$ tickets per hour. Based on the model, how many tickets were sold by 3 P.M. ($t = 3$), to the nearest whole number ?

785.

t (*seconds*)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

Rocket A has a positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t = 0$ seconds. The velocity of the rocket is recorded for selected valves of t over the interval $0 \le t \le 80$ seconds, as shown in the table.

a) Find the average acceleration of rocket A over the time interval $0 \le t \le 80$ seconds. Indicate units of measure.

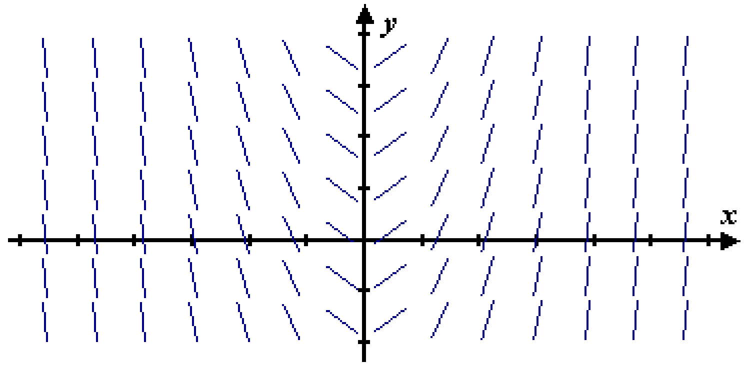
b) Using correct units, explain the meaning of $\int_{10}^{70} v(t)\,dt$ in terms of the rocket's flight.

Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t)\,dt$

c) Rocket B is launched upward with an acceleration of $a(t) = \dfrac{3}{\sqrt{t+1}}$ feet per second per second. At time $t = 0$ seconds, the initial height of the rocket is 0 feet, and the initial velocity is 2 feet per second. Which of the two rockets is traveling faster at time $t = 80$ seconds ? Explain your answer.

786.

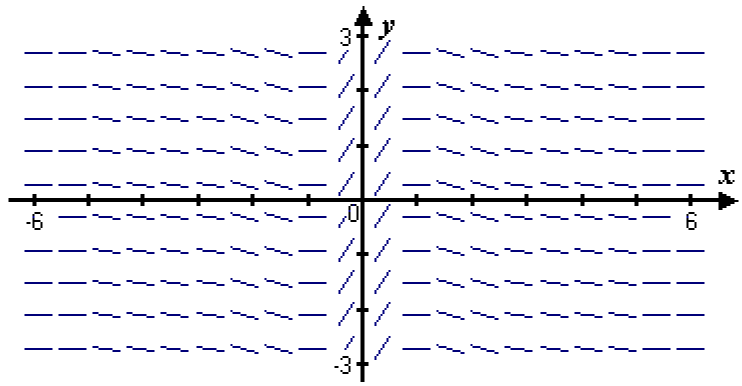
Which of the following could be a solution of the differential equation with the given slope field ?



A. $y = x + 1$ B. $y = x^2 + 2$ C. $y = x^3 - 2$ D. $y = \ln(x+1)$ E. $y = 2e^x$

787.

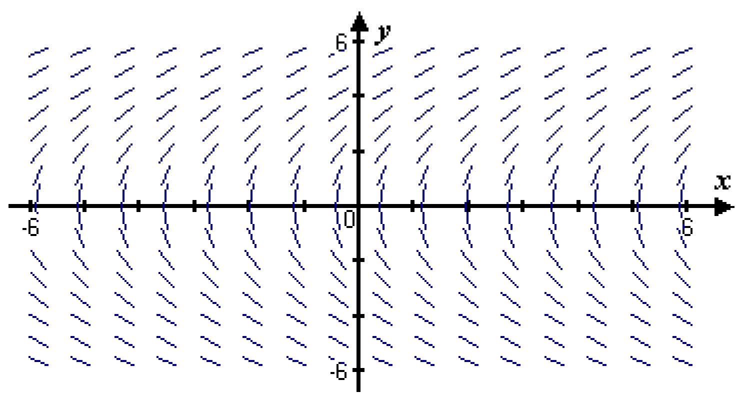
Which function could be a particular solution of the differential equation whose slope field is shown on the right ?



A. $y = x^3$ B. $y = \dfrac{2x}{x^2+1}$ C. $y = \dfrac{x^2}{x^2+1}$ D. $y = \sin x$ E. $y = e^{-x^2}$

788.

Which equation has the slope field shown on the right ?

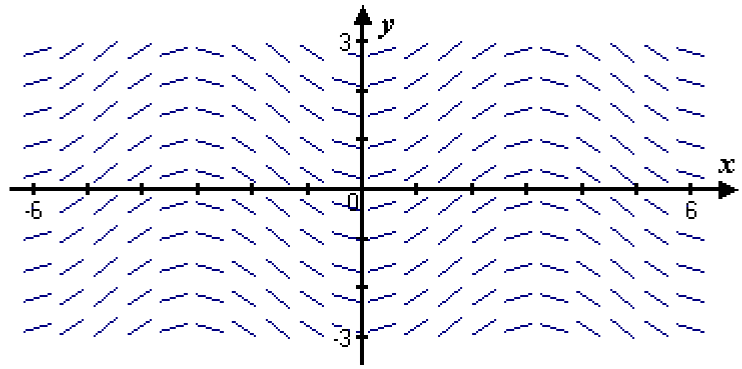


A. $\dfrac{dy}{dx} = \dfrac{5}{y}$ B. $\dfrac{dy}{dx} = \dfrac{5}{x}$ C. $\dfrac{dy}{dx} = \dfrac{x}{y}$ D. $\dfrac{dy}{dx} = 5y$ E. $\dfrac{dy}{dx} = x + y$

789. The slope field for $\dfrac{dy}{dx} = y$ shows that the solutions to the differential equation

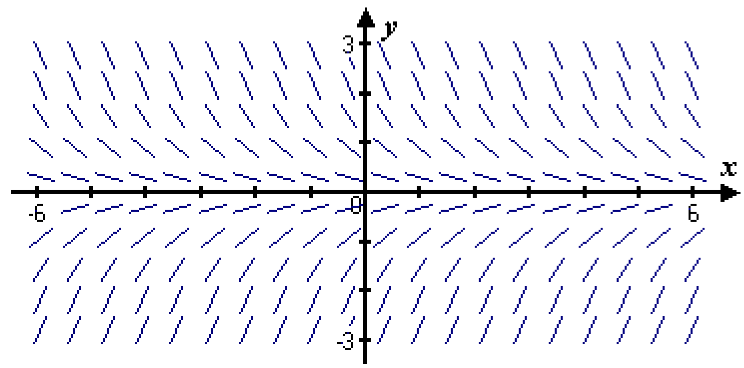
A. have y-intercept $(0, 1)$
B. have a positive y-intercept
C. have a horizontal asymptote
D. are even functions
E. are odd functions

790. The slope field for a differential equation $\frac{dy}{dx} = f(x, y)$ is given in the diagram. The slope field corresponds to which of the following differential equations ?



A. $\frac{dy}{dx} = \tan x \sec x$
B. $\frac{dy}{dx} = \sin x$
C. $\frac{dy}{dx} = \cos x$
D. $\frac{dy}{dx} = -\sin x$
E. $\frac{dy}{dx} = -\cos x$

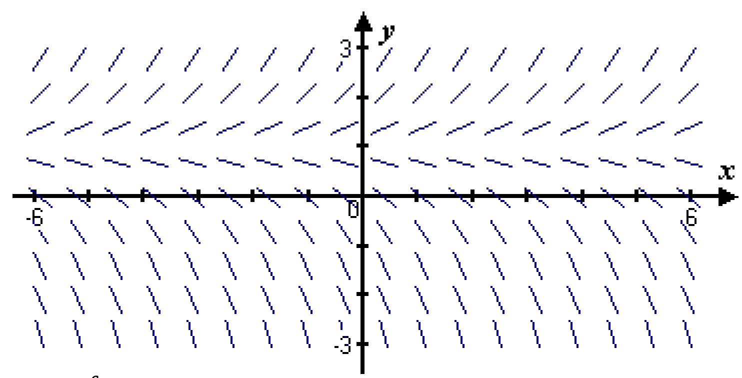
791. The slope field for a differential equation $\frac{dy}{dx} = f(x, y)$ is given in the diagram. The slope field corresponds to which of the following differential equations ?



A. $\frac{dy}{dx} = x + y$
B. $\frac{dy}{dx} = y^2$
C. $\frac{dy}{dx} = -y$
D. $\frac{dy}{dx} = e^{-x}$
E. $\frac{dy}{dx} = 1 - \ln x$

792. The slope field for a differential equation $\frac{dy}{dx} = f(x, y)$ is given in the diagram. Which of the following statements are true ?

I. A solution curve that contains the point **(0, 2)** also contains the point **(–2, 0)**



II. As y approaches **1**, the rate of change of y approaces zero.

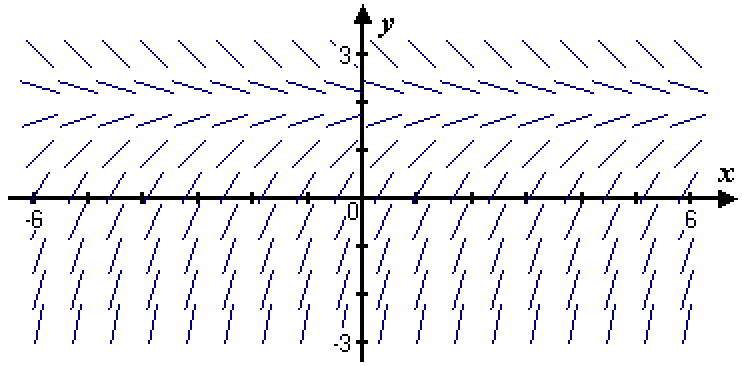
III. All solution curves for the differential equation have the same slope for a given value of y

A. **I** *only*
B. **II** *only*
C. **I** *and* **II** *only*
D. **II** *and* **III** *only*
E. **I**, **II** *and* **III**

793. On the positive y-axis, the slope field for the differential equation $\frac{dy}{dt} = \frac{t^2}{y}$ has

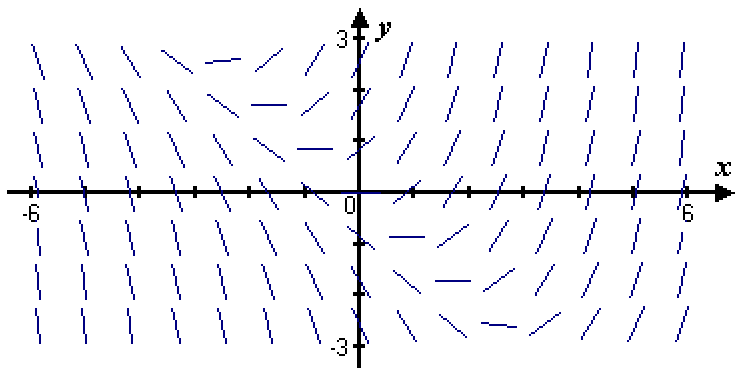
A. horizontal segments
B. vertical segments
C. segments with positive slope
D. segments with negative slope
E. segments with slope equal to **1**

794. The slope field for a differential equation $\dfrac{dy}{dx} = f(x,y)$ is given in the diagram. The slope field corresponds to which of the following differential equations ?



A. $\dfrac{dy}{dx} = 2 - \ln x$ B. $\dfrac{dy}{dx} = 2 - e^{-x}$ C. $\dfrac{dy}{dx} = y - 2y^2$ D. $\dfrac{dy}{dx} = 2 - y$ E. $\dfrac{dy}{dx} = -x^2$

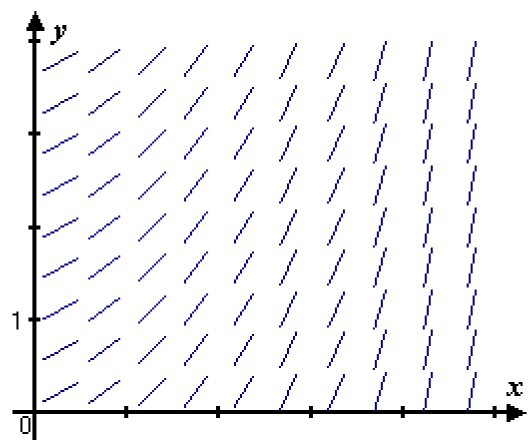
795. The slope field for a differential equation $\dfrac{dy}{dx} = f(x,y)$ is given in the diagram. The slope field corresponds to which of the following differential equations ?



A. $\dfrac{dy}{dx} = x + y$ B. $\dfrac{dy}{dx} = -y$ C. $\dfrac{dy}{dx} = y - \dfrac{1}{2}y^2$ D. $\dfrac{dy}{dx} = x^2 + y^2$ E. $\dfrac{dy}{dx} = y^2$

796.

The slope field shown in the figure at the right represents solutions to a certain differential equation. Which of the following could be a specific solution to that differential equation ?



A. $y = e^{-x}$ B. $y = \sin x$ C. $y = \sqrt{x}$ D. $y = \ln x$ E. $y = e^{0.5x}$

797. In a slope field, the line segments are

A. part of the graph of the solution to the differential equation

B. parts of the lines tangent to the graph of the solution to the differential equation

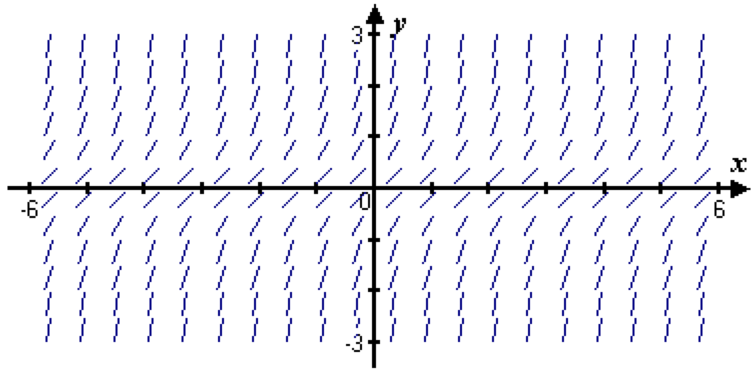
C. asymptotes to the graph of the solution of the differential equation

D. lines of the symmetry of the graph of the solution to the differential equation

E. ***none of these***

798.

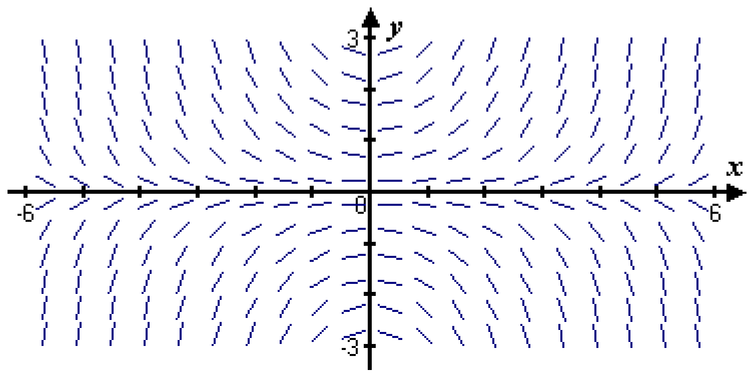
Which of the following differential equations could be represented by this slope field ?



A. $\dfrac{dy}{dx} = x^3 + 1$ B. $\dfrac{dy}{dx} = \tan x$ C. $\dfrac{dy}{dx} = \dfrac{1}{1+x^2}$ D. $\dfrac{dy}{dx} = 1 + y^2$ E. $\dfrac{dy}{dx} = \tan^{-1} y$

799.

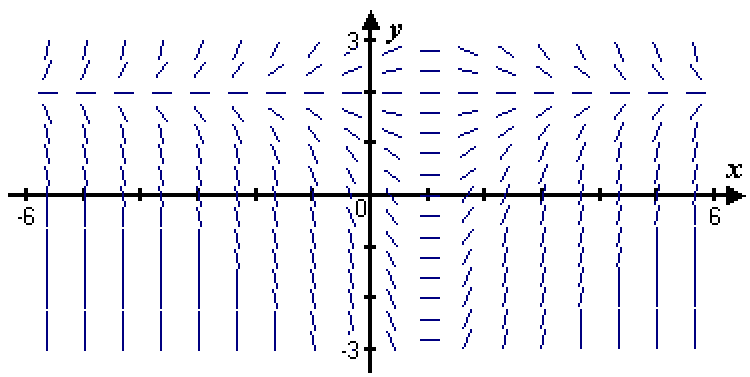
Which of the following differential equations corresponds to the slope field shown in the diagram ?



A. $\dfrac{dy}{dx} = \dfrac{xy}{2}$ B. $\dfrac{dy}{dx} = \dfrac{y}{x}$ C. $\dfrac{dy}{dx} = -\dfrac{y}{x}$ D. $\dfrac{dy}{dx} = \dfrac{x}{y}$ E. $\dfrac{dy}{dx} = -\dfrac{x}{y}$

800.

The graph is a slope field for which of the following differential equations ?



A. $\dfrac{dy}{dx} = y - 2x$ B. $\dfrac{dy}{dx} = 1 + x + y$ C. $\dfrac{dy}{dx} = (1-x)(y-2)$

D. $\dfrac{dy}{dx} = xy^2$ E. $\dfrac{dy}{dx} = (x-1)y^2$

801. Drawing a slope field

A. provides a way of visualizing the solution to a differential equation

B. can help find horizontal asymptotes to the graph of the solution of the differential equation

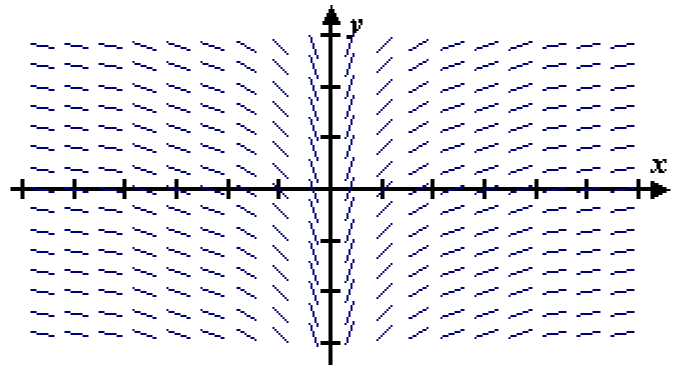
C. can serve as a check to the solution of a differential equation

D. can give evidence as to the symmetry of the graph of the solution to a differential equation

E. all of the above

802.

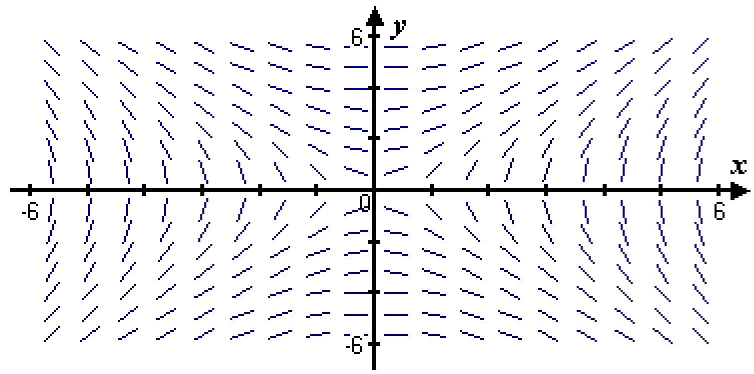
The slope field matches which differential equation ?



A. $\dfrac{dy}{dx}=\dfrac{1}{x}$ B. $\dfrac{dy}{dx}=\dfrac{1}{x^2}$ C. $\dfrac{dy}{dx}=\dfrac{y}{x}$ D. $\dfrac{dy}{dx}=\dfrac{\ln x}{x}$ E. $\dfrac{dy}{dx}=\dfrac{\sin x}{x}$

803.

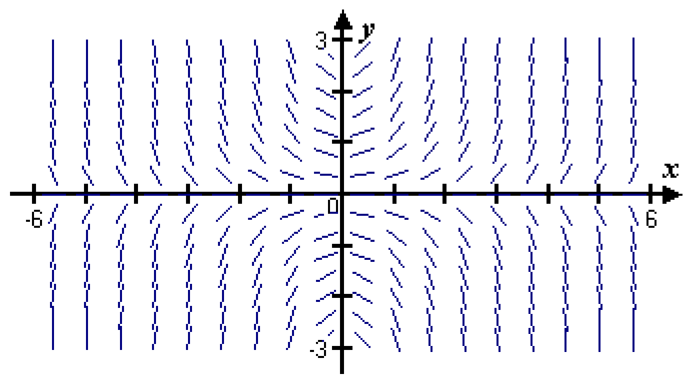
Which of the following equations has the slope field shown ?



A. $\dfrac{dy}{dx}=2x$ B. $\dfrac{dy}{dx}=2y$ C. $\dfrac{dy}{dx}=\dfrac{2x}{y}$ D. $\dfrac{dy}{dx}=xy$ E. $\dfrac{dy}{dx}=\dfrac{2y}{x}$

804.

Which of the following differential equations generates the slope field shown in the diagram ?



A. $\dfrac{dy}{dx}=xy$ B. $\dfrac{dy}{dx}=x$ C. $\dfrac{dy}{dx}=y$ D. $\dfrac{dy}{dx}=x+y$ E. $\dfrac{dy}{dx}=x^2$

805. The slope field for the differential equation $\dfrac{dy}{dx}=x$

A. has line segments symmetric to the y-axis

B. shows that the solutions to the differential equation are odd functions

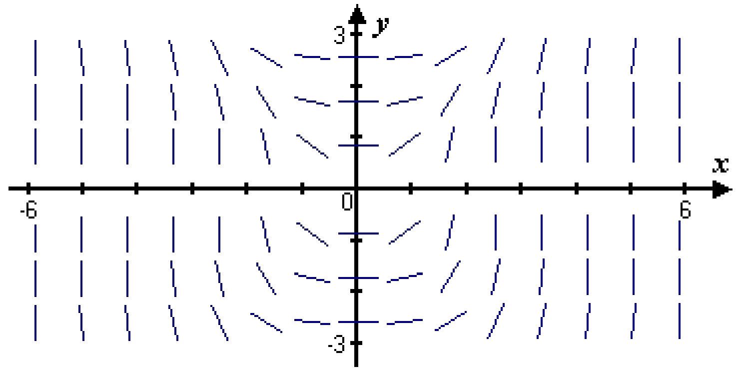
C. shows that the solutions to the differential are straight lines

D. shows that the solutions to the differential equation are decreasing for increasing x

E. shows that there is a horizontal asymptote

806.

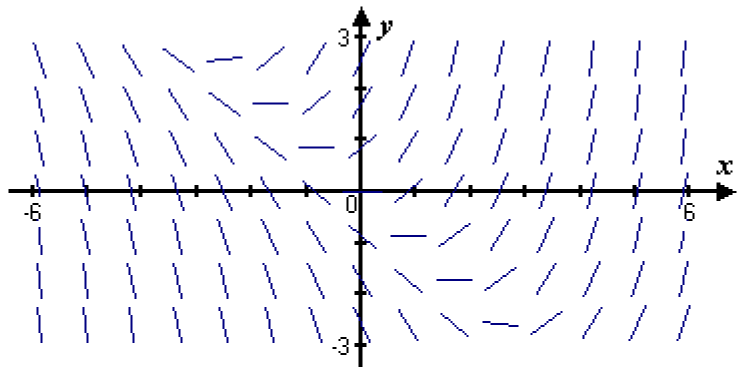
Shown in the diagram is a slope field for which of the following differential equations ?



A. $\frac{dy}{dx} = \frac{x}{y}$ B. $\frac{dy}{dx} = \frac{x^2}{y^2}$ C. $\frac{dy}{dx} = \frac{x^3}{y}$ D. $\frac{dy}{dx} = \frac{x^2}{y}$ E. $\frac{dy}{dx} = \frac{x^3}{y^2}$

807.

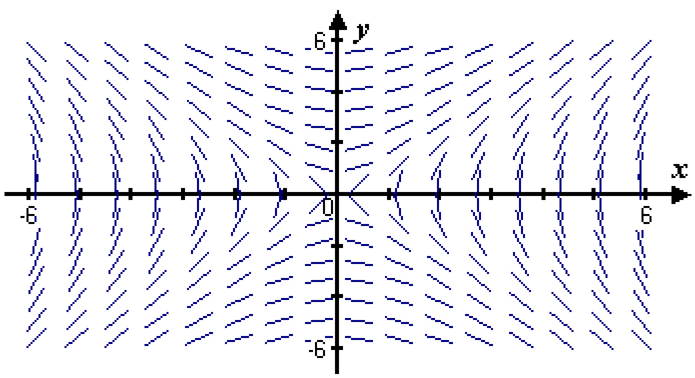
Shown on the right is a slope field for which of the following differential equations ?



A. $\frac{dy}{dx} = 1 + x$ B. $\frac{dy}{dx} = x^2$ C. $\frac{dy}{dx} = x + y$ D. $\frac{dy}{dx} = \frac{x}{y}$ E. $\frac{dy}{dx} = \ln y$

808.

Which of the following equations can be a solution of the differential equation whose slope field is shown on the right.



A. $2xy = 1$ B. $2x + y = 1$ C. $2x^2 + y^2 = 1$ D. $2x^2 - y^2 = 1$ E. $y = 2x^2 + 1$

809. The slope field for the differential equation $\frac{dy}{dx} = x^2$

A. has line segments symmetric to the y-axis

B. shows that the solutions to the differential equation are even functions

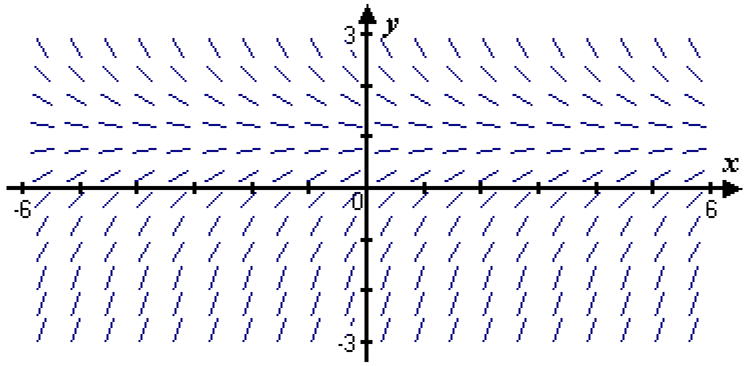
C. shows that the graphs of the solutions are increasing for increasing x

D. shows that the graphs of the solutions are decreasing for increasing x

E. shows that there are solutions that have a horizontal asymptote

810.

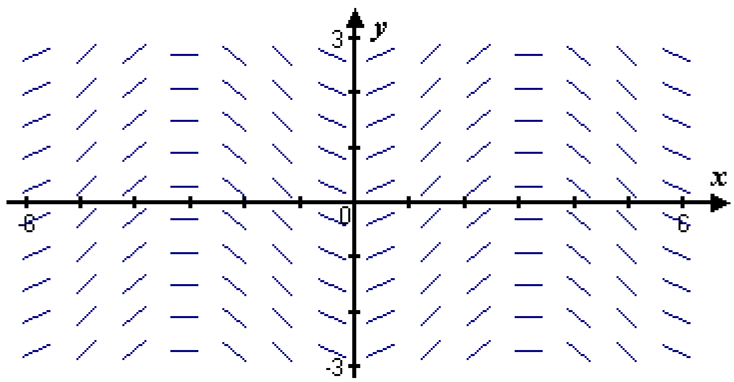
Shown on the right is the slope field for which differential equation ?



A. $\dfrac{dy}{dx} = 1 - x$ B. $\dfrac{dy}{dx} = x - y$ C. $\dfrac{dy}{dx} = -\dfrac{x}{y}$ D. $\dfrac{dy}{dx} = \dfrac{y}{x}$ E. $\dfrac{dy}{dx} = 1 - y$

811.

The slope field for a differential equation $\dfrac{dy}{dx} = f(x, y)$ is given in the figure. The slope field corresponds to which of the following differential equations ?

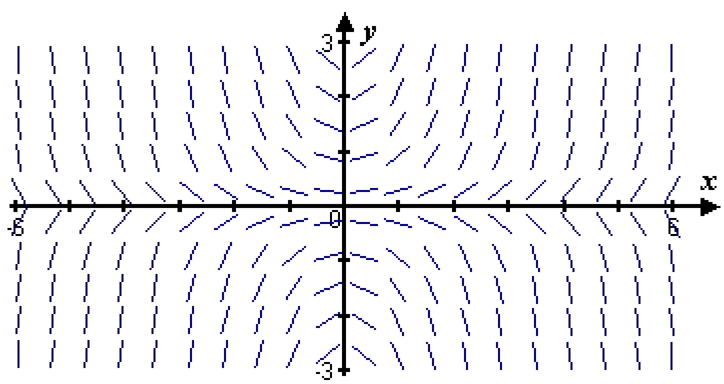


A. $\dfrac{dy}{dx} = \tan x \sec x$ B. $\dfrac{dy}{dx} = \sin x$ C. $\dfrac{dy}{dx} = \sec^2 x$

D. $\dfrac{dy}{dx} = \ln x$ E. $\dfrac{dy}{dx} = e^{2x}$

812.

This slope field is for which of the following differential equations ?



A. $\dfrac{dy}{dx} = x^2 + y^2$ B. $\dfrac{dy}{dx} = x(x-1)$ C. $\dfrac{dy}{dx} = xy$ D. $\dfrac{dy}{dx} = \dfrac{x}{2y}$ E. $\dfrac{dy}{dx} = y$

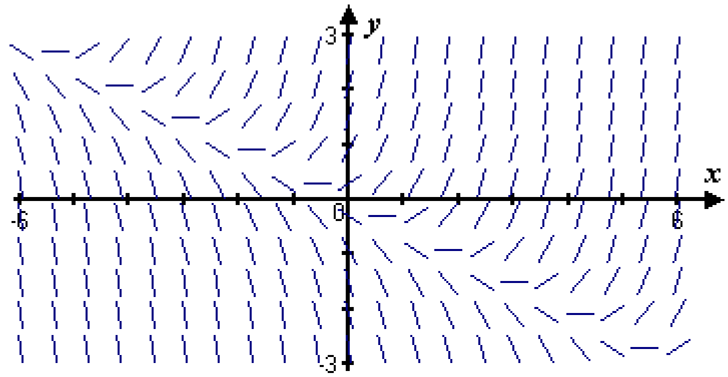
813. The slope field for the differential equation $\dfrac{dy}{dx} = \dfrac{3y}{xy + 5x}$ will have vertical segments when

A. $x = 0$ only B. $y = 0$ only C. $y = -5$ only

D. $y = 5$ only E. $x = 0$ or $y = -5$

814.

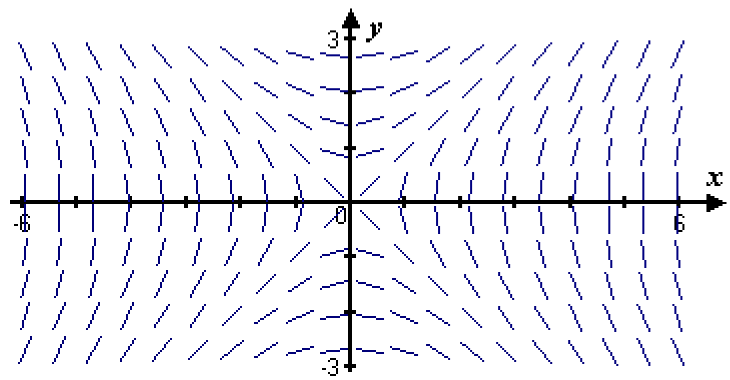
The slope field is for which of the following differential equations ?



A. $\frac{dy}{dx} = y - 2$ B. $\frac{dy}{dx} = x - 2$ C. $\frac{dy}{dx} = x + 2y$ D. $\frac{dy}{dx} = xy$ E. $\frac{dy}{dx} = x + y$

815.

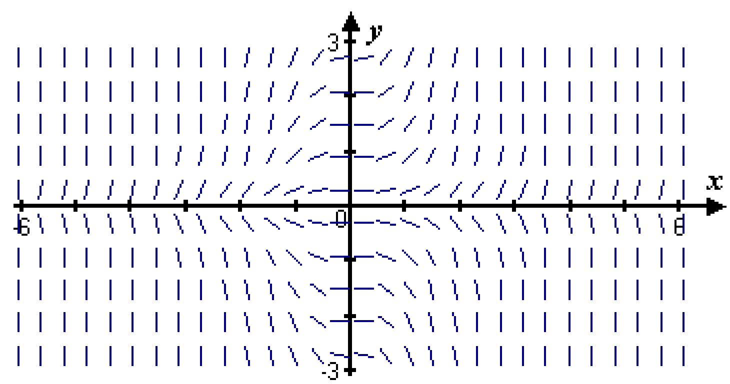
This is a slope field for which of the following differential equations ?



A. $\frac{dy}{dx} = xy$ B. $\frac{dy}{dx} = \frac{x^2}{y}$ C. $\frac{dy}{dx} = x^2 y$ D. $\frac{dy}{dx} = \frac{y}{x}$ E. $\frac{dy}{dx} = \frac{x}{y}$

816.

This is a slope field for which of the following differential equations ?

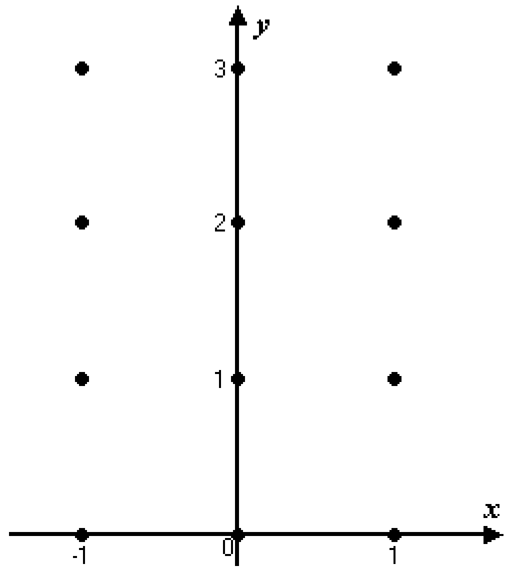


A. $\frac{dy}{dx} = xy^2$ B. $\frac{dy}{dx} = xy$ C. $\frac{dy}{dx} = \frac{x}{y}$ D. $\frac{dy}{dx} = \frac{x^2}{y}$ E. $\frac{dy}{dx} = x^2 y$

817. The slope field for the differential equation $\frac{dy}{dx} = \frac{x^2 y + y^2 x}{3x + y}$ will have horizontal segments when

A. $x = 0$ or $y = 0$ only B. $y = -x$ only C. $y = -3x$ only
D. $y = 5$ only E. $x = 0$ or $y = 0$ or $y = -x$

818. Consider the differential equation $\frac{dy}{dx} = x^2(y-1)$

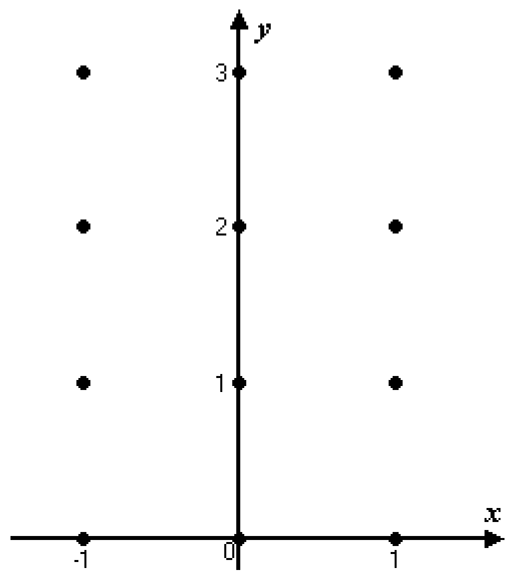


a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.

b) While the slope field in part (***a***) is drawn at only twelve points, it is defined at every point in the *xy*-plane. Describe all points in the *xy*-plane for which the slopes are positive.

c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$

819. Consider the differential equation $\frac{dy}{dx} = x^4(y-2)$

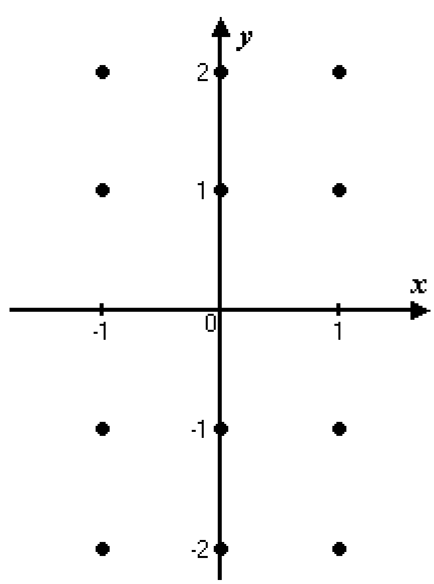


a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.

b) While the slope field in part (***a***) is drawn at only twelve points, it is defined at every point in the *xy*-plane. Describe all points in the *xy*-plane for which the slopes are negative.

c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 0$

820. Consider the differential equation $\frac{dy}{dx} = -\frac{2x}{y}$



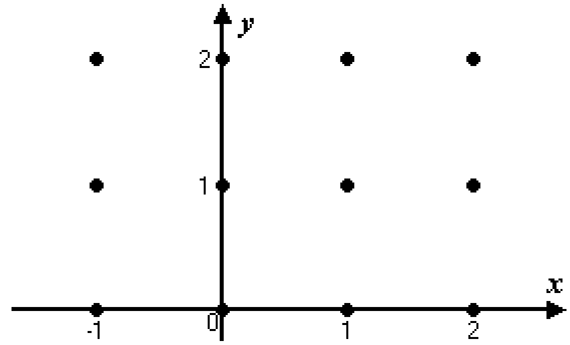
a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.

b) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = -1$ Write an equation for the line tangent to the graph of f at $(1, -1)$ and use it to approximate $f(1.1)$

c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(1) = -1$

821.
Consider the differential equation $\frac{dy}{dx} = -\frac{xy^2}{2}$

Let $y = f(x)$ be the particular solution to this differential equation with the initial condition $f(-1) = 2$



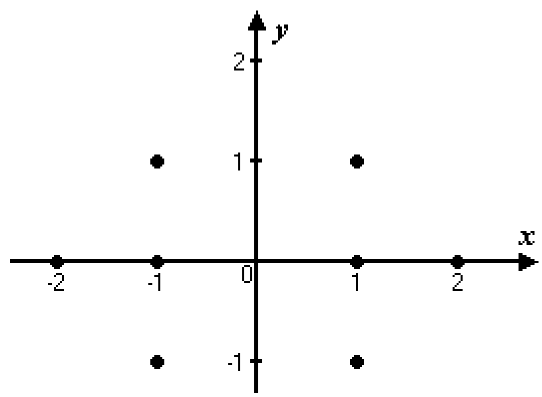
a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.

b) Write an equation for the line tangent to the graph of f at $x = -1$

c) Find the solution $y = f(x)$ to the given differential equation with the initial condition $f(-1) = 2$

822.
Consider the differential equation $\frac{dy}{dx} = \frac{1+y}{x}$

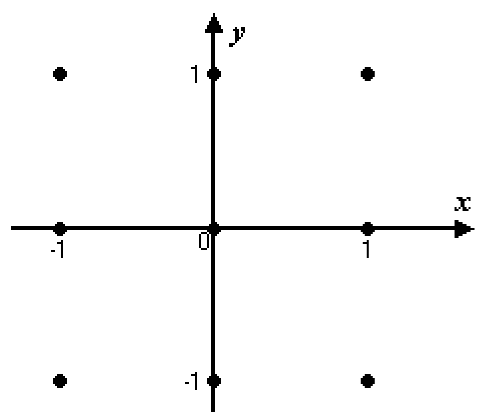
where $x \neq 0$



a) On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.

b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(-1) = 1$ and state its domain.

823.
Consider the differential equation $\frac{dy}{dx} = (y-1)^2 \cos(\pi x)$

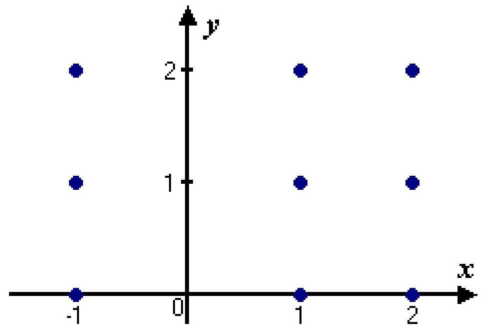


a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

b) There is a horizontal line with equation $y = c$ that satisfies this differential equation. Find the value of c

c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 0$

824. Consider the differential equation $\frac{dy}{dx} = \frac{y-1}{x^2}$ where $x \neq 0$

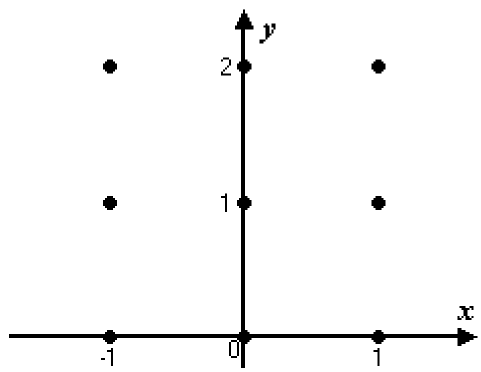


a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(2) = 0$

c) For the particular solution $y = f(x)$ described in part (*b*), find $\lim_{x\to\infty} f(x)$

825. Consider the differential equation $\frac{dy}{dx} = \frac{1}{2}x + y - 1$



a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

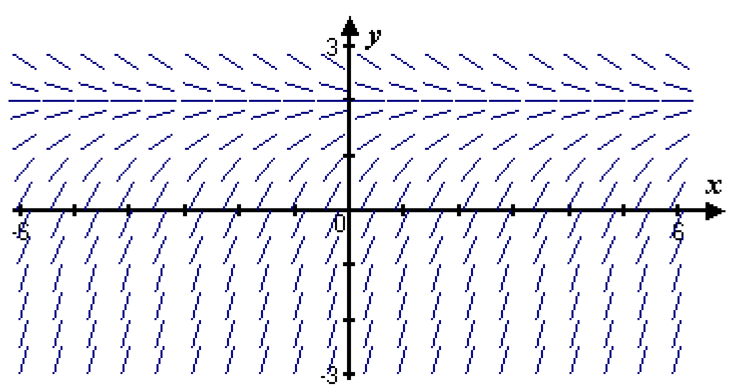
b) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Describe the region in the xy-plane in which all solution curves to the differential equation are concave up.

c) Let $y = f(x)$ be a particular solution to the differential equation with the initial condition $f(0) = 1$. Does f have a relative minimum, a relative maximum, or neither at $x = 0$ Justify your answer.

d) Find the values of the constants m and b, for which $y = mx + b$ is a solution to the differential equation.

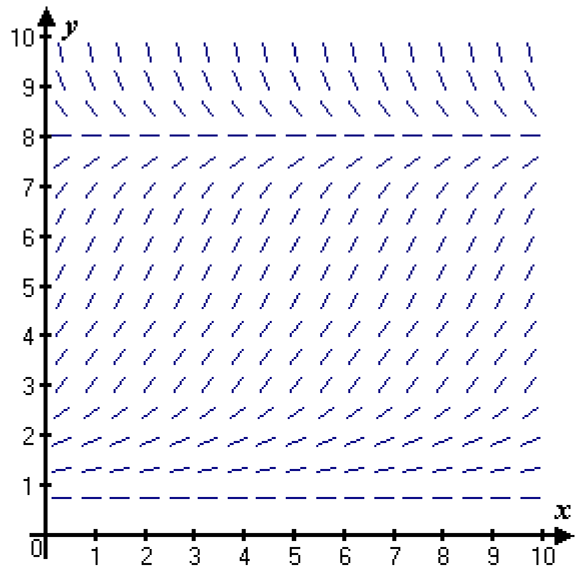
826.

A slope field for a differential equation $\frac{dy}{dx} = f(x, y)$ is given at the right. Which of the following could be a solution ?



A. $y = 2 + \ln x$ B. $y = 2 - \ln x$ C. $y = 2 - e^{x}$ D. $y = 2 - e^{-x}$ E. $y = 2 + e^{2x}$

827. A slope field for a differential equation $\dfrac{dy}{dx} = f(x, y)$ is given in the figure on the right. Which of the following statements are true ?



I. The value of $\dfrac{dy}{dx}$ at the point $(3,\ 3)$ is approximately $\mathbf{1}$

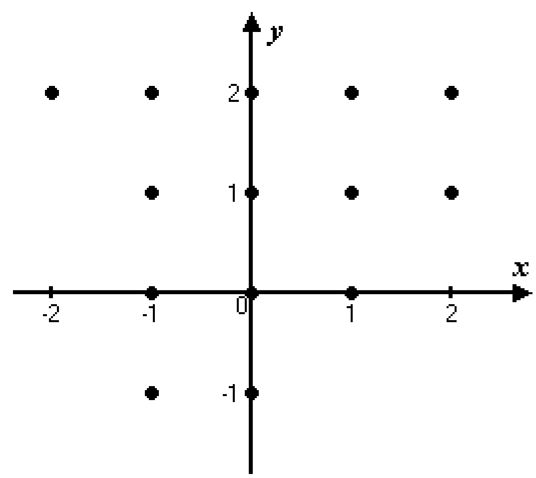
II. As y approaches $\mathbf{8}$ the rate of change of y approaches zero.

III. All solution curves for the differential equation have the same slope for a given value of x

A. **I *only*** B. **II *only*** C. **I *and* II *only***

D. **II *and* III *only*** E. **I, II *and* III**

828. Consider the differential equation $\dfrac{dy}{dx} = x - y$



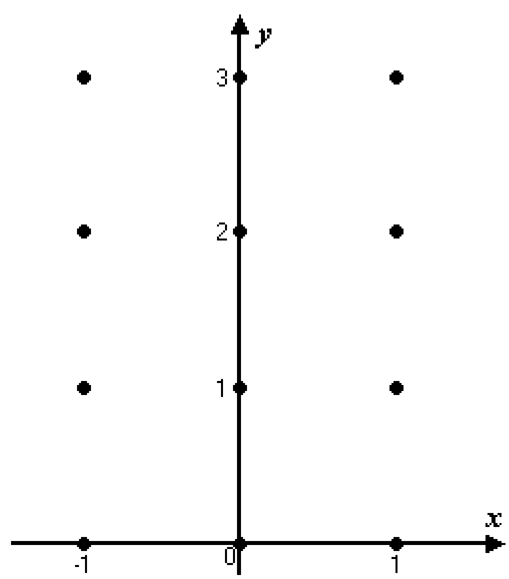
***a*)** On the axes provided, sketch a slope field for the given differential equation at the fourteen points indicated.

***b*)** Sketch the solution curve that contains the point $(-1,\ 1)$

***c*)** Find an equation for the straight line solution through the point $(1,\ 0)$

***d*)** Show that if **C** is a constant, then $y = x - 1 + Ce^{-x}$ is a solution of the differential equation

829. Consider the differential equation $\dfrac{dy}{dx} = x^2(2y+1)$



***a*)** On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.

b*)** Although the slope field in part **(*a*)** is drawn at only **12** points, it is defined at every point in the ***xy-plane. Describe all points in the ***xy***-plane for which the slopes are positive.

***c*)** Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 2$

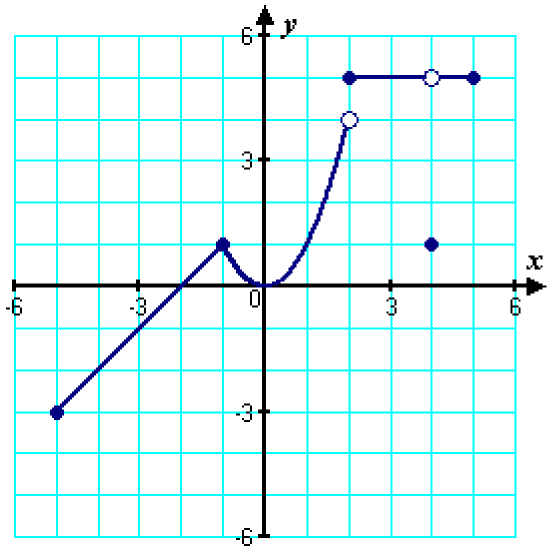
Calculus AB package #2 Feb/2009 Name: ______________________________

1.

Piecewise functions explained by example

$$f(x)=\begin{cases} x+2 & \text{for} \quad -5\le x<-1 \\ x^2 & \text{for} \quad -1\le x<2 \\ 5 & \text{for} \quad 2\le x\le 5 \quad x\ne 4 \\ 1 & \text{for} \quad x=4 \end{cases}$$

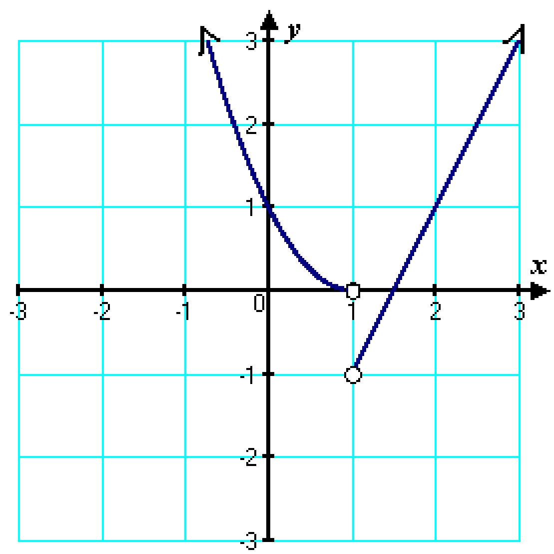
$f(-4)=-2$	$f'(-4)=1$	$\lim\limits_{x\to -1^+} f(x)=1$
$f(1)=1$	$f'(1)=2$	$\lim\limits_{x\to 2^-} f(x)=4$
$f(3)=5$	$f'(3)=0$	$\lim\limits_{x\to 4} f(x)=5$
$f(8)=$ undef	$f'(8)=$ undef	



2. *Answer is B.*

Find the range of the piecewise function defined by $f(x)=\begin{cases} (x-1)^2, & x<1 \\ 2x-3, & x>1 \end{cases}$

Sketch graph and range is $\boxed{y>-1}$



3. *Answer is D.*

Given $f(x) = \begin{cases} x+3 & \text{where } x<0 \\ x-3 & \text{where } x \geq 0 \end{cases}$

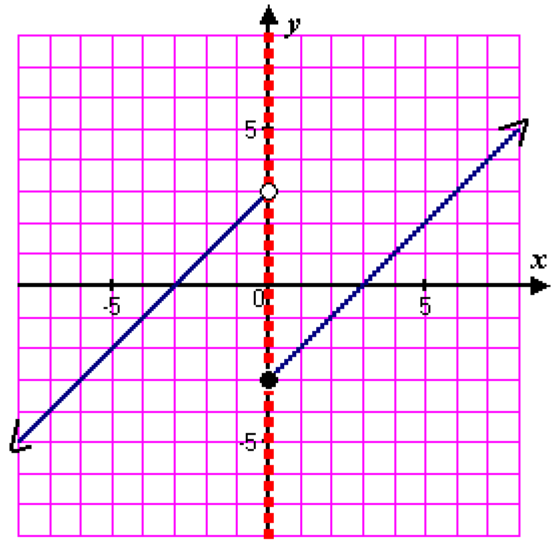
then $\lim\limits_{x \to 0^-} f(x) =$

Sketch graph of piecewise function

Domain all real x

with jump discontinuity at $x = 0$

From the ***negative*** side $\lim\limits_{x \to 0^-} f(x) = \boxed{3}$



4. *Answer is A.*

Given $f(x) = \begin{cases} x+3 & \text{where } x<0 \\ x-3 & \text{where } x \geq 0 \end{cases}$

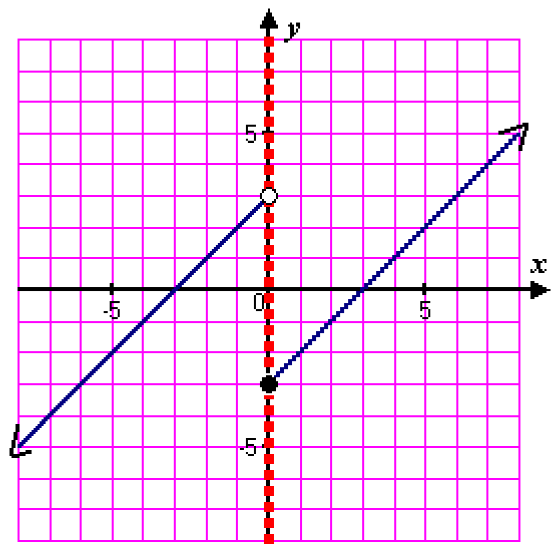
then $\lim\limits_{x \to 0^-} f(x) =$

Sketch graph of piecewise function

Domain all real x

with jump discontinuity at $x = 0$

From the ***positive*** side $\lim\limits_{x \to 0^+} f(x) = \boxed{-3}$



5. *Answer is E.*

Given $f(x) = \begin{cases} x+3 & \text{where } x<0 \\ x-3 & \text{where } x \geq 0 \end{cases}$

then $\lim\limits_{x \to 0^-} f(x) =$

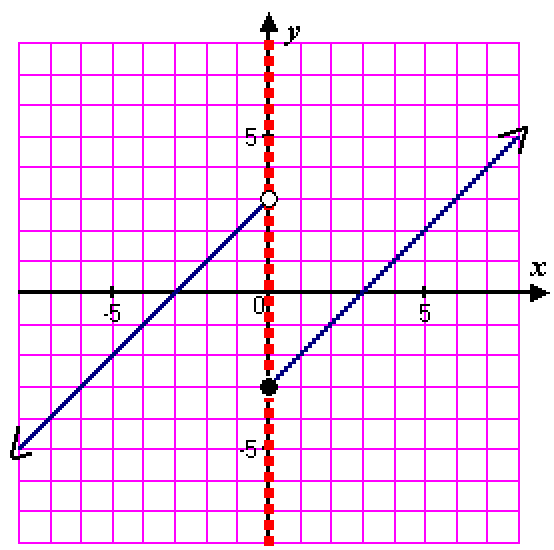
Sketch graph of piecewise function

Domain all real x

with jump discontinuity at $x = 0$

$\lim\limits_{x \to 0^-} f(x) \neq \lim\limits_{x \to 0^+} f(x)$

then $\lim\limits_{x \to 0} f(x) = \boxed{\textit{does not exist}}$



6. *Answer is A.*

Given $f(x)=\begin{cases} x+3 & \text{where } x<0 \\ x-3 & \text{where } x\ge 0 \end{cases}$

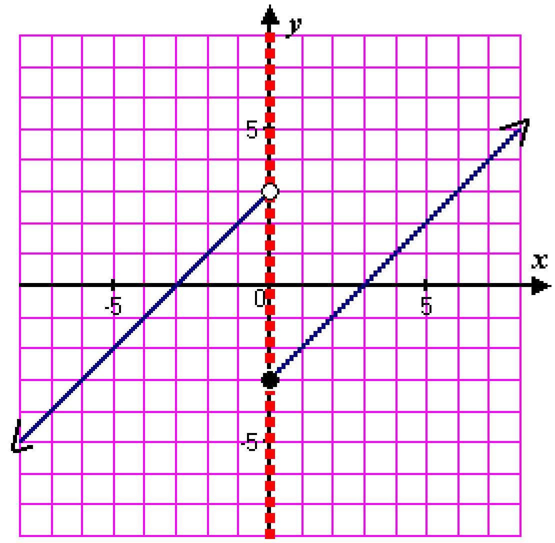
then $\lim\limits_{x\to 1} f(x)=$

Sketch graph of piecewise function

Domain all real x

with jump discontinuity at $x=0$

and $\lim\limits_{x\to 1} f(x)=\lim\limits_{x\to 1} x-3=\boxed{-2}$



7. *Answer is C.*

Given $f(x)=\begin{cases} x+3 & \text{where } x<0 \\ x-3 & \text{where } x\ge 0 \end{cases}$

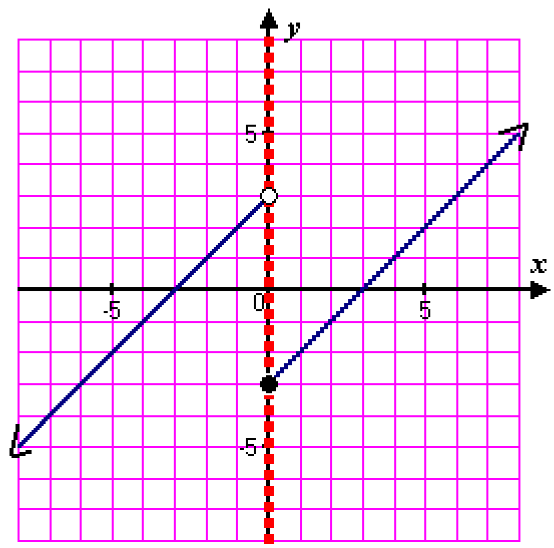
then $\lim\limits_{x\to -2} f(x)=$

Sketch graph of piecewise function

Domain all real x

with jump discontinuity at $x=0$

and $\lim\limits_{x\to -2} f(x)=\lim\limits_{x\to -2} x+3=\boxed{1}$



8. *Answer is C.*

Given $f(x)=\begin{cases} x^2 & \text{where } x\ne 2 \\ 2 & \text{where } x=2 \end{cases}$ then

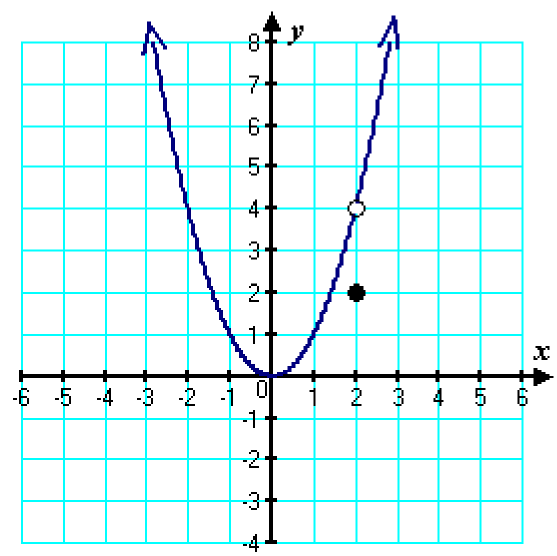
$f(2)=$

Sketch graph of piecewise function

Domain all real x

with ***hole*** at $x=0$

and $\lim\limits_{x\to 2} f(x)=4$ but $f(2)=\boxed{2}$



9. *Answer is D.*

Given $f(x)=\begin{cases} x^2 & \text{where } x \neq 2 \\ 2 & \text{where } x = 2 \end{cases}$

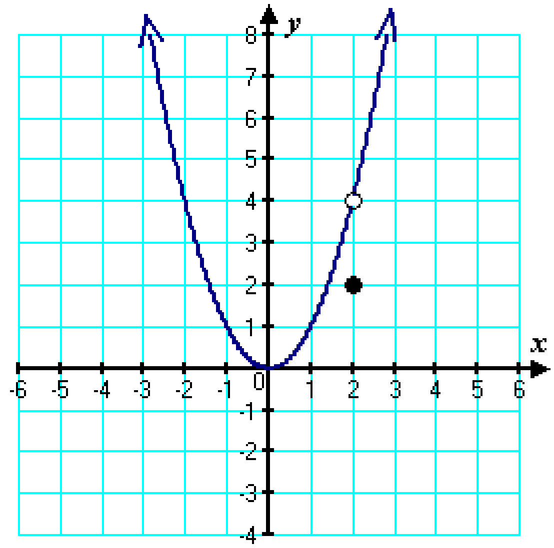
then $\lim\limits_{x\to 2^-} f(x) =$

Sketch graph of piecewise function

Domain all real x

with ***hole*** at $x = 0$

and $\lim\limits_{x\to 2^-} f(x) = 4$



10. *Answer is D.*

Given $f(x)=\begin{cases} x^2 & \text{where } x \neq 2 \\ 2 & \text{where } x = 2 \end{cases}$

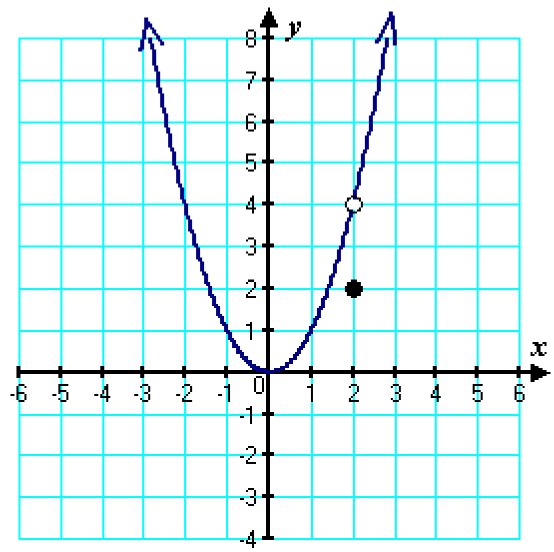
then $\lim\limits_{x\to 2^+} f(x) =$

Sketch graph of piecewise function

Domain all real x

with ***hole*** at $x = 0$

and $\lim\limits_{x\to 2^+} f(x) = \boxed{4}$



11. *Answer is D.*

Given $f(x)=\begin{cases} x^2 & \text{where } x \neq 2 \\ 2 & \text{where } x = 2 \end{cases}$

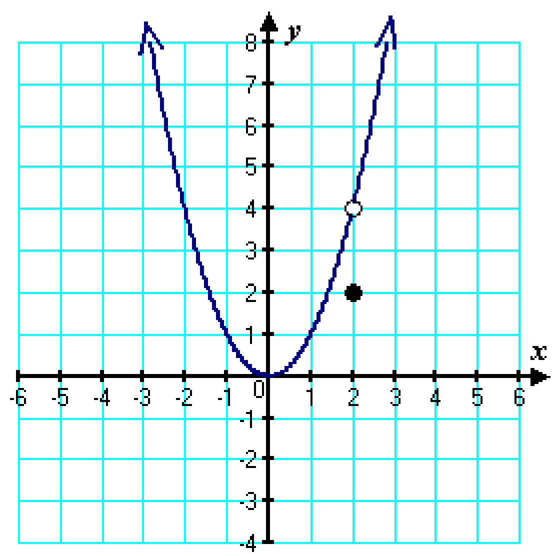
then $\lim\limits_{x\to 2} f(x) =$

Sketch graph of piecewise function

Domain all real x

with ***hole*** at $x = 0$

and $\lim\limits_{x\to 2} f(x) = \boxed{4}$



12. *Answer is A.*

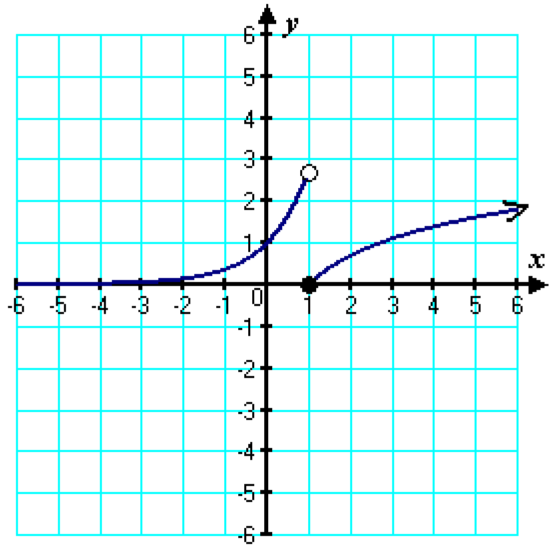
$$\boxed{\begin{array}{l}\text{Given } \ f(x)=\begin{cases} e^{x} & \text{where } \ x<1 \\ \ln x & \text{where } \ x\ge 1\end{cases} \\ \quad \text{then } \ f(1)= \end{array}}$$

$$\lim_{x\to 1^{+}} f(1)=0$$

$$\lim_{x\to 1^{-}} f(1)=e$$

$$\lim_{x\to 1} f(1)=\text{does not exist}$$

$$\boxed{f(1)=0}$$



13. *Answer is D.*

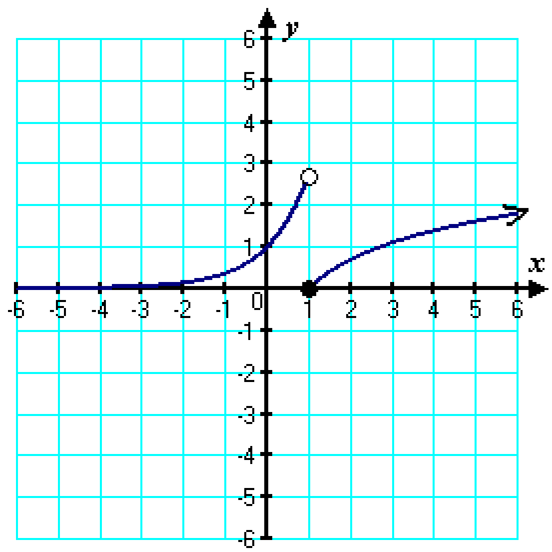
$$\boxed{\begin{array}{l}\text{Given } \ f(x)=\begin{cases} e^{x} & \text{where } \ x<1 \\ \ln x & \text{where } \ x\ge 1\end{cases} \\ \quad \text{then } \ \lim_{x\to 1^{-}} f(x)= \end{array}}$$

$$\lim_{x\to 1^{+}} f(1)=0$$

$$\boxed{\lim_{x\to 1^{-}} f(1)=e}$$

$$\lim_{x\to 1} f(1)=\text{does not exist}$$

$$f(1)=0$$



14. *Answer is A.*

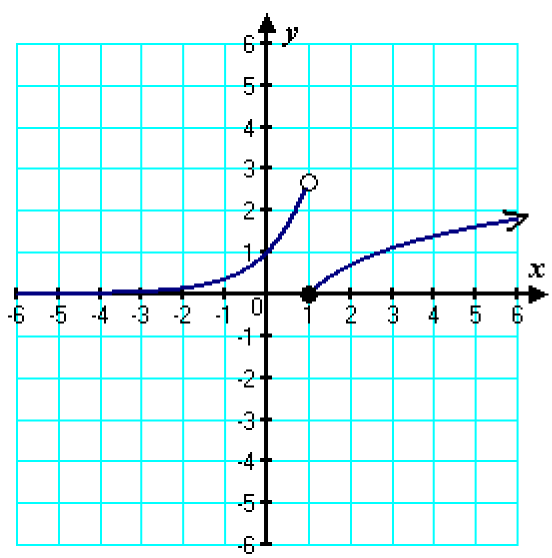
$$\boxed{\begin{array}{l}\text{Given } \ f(x)=\begin{cases} e^{x} & \text{where } \ x<1 \\ \ln x & \text{where } \ x\ge 1\end{cases} \\ \text{then } \ \lim_{x\to 1^{+}} f(x)= \end{array}}$$

$$\boxed{\lim_{x\to 1^{+}} f(1)=0}$$

$$\lim_{x\to 1^{-}} f(1)=e$$

$$\lim_{x\to 1} f(1)=\text{does not exist}$$

$$f(1)=0$$



15. *Answer is E.*

Given $f(x)=\begin{cases} e^x & \text{where } x<1 \\ \ln x & \text{where } x\geq 1 \end{cases}$

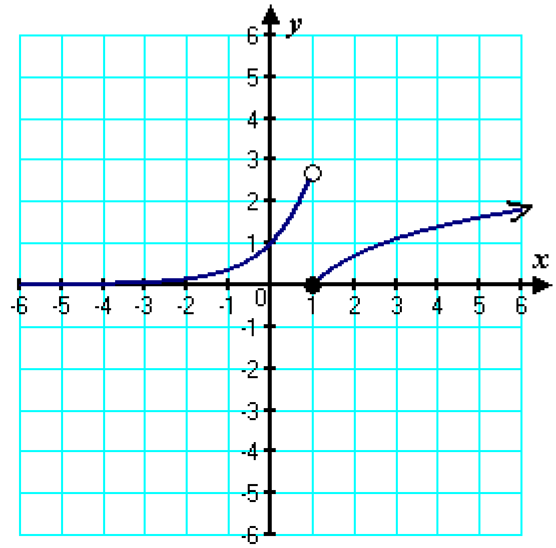
then $\lim\limits_{x\to 1} f(x)=$

Piecewise function, sketch graph and obvious graphs do not meet at $x=1$

$\lim\limits_{x\to 1^-} f(x)=e$

$\lim\limits_{x\to 1^+} f(x)=0$

$\boxed{\lim\limits_{x\to 1} f(x)=\textit{does not exist}}$



16. *Answer is D.*

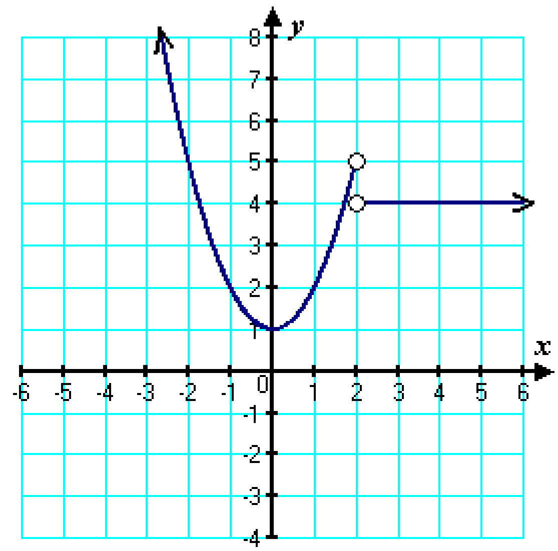
Given $f(x)=\begin{cases} x^2+1 & \text{where } x<2 \\ 4 & \text{where } x>2 \end{cases}$

then $\lim\limits_{x\to 2^-} f(x)=$

Sketch graph of piecewise function

Domain all real x where $x\neq 2$

and $\lim\limits_{x\to 2^-} f(x)=\boxed{5}$



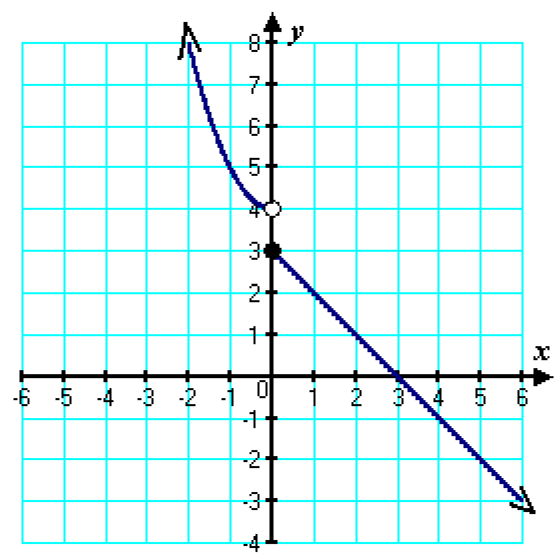
17. *Answer is A.*

Find $f(2)$ for

$f(x)=\begin{cases} x^2+4 & \text{where } x<0 \\ 3-x & \text{where } x\geq 0 \end{cases}$

Sketch graph of piecewise function

Domain all real x and $f(2)=\boxed{1}$



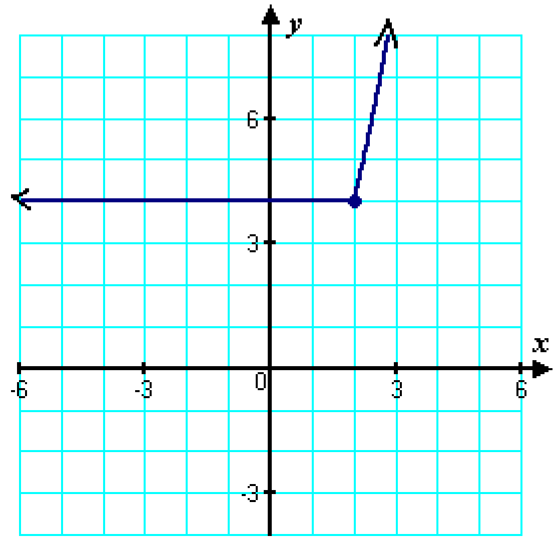
18. *Answer is B.*

$$f(x) = \begin{cases} \mathbf{4} & \text{where} \quad x < 2 \\ x^2 & \text{where} \quad x \geq 2 \end{cases}$$

is differentiable for

$f(x)$ is differentiable for all x where $x \neq 2$

(there is no derivative at sharp points)



19. *Answer is E.*

Consider the function

$$f(x) = \begin{cases} 2x^2 - x^3 & \text{for } x < 2 \\ e^{2x-4} & \text{for } x \geq 2 \end{cases}$$

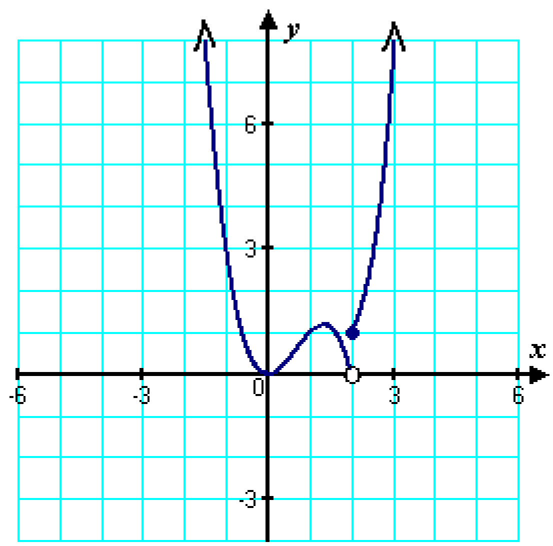
Find $\lim\limits_{x \to 2} f(x) =$

$\lim\limits_{x \to 2^-} f(x) = 0$

$\lim\limits_{x \to 2^+} f(x) = 1$

$\lim\limits_{x \to 2^-} f(x) \neq \lim\limits_{x \to 2^+} f(x)$

$\boxed{\lim\limits_{x \to 2} f(x) = \text{does not exist}}$



20. *Answer is D.*

$$\text{If } f(x) = \begin{cases} 1 & \text{if } x \leq -2 \\ x^2 - 4 & \text{if } -2 < x < 2 \\ x & \text{if } x \geq 2 \end{cases}$$

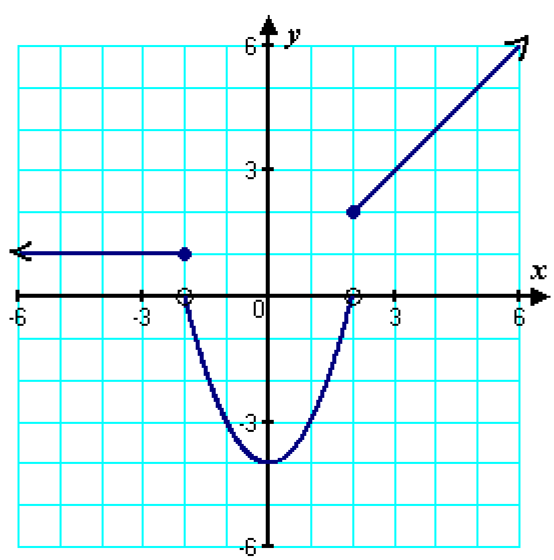
the range of f is

Sketch graph of piecewise function and the range is

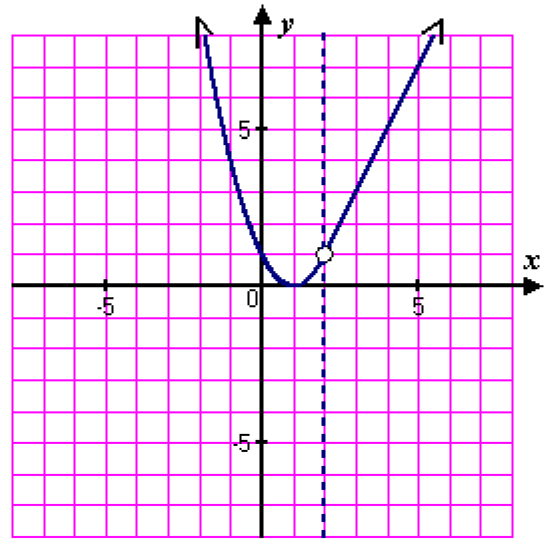
$-4 \leq y < 0$ (parabola section)

or $y = 1$ (horizontal line $y = 1$)

or $y \geq 2$ (diagonal line $y = x$)



21. *Answer is E.*

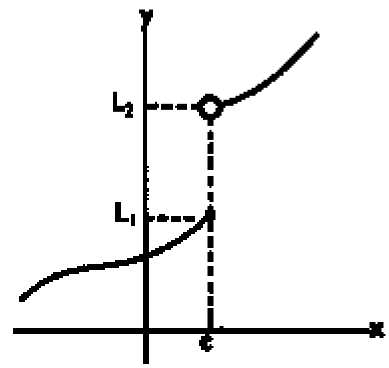


The ***range*** of the piecewise function defined by $f(x)=\begin{cases}(x-1)^2 & \text{for } x<2\\ 2x-3 & \text{for } x>2\end{cases}$ is

Sketch graph of $f(x)$ and observe

$\boldsymbol{Range} \rightarrow \boxed{y \geq 0}$

22. *Answer is E.*



Referring to the following figure showing the graph of $y=f(x)$, $\lim\limits_{x\to c} f(x)=$

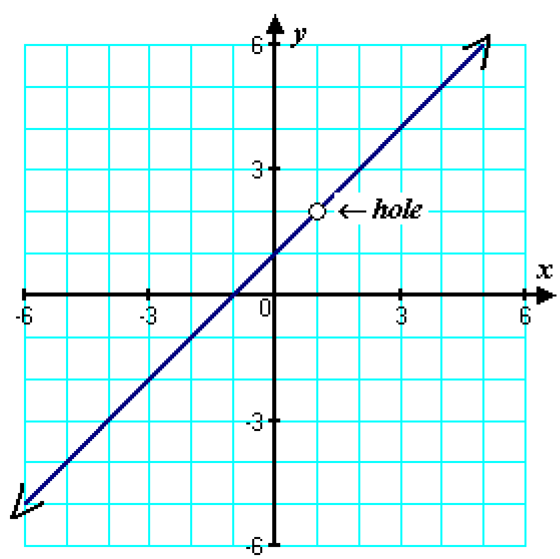
$$\left.\begin{array}{l}\lim\limits_{x\to c^-} f(x)=\mathrm{L}_1\\ \lim\limits_{x\to c^+} f(x)=\mathrm{L}_2\end{array}\right\}\text{ one-sided limits are } \textbf{\textit{not}} \text{ equal}$$

so $\lim\limits_{x\to c} f(x)=\boxed{\text{does not exist}}$

(behaviour differs from right and left)

see page 64 text

23. *Answer is A.*



The graph of $f(x)=\dfrac{x^2-1}{x-1}$ has a

$$\lim_{x\to 1}\frac{(x-1)(x+1)}{x-1}=\frac{0}{0} \leftarrow \text{indeterminant}$$

$$f(x)=\frac{x^2-1}{x-1}=\frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}}=(x+1)$$

$\underbrace{\textbf{\textit{hole}} \text{ at } x=1}_{point\,(1,2)}$ in the line $y=x+1$

24.

Limits → needed to handle ***holes***, ***asymptotes***, ***sharp points***, ***endpoints***,

→ **definition of derivative**

$\boxed{1}$ Direct substitution → polynomials $\lim\limits_{x\to c} f(x) = f(c)$ ← too easy (difficult to see usefulness)

$\boxed{2}$ Does ***not*** exist

a) left/right $\lim\limits_{x\to 3}\frac{|x-3|}{x-3}$ (jump discontinuity) $\lim\limits_{x\to 3^-}\frac{|x-3|}{x-3} \neq \lim\limits_{x\to 3^+}\frac{|x-3|}{x-3}$

b) unbounded $\lim\limits_{x\to 3}\frac{1}{x-3} = \frac{1}{3-3} = \frac{1}{0} = \pm\infty$ $\lim\limits_{x\to 3^-}\frac{1}{x-3} \neq \lim\limits_{x\to 3^+}\frac{1}{x-3}$

c) oscillating $\lim\limits_{x\to 3}\sin\left(\frac{1}{x}\right)$

$\boxed{3}$ Piecewise → learn and understand Given $f(x) = \begin{cases} x+3 & \text{where } x<0 \\ x-3 & \text{where } x\geq 0 \end{cases}$ then $\lim\limits_{x\to 0^-} f(x) =$

$\boxed{4a}$ Indeterminant form → $\lim\limits_{x\to 3}\frac{x^2-9}{x^2+x-12} = \frac{3^2-9}{3^2+3-12} = \boxed{\frac{0}{0}}$ ← factor

$$\lim_{x\to 3}\frac{x^2-9}{x^2+x-12} = \lim_{x\to 3}\frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}(x+4)} = \lim_{x\to 3}\frac{x+3}{x+4} = \frac{3+3}{3+4} = \boxed{\frac{6}{7}}$$ ☑

$\boxed{4b}$ Indeterminant form → $\lim\limits_{x\to 9}\frac{\sqrt{x}-3}{x-9} = \frac{\sqrt{9}-3}{9-9} = \boxed{\frac{0}{0}}$ ← conjugate (of numerator in this case)

$$\lim_{x\to 9}\frac{\sqrt{x}-3}{x-9} = \lim_{x\to 9}\frac{\sqrt{x}-3}{x-9}\left(\frac{\sqrt{x}+3}{\sqrt{x}+3}\right) = \lim_{x\to 9}\frac{\cancel{x-9}}{\cancel{(x-9)}\left(\sqrt{x}+3\right)} = \lim_{x\to 9}\frac{1}{\sqrt{x}+3} = \frac{1}{\sqrt{9}+3} = \boxed{\frac{1}{6}}$$

$\boxed{4c}$ Indeterminant form → $\lim\limits_{x\to 1}\frac{\ln x}{x^4-1} = \frac{\ln 1}{1^4-1} = \boxed{\frac{0}{0}}$ ← L'hopital rule

$$\lim_{x\to 1}\frac{\frac{1}{x}}{4x^3} = \frac{\frac{1}{1}}{4(1)^3} = \boxed{\frac{1}{4}}$$

$\boxed{5}$ Infinity → horizontal asymptotes (divide by the variable with highest exponent in denominator

$$\lim_{x\to\infty}\frac{4x-5x^2}{2x^2+3x-1} = \lim_{x\to\infty}\frac{\frac{4x}{x^2}-\frac{5x^2}{x^2}}{\frac{2x^2}{x^2}+\frac{3x}{x^2}-\frac{1}{x^2}} = \lim_{x\to\infty}\frac{\frac{4}{x}-5}{2+\frac{3}{x}-\frac{1}{x^2}} = \frac{0-5}{2+0-0} = \boxed{-\frac{5}{2}}$$

$\boxed{6a}$ Trig → basic $\lim\limits_{x\to 0}\frac{\sin x}{x} = 1$ $\lim\limits_{x\to 0}\frac{\cos x - 1}{x} = 0$ ← squeeze theorem proof

$\boxed{6b}$ Trig → advanced questions based on basics

$\boxed{7}$ Definition of derivative $f'(x) = \lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$ ← secant changes into a tangent

25. *Answer is C.*

Difficulty = **0.88 U**

$$\lim_{x\to 3}(x^2-2x+2)=$$

$$\lim_{x\to 3}(x^2-2x+2)=\underbrace{3^2-2(3)+2}_{\textit{direct substitution}}=\boxed{5}$$

26. *Answer is B.*

$$\lim_{x\to 3}\frac{x^2-9}{x^2+9}=$$

$$\lim_{x\to 3}\frac{x^2-9}{x^2+9}=\lim_{x\to 3}\frac{3^2-9}{3^2+9}=\frac{0}{18}=\boxed{0}$$

27. *Answer is B.*

$$\lim_{x\to 5}\frac{x-5}{x-5}=$$

$$\lim_{x\to 5}\frac{x-5}{x-5}=\lim_{x\to 5}\frac{\cancel{x-5}}{\cancel{x-5}}=\boxed{1}$$

28. *Answer is B.*

$$\lim_{x\to 2}\frac{x^2-4}{x^2+4}=$$

$$\lim_{x\to 2}\frac{x^2-4}{x^2+4}=\lim_{x\to 2}\frac{2^2-4}{2^2+4}=\frac{0}{8}=\boxed{0}$$

29. *Answer is E.*

Difficulty = **0.34**

$$\lim_{x\to 1}\frac{x}{\ln x}=$$

$$\lim_{x\to 1}\frac{x}{\ln x}=\frac{1}{\ln 1}=\frac{1}{0}=\boxed{\infty}\quad \text{undefined (does not exist)}$$

30. *Answer is C.*

$$\lim_{x\to 10}\frac{4x^2-6x+10}{50+4x^2}=$$

$$\lim_{x\to 10}\frac{4x^2-6x+10}{4x^2+50}=\frac{4(10)^2-6(10)+10}{4(10)^2+50}=\frac{400-60+10}{400+50}=\frac{35}{45}=\boxed{\frac{7}{9}}$$

31. *Answer is B.*

$$\lim_{x\to 10}\frac{3x^2-7x+10}{60+3x^2}=$$

$$\lim_{x\to 10}\frac{3x^2-7x+10}{60+3x^2}=\lim_{x\to 10}\frac{3(10)^2-7(10)+10}{60+3(10)^2}=\frac{300-70+10}{60+300}=\frac{240}{360}=\boxed{\frac{2}{3}}$$

32. *Answer is A.*

$$\lim_{x\to 0}\left(\frac{\frac{1}{x-1}+1}{x}\right)=$$

$$\lim_{x\to 0}\left(\frac{\frac{1}{x-1}+\frac{x-1}{x-1}}{\frac{x}{1}}\right)=\lim_{x\to 0}\left(\frac{\frac{x}{x-1}}{\frac{x}{1}}\right)=\lim_{x\to 0}\left(\frac{\cancel{x}}{x-1}\cdot\frac{1}{\cancel{x}}\right)=\lim_{x\to 0}\left(\frac{1}{x-1}\right)=\lim_{x\to 0}\left(\frac{1}{0-1}\right)=\boxed{-1}$$

33. *Answer is B.*

$$\lim_{x\to 1}\frac{\ln x}{x}=$$

$$\lim_{x\to 1}\frac{\ln x}{x}=\frac{\ln 1}{1}=\frac{0}{1}=\boxed{0}$$

34. *Answer is C.*

$$\lim_{x\to 3}\frac{6x^2-5}{4x^2+1}=$$

$$\lim_{x\to 3}\frac{6x^2-5}{4x^2+1}=\frac{6(3)^2-5}{4(3)^2+1}=\boxed{\frac{49}{37}}$$

35. *Answer is B.*

$$\lim_{x\to 2}(3x^2+5)=$$

$$\lim_{x\to 2}(3x^2+5)=(3(2)^2+5)=\boxed{17}$$

36. *Answer is C.*

$$\lim_{x\to -3}(-2x^2+1)=$$

$$\lim_{x\to -3}(-2x^2+1)=(-2(-3)^2+1)=\boxed{-17}$$

37. *Answer is A.*

$$\lim_{x\to -1}\frac{x^2+3x+2}{x^2+1}=$$

$$\lim_{x\to -1}\frac{x^2+3x+2}{x^2+1}=\frac{(-1)^2+3(-1)+2}{(-1)^2+1}=\frac{1-3+2}{1+1}=\boxed{0}$$

38. *Answer is B.*

$$\lim_{x\to -1}\frac{x^2+2x+3}{x^2+1}=$$

$$\lim_{x\to -1}\frac{x^2+2x+3}{x^2+1}=\frac{(-1)^2+2(-1)+3}{(-1)^2+1}=\frac{1-2+3}{1+1}=\boxed{1}$$

39. *Answer is D.*

$$\lim_{x\to 3}\sqrt{x^2-4}=$$

$$\lim_{x\to 3}\sqrt{x^2-4}=\sqrt{(3)^2-4}=\boxed{\sqrt{5}}$$

40. *Answer is A.*

$$\lim_{x\to 3}\sqrt{9-x^2}=$$

$$\lim_{x\to 3}\sqrt{9-x^2}=\sqrt{9-(3)^2}=\boxed{0}$$

41. *Answer is B.*

$$\lim_{x\to 2^-}\sqrt{2x-3}=$$

$$\lim_{x\to 2^-}\sqrt{2x-3}=\sqrt{2(2)-3}=\sqrt{1}=1$$

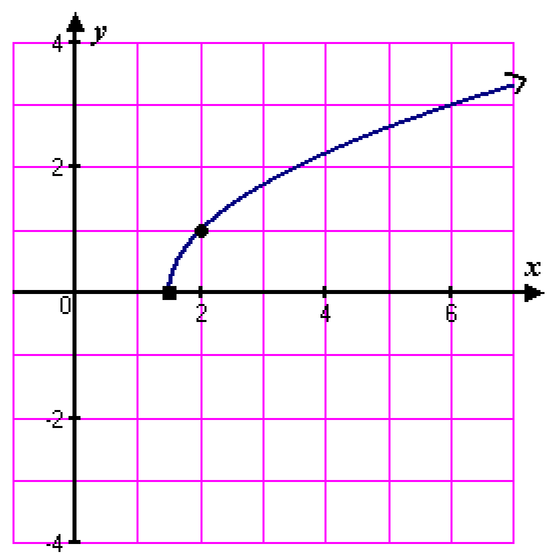
$$f(x)=\sqrt{2x-3}$$

$$2x-3\ge 0$$

$$2x\ge 3$$

$$x\ge \tfrac{3}{2}$$

In its domain, $x\ge \frac{3}{2}$ $f(x)$ is continuous



42. *Answer is D.*

$$\lim_{x\to 0}\frac{x-1}{x^2-1}=$$

$$\lim_{x\to 0}\frac{x-1}{x^2-1}=\frac{0-1}{0^2-1}=\boxed{1}$$

43. *Answer is A.*

Difficulty $= \mathbf{0.67}$

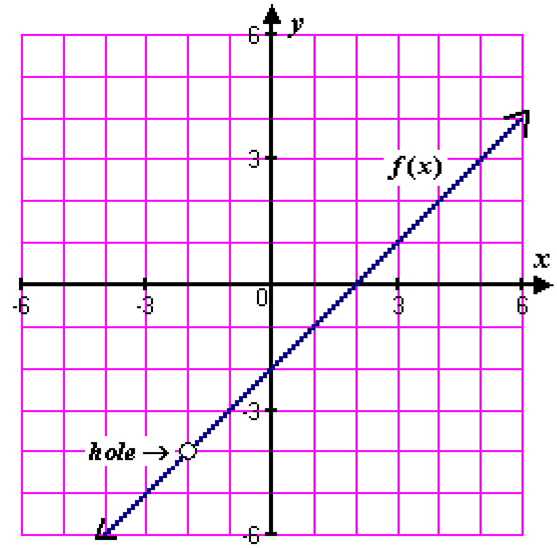
If the function f is continuous for all real numbers and if $f(x) = \dfrac{x^2-4}{x+2}$ when $x \neq -2$, then $f(-2) =$

If $x \neq -2$ then

$$f(x) = \frac{x^2-4}{x+2} = \frac{(x-2)(x+2)}{x+2} = x-2$$

The graph of $f(x)$ is the line $y = x-2$ with a ***hole*** at $x = -2$

If $\boxed{f(-2) = -4}$ then the hole is filled and the function is ***continuous***



44. *Answer is B.*

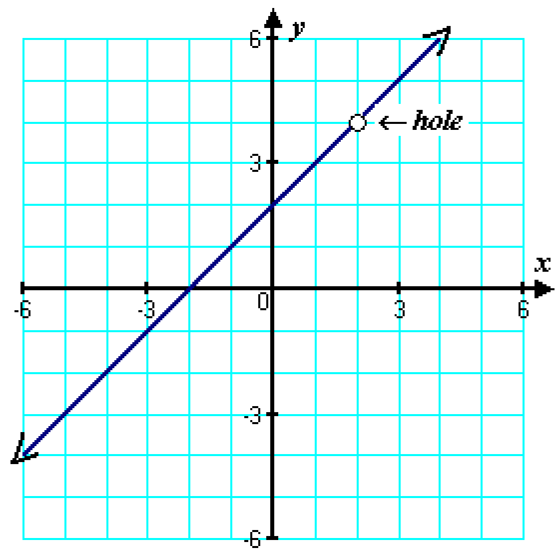
If $f(x) = \begin{cases} \dfrac{x^2-4}{x-2} & \text{if } x \neq 2 \\ k & \text{if } x = 2 \end{cases}$, for what value(s) of k is $f(x)$ continuous at $x = 2$

$$\lim_{x\to 2} \frac{x^2-4}{x-2} = \lim_{x\to 2} \frac{(x-2)(x+2)}{x-2} = \frac{0}{0}$$

indeterminant form (hole at $x = 2$)

$$\lim_{x\to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}} = \lim_{x\to 2}(x+2) = \boxed{4}$$

If $k = 4$ then the function $f(x)$ will be continuous (the hole will be filled by $k = 4$)



45. *Answer is B.*

If $f(x)=\begin{cases} \dfrac{x^2-x}{2x} & \text{for } x\neq 0 \\ k & \text{for } x=0 \end{cases}$

and if f is continuous at $x=0$ then $k=$

for $x\neq 0$ $\quad \dfrac{x^2-x}{2x}=\dfrac{x(x-1)}{2x}=\dfrac{x-1}{2}=\frac{1}{2}x-\frac{1}{2}$

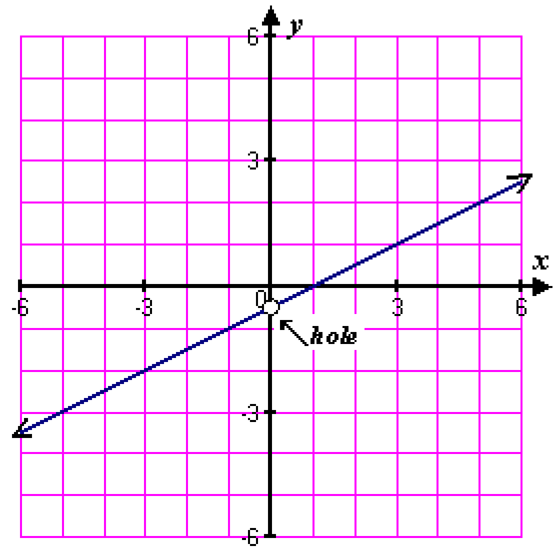
The function $f(x)$ is the graph of $y=\frac{1}{2}x-\frac{1}{2}$

with at ***hole*** in it at $(0,-\frac{1}{2})$

What value of k at $(0,k)$ will fill the hole to

to make the function f ***continuous*** at $x=0$

So if $\boxed{k=-\frac{1}{2}}$ $\rightarrow$ then hole is filled



46.

$\boxed{\text{Limits}}$ $\rightarrow$ needed to handle ***holes***, ***asymptotes***, ***sharp points***, ***endpoints***,

$\boxed{4a}$ Indeterminant form $\rightarrow$ $\displaystyle\lim_{x\to 3}\frac{x^2-9}{x^2+x-12}=\frac{3^2-9}{3^2+3-12}=\boxed{\frac{0}{0}}$ $\leftarrow$ factor

$$\lim_{x\to 3}\frac{x^2-9}{x^2+x-12}=\lim_{x\to 3}\frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}(x+4)}=\lim_{x\to 3}\frac{x+3}{x+4}=\frac{3+3}{3+4}=\boxed{\frac{6}{7}}\quad ☑$$

$\boxed{4b}$ Indeterminant form $\rightarrow$ $\displaystyle\lim_{x\to 9}\frac{\sqrt{x}-3}{x-9}=\frac{\sqrt{9}-3}{9-9}=\boxed{\frac{0}{0}}$ $\leftarrow$ conjugate (of numerator in this case)

$$\lim_{x\to 9}\frac{\sqrt{x}-3}{x-9}=\lim_{x\to 9}\frac{\sqrt{x}-3}{x-9}\left(\frac{\sqrt{x}+3}{\sqrt{x}+3}\right)=\lim_{x\to 9}\frac{\cancel{x-9}}{\cancel{(x-9)}\left(\sqrt{x}+3\right)}=\lim_{x\to 9}\frac{1}{\sqrt{x}+3}=\frac{1}{\sqrt{9}+3}=\boxed{\frac{1}{6}}$$

$\boxed{4c}$ Indeterminant form $\rightarrow$ $\displaystyle\lim_{x\to 1}\frac{\ln x}{x^4-1}=\frac{\ln 1}{1^4-1}=\boxed{\frac{0}{0}}$ $\leftarrow$ L'hopital rule

$$\lim_{x\to 1}\frac{\frac{1}{x}}{4x^3}=\frac{\frac{1}{1}}{4(1)^3}=\boxed{\frac{1}{4}}$$

47. *Answer is C.*

Difficulty = **0.81 U**

$$\lim_{x\to 7}\frac{x^2-6x-7}{x^2-5x-14}=$$

$$\lim_{x\to 7}\frac{x^2-6x-7}{x^2-5x-14}=\frac{7^2-6(7)-7}{7^2-5(7)-14}=\frac{0}{0} \quad \leftarrow \text{indeterminate form}$$

$$\lim_{x\to 7}\frac{x^2-6x-7}{x^2-5x-14}=\lim_{x\to 7}\frac{\cancel{(x-7)}(x+1)}{\cancel{(x-7)}(x+2)}=\lim_{x\to 7}\frac{(x+1)}{(x+2)}=\frac{7+1}{7+2}=\boxed{\frac{8}{9}}$$

48. *Answer is B.*

Difficulty = **0.76 U**

$$\lim_{x\to -3}\frac{x^2+x-6}{x^2-9}=$$

$$\lim_{x\to -3}\frac{x^2+x-6}{x^2-9}=\frac{(-3)^2+(-3)-6}{(-3)^2-9}=\frac{0}{0} \quad \leftarrow \text{indeterminate form}$$

$$\lim_{x\to -3}\frac{x^2+x-6}{x^2-9}=\lim_{x\to -3}\frac{\cancel{(x+3)}(x-2)}{\cancel{(x+3)}(x-3)}=\lim_{x\to -3}\frac{(x-2)}{(x-3)}=\frac{-3-2}{-3-3}=\boxed{\frac{5}{6}}$$

49. *Answer is C.*

Difficulty = **0.75 U**

$$\lim_{x\to 4}\frac{x^2-2x-8}{x^2-6x+8}=$$

$$\lim_{x\to 4}\frac{x^2-2x-8}{x^2-6x+8}=\frac{4^2-2(4)-8}{(4)^2-6(4)+8}=\frac{0}{0} \quad \leftarrow \text{indeterminate form}$$

$$\lim_{x\to 4}\frac{x^2-2x-8}{x^2-6x+8}=\lim_{x\to 4}\frac{\cancel{(x-4)}(x+2)}{\cancel{(x-4)}(x-2)}=\lim_{x\to 4}\frac{(x+2)}{(x-2)}=\frac{4+2}{4-2}=\frac{6}{2}=\boxed{3}$$

50. *Answer is B.*

Difficulty = **0.73 U**

$$\lim_{x\to 10}\frac{x^2-9x-10}{x^2-100}=$$

$$\lim_{x\to 10}\frac{x^2-9x-10}{x^2-100}=\frac{10^2-9(10)-10}{10^2-100}=\frac{0}{0} \quad \leftarrow \text{indeterminate form}$$

$$\lim_{x\to 10}\frac{x^2-9x-10}{x^2-100}=\lim_{x\to 10}\frac{\cancel{(x-10)}(x+1)}{\cancel{(x-10)}(x+10)}=\lim_{x\to 10}\frac{x+1}{x+10}=\frac{10+1}{10+10}=\boxed{\frac{11}{20}}$$

51. *Answer is B.*

$\boxed{\text{Difficulty} = \mathbf{0.72\ U}}$

$$\boxed{\lim_{x\to3}\frac{x^2-9}{x^2+x-12}=}$$

$$\lim_{x\to3}\frac{x^2-9}{x^2+x-12}=\frac{3^2-9}{3^2+3-12}=\frac{0}{0}\quad\leftarrow\text{indeterminate form}$$

$$\lim_{x\to3}\frac{x^2-9}{x^2+x-12}=\lim_{x\to3}\frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}(x+4)}=\lim_{x\to3}\frac{x+3}{x+4}=\frac{3+3}{3+4}=\boxed{\frac{6}{7}}$$

52. *Answer is B.*

$\boxed{\text{Difficulty} = \mathbf{0.71\ U}}$

$$\boxed{\lim_{x\to2}\frac{x^2-x-2}{x^2+x-6}=}$$

$$\lim_{x\to2}\frac{x^2-x-2}{x^2+x-6}=\frac{2^2-2-2}{2^2+2-6}=\frac{0}{0}\quad\leftarrow\text{indeterminate form}$$

$$\lim_{x\to2}\frac{x^2-x-2}{x^2+x-6}=\lim_{x\to2}\frac{\cancel{(x-2)}(x+1)}{\cancel{(x-2)}(x+3)}=\lim_{x\to2}\frac{x+1}{x+3}=\frac{2+1}{2+3}=\boxed{\frac{3}{5}}$$

53. *Answer is C.*

$\boxed{\text{Difficulty} = \mathbf{0.70\ U}}$

$$\boxed{\lim_{x\to1}\frac{x^2+x-2}{x^2-1}=}$$

$$\lim_{x\to1}\frac{x^2+x-2}{x^2-1}=\frac{1^2+1-2}{1^2-1}=\frac{0}{0}\quad\leftarrow\text{indeterminate form}$$

$$\lim_{x\to1}\frac{x^2+x-2}{x^2-1}=\lim_{x\to1}\frac{\cancel{(x-1)}(x+2)}{\cancel{(x-1)}(x+1)}=\lim_{x\to1}\frac{x+2}{x+1}=\frac{1+2}{1+1}=\boxed{\frac{3}{2}}$$

54. *Answer is B.*

$$\boxed{\lim_{x\to-2}\frac{x^2-4}{x^2+x-2}=}$$

$$\lim_{x\to-2}\frac{x^2-4}{x^2+x-2}=\frac{(-2)^2-4}{(-2)^2+(-2)-2}=\frac{0}{0}\quad\leftarrow\text{indeterminate form}$$

$$\lim_{x\to-2}\frac{x^2-4}{x^2+x-2}=\lim_{x\to-2}\frac{\cancel{(x+2)}(x-2)}{\cancel{(x+2)}(x-1)}=\lim_{x\to-2}\frac{x-2}{x-1}=\frac{-2-2}{-2-1}=\boxed{\frac{4}{3}}$$

55. *Answer is C.*

$$\boxed{\lim_{x\to5}\frac{x^2-2x-15}{x^2-7x+10}=}$$

$$\lim_{x\to5}\frac{x^2-2x-15}{x^2-7x+10}=\frac{5^2-2(5)-15}{5^2-7(5)+10}=\frac{0}{0}\quad\leftarrow\text{indeterminate form}$$

$$\lim_{x\to5}\frac{x^2-2x-15}{x^2-7x+10}=\lim_{x\to5}\frac{\cancel{(x-5)}(x+3)}{\cancel{(x-5)}(x-2)}=\lim_{x\to5}\frac{x+3}{x-2}=\frac{5+3}{5-2}=\boxed{\frac{8}{3}}$$

56. *Answer is B.*

$$\lim_{x\to 2}\frac{x-2}{x^2-x-2}=$$

$$\lim_{x\to 2}\frac{x-2}{x^2-x-2}=\frac{2-2}{2^2-2-2}=\frac{0}{0}\quad\leftarrow \text{indeterminate form}$$

$$\lim_{x\to 2}\frac{x-2}{x^2-x-2}=\lim_{x\to 2}\frac{\cancel{(x-2)}}{\cancel{(x-2)}(x+1)}=\lim_{x\to 2}\frac{1}{x+1}=\frac{1}{2+1}=\boxed{\frac{1}{3}}$$

57. *Answer is A.*

$$\lim_{x\to 7}\frac{x^2-6x-7}{x^2-49}=$$

$$\lim_{x\to 7}\frac{x^2-6x-7}{x^2-49}=\lim_{x\to 7}\frac{7^2-6(7)-7}{7^2-49}=\frac{0}{0}\quad\leftarrow \text{indeterminant form}$$

$$\lim_{x\to 7}\frac{x^2-6x-7}{x^2-49}=\lim_{x\to 7}\frac{\cancel{(x-7)}(x+1)}{\cancel{(x-7)}(x+7)}=\lim_{x\to 7}\frac{x+1}{x+7}=\frac{7+1}{7+7}=\boxed{\frac{4}{7}}$$

58. *Answer is C.*

$$\lim_{x\to -1}\frac{x^2-5x-6}{x^2-1}=$$

$$\lim_{x\to -1}\frac{x^2-5x-6}{x^2-1}=\lim_{x\to -1}\frac{(-1)^2-5(-1)-6}{(-1)^2-1}=\frac{0}{0}\quad\leftarrow \text{indeterminant form}$$

$$\lim_{x\to -1}\frac{x^2-5x-6}{x^2-1}=\lim_{x\to -1}\frac{\cancel{(x+1)}(x-6)}{\cancel{(x+1)}(x-1)}=\lim_{x\to -1}\frac{x-6}{x-1}=\frac{-1-6}{-1-1}=\boxed{\frac{7}{2}}$$

59. *Answer is C.*

$$\lim_{x\to 5}\frac{x-5}{x^2-25}=$$

$$\lim_{x\to 5}\frac{x-5}{x^2-25}=\frac{5-5}{5^2-25}=\frac{0}{0}\quad\leftarrow \text{indeterminant form}$$

$$\lim_{x\to 5}\frac{x-5}{x^2-25}=\lim_{x\to 5}\frac{\cancel{x-5}}{\cancel{(x-5)}(x+5)}=\lim_{x\to 5}\frac{1}{x+5}=\frac{1}{5+5}=\boxed{\frac{1}{10}}$$

60. *Answer is B.*

$$\lim_{x\to 2}\frac{2-x}{x^2-4}=$$

$$\lim_{x\to 2}\frac{2-x}{x^2-4}=\frac{2-2}{2^2-4}=\frac{0}{0}\quad\leftarrow \text{indeterminant form}$$

$$\lim_{x\to 2}\frac{2-x}{x^2-4}=\lim_{x\to 2}\frac{-\cancel{(x-2)}}{\cancel{(x-2)}(x+2)}=\lim_{x\to 2}\frac{-1}{(x+2)}=\frac{-1}{2+2}=\boxed{\frac{-1}{4}}$$

61. *Answer is A.*

$$\lim_{x\to 1}\left(\frac{x^5-1}{x^2-x}\right)=$$

$$\lim_{x\to 1}\left(\frac{x^5-1}{x^2-x}\right)=\lim_{x\to 1}\frac{1^5-1}{1^2-1}=\frac{0}{0}\quad \leftarrow \text{indeterminant form}$$

$$\lim_{x\to 1}\left(\frac{x^5-1}{x^2-x}\right)=\lim_{x\to 1}\frac{\cancel{(x-1)}(x^4+x^3+x^2+x+1)}{x\,\cancel{(x-1)}}=\lim_{x\to 1}\frac{(x^4+x^3+x^2+x+1)}{x}=\boxed{5}$$

62. *Answer is B.*

$$\lim_{x\to 0}\frac{\sqrt{x+4}-2}{x}=$$

$$\lim_{x\to 0}\frac{\sqrt{x+4}-2}{x}=\frac{\sqrt{0+4}-2}{0}=\frac{2-2}{0}=\frac{0}{0}\quad \leftarrow \text{indeterminate form}$$

$$\lim_{x\to 0}\frac{\sqrt{x+4}-2}{x}\left(\frac{\sqrt{x+4}+2}{\sqrt{x+4}+2}\right)=\lim_{x\to 0}\frac{\cancel{x}\cancel{+4}\cancel{-4}}{\cancel{x}\left(\sqrt{x+4}+2\right)}=\lim_{x\to 0}\frac{1}{\sqrt{x+4}+2}=\frac{1}{2+2}=\boxed{\frac{1}{4}}$$

63. *Answer is C.*

$$\lim_{x\to 3}\frac{x-3}{x^2-2x-3}=$$

$$\lim_{x\to 3}\frac{x-3}{x^2-2x-3}=\lim_{x\to 3}\frac{3-3}{3^2-2(3)-3}=\frac{0}{0}\quad \leftarrow \text{indeterminate form}$$

$$\lim_{x\to 3}\frac{x-3}{x^2-2x-3}=\lim_{x\to 3}\frac{\cancel{x-3}}{\cancel{(x-3)}(x+1)}=\lim_{x\to 3}\frac{1}{x+1}=\frac{1}{3+1}=\boxed{\frac{1}{4}}$$

64. *Answer is A.*

$$\lim_{x\to 0}\frac{x}{x}=$$

$$\lim_{x\to 0}\frac{x}{x}=\frac{0}{0}\quad \leftarrow \text{indeterminate form}$$

$$\lim_{x\to 0}\frac{x}{x}=\lim_{x\to 0}1=\boxed{1}$$

65. *Answer is D.*

$$\lim_{x\to 2}\frac{x^3-8}{x^2-4}=$$

$$\lim_{x\to 2}\frac{x^3-8}{x^2-4}=\frac{2^3-8}{2^2-4}=\frac{0}{0}\quad \leftarrow \text{indeterminate form}$$

$$\lim_{x\to 2}\frac{x^3-8}{x^2-4}=\lim_{x\to 2}\frac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{(x-2)}(x+2)}=\lim_{x\to 2}\frac{x^2+2x+4}{x+2}=\frac{2^2+2(2)+4}{2+2}=\frac{12}{4}=\boxed{3}$$

66. *Answer is E.*

$$\lim_{x\to 2}\frac{x^2-2}{4-x^2}=$$

$$\lim_{x\to 2}\frac{x^2-2}{4-x^2}=\frac{2^2-2}{4-2^2}=\frac{2}{0}=\boxed{\textit{undefined}}$$

67. *Answer is B.*

$$\lim_{h\to 0}\frac{\sqrt{25+h}-5}{h}=$$

$$\lim_{h\to 0}\frac{\sqrt{25+h}-5}{h}=\frac{\sqrt{25+0}-5}{0}=\frac{0}{0}\quad \leftarrow \text{indeterminate form}$$

$$\lim_{h\to 0}\frac{\sqrt{25+h}-5}{h}\left(\frac{\sqrt{25+h}+5}{\sqrt{25+h}+5}\right)=\lim_{h\to 0}\frac{\cancel{h}}{\cancel{h}(\sqrt{25+h}+5)}=\lim_{h\to 0}\frac{1}{\sqrt{25+h}+5}=\frac{1}{\sqrt{25+0}+5}=\boxed{\frac{1}{10}}$$

68. *Answer is B.*

Difficulty $=$ **0.45**

If $a\neq 0$, then $\displaystyle\lim_{x\to a}\frac{x^2-a^2}{x^4-a^4}=$

$$\lim_{x\to a}\frac{x^2-a^2}{x^4-a^4}=\frac{a^2-a^2}{a^4-a^4}=\frac{0}{0}\quad \leftarrow \text{indeterminate form}$$

$$\lim_{x\to a}\frac{x^2-a^2}{x^4-a^4}=\lim_{x\to a}\frac{\cancel{x^2-a^2}}{\cancel{(x^2-a^2)}(x^2+a^2)}=\lim_{x\to a}\frac{1}{(x^2+a^2)}=\frac{1}{a^2+a^2}=\boxed{\frac{1}{2a^2}}$$

69. *Answer is B.*

$$\lim_{x\to \pi}\frac{\pi x-\pi^2}{2x-2\pi}=$$

$$\lim_{x\to \pi}\frac{\pi x-\pi^2}{2x-2\pi}=\frac{\pi\pi-\pi^2}{2\pi-2\pi}=\frac{0}{0}\quad \leftarrow \text{indeterminate form}$$

$$\lim_{x\to \pi}\frac{\pi\,\cancel{(x-\pi)}}{2\,\cancel{(x-\pi)}}=\lim_{x\to \pi}\frac{\pi}{2}=\boxed{\frac{\pi}{2}}$$

70. *Answer is E.*

$$\lim_{x\to 0}\frac{x^5-16x}{x^3-4x}=$$

$$\lim_{x\to 0}\frac{x^5-16x}{x^3-4x}=\frac{0^5-16(0)}{0^3-4(0)}=\frac{0}{0}\quad \leftarrow \text{indeterminate form}$$

$$\lim_{x\to 0}\frac{x^5-16x}{x^3-4x}=\lim_{x\to 0}\frac{x(x^4-16)}{x(x^2-4)}=\lim_{x\to 0}\frac{\cancel{x}\,\cancel{(x^2-4)}(x^2+4)}{\cancel{x}\,\cancel{(x^2-4)}}=\lim_{x\to 0}(x^2+4)=0^2+4=\boxed{4}$$

71. *Answer is B.*

$$\lim_{x\to 3}\frac{x^3-2x^2-3x}{x^3-9x}=$$

$$\lim_{x\to 3}\frac{x^3-2x^2-3x}{x^3-9x}=\lim_{x\to 3}\frac{(0)^3-2(0)^2-3(0)}{(0)^3-9(0)}=\frac{0}{0}\quad\leftarrow \text{indeterminante form}$$

$$\lim_{x\to 3}\frac{x^3-2x^2-3x}{x^3-9x}=\lim_{x\to 3}\frac{x(x^2-2x-3)}{x(x^2-9)}=\lim_{x\to 3}\frac{\cancel{x}\,\cancel{(x-3)}\,(x+1)}{\cancel{x}\,\cancel{(x-3)}\,(x+3)}=\lim_{x\to 3}\frac{x+1}{x+3}=\frac{3+1}{3+3}=\boxed{\frac{2}{3}}$$

72. *Answer is C.*

$$\lim_{x\to 1}\frac{x^3-1}{x^2-1}=$$

$$\lim_{x\to 1}\frac{x^3-1}{x^2-1}=\frac{1^3-1}{1^2-1}=\frac{0}{0}\quad\leftarrow \text{indeterminante form}$$

$$\lim_{x\to 1}\frac{x^3-1}{x^2-1}=\lim_{x\to 1}\frac{\cancel{(x-1)}\,(x^2+x+1)}{\cancel{(x-1)}\,(x+1)}=\lim_{x\to 1}\frac{x^2+x+1}{x+1}=\frac{1^2+1+1}{1+1}=\boxed{\frac{3}{2}}$$

73. *Answer is C.*

$$\lim_{x\to 9}\frac{\sqrt{x}-3}{x-9}=$$

$$\lim_{x\to 9}\frac{\sqrt{x}-3}{x-9}=\lim_{x\to 9}\frac{\sqrt{9}-3}{9-9}=\frac{0}{0}\quad\leftarrow \text{indeterminate form}$$

$$\lim_{x\to 9}\frac{\sqrt{x}-3}{x-9}=\lim_{x\to 9}\frac{\sqrt{x}-3}{x-9}\left(\frac{\sqrt{x}+3}{\sqrt{x}+3}\right)=\lim_{x\to 9}\frac{\cancel{x-9}}{\cancel{(x-9)}\left(\sqrt{x}+3\right)}=\lim_{x\to 9}\frac{1}{\sqrt{x}+3}=\frac{1}{3+3}=\boxed{\frac{1}{6}}$$

74. *Answer is C.*

$$\lim_{a\to b}\left(\frac{a^2-b^2}{a-b}+3ab\right)=$$

$$\lim_{a\to b}\left(\frac{a^2-b^2}{a-b}+3ab\right)=\lim_{a\to b}\frac{a^2-b^2+3ab(a-b)}{a-b}=\frac{b^2-b^2+3bb(b-b)}{b-b}=\frac{0}{0}\quad\leftarrow \text{indeterminant form}$$

$$\lim_{a\to b}\left(\frac{a^2-b^2}{a-b}+3ab\right)=\lim_{a\to b}\left(\frac{(a+b)\,\cancel{(a-b)}}{\cancel{a-b}}+3ab\right)=\left((b+b)+3bb\right)=\boxed{2b+3b^2}$$

75. *Answer is A.*

$$\lim_{x\to a}\frac{x^2-a^2}{a-x}=$$

$$\lim_{x\to a}\frac{x^2-a^2}{a-x}=\frac{a^2-a^2}{a-a}=\frac{0}{0}\quad\leftarrow \text{indeterminant form}$$

$$\lim_{x\to a}\frac{x^2-a^2}{a-x}=\lim_{x\to a}\frac{\cancel{(x-a)}\,(x+a)}{-\cancel{(x-a)}}=\lim_{x\to a}\left[-(x+a)\right]=-(a+a)=\boxed{-2a}$$

76. *Answer is D.*

$$\boxed{\lim_{x\to 0}\frac{1-2^{2x}}{1-2^{x}}=}$$

$$\lim_{x\to 0}\frac{1-2^{2x}}{1-2^{x}}=\frac{1-2^{2(0)}}{1-2^{0}}=\frac{0}{0}\quad \leftarrow \text{indeterminant form}$$

$$\lim_{x\to 0}\frac{1-2^{2x}}{1-2^{x}}=\lim_{x\to 0}\frac{1-(2^{x})^{2}}{1-2^{x}}=\lim_{x\to 0}\frac{\cancel{(1-2^{x})}(1+2^{x})}{\cancel{1-2^{x}}}=\lim_{x\to 0}(1+2^{x})=(1+2^{0})=\boxed{2}$$

77. *Answer is D.*

$$\boxed{\lim_{x\to 0}\frac{x^{3}+x^{2}-2x}{x^{3}-x}=}$$

$$\lim_{x\to 0}\frac{x^{3}+x^{2}-2x}{x^{3}-x}=\frac{0^{3}+0^{2}-2(0)}{0^{3}-0}=\frac{0}{0}\quad \leftarrow \text{indeterminate form}$$

$$\lim_{x\to 0}\frac{x^{3}+x^{2}-2x}{x^{3}-x}=\lim_{x\to 0}\frac{x(x^{2}+x-2)}{x(x^{2}-1)}=\lim_{x\to 0}\frac{\cancel{x}(x+2)\cancel{(x-1)}}{\cancel{x}\cancel{(x^{2}-1)}}=\lim_{x\to 0}(x+2)=(0+2)=\boxed{2}$$

78. *Answer is C.*

$$\boxed{\lim_{x\to 3}\frac{(3-x)^{2}}{(x-3)}=}$$

$$\lim_{x\to 3}\frac{(3-x)^{2}}{(x-3)}=\frac{(3-3)^{2}}{(3-3)}=\frac{0}{0}\quad \leftarrow \text{indeterminant form}$$

$$\lim_{x\to 3}\frac{(3-x)^{2}}{(x-3)}=\lim_{x\to 3}\frac{(3-x)(3-x)}{-(3-x)}=\lim_{x\to 3}\frac{(3-x)\cancel{(3-x)}}{-\cancel{(3-x)}}=\lim_{x\to 3}(x-3)=3-3=\boxed{0}$$

79. *Answer is A.*

$$\boxed{\lim_{x\to b}\frac{b-x}{\sqrt{x}-\sqrt{b}}=}$$

$$\lim_{x\to b}\frac{b-x}{\sqrt{x}-\sqrt{b}}=\frac{b-b}{\sqrt{b}-\sqrt{b}}=\frac{0}{0}\quad \leftarrow \text{indeterminant form}$$

$$\lim_{x\to b}\frac{b-x}{\sqrt{x}-\sqrt{b}}=\lim_{x\to b}\frac{-(x-b)}{\sqrt{x}-\sqrt{b}}=\lim_{x\to b}\frac{-\cancel{(\sqrt{x}-\sqrt{b})}(\sqrt{x}+\sqrt{b})}{\cancel{\sqrt{x}-\sqrt{b}}}=\lim_{x\to b}(-\sqrt{x}-\sqrt{b})$$

$$=(-\sqrt{b}-\sqrt{b})=\boxed{-2\sqrt{b}}$$

80. *Answer is E.*

$$\lim_{x\to 1}\frac{x^2+2x-3}{x^2-1}=$$

$$\lim_{x\to 1}\frac{x^2+2x-3}{x^2-1}=\frac{1^2+2(1)-3}{(1)^2-1}=\frac{0}{0}\quad\leftarrow \text{indeterminant form}$$

$$\lim_{x\to 1}\frac{x^2+2x-3}{x^2-1}=\lim_{x\to 1}\frac{\cancel{(x-1)}(x+3)}{\cancel{(x-1)}(x+1)}=\lim_{x\to 1}\frac{x+3}{x+1}=\frac{1+3}{1+1}=\boxed{2}$$

81. *Answer is B.*

$$\lim_{x\to 9}\frac{x-9}{3-\sqrt{x}}=$$

$$\lim_{x\to 9}\frac{x-9}{3-\sqrt{x}}=\frac{9-9}{3-\sqrt{9}}=\frac{0}{0}\quad\leftarrow \text{indeterminant form}$$

$$\lim_{x\to 9}\frac{x-9}{3-\sqrt{x}}=\lim_{x\to 9}\frac{x-9}{3-\sqrt{x}}\left(\frac{3+\sqrt{x}}{3+\sqrt{x}}\right)=\lim_{x\to 9}\frac{(x-9)(3+\sqrt{x})}{9-x}=\lim_{x\to 9}\frac{-\cancel{(9-x)}(3+\sqrt{x})}{\cancel{9-x}}$$

$$=\lim_{x\to 9}\left[-(3+\sqrt{x})\right]=\left[-(3+\sqrt{9})\right]=\boxed{-6}$$

82. *Answer is B.*

If $k\neq 0$ then $\lim_{x\to k}\frac{x^2-k^2}{x^2-kx}=$

$$\lim_{x\to k}\frac{x^2-k^2}{x^2-kx}=\frac{k^2-k^2}{k^2-k(k)}=\frac{0}{0}\quad\leftarrow \text{indeterminant form}$$

$$\lim_{x\to k}\frac{x^2-k^2}{x^2-kx}=\lim_{x\to k}\frac{\cancel{(x-k)}(x+k)}{x\cancel{(x-k)}}=\lim_{x\to k}\frac{x+k}{x}=\frac{k+k}{k}=\boxed{2}$$

83. *Answer is A.*

$$\lim_{x\to -2}\frac{x+2}{x^2-4}=$$

$$\lim_{x\to -2}\frac{x+2}{x^2-4}=\frac{-2+2}{(-2)^2-4}=\frac{0}{0}\quad\leftarrow \text{indeterminant form}$$

$$\lim_{x\to -2}\frac{x+2}{x^2-4}=\lim_{x\to -2}\frac{\cancel{x+2}}{\cancel{(x+2)}(x-2)}=\lim_{x\to -2}\frac{1}{x-2}=\frac{1}{-2-2}=\boxed{\frac{-1}{4}}$$

84. *Answer is B.*

$$\lim_{x\to 2}\frac{x^2-4}{x^3-8}=$$

$$\lim_{x\to 2}\frac{x^2-4}{x^3-8}=\frac{2^2-4}{2^3-8}=\frac{0}{0}\quad\leftarrow \text{indeterminant form}$$

$$\lim_{x\to 2}\frac{x^2-4}{x^3-8}=\lim_{x\to 2}\frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(x^2+2x+4)}=\lim_{x\to 2}\frac{x+2}{x^2+2x+4}=\frac{2+2}{2^2+2(2)+4}=\boxed{\frac{1}{3}}$$

85. *Answer is E.*

$$\lim_{x\to 0}\frac{x^2-x}{x^4+x^3}=$$

$$\lim_{x\to 0}\frac{x^2-x}{x^4+x^3}=\frac{0^2-0}{0^4+0^3}=\frac{0}{0}\quad\leftarrow \text{indeterminant form}$$

$$\lim_{x\to 0}\frac{x^2-x}{x^4+x^3}=\lim_{x\to 0}\frac{x(x-1)}{x^3(x+1)}=\lim_{x\to 0}\frac{(x-1)}{x^2(x+1)}=\frac{0-1}{0^2(0+1)}=\frac{-1}{0}=\boxed{\textit{undefined}}$$

86. *Answer is A.*

$$\lim_{x\to 5}\frac{2x^2-50}{x^2-15x+50}=$$

$$\lim_{x\to 5}\frac{2x^2-50}{x^2-15x+50}=\frac{2(5)^2-50}{(5)^2-15(5)+50}=\frac{0}{0}\quad\leftarrow \text{indeterminant form}$$

$$\lim_{x\to 5}\frac{2x^2-50}{x^2-15x+50}=\lim_{x\to 5}\frac{2(x+5)\cancel{(x-5)}}{(x-10)\cancel{(x-5)}}=\lim_{x\to 5}\frac{2(x+5)}{x-10}=\frac{2(5+5)}{5-10}=\frac{20}{-5}=\boxed{-4}$$

87. *Answer is E.*

$$\lim_{x\to 2}\frac{4x^2-16}{x-2}=$$

$$\lim_{x\to 2}\frac{4x^2-16}{x-2}=\frac{4(2)^2-16}{2-2}=\frac{0}{0}\quad\leftarrow \text{indeterminate form}$$

$$\lim_{x\to 2}\frac{4x^2-16}{x-2}=\lim_{x\to 2}\frac{4\cancel{(x-2)}(x+2)}{\cancel{x-2}}=\lim_{x\to 2}4(x+2)=4(2+2)=\boxed{16}$$

88. *Answer is C.*

$$\lim_{x\to 5}\frac{x^2-x-20}{x-5}=$$

$$\lim_{x\to 5}\frac{x^2-x-20}{x-5}=\frac{5^2-5-20}{5-5}=\frac{0}{0}\quad\leftarrow \text{indeterminate form}$$

$$\lim_{x\to 5}\frac{x^2-x-20}{x-5}=\lim_{x\to 5}\frac{(x+4)\cancel{(x-5)}}{\cancel{x-5}}=\lim_{x\to 5}(x+4)=(5+4)=\boxed{9}$$

89. *Answer is D.*

$$\lim_{x\to -1}\frac{x+x^2}{x^2-1}=$$

$$\lim_{x\to -1}\frac{x+x^2}{x^2-1}=\frac{-1+(-1)^2}{(-1)^2-1}=\frac{0}{0}\quad\leftarrow \text{indeterminate form}$$

$$\lim_{x\to -1}\frac{x+x^2}{x^2-1}=\lim_{x\to -1}\frac{x\,\cancel{(x+1)}}{(x-1)\,\cancel{(x+1)}}=\lim_{x\to -1}\frac{x}{x-1}=\frac{-1}{-1-1}=\boxed{\frac{1}{2}}$$

90. *Answer is B.*

$$\lim_{x\to 1}\frac{2x-2}{x^3+2x^2-x-2}=$$

$$\lim_{x\to 1}\frac{2x-2}{x^3+2x^2-x-2}=\frac{2(1)-2}{1^3+2(1)^2-1-2}=\frac{0}{0}\quad\leftarrow \text{indeterminate form}$$

$$\lim_{x\to 1}\frac{2x-2}{x^3+2x^2-x-2}=\lim_{x\to 1}\frac{2\,\cancel{(x-1)}}{(x+2)(x+1)\,\cancel{(x-1)}}=\frac{2}{(1+2)(1+1)}=\frac{2}{6}=\boxed{\frac{1}{3}}$$

91. *Answer is B.*

$$\lim_{x\to 2}\frac{x-2}{x^2-4}=$$

$$\lim_{x\to 2}\frac{x-2}{x^2-4}=\frac{2-2}{2^2-4}=\frac{0}{0}\quad\leftarrow \text{indeterminate form}$$

$$\lim_{x\to 2}\frac{x-2}{x^2-4}=\lim_{x\to 2}\frac{\cancel{x-2}}{\cancel{(x-2)}\,(x+2)}=\lim_{x\to 2}\frac{1}{(x+2)}=\frac{1}{2+2}=\boxed{\frac{1}{4}}$$

92. *Answer is C.*

$$\lim_{x\to 4}\frac{x^2-5x+4}{x-4}=$$

$$\lim_{x\to 4}\frac{x^2-5x+4}{x-4}=\frac{4^2-5(4)+4}{4-4}=\frac{0}{0}\quad\leftarrow \text{indeterminate form}$$

$$\lim_{x\to 4}\frac{x^2-5x+4}{x-4}=\lim_{x\to 4}\frac{(x-1)\,\cancel{(x-4)}}{\cancel{x-4}}=\lim_{x\to 4}(x-1)=4-1=\boxed{3}$$

93. *Answer is A.*

$$\lim_{x\to2}\frac{\sqrt{3-x}-\sqrt{x-1}}{6-3x}=$$

$$\lim_{x\to2}\frac{\sqrt{3-x}-\sqrt{x-1}}{6-3x}=\frac{\sqrt{3-2}-\sqrt{2-1}}{6-3(2)}=\frac{0}{0}\quad\leftarrow\text{indeterminate form}$$

$$\lim_{x\to2}\frac{\sqrt{3-x}-\sqrt{x-1}}{6-3x}=\lim_{x\to2}\frac{\sqrt{3-x}-\sqrt{x-1}}{3(2-x)}\left(\frac{\sqrt{3-x}+\sqrt{x-1}}{\sqrt{3-x}+\sqrt{x-1}}\right)$$

$$=\lim_{x\to2}\frac{(3-x)-(x-1)}{3(2-x)(\sqrt{3-x}+\sqrt{x-1})}=\lim_{x\to2}\frac{2\cancel{(2-x)}}{3\cancel{(2-x)}(\sqrt{3-x}+\sqrt{x-1})}$$

$$=\lim_{x\to2}\frac{2}{3(\sqrt{3-x}+\sqrt{x-1})}=\frac{2}{3(\sqrt{3-2}+\sqrt{2-1})}=\boxed{\frac{1}{3}}$$

94. *Answer is B.*

$$\lim_{x\to1}\frac{\sqrt{x}-1}{x-1}=$$

$$\lim_{x\to1}\frac{\sqrt{x}-1}{x-1}=\lim_{x\to1}\frac{\sqrt{1}-1}{1-1}=\frac{0}{0}\quad\leftarrow\text{indeterminant form}$$

$$\lim_{x\to1}\frac{\sqrt{x}-1}{x-1}=\lim_{x\to1}\frac{\sqrt{x}-1}{x-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}+1}\right)=\lim_{x\to1}\frac{\cancel{x-1}}{\cancel{(x-1)}\left(\sqrt{x}+1\right)}=\lim_{x\to1}\frac{1}{\sqrt{x}+1}=\frac{1}{\sqrt{1}+1}=\boxed{\frac{1}{2}}$$

95. *Answer is D.*

$$\lim_{x\to1}\left(\frac{\sqrt{x+3}-2}{1-x}\right)=$$

$$\lim_{x\to1}\left(\frac{\sqrt{x+3}-2}{1-x}\right)=\left(\frac{\sqrt{1+3}-2}{1-1}\right)=\frac{0}{0}\quad\leftarrow\text{indeterminant form}$$

$$\lim_{x\to1}\left(\frac{\sqrt{x+3}-2}{1-x}\right)=\lim_{x\to1}\left(\frac{\sqrt{x+3}-2}{1-x}\right)\left(\frac{\sqrt{x+3}+2}{\sqrt{x+3}+2}\right)=\lim_{x\to1}\frac{x-1}{(1-x)(\sqrt{x+3}+2)}$$

$$=\lim_{x\to1}\frac{-\cancel{(1-x)}}{\cancel{(1-x)}(\sqrt{x+3}+2)}=\frac{-1}{\sqrt{1+3}+2}=\boxed{\frac{-1}{4}}$$

96. *Answer is C.*

$$\lim_{x\to -1}\frac{\sqrt{x^2+3}-2}{x+1}=$$

$$\lim_{x\to -1}\frac{\sqrt{x^2+3}-2}{x+1}=\frac{\sqrt{(-1)^2+3}-2}{-1+1}=\frac{0}{0}\quad\leftarrow\text{indeterminant form}$$

$$\lim_{x\to -1}\frac{\sqrt{x^2+3}-2}{x+1}=\lim_{x\to -1}\frac{\sqrt{x^2+3}-2}{x+1}\left(\frac{\sqrt{x^2+3}+2}{\sqrt{x^2+3}+2}\right)=\lim_{x\to -1}\frac{x^2-1}{(x+1)(\sqrt{x^2+3}+2)}$$

$$=\lim_{x\to -1}\frac{\cancel{(x+1)}(x-1)}{\cancel{(x+1)}(\sqrt{x^2+3}+2)}=\frac{-1-1}{\sqrt{(-1)^2+3}+2}=\frac{-2}{4}=\boxed{\frac{-1}{2}}$$

97. *Answer is D.*

$$\lim_{h\to 0}\frac{e^{1+h}-e}{h}=$$

$$\lim_{h\to 0}\frac{e^{1+h}-e}{h}=\lim_{h\to 0}\frac{e^{1+0}-e}{0}=\lim_{h\to 0}\frac{e-e}{0}=\frac{0}{0}\quad\leftarrow\text{indeterminant form}$$

$$\lim_{h\to 0}\frac{e^{1+h}-e}{h}=\lim_{h\to 0}\frac{e^{1+h}(1)-0}{1}=\lim_{h\to 0}e^{1+h}=e^{1+0}=\boxed{e}$$

98.

$$\lim_{x\to\frac{\pi}{2}}\frac{2x-\pi}{x-\frac{\pi}{2}}=$$

$$\lim_{x\to\frac{\pi}{2}}\frac{2(\frac{\pi}{2})-\pi}{\frac{\pi}{2}-\frac{\pi}{2}}=\frac{0}{0}\quad\leftarrow\text{indeterminate form (now use L'Hopitals rule)}$$

$$\lim_{x\to\frac{\pi}{2}}\frac{2x-\pi}{x-\frac{\pi}{2}}=\lim_{x\to\frac{\pi}{2}}\frac{2}{1}=\boxed{2}$$

99. *Answer is A.*

$$\lim_{x\to 1}\frac{\frac{1}{x+1}-\frac{1}{2}}{x-1}=$$

$$\lim_{x\to 1}\frac{\frac{1}{x+1}-\frac{1}{2}}{x-1}=\lim_{x\to 1}\frac{\frac{1}{1+1}-\frac{1}{2}}{1-1}=\frac{0}{0}\quad\leftarrow\text{indeterminant form now simplify or use L'Hopitals rule)}$$

$$\lim_{x\to 1}\frac{(\frac{2}{2})(\frac{1}{x+1})-(\frac{1}{2})(\frac{x+1}{x+1})}{x-1}=\lim_{x\to 1}\frac{\frac{2-(x+1)}{2(x+1)}}{x-1}=\lim_{x\to 1}\frac{\frac{1-x}{2(x+1)}}{x-1}=\lim_{x\to 1}\frac{\frac{1-x}{2(x+1)}}{\frac{x-1}{1}}$$

$$=\lim_{x\to 1}\frac{-\cancel{(x-1)}}{2(x+1)}\left(\frac{1}{\cancel{x-1}}\right)=\lim_{x\to 1}\frac{-1}{2(x+1)}=\boxed{\frac{-1}{4}}$$

100.

Vertical asymptotes

Let f and g be continuous on an open interval containing c. If $f(c) \neq 0$, $g(c) = 0$ then the function $h(x) = \dfrac{f(x)}{g(x)}$ $\lim\limits_{x \to c} h(x) = \dfrac{k}{0} = \textbf{\textit{undefined}}$ (has a ***vertical*** asymptote at $x = c$)

101. *Answer is D.*

Difficulty = **0.79 U**

$$\boxed{\lim_{x \to -2} \frac{x-2}{x^2-4} =}$$

$$\lim_{x \to 2} \frac{(x-2)}{(x-2)(x+2)} = \frac{0}{0} = \text{indeterminant form}$$

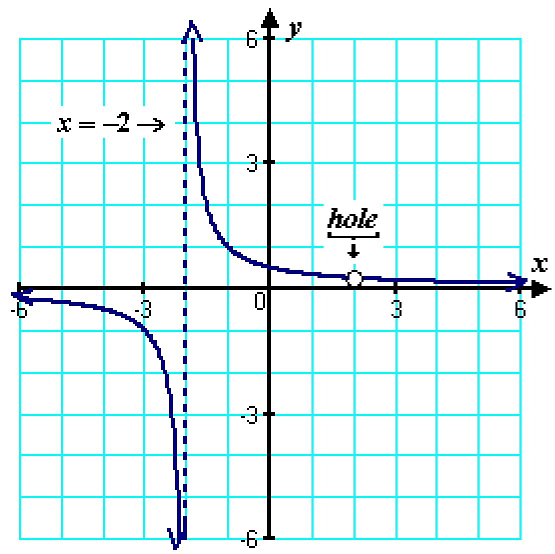
$\therefore$ $x = 2$ is a ***hole*** in the graph

$$\lim_{x \to -2} \frac{(x-2)}{(x-2)(x+2)} = \frac{-4}{0} = \infty = \textbf{\textit{undefined}}$$

$\therefore$ $\boxed{x = -2}$ is a ***vertical*** asymptote

$$\lim_{x \to \infty} \frac{x-2}{x^2-4} = \lim_{x \to \infty} \frac{\frac{x}{x^2} - \frac{2}{x^2}}{\frac{x^2}{x^2} - \frac{4}{x^2}} = \lim_{x \to \infty} \frac{0-0}{1-0} = 0$$

$\therefore$ $y = 0$ is a ***horizontal*** asymptote



102. *Answer is D.*

Difficulty = **0.77 U**

$$\boxed{\lim_{x \to 2} \frac{x+6}{x-2} =}$$

$$\lim_{x \to 2} \frac{x+6}{x-2} = \frac{8}{0} = \infty = \textbf{\textit{undefined}}$$

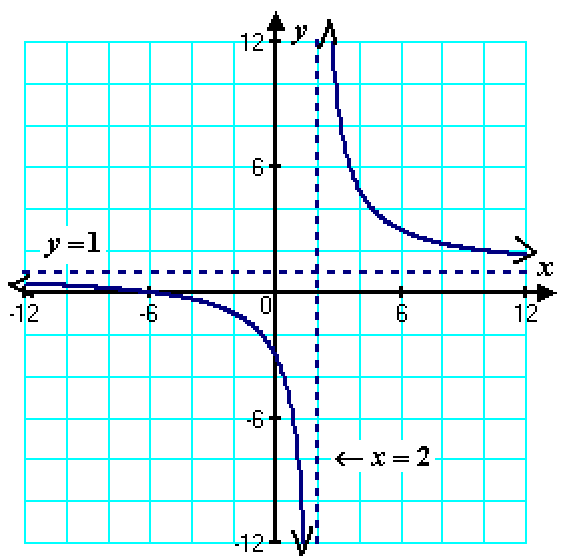
$\therefore$ $\boxed{x = 2}$ is a ***vertical*** asymptote

$$\lim_{x \to \infty} \frac{x+6}{x-2} = \lim_{x \to \infty} \frac{\frac{x}{x} + \frac{6}{x}}{\frac{x}{x} - \frac{2}{x}} = \lim_{x \to \infty} \frac{1+0}{1-0} = 0$$

Notice there is a horizontal asymptote at $y = 1$

$$y = \frac{x+6}{x-2} \rightarrow \frac{x+6}{x-2} = 0 \rightarrow x+6 = 0 \rightarrow \underbrace{x = -6}_{x\text{-intercept}}$$

$$x = 0 \rightarrow y = \frac{0+6}{0-2} \rightarrow \underbrace{y = -3}_{y\text{-intercept}}$$



103. *Answer is E.*

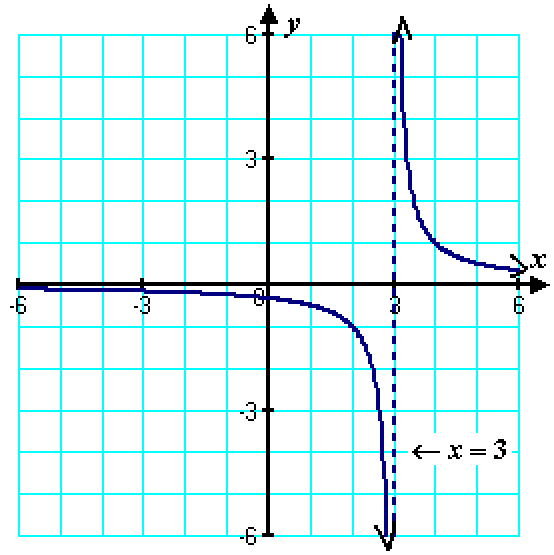
$$\lim_{x\to 3}\frac{1}{x-3}=$$

$$\lim_{x\to 3}\frac{1}{x-3}=\frac{1}{0}=\infty=\textit{undefined}=\textit{nonexistant}$$

$\therefore$ $\boxed{x=3}$ is a ***vertical*** asymptote

$$\lim_{x\to\infty}\frac{1}{x-3}=\lim_{x\to\infty}\frac{\frac{1}{x}}{\frac{x}{x}-\frac{3}{x}}=\frac{0}{1-0}0$$

$\therefore$ $y=0$ is a ***horizontal*** asymptote



104. *Answer is C.*

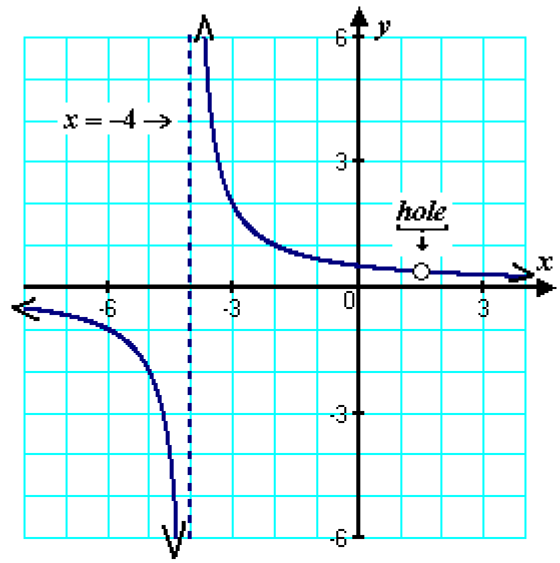
$$\lim_{x\to -4^+}\frac{4x-6}{2x^2+5x-12}=$$

$$\lim_{x\to -4^+}\frac{2(2x-3)}{(2x-3)(x+4)}=\frac{-22}{0}=+\infty=\textit{undefined}$$

$\therefore$ $\boxed{x=-4}$ is a ***vertical*** asymptote

$$\lim_{x\to \frac{3}{2}}\frac{2(2x-3)}{(2x-3)(x+4)}=\frac{0}{0}=\text{indeterminant form}$$

$\therefore$ $x=\frac{3}{2}$ is a ***hole*** in the graph



105. *Answer is C.*

Find the equation of the vertical asymptote of

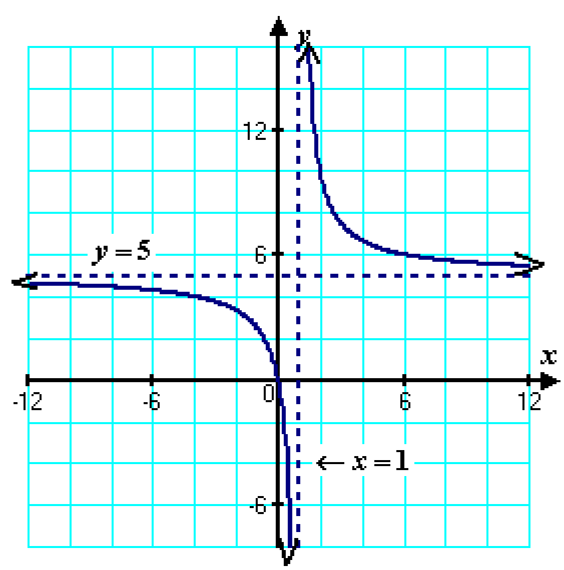
$$y=\frac{5x}{x-1}$$

$$\lim_{x\to 1}\frac{5x}{x-1}=\frac{5}{0}=\infty=\textit{undefined}$$

$\therefore$ $\boxed{x=1}$ is a ***vertical*** asymptote

$$\lim_{x\to\infty}\frac{5x}{x-1}=\lim_{x\to\infty}\frac{\frac{5x}{x}}{\frac{x}{x}-\frac{1}{x}}=\frac{5}{1-0}=5$$

$\therefore$ $y=5$ is a ***horizontal*** asymptote

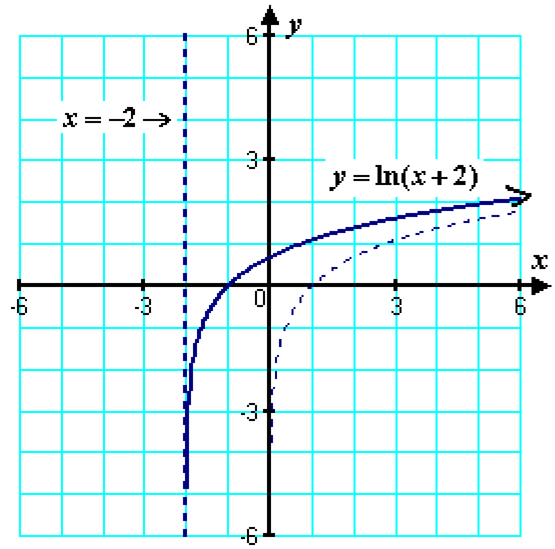


106. *Answer is A*

The graph of $y = \ln(x+2)$ has a vertical asymptote with equation

From the log unit, this is the basic $y = \ln x$ graph moved left $\mathbf{2}$ units, so

so has a vertical asymptote $\boxed{x = -2}$



107. *Answer is D.*

Find the equation of the vertical asymptote(s) of $f(x) = \dfrac{2x}{x^2 - 4}$

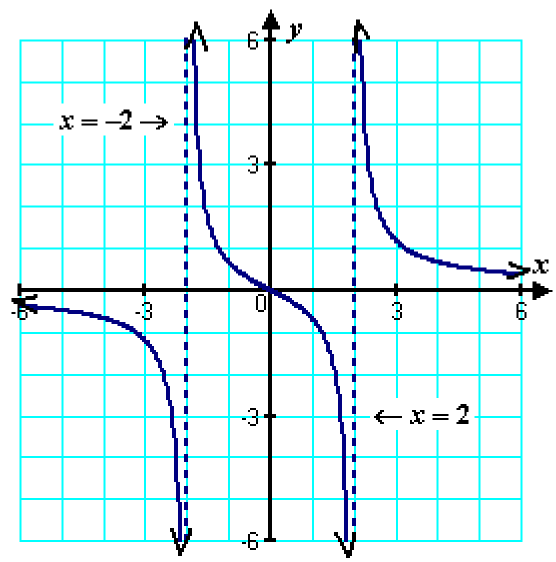
$$\lim_{x \to 2} \frac{2x}{(x-2)(x+2)} = \frac{4}{0} = \infty = \textit{undefined}$$

$$\lim_{x \to -2} \frac{2x}{(x-2)(x+2)} = \frac{-4}{0} = \infty = \textit{undefined}$$

$\therefore$ $\boxed{x = -2,\ 2}$ are ***vertical*** asymptotes

$$\lim_{x \to \infty} \frac{2x}{x^2 - 4} = \lim_{x \to \infty} \frac{\frac{2x}{x^2}}{\frac{x^2}{x^2} - \frac{4}{x^2}} = \frac{0}{1 - 0} = 0$$

$\therefore$ $y = 0$ is a ***horizontal*** asymptote



108. *Answer is B.*

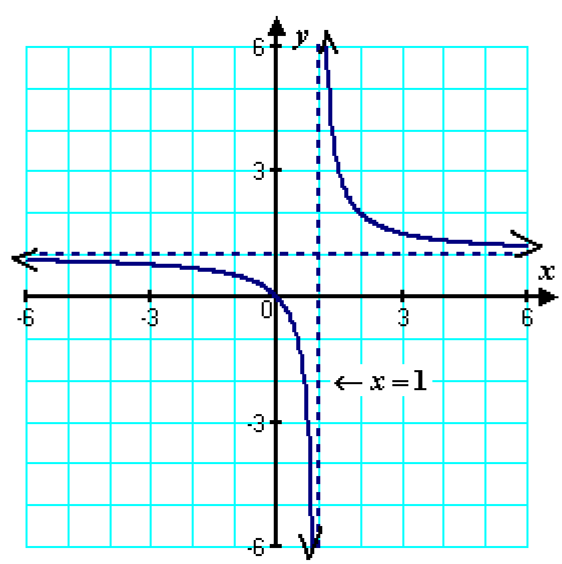
The vertical asymptote of $f(x) = \dfrac{x}{x-1}$ has equation

$$\lim_{x \to 1} \frac{x}{x-1} = \frac{1}{0} = \infty = \textit{undefined}$$

$\therefore$ $\boxed{x = 1}$ is a ***vertical*** asymptote

$$\lim_{x \to \infty} \frac{x}{x-1} = \lim_{x \to \infty} \frac{\frac{x}{x}}{\frac{x}{x} - \frac{1}{x}} = \frac{1}{1 - 0} = 1$$

$\therefore$ $y = 1$ is a ***horizontal*** asymptote



109. *Answer is A.*

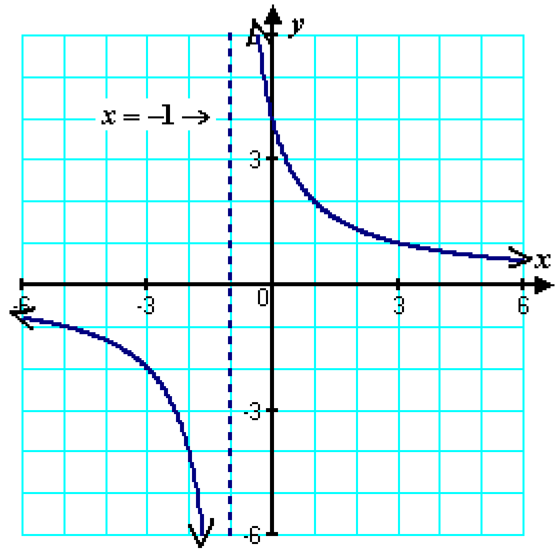
The vertical asymptote of $f(x)=\dfrac{4}{x+1}$ is

$$\lim_{x\to -1}\frac{4}{x+1}=\frac{4}{0}=\infty=\textit{undefined}$$

$\therefore\ \boxed{x=-1}$ is a ***vertical*** asymptote

$$\lim_{x\to\infty}\frac{4}{x+1}=\lim_{x\to\infty}\frac{\frac{4}{x}}{\frac{x}{x}+\frac{1}{x}}=\frac{0}{1+0}=0$$

$\therefore\ y=0$ is a ***horizontal*** asymptote



110. *Answer is C.*

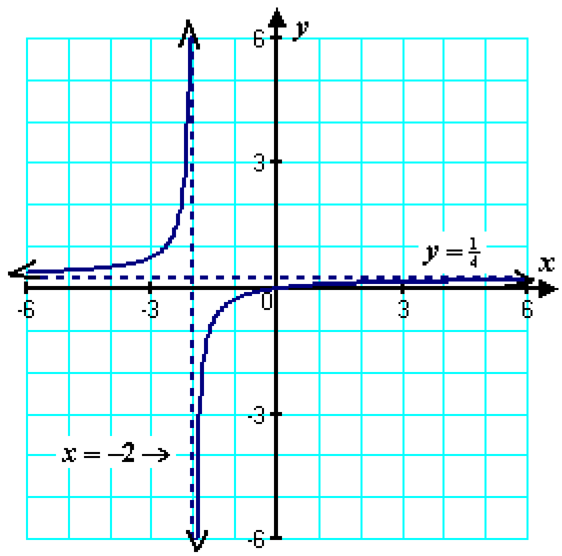
Find the equation of the vertical asymptote of $f(x)=\dfrac{x}{4x+8}$

$$\lim_{x\to -2}\frac{x}{4(x+2)}=\frac{-2}{0}=\infty=\textit{undefined}$$

$\therefore\ \boxed{x=-2}$ is a ***vertical*** asymptote

$$\lim_{x\to\infty}\frac{x}{4x+8}=\lim_{x\to\infty}\frac{\frac{x}{x}}{\frac{4x}{x}+\frac{8}{x}}=\frac{1}{4+0}=\frac{1}{4}$$

$\therefore\ y=\frac{1}{4}$ is a ***horizontal*** asymptote



111. *Answer is B.*

The graph of $f(x)=\dfrac{x^2-5x+6}{x^2-4}$ has vertical asymptotes at

$$\lim_{x\to 2}\frac{(x-3)(x-2)}{(x+2)(x-2)}=\frac{0}{0}=\text{indeterminant form}$$

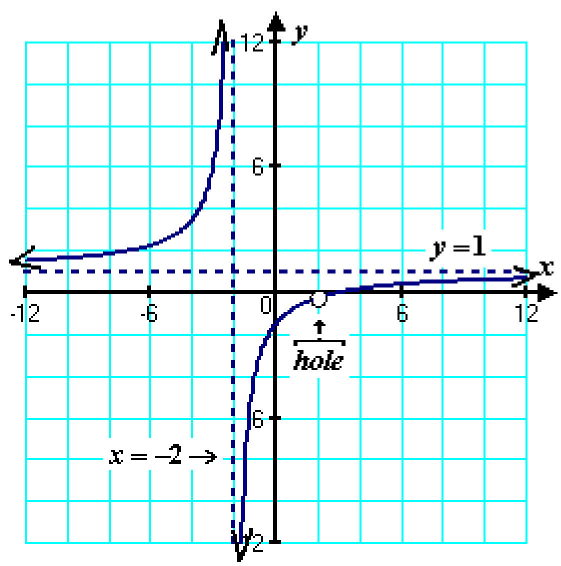
$\therefore\ x=2$ is a ***hole*** in the graph

$$\lim_{x\to -2}\frac{(x-3)(x-2)}{(x+2)(x-2)}=\frac{4}{0}=\infty=\textit{undefined}$$

$\therefore\ \boxed{x=-2}$ is a ***vertical*** asymptote

$$\lim_{x\to\infty}\frac{x^2-5x+6}{x^2-4}=\lim_{x\to\infty}\frac{\frac{x^2}{x^2}-\frac{5x}{x^2}+\frac{6}{x^2}}{\frac{x^2}{x^2}-\frac{4}{x^2}}=\frac{1-0+0}{1-0}=1$$

$\therefore\ y=1$ is a ***horizontal*** asymptote



112. *Answer is A.*

The graph of $f(x)=\dfrac{x^2-4}{x^3+3x^2-4x-12}$
has a vertical asymptote at $x=$

$$P(2)=2^3+3(2)^2-4(2)-12=0$$

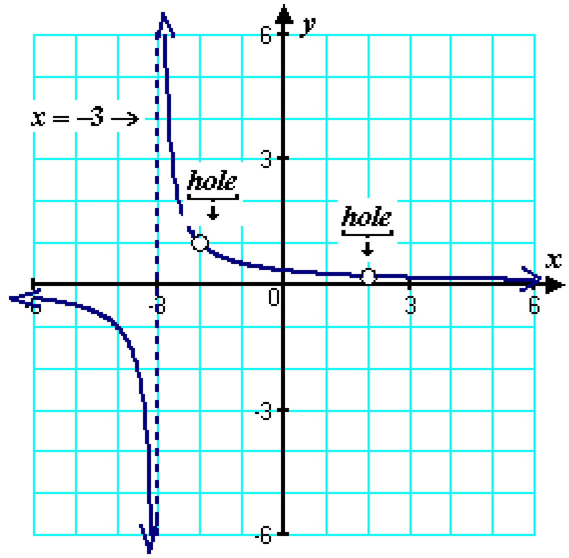
$$\lim_{x\to 2}\frac{(x-2)(x+2)}{(x-2)(x+2)(x+3)}=\frac{0}{0}=\text{indeterminant}$$

$$\lim_{x\to -2}\frac{(x-2)(x+2)}{(x-2)(x+2)(x+3)}=\frac{0}{0}=\text{indeterminant}$$

$\therefore\ x=2,\ -2$ are ***holes*** in the graph

$$\lim_{x\to -3}\frac{(x-2)(x+2)}{(x-2)(x+2)(x+3)}=\frac{5}{0}=\infty=\textit{undefined}$$

$\therefore\ \boxed{x=-3}$ is a ***vertical*** asymptote



113. *Answer is C.*

A function $f(x)$ has a vertical asymptote at $x=2$
The derivative of $f(x)$ is positive for all $x\neq 2$
Which of the following statements are true ?

$$\lim_{x\to 2} f(x)=\frac{k}{0}=\infty=\textit{undefined}\qquad (k\neq 0)$$

$\therefore\ \boxed{x=2}$ is a ***vertical*** asymptote

$f'(x)$ is positive for all $x\neq 2$

$f(x)$ is ***increasing*** for all $x\neq 2$

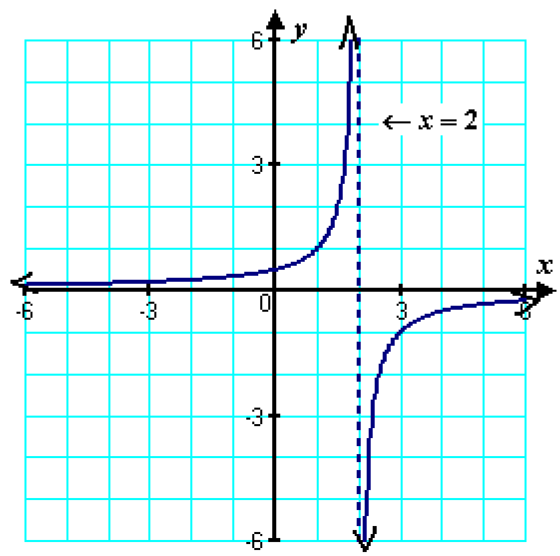
Example $\rightarrow$ $f(x)=\dfrac{-1}{x-2}$

$$f'(x)=\frac{1}{(x-2)^2}$$

I. $\lim\limits_{x\to 2} f(x)=+\infty$ ☒ (does not exist)

II. $\lim\limits_{x\to 2^+} f(x)=+\infty$ ☒ (does not exist)

III. $\lim\limits_{x\to 2^-} f(x)=+\infty$ ☑ (does not exist)



114. *Answer is C.*

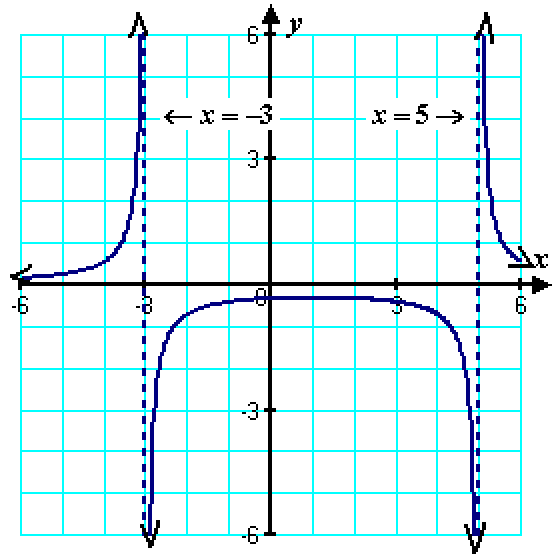
Where is the function $f(x) = \dfrac{5}{x^2 - 2x - 15}$ discontinuous ?

$$\lim_{x \to 5} \frac{5}{(x-5)(x+3)} = \frac{5}{0} = \infty = \textbf{\textit{undefined}}$$

$$\lim_{x \to -3} \frac{5}{(x-5)(x+3)} = \frac{5}{0} = \infty = \textbf{\textit{undefined}}$$

$\therefore$ $\boxed{x = -3,\ 5}$ are ***vertical*** asymptotes

$f(x)$ is ***discontinuous*** for $\boxed{x = -3,\ 5}$



115. *Answer is D.*

The graph of a function f whose domain is the closed interval $[\ 1,\ 7\ ]$ is shown. Which of the following statements about $f(x)$ is true ?

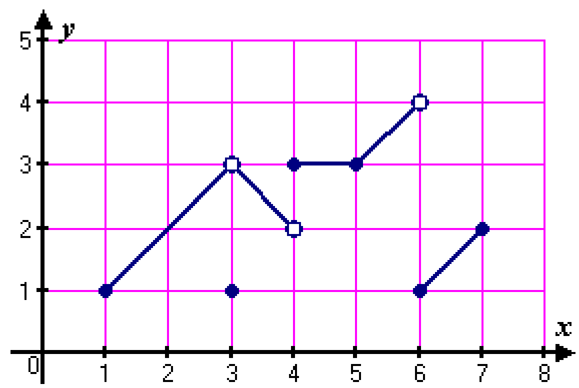
$\lim_{x \to 3} f(x) = 1$ ☒ $\to \lim_{x \to 3} f(x) = 3$

$\lim_{x \to 4} f(x) = 3$ ☒ $\to$ ***undefined*** (jump)

$f(x)$ is continuous at $x = 3$ ☒ $\to$ hole

$f(x)$ is continuous at $x = 5$ ☑

$\lim_{x \to 6} f(x) = f(6)$ ☒ $\to$ ***undefined*** (jump)



116.

Definition of a derivative

$$\frac{f(x+h)-f(x)}{h} \leftarrow \text{slope of a secant}$$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \leftarrow \text{secant changes into a tangent by limit process}$$

117. *Answer is A.*

Difficulty = **0.79 K**

Which of the following represents the derivative of $f(x)=x^3$

$f(x)=x^3$ $f'(x)=\lim_{\triangle x\to 0}\frac{f(x+\triangle x)-f(x)}{\triangle x}$ *or* $f'(x)=\lim_{\triangle x\to 0}\frac{(x+\triangle x)^3-x^3}{\triangle x}$	$f(x)=x^3$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $f'(x)=\lim_{h\to 0}\frac{(x+h)^3-x^3}{h}$

118. *Answer is B.*

Difficulty = **0.79 K**

Which expression represents the derivative of $f(x)=\frac{1}{x^3}$

$f(x)=\frac{1}{x^3}$ $f'(x)=\lim_{\triangle x\to 0}\frac{f(x+\triangle x)-f(x)}{\triangle x}$ *or* $f'(x)=\lim_{\triangle x\to 0}\frac{\frac{1}{(x+\triangle x)^3}-\frac{1}{x^3}}{\triangle x}$	$f(x)=\frac{1}{x^3}$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $f'(x)=\lim_{h\to 0}\frac{\frac{1}{(x+h)^3}-\frac{1}{x^3}}{h}$

119. *Answer is D.*

Difficulty = **0.78 K**

Which expression represents the derivative of $f(x)=x^4$

$f(x)=x^4$ $f'(x)=\lim_{\triangle x\to 0}\frac{f(x+\triangle x)-f(x)}{\triangle x}$ *or* $f'(x)=\lim_{\triangle x\to 0}\frac{(x+\triangle x)^4-x^4}{\triangle x}$	$f(x)=x^4$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $f'(x)=\lim_{h\to 0}\frac{(x+h)^4-x^4}{h}$

120. *Answer is C.*

Difficulty = **0.78 K**

Which of the following is the derivative of $f(x)$

Must know !!!

$$f'(x)=\lim_{\triangle x\to 0}\frac{f(x+\triangle x)-f(x)}{\triangle x}$$

or

Must know !!!

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

121. *Answer is C.*

Difficulty = **0.77 K**

Which of the following limits represents the derivative of the function $f(x)=x^2-3x+1$

$$f(x)=x^2-3x+1$$

$$f'(x)=\lim_{\triangle x\to 0}\frac{f(x+\triangle x)-f(x)}{\triangle x}$$

$$f'(x)=\lim_{\triangle x\to 0}\frac{\left[(x+\triangle x)^2-3(x+\triangle x)+1\right]-\left[x^2-3x+1\right]}{\triangle x}$$

$$f(x)=x^2-3x+1$$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{\left[(x+h)^2-3(x+h)+1\right]-\left[x^2-3x+1\right]}{h}$$

122. *Answer is A.*

Difficulty = **0.75 U**

Which expression represents the derivative of $f(x)=x^2+3x$

$$f(x)=x^2+3x$$

$$f'(x)=\lim_{\triangle x\to 0}\frac{f(x+\triangle x)-f(x)}{\triangle x}$$

$$f'(x)=\lim_{\triangle x\to 0}\frac{\left[(x+\triangle x)^2+3(x+\triangle x)\right]-\left[x^2+3x\right]}{\triangle x}$$

$$f(x)=x^2+3x$$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{\left[(x+h)^2+3(x+h)\right]-\left[x^2+3x\right]}{h}$$

123. *Answer is D.*

Difficulty = **0.69 K**

For the polynomial function $y = f(x)$, the expression $\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ represents:

Definition of $f'(x) = \boxed{\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}}$ $\leftarrow$ must know and understand **!!!**

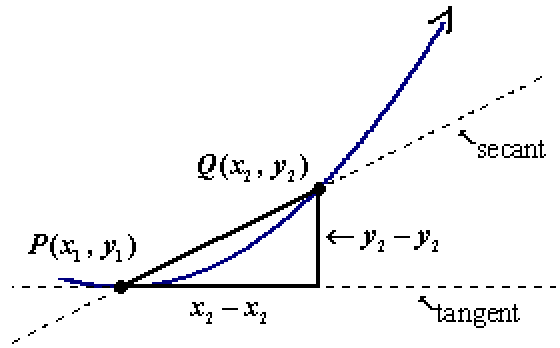
124. *Answer is D.*

Difficulty = **0.46 H**

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two points on the graph of a polynomial function. Which expression represents the derivative at point P

At point **P** slope of tangent is

$$\text{slope} = \frac{rise}{run} = \lim_{x_2 \to x_1} \frac{y_2 - y_1}{x_2 - x_1} = f'(x_1)$$



125. *Answer is D.*

Which of the following represents the slope of the tangent to the function $f(x) = \sqrt{x}$

$$f(x) = \sqrt{x}$$
$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$f'(x) = \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x} - \sqrt{x}}{\Delta x}$$

or

$$f(x) = \sqrt{x}$$
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$f'(x) = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

126. *Answer is D.*

If $f(x) = x^2$ determine the value of $\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$$f(x) = x^2$$
$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$f'(x) = \lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 - x^2}{\Delta x}$$
$$f'(x) = \lim_{\Delta x \to 0} \frac{\cancel{x^2} + 2x\Delta x + (\Delta x)^2 - \cancel{x^2}}{\Delta x}$$
$$f'(x) = \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(2x + \Delta x)}{\cancel{\Delta x}}$$
$$f'(x) = \lim_{\Delta x \to 0} 2x + \Delta x = \boxed{2x}$$

or

$$f(x) = x^2$$
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$
$$f'(x) = \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h}$$
$$f'(x) = \lim_{h \to 0} \frac{\cancel{h}(2x + h)}{\cancel{h}}$$
$$f'(x) = \lim_{h \to 0} 2x + h = \boxed{2x}$$

127. *Answer is A.*

Which one of the following is equal to $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

Definition of $f'(x)=\boxed{\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}}$ ← must know and understand **!!!**

128. *Answer is B.*

Which one of the following limits represents the derivative of the function $f(x)=x^2-2x+3$

$$f(x)=x^2-2x+3$$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{\left[(x+h)^2-2(x+h)+3\right]-\left[x^2-2x+3\right]}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{(x+h)^2-2(x+h)+3-x^2+2x-3}{h}$$

129.

Given $f(x)=3x^2$, use the **definition of the derivative** to show that $f'(x)=6x$

Solution:

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$=\lim_{h\to 0}\frac{3(x+h)^2-3x^2}{h} \qquad \leftarrow 1\ \textbf{mark}$$

$$=\lim_{h\to 0}\frac{3\left(x^2+2xh+h^2\right)-3x^2}{h}$$

$$=\lim_{h\to 0}\frac{3x^2+6xh+3h^2-3x^2}{h}$$

$$=\lim_{h\to 0}\frac{6xh+3h^2}{h} \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$\left.\begin{aligned}&=\lim_{h\to 0}\frac{h(6x+3h)}{h}\\ &=\lim_{h\to 0}\,(6x+3h)\end{aligned}\right\}\leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$=6x \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

130. $\boxed{\text{Given } f(x)=x^2-3x, \text{ use the } \textbf{definition of the derivative} \text{ to show that } f'(x)=2x-3}$

$$\boxed{f'(x)=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}}$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{\left[(x+\Delta x)^2-3(x+\Delta x)\right]-(x^2-3x)}{\Delta x}$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{\cancel{x^2}+2x\Delta x+(\Delta x)^2-\cancel{3x}-3\Delta x-\cancel{x^2}+\cancel{3x}}{\Delta x}$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{\cancel{\Delta x}(2x+\Delta x-3)}{\cancel{\Delta x}}$$

$$f'(x)=\lim_{\Delta x\to 0}(2x+\Delta x-3)=\boxed{2x-3}$$

131. $\boxed{\text{Given } f(x)=x^2+5x, \text{ use the } \textbf{definition of the derivative} \text{ to show that } f'(x)=2x+5}$

$$\boxed{f'(x)=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}}$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{\left[(x+\Delta x)^2+5(x+\Delta x)\right]-(x^2+5x)}{\Delta x}$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{\cancel{x^2}+2x\Delta x+(\Delta x)^2+\cancel{5x}+5\Delta x-\cancel{x^2}-\cancel{5x}}{\Delta x}$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{\cancel{\Delta x}(2x+\Delta x+5)}{\cancel{\Delta x}}=\lim_{\Delta x\to 0}(2x+\Delta x+5)=2x+0+5=\boxed{2x+5}$$

132. *Answer is C.*

$$\boxed{\text{If } f(x)=\sqrt{x-2} \quad \text{then} \quad \frac{f(x+h)-f(x)}{h}=}$$

$$\frac{f(x+h)-f(x)}{h}=\frac{\sqrt{(x+h)-2}-\sqrt{x-2}}{h}=\boxed{\frac{\sqrt{x+h-2}-\sqrt{x-2}}{h}}$$

133. *Answer is B.*

$$\boxed{\text{If } f(x)=\frac{1}{x+2} \quad \text{then} \quad \frac{f(x+h)-f(x)}{h}=}$$

$$\frac{f(x+h)-f(x)}{h}=\frac{\frac{1}{(x+h)+2}-\frac{1}{x+2}}{h}=\frac{\frac{1}{x+h+2}\left(\frac{x+2}{x+2}\right)-\frac{1}{x+2}\left(\frac{x+h+2}{x+h+2}\right)}{h}=\frac{\frac{(x+2)-(x+h+2)}{(x+h+2)(x+2)}}{h}$$

$$=\frac{\frac{x+2-x-h-2}{(x+h+2)(x+2)}}{h}=\frac{\frac{-h}{(x+h+2)(x+2)}}{\frac{h}{1}}=\left(\frac{-\cancel{h}}{(x+h+2)(x+2)}\right)\left(\frac{1}{\cancel{h}}\right)=\boxed{\frac{-1}{(x+2)(x+h+2)}}$$

134. *Answer is B.*

$$\lim_{h\to 0}\frac{3(x+h)^{37}-3x^{37}}{h}=$$

$$\lim_{h\to 0}\frac{3(x+h)^{37}-3x^{37}}{h}= \quad \leftarrow \text{means the derivative of } \; f(x)=3x^{37}$$

135. *Answer is D.*

$$\lim_{h\to 0}\frac{(x+h)^{\frac{1}{2}}-x^{\frac{1}{2}}}{h}=$$

$$\lim_{h\to 0}\frac{(x+h)^{\frac{1}{2}}-x^{\frac{1}{2}}}{h}= \quad \leftarrow \text{means the derivative of } \; f(x)=\sqrt{x}=x^{\frac{1}{2}}$$

$$f'(x)=\tfrac{1}{2}x^{-\frac{1}{2}}=\boxed{\frac{1}{2\sqrt{x}}}$$

136. *Answer is C.*

$$\lim_{h\to 0}\frac{(1+h)^{6}-1}{h}=$$

$$\lim_{h\to 0}\frac{(1+h)^{6}-1}{h}= \quad \leftarrow \text{means the derivative of } \; f(x)=x^{6} \; \text{ evaluated at } \; x=1$$

$$f'(x)=6x^{5}$$

$$f'(1)=6(1)^{5}=\boxed{6}$$

137. *Answer is B.*

$$\lim_{h\to 0}\frac{\sqrt[3]{8+h}-2}{h} \quad \text{is}$$

$$\lim_{h\to 0}\frac{\sqrt[3]{8+h}-2}{h} \quad \text{is} \quad \leftarrow \text{means the derivative of } \; f(x)=\sqrt[3]{x}=x^{\frac{1}{3}} \; \text{ evaluated at } \; x=8$$

$$f'(x)=\tfrac{1}{3}x^{-\frac{2}{3}}=\frac{1}{3(\sqrt[3]{x})^{2}}$$

$$f'(8)=\frac{1}{3(\sqrt[3]{8})^{2}}=\frac{1}{3(2)^{2}}=\boxed{\frac{1}{12}}$$

138. *Answer is C.*

$$\lim_{h\to 0}\frac{(2+h)^3-2^3}{h}=$$

$$\lim_{h\to 0}\frac{(2+h)^3-2^3}{h}=\lim_{h\to 0}\frac{(8+12h+6h^2+h^3)-8}{h}=\lim_{h\to 0}\frac{\cancel{h}(12+6h+h^2)}{\cancel{h}}$$

$$=\lim_{h\to 0}12+6h+h^2=\boxed{12}$$

139. *Answer is B.*

$$\lim_{h\to 0}\frac{\ln(e+h)-1}{h}=$$

$$\lim_{h\to 0}\frac{\ln(e+h)-1}{h}= \quad \leftarrow \text{means the derivative of } f(x)=\ln(x) \text{ evaluated at } x=e$$

$$f'(x)=\frac{1}{x}$$

$$f'(e)=\boxed{\frac{1}{e}}$$

140. *Answer is B.*

$$\lim_{h\to 0}\frac{\frac{1}{3+h}-\frac{1}{3}}{h}=$$

$$\lim_{h\to 0}\frac{\frac{1}{3+h}-\frac{1}{3}}{h}=\lim_{h\to 0}\frac{\left(\frac{3}{3}\right)\left(\frac{1}{3+h}\right)-\left(\frac{1}{3}\right)\left(\frac{3+h}{3+h}\right)}{h}=\lim_{h\to 0}\frac{\frac{\cancel{3}-\cancel{3}-h}{3(3+h)}}{\frac{h}{1}}=\lim_{h\to 0}\left(\frac{-\cancel{h}}{3(3+h)}\right)\left(\frac{1}{\cancel{h}}\right)=\boxed{-\frac{1}{9}}$$

141. *Answer is C.*

$$\lim_{h\to 0}\frac{\ln(2+h)-\ln 2}{h}=$$

$$\lim_{h\to 0}\frac{\ln(2+h)-\ln 2}{h}= \quad \leftarrow \text{means the derivative of } f(x)=\ln(x) \text{ evaluated at } x=2$$

$$f'(x)=\frac{1}{x}$$

$$f'(2)=\boxed{\frac{1}{2}}$$

142. *Answer is B.*

$$\lim_{h\to 0}\frac{8(\frac{1}{2}+h)^8-8(\frac{1}{2})^8}{h}=$$

$$\lim_{h\to 0}\frac{8(\frac{1}{2}+h)^8-8(\frac{1}{2})^8}{h}= \quad \leftarrow \text{means the derivative of } f(x)=8x^8 \text{ evaluated at } x=\tfrac{1}{2}$$

$$f'(x)=64x^7$$

$$f'(\tfrac{1}{2})=2^6(\tfrac{1}{2})^7=\boxed{\frac{1}{2}}$$

143. *Answer is B.*

$$\lim_{h\to 0}\frac{e^{4+h}-e^4}{h}=$$

$$\lim_{h\to 0}\frac{e^{4+h}-e^4}{h}= \quad \leftarrow \text{means the derivative of } f(x)=e^x \text{ evaluated at } x=4$$

$$f'(x)=e^x$$

$$f'(4)=\boxed{e^4}$$

144. *Answer is D.*

$$\lim_{h\to 0}\frac{8\sqrt{16+h}-32}{h}=$$

$$\lim_{h\to 0}\frac{8\sqrt{16+h}-32}{h}= \quad \leftarrow \text{means the derivative of } f(x)=8\sqrt{x}=8x^{\frac{1}{2}} \text{ evaluated at } x=16$$

$$f'(x)=8(\tfrac{1}{2})x^{-\frac{1}{2}}=\frac{4}{\sqrt{x}}$$

$$f'(16)=\frac{4}{\sqrt{16}}=\boxed{1}$$

145. *Answer is E.*

$$\lim_{h\to 0}\frac{(2+h)^3+(2+h)-10}{h}=$$

$$\lim_{h\to 0}\frac{(2+h)^3+(2+h)-10}{h}= \quad \leftarrow \text{means the derivative of } f(x)=x^3+x \text{ evaluated at } x=2$$

$$f'(x)=3x^2+1$$

$$f'(2)=3(2)^2+1=\boxed{13}$$

146. *Answer is E.*

$$\lim_{h\to 0}\frac{4(2+h)^3-2(2+h)-28}{h}=$$

$$\lim_{h\to 0}\frac{4(2+h)^3-2(2+h)-28}{h}= \quad \leftarrow \text{ derivative of } f(x)=4x^3-2x \text{ evaluated at } x=2$$

$$f'(x)=12x^2-2$$

$$f'(2)=12(2)^2-2=\boxed{46}$$

147. *Answer is B.*

$$\lim_{h\to 0}\frac{\sqrt{9+h}-3}{h}=$$

$$\lim_{h\to 0}\frac{\sqrt{9+h}-3}{h}= \quad \leftarrow \text{means the derivative of } f(x)=\sqrt{x}=x^{\frac{1}{2}} \text{ evaluated at } x=9$$

$$f'(x)=\frac{1}{2\sqrt{x}}$$

$$f'(9)=\frac{1}{2\sqrt{9}}=\boxed{\frac{1}{6}}$$

148. *Answer is D.*

$$\lim_{h\to 0}\frac{(2+h)^4-3(2+h)-10}{h}=$$

$$\lim_{h\to 0}\frac{(2+h)^4-3(2+h)-10}{h}= \quad \leftarrow \text{ derivative of } f(x)=x^4-3x \text{ evaluated at } x=2$$

$$f'(x)=4x^3-3$$

$$f'(2)=4(2)^3-3=\boxed{29}$$

149. *Answer is E.*

$$\lim_{h\to 0}\frac{e^{x+h}-e^x}{h}=$$

$$\lim_{h\to 0}\frac{e^{x+h}-e^x}{h}= \quad \leftarrow \text{means the derivative of } f(x)=e^x$$

$$f'(x)=\boxed{e^x}$$

150. *Answer is C.*

$$\lim_{h\to 0}\frac{(3+h)^3+(3+h)-30}{h}=$$

$$\lim_{h\to 0}\frac{(3+h)^3+(3+h)-30}{h}= \quad \leftarrow \text{ derivative of } f(x)=x^3+x \text{ evaluated at } x=3$$

$$f'(x)=3x^2+1$$

$$f'(3)=3(3)^2+1=\boxed{28}$$

151. *Answer is C.*

$$\lim_{h\to 0}\frac{\sqrt{2(6+h)-3}-\sqrt{2(6)-3}}{h}=$$

$$\lim_{h\to 0}\frac{\sqrt{2(6+h)-3}-\sqrt{2(6)-3}}{h}= \quad \leftarrow \text{ derivative of } f(x)=\sqrt{2x-3} \text{ evaluated at } x=6$$

$$f'(x)=\frac{1}{\sqrt{2x-3}}$$

$$f'(6)=\frac{1}{\sqrt{2(6)-3}}=\boxed{\frac{1}{3}}$$

152. *Answer is D.*

If $f(x)=\sqrt{x+2}$, then $\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}=$

$$\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}= \quad \leftarrow \text{ derivative of } f(x)=\sqrt{x+2} \text{ evaluated at } x=2$$

$$f'(x)=\frac{1}{2}(x+2)^{-\frac{1}{2}}=\frac{1}{2\sqrt{x+2}}$$

$$f'(2)=\frac{1}{2\sqrt{2+2}}=\boxed{\frac{1}{4}}$$

153. *Answer is D.*

$$\lim_{h\to 0}\frac{2(x+h)^5-5(x+h)^3-2x^5+5x^3}{h}=$$

$$\lim_{h\to 0}\frac{2(x+h)^5-5(x+h)^3-2x^5+5x^3}{h}= \quad \leftarrow \text{ derivative of } f(x)=2x^5-5x^3$$

$$f'(x)=\boxed{10x^4-15x^2}$$

154. *Answer is C.*

$$\lim_{h\to 0}\frac{3(\frac{1}{2}+h)^5-3(\frac{1}{2})^5}{h}=$$

$$\lim_{h\to 0}\frac{3(\frac{1}{2}+h)^5-3(\frac{1}{2})^5}{h}= \quad \leftarrow \text{ derivative of } f(x)=3x^5 \text{ evaluated at } x=(\tfrac{1}{2})$$

$$f'(x)=15x^4$$

$$f'(\tfrac{1}{2})=15(\tfrac{1}{2})^4=\boxed{\frac{15}{16}}$$

155. *Answer is D.*

$$\lim_{h\to 0}\frac{e^{1+h}-e}{h}=$$

$$\lim_{h\to 0}\frac{e^{1+h}-e}{h}= \quad \leftarrow \text{means the derivative of } f(x)=e^x \text{ when } x=1$$

$$f'(x)=e^x$$

$$f'(1)=\boxed{e}$$

156. *Answer is B.*

$$\lim_{h\to 0}\frac{\sqrt{1+2h}-1}{h}=$$

$$\lim_{h\to 0}\frac{\sqrt{1+2h}-1}{h}=\frac{2}{2}\lim_{h\to 0}\frac{\sqrt{1+2h}-1}{h}=2\lim_{h\to 0}\frac{\sqrt{1+2h}-1}{2h}=2\lim_{m\to 0}\frac{\sqrt{1+m}-1}{m}$$

$$2\lim_{m\to 0}\frac{\sqrt{1+m}-1}{m}= \quad \leftarrow \text{means the derivative of } 2f(x)=\sqrt{x} \text{ evaluated at } x=1$$

$$2f'(x)=\frac{1}{2\sqrt{x}}$$

$$2f'(1)=\frac{1}{2\sqrt{1}}=\boxed{1}$$

157. *Answer is A.*

Difficulty = **0.71**

$$\lim_{h\to 0}\frac{\ln(e+h)-1}{h}=$$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \quad \leftarrow \text{means the derivative of } f(x) \text{ at any point } x$$

$$f'(x)=\lim_{h\to 0}\frac{\ln(x+h)-\ln(x)}{h} \quad \leftarrow \text{means the derivative of } f(x)=\ln x \text{ at any point } x$$

$$f'(e)=\lim_{h\to 0}\frac{\ln(e+h)-1}{h}= \quad \leftarrow \text{means the derivative of } f(x)=\ln x \text{ evaluated at } x=e$$

158. *Answer is C.*

$$\boxed{\lim_{h\to 0}\frac{1}{h}\ln\left(\frac{2+h}{2}\right)=}$$

$$\lim_{h\to 0}\frac{1}{h}\ln\left(\frac{2+h}{2}\right)=\lim_{h\to 0}\frac{1}{h}\left[\ln(2+h)-\ln 2\right]=\lim_{h\to 0}\frac{\ln(2+h)-\ln 2}{h}$$

$$\lim_{h\to 0}\frac{\ln(2+h)-\ln 2}{h}= \quad \leftarrow \text{means the derivative of } f(x)=\ln(x) \text{ evaluated at } x=2$$

$$f'(x)=\frac{1}{x}$$

$$f'(2)=\boxed{\frac{1}{2}}$$

159.

$$\boxed{\lim_{h\to 0}\frac{(4+h)^3+(4+h)-68}{h}=}$$

$$\lim_{h\to 0}\frac{(4+h)^3+(4+h)-68}{h}= \quad \leftarrow \text{ derivative of } f(x)=x^3+x \text{ evaluated at } x=4$$

$$f'(x)=3x^2+1$$

$$f'(4)=3(4)^2+1=\boxed{49}$$

160. *Answer is C.*

$\boxed{\text{Difficulty}=\mathbf{0.24}}$

If f is a differentiable function, then $f'(a)$ is given by which of the following ?

I. $\displaystyle\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$ **II.** $\displaystyle\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$ **III.** $\displaystyle\lim_{x\to a}\frac{f(x+h)-f(x)}{h}$

$$\boxed{\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}} \quad \leftarrow \text{formal definition of derivative of } f(x) \text{ at point } \textbf{\textit{any}} \text{ point } x$$

I. $\displaystyle\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$ $\leftarrow$ formal definition of derivative of $f(x)$ at point $x=a$

II. $\displaystyle\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$ $\leftarrow$ alternative definition of derivative of $f(x)$ at point $x=a$

III. $\displaystyle\lim_{x\to a}\frac{f(x+h)-f(x)}{h}$ $\leftarrow$ distractor trying to confuse you with the $x\to a$

161. *Answer is B.*

If $\displaystyle\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=g(x)$ then

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \quad \leftarrow \text{definition of derivative of } f(x) \text{ at any point } x$$

$$\boxed{f'(x)=g(x)}$$

162. *Answer is E.*

If $f(x)=e^x$ which of the following is equal to $f'(e)$

$f'(x)=\lim_{x\to 0}\dfrac{f(x+h)-f(x)}{h}$ $\leftarrow$ definition of a derivative general function at any point x

$f'(x)=\lim_{x\to 0}\dfrac{e^{x+h}-e^x}{h}$ $\leftarrow$ definition of a derivative of $y=e^x$ at any point x

$f'(e)=\boxed{\lim_{x\to 0}\dfrac{e^{e+h}-e^e}{h}}$ $\leftarrow$ definition of a derivative of $y=e^x$ at specific point $x=e$

163. *Answer is B.*

If $f(x)=6x^2+\dfrac{16}{x^2}$ then $\lim_{h\to 0}\dfrac{f(2+h)-f(2)}{h}=$

$\lim_{h\to 0}\dfrac{f(2+h)-f(2)}{h}=$ $\leftarrow$ derivative of $f(x)=6x^2+\dfrac{16}{x^2}$ evaluated at $x=2$

$f'(x)=12x-\dfrac{32}{x^3}$

$f'(2)=12(2)-\dfrac{32}{(2)^3}=\boxed{20}$

164. *Answer is C.*

If $\lim_{h\to 0}\dfrac{f(4+h)-f(4)}{h}=6$ which of the following must be true ?

$f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $\leftarrow$ definition of a derivative of $f'(x)$ at any point x

$f'(4)=\lim_{h\to 0}\dfrac{f(4+h)-f(4)}{h}$ $\leftarrow$ derivative of $f(x)$ when $x=4$ $\rightarrow$ $f'(4)$

$f'(4)=\boxed{\lim_{h\to 0}\dfrac{f(4+h)-f(4)}{h}}=6$ $\leftarrow$ derivative of $f(x)$ when $x=4$ equals 6

$\boxed{f'(4)=6}$

165. *Answer is B.*

If the function f is continuous for all real numbers and $\lim_{h\to 0}\dfrac{f(a+h)-f(a)}{h}=7$ then which of the following statements must be true ?

$\lim_{h\to 0}\dfrac{f(a+h)-f(a)}{h}$ $\leftarrow$ means the derivative of $f(x)=?$ evaluated at $x=a$

$\lim_{h\to 0}\dfrac{f(a+h)-f(a)}{h}=7$ $\leftarrow$ means the derivative of $f'(a)=7$

$\boxed{f \text{ is differentiable at } x=a}$ $\leftarrow$ ***must*** be true from the given information

All other choices ***could*** be true from the given information

166. *Answer is B.*

Given $\lim\limits_{h\to 0}\dfrac{f(6+h)-f(6)}{h}=-2$ which of the following must be true ?

I. $f'(6)$ exists **II.** $f(x)$ is continuous at $x=6$ **III.** $f(6)<0$

$$\lim_{h\to 0}\frac{f(6+h)-f(6)}{h}=-2 \leftarrow \text{means the derivative of } f(x)=? \text{ evaluated at } x=6$$

$$f'(x)=?$$

$$f'(6)=-2$$

I. $f'(6)$ exists ☑ True $\leftarrow f'(6)=-2$

II. $f(x)$ is continuous at $x=6$ ☑ True $\leftarrow$ differentiability implies continuity

III. $f(6)<0$ ☒ False $\leftarrow$ could be true but need not be

167. *Answer is D.*

Suppose $\lim\limits_{x\to 0}\dfrac{g(x)-g(0)}{x}=1$ It follows necessarily that

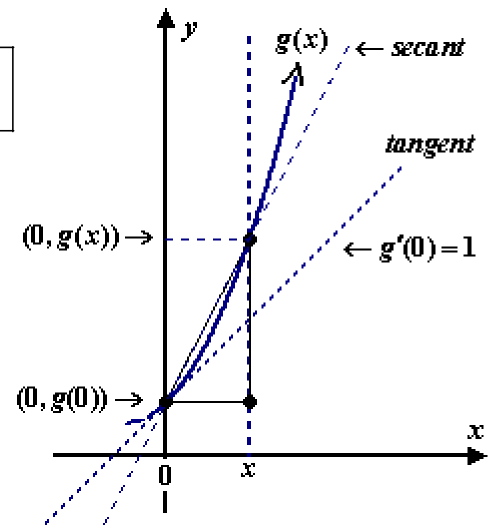
$$\lim_{x\to 0}\frac{g(x)-g(0)}{x-0}=1$$

$\rightarrow$ the derivative of $g(x)=?$ evaluated at $x=0$

$$g'(x)=?$$

$$g'(0)=1$$

Slope of tangent $m=1$ through $(\ 0,\ g(0))$



168. *Answer is B.*

If f is a function such that $\lim\limits_{x\to 5}\dfrac{f(x)-f(5)}{x-5}=0$ which of the following must be true ?

Definition of derivitive at any point $(\ x,\ f(x))$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

Definition of derivitive at specific point $(\ a,\ f(a))$

$$f'(a)=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

$$f'(5)=\lim_{x\to 5}\frac{f(x)-f(5)}{x-5}=0$$

derivative at ***specific*** point $(\ 5,\ f(5))$ is $\underbrace{0}_{\textit{horizontal tangent}}$

169. *Answer is D.*

Let f be a function $\underbrace{\textbf{defined} \text{ for all real numbers}}_{\text{did not say } \textbf{continuous}}$. Which of the following statements about f ***must*** be true ?

If $\lim\limits_{x\to2} f(x)=7$ then $f(2)=7$	If $\lim\limits_{x\to5} f(x)=-3$ then -3 in range of f	If $\lim\limits_{x\to1^-} f(x)=\lim\limits_{x\to1^+} f(x)$ then $f(1)$ exists	If $\lim\limits_{x\to3^-} f(x)\neq\lim\limits_{x\to3^+} f(x)$ $\lim\limits_{x\to3} f(x)$ does not exist	If $\lim\limits_{x\to4} f(x)$ does not exist, then $f(4)$ does not exist
could be true, but hole at $(2, 7)$ with $f(2)=1$ ***could*** $\neq$ ***must***	***could*** be true, but hole at $(5,-3)$ with $f(5)=1$ ***could*** $\neq$ ***must***	***could*** be true, but hole at $\lim\limits_{x\to1} f(x)$ with $f(1)\neq\lim\limits_{x\to1} f(x)$ ***could*** $\neq$ ***must***	**TRUE** ***must*** be true	***could*** be true, but jump at $x=4$ with $f(4)=5$ ***could*** $\neq$ ***must***

170. *Answer is C.*

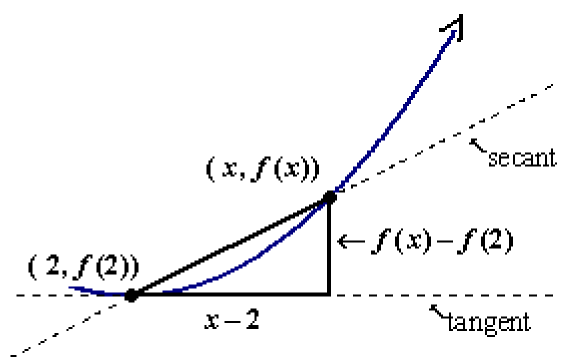
If f is a function such that $\lim\limits_{x\to2}\dfrac{f(x)-f(2)}{x-2}=0$ which of the following must be true ?

$$\lim_{x\to2}\frac{f(x)-f(2)}{x-2} \leftarrow \begin{cases} \text{definition of derivative of } f(x)=? \\ \quad(\text{evaluated at } x=2) \\ \quad f'(x)=? \\ \quad f'(2)=0 \end{cases}$$

$$\textbf{slope} \text{ of } \textbf{secant} = \frac{\textbf{rise}}{\textbf{run}} = \frac{f(x)-f(2)}{x-2}$$

$$f'(2)=\underbrace{\lim_{x\to2}\frac{f(x)-f(2)}{x-2}}_{\text{slope of } \textbf{tangent} \text{ at } x=2}=0$$

$f'(2)=0 \leftarrow$ horizontal tangent at $x=2$



171. *Answer is C.*

$$\lim_{a\to4}\frac{2-\sqrt{a}}{4-a}=$$

$$\lim_{a\to4}\frac{2-\sqrt{a}}{4-a}=\lim_{a\to4}\frac{\sqrt{a}-2}{a-4}= \quad \leftarrow \text{ derivative of } f(x)=\sqrt{x} \text{ evaluated at } x=4$$

$$f'(x)=\frac{1}{2}(x)^{-\frac{1}{2}}=\frac{1}{2\sqrt{x}}$$

$$f'(4)=\frac{1}{2\sqrt{4}}=\boxed{\frac{1}{4}}$$

$$\boxed{f'(4),\ \text{where } f(x)=\sqrt{x}}$$

172. *Answer is D.*

$$\lim_{x\to a}\frac{\sqrt[3]{x}-\sqrt[3]{a}}{x-a}=$$

$$\lim_{x\to a}\frac{\sqrt[3]{x}-\sqrt[3]{a}}{x-a}= \quad \leftarrow \text{means the derivative of } f(x)=\sqrt[3]{x}=x^{\frac{1}{3}} \text{ evaluated at } x=a$$

$$f'(x)=\frac{1}{3}x^{-\frac{2}{3}}=\frac{1}{3\sqrt[3]{x^2}}$$

$$f'(a)=\frac{1}{3\sqrt[3]{a^2}}\left(\frac{\sqrt[3]{a}}{\sqrt[3]{a}}\right)=\boxed{\frac{\sqrt[3]{a}}{3a}}$$

173.

$\boxed{\text{Limits}}$ $\rightarrow$ needed to handle ***holes***, ***asymptotes***, ***sharp points***, ***endpoints***,

Infinity $\rightarrow$ horizontal asymptotes (divide by the variable with highest exponent in denominator

$$\lim_{x\to\infty}\frac{4x-5x^2}{2x^2+3x-1}=\lim_{x\to\infty}\frac{\frac{4x}{x^2}-\frac{5x^2}{x^2}}{\frac{2x^2}{x^2}+\frac{3x}{x^2}-\frac{1}{x^2}}=\lim_{x\to\infty}\frac{\frac{4}{x}-5}{2+\frac{3}{x}-\frac{1}{x^2}}=\frac{0-5}{2+0-0}=\boxed{-\frac{5}{2}}$$

174. *Answer is C.*

Difficulty = **0.86 K**

$$\lim_{x\to\infty}\frac{2x-7}{4x+3}=$$

$$\lim_{x\to\infty}\frac{2x-7}{4x+3}=\lim_{x\to\infty}\frac{\frac{2x}{x}-\frac{7}{x}}{\frac{4x}{x}+\frac{3}{x}}=\frac{2+0}{4+0}=\boxed{\frac{1}{2}}$$

175. *Answer is C.*

Difficulty = **0.83 U**

$$\lim_{n\to\infty}\frac{2n^3-1}{3n^3}=$$

$$\lim_{n\to\infty}\frac{2n^3-1}{3n^3}=\lim_{n\to\infty}\frac{\frac{2n^3}{n^3}-\frac{1}{n^3}}{\frac{3n^3}{n^3}}=\frac{2-0}{3}=\boxed{\frac{2}{3}}$$

176. *Answer is C.*

Difficulty = **0.82 U**

$$\lim_{n\to\infty}\frac{6n^2-4n}{3n^2+5n}=$$

$$\lim_{n\to\infty}\frac{6n^2-4n}{3n^2+5n}=\lim_{n\to\infty}\frac{\frac{6n^2}{n^2}-\frac{4n}{n^2}}{\frac{3n^2}{n^2}+\frac{5n}{n^2}}=\lim_{n\to\infty}\frac{6-0}{3+0}=\frac{6}{3}=\boxed{2}$$

177. *Answer is C.*

$\boxed{\text{Difficulty} = \mathbf{0.82\ U}}$

$$\lim_{x\to\infty}\frac{3x-2}{9x+8}=$$

$$\lim_{x\to\infty}\frac{3x-2}{9x+8}=\lim_{x\to\infty}\frac{\frac{3x}{x}-\frac{2}{x}}{\frac{9x}{x}+\frac{8}{x}}=\lim_{x\to\infty}\frac{3-\frac{2}{x}}{9+\frac{8}{x}}=\frac{3-0}{9+0}=\boxed{\frac{1}{3}}$$

178. *Answer is A.*

$\boxed{\text{Difficulty} = \mathbf{0.80\ U}}$

$$\lim_{x\to\infty}\frac{4x-5x^2}{2x^2+3x-1}=$$

$$\lim_{x\to\infty}\frac{4x-5x^2}{2x^2+3x-1}=\lim_{x\to\infty}\frac{\frac{4x}{x^2}-\frac{5x^2}{x^2}}{\frac{2x^2}{x^2}+\frac{3x}{x^2}-\frac{1}{x^2}}=\lim_{x\to\infty}\frac{\frac{4}{x}-5}{2+\frac{3}{x}-\frac{1}{x^2}}=\frac{0-5}{2+0-0}=\boxed{-\frac{5}{2}}$$

179. *Answer is C.*

$\boxed{\text{Difficulty} = \mathbf{0.79\ U}}$

$$\lim_{x\to\infty}\frac{5x^2+3x-2}{3x^2-4x+7}=$$

$$\lim_{x\to\infty}\frac{5x^2+3x-2}{3x^2-4x+7}=\lim_{x\to\infty}\frac{\frac{5x^2}{x^2}+\frac{3x}{x^2}-\frac{2}{x^2}}{\frac{3x^2}{x^2}-\frac{4x}{x^2}+\frac{7}{x^2}}=\lim_{x\to\infty}\frac{5+\frac{3}{x}-\frac{2}{x^2}}{3-\frac{4}{x}+\frac{7}{x^2}}=\frac{5+0-0}{3-0+0}=\boxed{\frac{5}{3}}$$

180. *Answer is C.*

$\boxed{\text{Difficulty} = \mathbf{0.76\ U}}$

$$\lim_{x\to\infty}\frac{5x^2+4x-1}{5x^2-3x+2}=$$

$$\lim_{x\to\infty}\frac{5x^2+4x-1}{5x^2-3x+2}=\lim_{x\to\infty}\frac{\frac{5x^2}{x^2}+\frac{4x}{x^2}-\frac{1}{x^2}}{\frac{5x^2}{x^2}-\frac{3x}{x^2}+\frac{2}{x^2}}=\lim_{x\to\infty}\frac{5+\frac{4}{x}-\frac{1}{x^2}}{5-\frac{3}{x}+\frac{2}{x^2}}=\frac{5+0-0}{5-0+0}=\boxed{1}$$

181. *Answer is A.*

$\boxed{\text{Difficulty} = \mathbf{0.73\ U}}$

$$\lim_{n\to\infty}\frac{3n-1}{4-2n}=$$

$$\lim_{n\to\infty}\frac{3n-1}{4-2n}=\lim_{n\to\infty}\frac{\frac{3n}{n}-\frac{1}{n}}{\frac{4}{n}-\frac{2n}{n}}=\lim_{n\to\infty}\frac{3-\frac{1}{n}}{\frac{4}{n}-2}=\frac{3-0}{0-2}=\boxed{-\frac{3}{2}}$$

182. *Answer is C.*

$\boxed{\text{Difficulty} = \mathbf{0.73\ U}}$

$$\lim_{n\to\infty}\frac{n^2+2n-3}{5-3n+6n^2}=$$

$$\lim_{n\to\infty}\frac{n^2+2n-3}{5-3n+6n^2}=\lim_{n\to\infty}\frac{\frac{n^2}{n^2}+\frac{2n}{n^2}-\frac{3}{n^2}}{\frac{5}{n^2}-\frac{3n}{n^2}+\frac{6n^2}{n^2}}=\lim_{n\to\infty}\frac{1+\frac{2}{n}-\frac{3}{n^2}}{\frac{5}{n^2}-\frac{3}{n}+6}=\frac{1+0-0}{0-0+6}=\boxed{\frac{1}{6}}$$

183. *Answer is C.*

$$\text{Difficulty} = \mathbf{0.69\ U}$$

$$\lim_{n\to\infty}\frac{n-1}{n} =$$

$$\lim_{n\to\infty}\frac{n-1}{n} = \lim_{n\to\infty}\frac{\frac{n}{n}-\frac{1}{n}}{\frac{n}{n}} = \lim_{n\to\infty}\frac{1-\frac{1}{n}}{1} = \frac{1-0}{1} = \boxed{1}$$

184. *Answer is D.*

$$\text{Difficulty} = \mathbf{0.69\ U}$$

$$\lim_{x\to\infty}\frac{2x^4-5x^2}{3x^3+8x^2} =$$

$$\lim_{x\to\infty}\frac{2x^4-5x^2}{3x^3+8x^2} = \lim_{x\to\infty}\frac{\frac{2x^4}{x^3}-\frac{5x^2}{x^3}}{\frac{3x^3}{x^3}+\frac{8x^2}{x^3}} = \lim_{x\to\infty}\frac{2x-\frac{5}{x}}{3+\frac{8}{x}} = \lim_{x\to\infty}\frac{2x-0}{3+0} = \boxed{\infty}$$

185. *Answer is B.*

$$\text{Difficulty} = \mathbf{0.69\ U}$$

$$\lim_{x\to\infty}\frac{x+5}{x+2} =$$

$$\lim_{x\to\infty}\frac{x+5}{x+2} = \lim_{x\to\infty}\frac{\frac{x}{x}+\frac{5}{x}}{\frac{x}{x}+\frac{2}{x}} = \lim_{x\to\infty}\frac{1+\frac{5}{x}}{1+\frac{2}{x}} = \frac{1+0}{1+0} = \boxed{1}$$

186. *Answer is B.*

$$\text{Difficulty} = \mathbf{0.62\ U}$$

$$\lim_{x\to\infty}\frac{2x^2+3x+4}{2-5x^3} =$$

$$\lim_{x\to\infty}\frac{2x^2+3x+4}{2-5x^3} = \lim_{x\to\infty}\frac{\frac{2x^2}{x^3}+\frac{3x}{x^3}+\frac{4}{x^3}}{\frac{2}{x^3}-\frac{5x^3}{x^3}} = \lim_{x\to\infty}\frac{\frac{2}{x}+\frac{3}{x^2}+\frac{4}{x^3}}{\frac{2}{x^3}-5} = \lim_{x\to\infty}\frac{0+0+0}{0-5} = \boxed{0}$$

187. *Answer is B.*

$$\text{Difficulty} = \mathbf{0.56\ U}$$

$$\lim_{n\to\infty}\frac{2-n^2}{3n^3+6} =$$

$$\lim_{n\to\infty}\frac{2-n^2}{3n^3+6} = \lim_{n\to\infty}\frac{\frac{2}{n^3}-\frac{n^2}{n^3}}{\frac{3n^3}{n^3}+\frac{6}{n^3}} = \lim_{n\to\infty}\frac{\frac{2}{n^3}-\frac{1}{n}}{3+\frac{6}{n^3}} = \frac{0-0}{3+0} = \boxed{0}$$

188. *Answer is B.*

$$\lim_{x\to\infty}\frac{x^3-1}{3-5x^3} =$$

$$\lim_{x\to\infty}\frac{x^3-1}{3-5x^3} = \lim_{x\to\infty}\frac{\frac{x^3}{x^3}-\frac{1}{x^3}}{\frac{3}{x^3}-\frac{5x^3}{x^3}} = \lim_{x\to\infty}\frac{1-\frac{1}{x^3}}{\frac{3}{x^3}-5} = \frac{1-0}{0-5} = \boxed{\frac{-1}{5}}$$

189. *Answer is C.*

$$\lim_{x\to\infty}\frac{6x^2+5x-7}{4x^2-8x+3}=$$

$$\lim_{x\to\infty}\frac{6x^2+5x-7}{4x^2-8x+3}=\lim_{x\to\infty}\frac{\frac{6x^2}{x^2}+\frac{5x}{x^2}-\frac{7}{x^2}}{\frac{4x^2}{x^2}-\frac{8x}{x^2}+\frac{3}{x^2}}=\lim_{x\to\infty}\frac{6+\frac{5}{x}-\frac{7}{x^2}}{4-\frac{8}{x}+\frac{3}{x^2}}=\frac{6+0-0}{4-0+0}=\boxed{\frac{3}{2}}$$

190. *Answer is D.*

$$\lim_{x\to\infty}\left(2x+\frac{5}{x}\right)=$$

$$\lim_{x\to\infty}\left(2x+\frac{5}{x}\right)=\lim_{x\to\infty}2x+\lim_{x\to\infty}\frac{5}{x}=2(\infty)+\frac{5}{\infty}=\infty+0=\infty\quad\leftarrow\text{means there is } \textbf{\textit{no}} \text{ limit}$$

191. *Answer is C.*

$$\lim_{x\to\infty}\frac{2x^3-4x^2+3x-1}{5-2x^2+6x^3}=$$

$$\lim_{x\to\infty}\frac{2x^3-4x^2+3x-1}{5-2x^2+6x^3}=\lim_{x\to\infty}\frac{\frac{2x^3}{x^3}-\frac{4x^2}{x^3}+\frac{3x}{x^3}-\frac{1}{x^3}}{\frac{5}{x^3}-\frac{2x^2}{x^3}+\frac{6x^3}{x^3}}=\lim_{x\to\infty}\frac{2-\frac{4}{x}+\frac{3}{x^2}-\frac{1}{x^3}}{\frac{5}{x^3}-\frac{2}{x}+6}=\frac{2-0+0-0}{0-0+6}=\boxed{\frac{1}{3}}$$

192. *Answer is C.*

$$\lim_{x\to\infty}\frac{5x-1}{2x}=$$

$$\lim_{x\to\infty}\frac{5x-1}{2x}=\lim_{x\to\infty}\frac{\frac{5x}{x}-\frac{1}{x}}{\frac{2x}{x}}=\lim_{x\to\infty}\frac{5-\frac{1}{x}}{2}=\frac{5-0}{2}=\boxed{\frac{5}{2}}$$

193. *Answer is C.*

$$\lim_{x\to\infty}\frac{3x^2+2x-7}{5-3x+2x^2}=$$

$$\lim_{x\to\infty}\frac{3x^2+2x-7}{5-3x+2x^2}=\lim_{x\to\infty}\frac{\frac{3x^2}{x^2}+\frac{2x}{x^2}-\frac{7}{x^2}}{\frac{5}{x^2}-\frac{3x}{x^2}+\frac{2x^2}{x^2}}=\lim_{x\to\infty}\frac{3+\frac{2}{x}-\frac{7}{x^2}}{\frac{5}{x^2}-\frac{3}{x}+2}=\frac{3+0-0}{0-0+2}=\boxed{\frac{3}{2}}$$

194. *Answer is D.*

$$\lim_{x\to\infty}\frac{1-2x+5x^4}{3+3x^2-2x^3}=$$

$$\lim_{x\to\infty}\frac{1-2x+5x^4}{3+3x^2-2x^3}=\lim_{x\to\infty}\frac{\frac{1}{x^3}-\frac{2x}{x^3}+\frac{5x^4}{x^3}}{\frac{3}{x^3}+\frac{3x^2}{x^3}-\frac{2x^3}{x^3}}=\lim_{x\to\infty}\frac{\frac{1}{x^3}-\frac{2}{x^2}+5x}{\frac{3}{x^3}+\frac{3}{x}-2}=\frac{0-0+\infty}{0+0-2}=\boxed{\infty}$$

195. *Answer is C.*

$$\lim_{x\to\infty}\frac{2x-7}{4x+3}=$$

$$\lim_{x\to\infty}\frac{2x-7}{4x+3}=\lim_{x\to\infty}\frac{\frac{2x}{x}-\frac{7}{x}}{\frac{4x}{x}+\frac{3}{x}}=\lim_{x\to\infty}\frac{2-\frac{7}{x}}{4+\frac{3}{x}}=\frac{2-0}{4+0}=\boxed{\frac{1}{2}}$$

196. *Answer is B.*

$$\lim_{x\to\infty}\frac{1}{x-1}=$$

$$\lim_{x\to\infty}\frac{1}{x-1}=\lim_{x\to\infty}\frac{\frac{1}{x}}{\frac{x}{x}-\frac{1}{x}}=\frac{0}{1-0}=\boxed{0}$$

197. *Answer is D.*

$$\lim_{x\to\infty}\frac{2x}{x+2}=$$

$$\lim_{x\to\infty}\frac{2x}{x+2}=\lim_{x\to\infty}\frac{\frac{2x}{x}}{\frac{x}{x}+\frac{2}{x}}=\lim_{x\to\infty}\frac{2}{1+\frac{2}{x}}=\frac{2}{1+0}=\boxed{2}$$

198. *Answer is B.*

$$\lim_{x\to\infty}\frac{x^2-5}{2x^2+1}=$$

$$\lim_{x\to\infty}\frac{x^2-5}{2x^2+1}=\lim_{x\to\infty}\frac{\frac{x^2}{x^2}-\frac{5}{x^2}}{\frac{2x^2}{x^2}+\frac{1}{x^2}}=\lim_{x\to\infty}\frac{1-\frac{5}{x^2}}{2+\frac{1}{x^2}}=\frac{1-0}{2+0}=\boxed{\frac{1}{2}}$$

199. *Answer is E.*

$$\lim_{x\to\infty}\frac{x^2-5}{2x+1}=$$

$$\lim_{x\to\infty}\frac{x^2-5}{2x+1}=\lim_{x\to\infty}\frac{\frac{x^2}{x}-\frac{5}{x}}{\frac{2x}{x}+\frac{1}{x}}=\lim_{x\to\infty}\frac{x-\frac{5}{x}}{2+\frac{1}{x}}=\frac{\infty-0}{2+0}=\boxed{\infty}$$

200. *Answer is D.*

$$\lim_{x\to-\infty}\frac{2^{-x}}{2^{x}}=$$

$$\lim_{x\to-\infty}\frac{2^{-x}}{2^{x}}=\lim_{x\to-\infty}\frac{2^{-x-x}}{1}=\lim_{x\to-\infty}2^{-2x}=2^{-2(-\infty)}=2^{2(\infty)}=\boxed{\infty}$$

201. *Answer is C.*

$$\lim_{x\to\infty}\frac{x-5}{2x^2+1}=$$

$$\lim_{x\to\infty}\frac{x-5}{2x^2+1}=\lim_{x\to\infty}\frac{\frac{x}{x^2}-\frac{5}{x^2}}{\frac{2x^2}{x^2}+\frac{1}{x^2}}=\lim_{x\to\infty}\frac{\frac{1}{x}-\frac{5}{x^2}}{2+\frac{1}{x^2}}=\frac{0-0}{2+0}=\boxed{0}$$

202. *Answer is B.*

$$\lim_{x\to\infty}\frac{100x}{x^2-1}=$$

$$\lim_{x\to\infty}\frac{100x}{x^2-1}=\lim_{x\to\infty}\frac{\frac{100x}{x^2}}{\frac{x^2}{x^2}-\frac{1}{x^2}}=\lim_{x\to\infty}\frac{\frac{100}{x}}{1-\frac{1}{x^2}}=\frac{0}{1+0}=\boxed{0}$$

203. *Answer is A.*

$$\lim_{x\to\infty}\frac{x^2-4x+4}{4x^2-1}=$$

$$\lim_{x\to\infty}\frac{x^2-4x+4}{4x^2-1}=\lim_{x\to\infty}\frac{\frac{x^2}{x^2}-\frac{4x}{x^2}+\frac{4}{x^2}}{\frac{4x^2}{x^2}-\frac{1}{x^2}}=\lim_{x\to\infty}\frac{1-\frac{4}{x}+\frac{4}{x^2}}{4-\frac{1}{x^2}}=\frac{1-0+0}{4-0}=\boxed{\frac{1}{4}}$$

204. *Answer is A.*

$$\lim_{x\to\infty}\frac{1-x}{1+x}=$$

$$\lim_{x\to\infty}\frac{1-x}{1+x}=\lim_{x\to\infty}\frac{\frac{1}{x}-\frac{x}{x}}{\frac{1}{x}+\frac{x}{x}}=\lim_{x\to\infty}\frac{\frac{1}{x}-1}{\frac{1}{x}+1}=\frac{0-1}{0+1}=\boxed{-1}$$

205. *Answer is D.*

$$\lim_{x\to\infty}\frac{4-x^2}{x^2-1}=$$

$$\lim_{x\to\infty}\frac{4-x^2}{x^2-1}=\lim_{x\to\infty}\frac{\frac{4}{x^2}-\frac{x^2}{x^2}}{\frac{x^2}{x^2}-\frac{1}{x^2}}=\lim_{x\to\infty}\frac{\frac{4}{x^2}-1}{1-\frac{1}{x^2}}=\frac{0-1}{1-0}=\boxed{-1}$$

206. *Answer is B.*

$$\lim_{x\to\infty}\frac{4-x^2}{4x^2-x-2}=$$

$$\lim_{x\to\infty}\frac{4-x^2}{4x^2-x-2}=\lim_{x\to\infty}\frac{\frac{4}{x^2}-\frac{x^2}{x^2}}{\frac{4x^2}{x^2}-\frac{x}{x^2}-\frac{2}{x^2}}=\lim_{x\to\infty}\frac{\frac{4}{x^2}-1}{4-\frac{1}{x}-\frac{2}{x^2}}=\frac{0-1}{4-0-0}=\boxed{\frac{-1}{4}}$$

207. *Answer is A.*

$$\lim_{x\to -\infty} \frac{5x^3+27}{20x^2+10x+9} =$$

$$\lim_{x\to -\infty} \frac{5x^3+27}{20x^2+10x+9} = \lim_{x\to -\infty} \frac{\frac{5x^3}{x^2}+\frac{27}{x^2}}{\frac{20x^2}{x^2}+\frac{10x}{x^2}+\frac{9}{x^2}} = \lim_{x\to -\infty} \frac{5x+\frac{27}{x^2}}{20+\frac{10}{x}+\frac{9}{x^2}} = \frac{5x}{20+0+0} = \boxed{-\infty}$$

208. *Answer is E.*

$$\lim_{x\to\infty} \frac{3x^2+27}{x^3-27} =$$

$$\lim_{x\to\infty} \frac{3x^2+27}{x^3-27} = \lim_{x\to\infty} \frac{\frac{3x^2}{x^3}+\frac{27}{x^3}}{\frac{x^3}{x^3}-\frac{27}{x^3}} = \lim_{x\to\infty} \frac{\frac{3}{x}+\frac{27}{x^3}}{1-\frac{27}{x^3}} = \frac{0+0}{1-0} = \boxed{0}$$

209. *Answer is C.*

$$\lim_{x\to\infty} \frac{2^{-x}}{2^x} =$$

$$\lim_{x\to\infty} \frac{2^{-x}}{2^x} = \lim_{x\to\infty} \frac{1}{2^{x+x}} = \lim_{x\to\infty} \frac{1}{2^{2x}} = \frac{1}{2^{2(\infty)}} = \boxed{0}$$

210. *Answer is B.*

$$\lim_{x\to\infty} \frac{2x^2+1}{(2-x)(2+x)} =$$

$$\lim_{x\to\infty} \frac{2x^2+1}{(2-x)(2+x)} = \lim_{x\to\infty} \frac{2x^2+1}{4-x^2} = \lim_{x\to\infty} \frac{\frac{2x^2}{x^2}+\frac{1}{x^2}}{\frac{4}{x^2}-\frac{x^2}{x^2}} = \lim_{x\to\infty} \frac{2+\frac{1}{x^2}}{\frac{4}{x^2}-1} = \frac{2+0}{0-1} = \boxed{-2}$$

211. *Answer is C.*

$$\lim_{x\to\infty} \frac{3x^2-4}{2-7x-x^2} =$$

$$\lim_{x\to\infty} \frac{3x^2-4}{2-7x-x^2} = \lim_{x\to\infty} \frac{\frac{3x^2}{x^2}-\frac{4}{x^2}}{\frac{2}{x^2}-\frac{7x}{x^2}-\frac{x^2}{x^2}} = \lim_{x\to\infty} \frac{3-\frac{4}{x^2}}{\frac{2}{x^2}-\frac{7}{x}-1} = \frac{3-0}{0-0-1} = \boxed{-3}$$

212. *Answer is C.*

$$\lim_{x\to\infty} \frac{20x^2-13x+5}{5-4x^3} =$$

$$\lim_{x\to\infty} \frac{20x^2-13x+5}{5-4x^3} = \lim_{x\to\infty} \frac{\frac{20x^2}{x^3}-\frac{13x}{x^3}+\frac{5}{x^3}}{\frac{5}{x^3}-\frac{4x^3}{x^3}} = \lim_{x\to\infty} \frac{\frac{20}{x}-\frac{13}{x^2}+\frac{5}{x^3}}{\frac{5}{x^3}-4} = \frac{0-0+0}{0-4} = \boxed{0}$$

213. *Answer is A.*

$$\lim_{x\to\infty}\frac{\sqrt{x}-4}{4-3\sqrt{x}}=$$

$$\lim_{x\to\infty}\frac{\sqrt{x}-4}{4-3\sqrt{x}}=\lim_{x\to\infty}\frac{\frac{\sqrt{x}}{\sqrt{x}}-\frac{4}{\sqrt{x}}}{\frac{4}{\sqrt{x}}-\frac{3\sqrt{x}}{\sqrt{x}}}=\lim_{x\to\infty}\frac{1-\frac{4}{\sqrt{x}}}{\frac{4}{\sqrt{x}}-3}=\frac{1-0}{0-3}=\boxed{-\frac{1}{3}}$$

214. *Answer is B.*

$$\lim_{x\to\infty}\frac{x^2-4}{2+x-4x^2}=$$

$$\lim_{x\to\infty}\frac{x^2-4}{2+x-4x^2}=\lim_{x\to\infty}\frac{\frac{x^2}{x^2}-\frac{4}{x^2}}{\frac{2}{x^2}+\frac{x}{x^2}-\frac{4x^2}{x^2}}=\lim_{x\to\infty}\frac{1-\frac{4}{x^2}}{\frac{2}{x^2}+\frac{1}{x}-4}=\frac{1-0}{0+0-4}=\boxed{\frac{-1}{4}}$$

215. *Answer is C.*

$\boxed{\text{Difficulty}=\mathbf{0.82}}$

$$\lim_{x\to\infty}\frac{x^3-2x^2+3x-4}{4x^3-3x^2+2x-1}=$$

$$\lim_{x\to\infty}\frac{x^3-2x^2+3x-4}{4x^3-3x^2+2x-1}=\lim_{x\to\infty}\frac{\frac{x^3}{x^3}-\frac{2x^2}{x^3}+\frac{3x}{x^3}-\frac{4}{x^3}}{\frac{4x^3}{x^3}-\frac{3x^2}{x^3}+\frac{2x}{x^3}-\frac{1}{x^3}}=\lim_{x\to\infty}\frac{1-\frac{2}{x}+\frac{3}{x^2}-\frac{4}{x^3}}{4-\frac{3}{x}+\frac{2}{x^2}-\frac{1}{x^3}}=\frac{1-0+0-0}{4-0+0-0}=\boxed{\frac{1}{4}}$$

216. *Answer is D.*

$$\lim_{n\to\infty}\frac{4n^2}{n^2+10000n}=$$

$$\lim_{n\to\infty}\frac{\frac{4n^2}{n^2}}{\frac{n^2}{n^2}+\frac{10000n}{n^2}}=\lim_{n\to\infty}\frac{4}{1+\frac{10000}{n}}=\frac{4}{1+0}=\boxed{4}$$

217. *Answer is D.*

$\boxed{\text{Difficulty}=\mathbf{0.85}}$

$$\lim_{n\to\infty}\frac{3n^3-5n}{n^3-2n^2+1}=$$

$$\lim_{n\to\infty}\frac{3n^3-5n}{n^3-2n^2+1}=\lim_{n\to\infty}\frac{\frac{3n^3}{n^3}-\frac{5n}{n^3}}{\frac{n^3}{n^3}-\frac{2n^2}{n^3}+\frac{1}{n^3}}=\lim_{n\to\infty}\frac{3-\frac{5}{n^2}}{1-\frac{2}{n}+\frac{1}{n^3}}=\frac{3-0}{1-0+0}=\boxed{3}$$

218. *Answer is B.*

$$\lim_{x\to\infty}\frac{x^2-4}{2+x-4x^2}=$$

$$\lim_{x\to\infty}\frac{x^2-4}{2+x-4x^2}=\lim_{x\to\infty}\frac{\frac{x^2}{x^2}-\frac{4}{x^2}}{\frac{2}{x^2}+\frac{x}{x^2}-\frac{4x^2}{x^2}}=\lim_{x\to\infty}\frac{1-\frac{4}{x^2}}{\frac{2}{x^2}+\frac{x}{x^2}-4}=\lim_{x\to\infty}\frac{1-0}{0+0-4}=\boxed{-\frac{1}{4}}$$

219. *Answer is B.*

$$\boxed{\lim_{x\to-\infty}\frac{10-2^{x}}{10+2^{-x}}=}$$

$$\lim_{x\to-\infty}\frac{10-2^{x}}{10+2^{-x}}=\lim_{x\to-\infty}\frac{10-2^{-\infty}}{10+2^{-(-\infty)}}=\lim_{x\to-\infty}\frac{10-\frac{1}{2^{\infty}}}{10+2^{\infty}}=\frac{10-0}{10+\infty}=\frac{10}{\infty}=\boxed{0}$$

220. *Answer is A.*

$$\boxed{\lim_{x\to\infty}\frac{3x^{5}-4}{x-2x^{5}}=}$$

$$\lim_{x\to\infty}\frac{3x^{5}-4}{x-2x^{5}}=\lim_{x\to\infty}\frac{\frac{3x^{5}}{x^{5}}-\frac{4}{x^{5}}}{\frac{x}{x^{5}}-\frac{2x^{5}}{x^{5}}}=\lim_{x\to\infty}\frac{3-\frac{4}{x^{5}}}{\frac{1}{x^{4}}-2}=\frac{3-0}{0-2}=\boxed{-\frac{3}{2}}$$

221. *Answer is B.*

$$\boxed{\lim_{x\to\infty}\frac{x^{2}-1}{1-2x^{2}}=}$$

$$\lim_{x\to\infty}\frac{x^{2}-1}{1-2x^{2}}=\lim_{x\to\infty}\frac{\frac{x^{2}}{x^{2}}-\frac{1}{x^{2}}}{\frac{1}{x^{2}}-\frac{2x^{2}}{x^{2}}}=\lim_{x\to\infty}\frac{1-\frac{1}{x^{2}}}{\frac{1}{x^{2}}-2}=\frac{1-0}{0-2}=\boxed{-\frac{1}{2}}$$

222. *Answer is B.*

$$\boxed{\lim_{x\to\infty}\frac{6\sqrt[3]{x}+3\sqrt[6]{x}+4}{\sqrt[3]{8x}-\sqrt[6]{x}-10}=}$$

$$\lim_{x\to\infty}\frac{6\sqrt[3]{x}+3\sqrt[6]{x}+4}{\sqrt[3]{8x}-\sqrt[6]{x}-10}=\lim_{x\to\infty}\frac{\frac{6x^{\frac{1}{3}}}{x^{\frac{1}{3}}}+\frac{3x^{\frac{1}{6}}}{x^{\frac{1}{3}}}+\frac{4}{x^{\frac{1}{3}}}}{\frac{2x^{\frac{1}{3}}}{x^{\frac{1}{3}}}-\frac{x^{\frac{1}{6}}}{x^{\frac{1}{3}}}-\frac{10}{x^{\frac{1}{3}}}}=\lim_{x\to\infty}\frac{6+\frac{3}{x^{\frac{1}{6}}}+\frac{4}{x^{\frac{1}{3}}}}{2-\frac{1}{x^{\frac{1}{6}}}-\frac{10}{x^{\frac{1}{3}}}}=\frac{6+0+0}{2-0-0}=\boxed{3}$$

223. *Answer is A.*

$$\boxed{\lim_{x\to\infty}\frac{10^{8}x^{5}+10^{6}x^{4}+10^{4}x^{2}}{10^{9}x^{6}+10^{7}x^{5}+10^{5}x^{3}}=}$$

$$\lim_{x\to\infty}\frac{10^{8}x^{5}+10^{6}x^{4}+10^{4}x^{2}}{10^{9}x^{6}+10^{7}x^{5}+10^{5}x^{3}}=\lim_{x\to\infty}\frac{\frac{10^{8}x^{5}}{x^{6}}+\frac{10^{6}x^{4}}{x^{6}}+\frac{10^{4}x^{2}}{x^{6}}}{\frac{10^{9}x^{6}}{x^{6}}+\frac{10^{7}x^{5}}{x^{6}}+\frac{10^{5}x^{3}}{x^{6}}}=\lim_{x\to\infty}\frac{\frac{10^{8}}{x}+\frac{10^{6}}{x^{2}}+\frac{10^{4}}{x^{4}}}{10^{9}+\frac{10^{7}}{x}+\frac{10^{5}}{x^{3}}}=\lim_{x\to\infty}\frac{0+0+0}{10^{9}+0+0}=\boxed{0}$$

224. *Answer is D.*

$$\boxed{\lim_{x\to+\infty}\frac{x-\frac{1}{2x}}{2x+\frac{1}{6x}}=}$$

$$\lim_{x\to+\infty}\frac{x-\frac{1}{2x}}{2x+\frac{1}{6x}}=\lim_{x\to+\infty}\frac{\frac{2x^{2}-1}{2x}}{\frac{12x^{2}+1}{6x}}=\lim_{x\to+\infty}\frac{2x^{2}-1}{\cancel{2x}}\times\frac{\overset{3}{\cancel{6x}}}{12x^{2}+1}=\lim_{x\to+\infty}\frac{6x^{2}-3}{12x^{2}+1}=\lim_{x\to+\infty}\frac{\frac{6x^{2}}{x^{2}}-\frac{3}{x^{2}}}{\frac{12x^{2}}{x^{2}}+\frac{1}{x^{2}}}$$

$$=\lim_{x\to+\infty}\frac{6-\frac{3}{x^{2}}}{12+\frac{1}{x^{2}}}=\frac{6-0}{12+0}=\boxed{\frac{1}{2}}$$

225. *Answer is B.*

$$\lim_{x\to\infty}\frac{x^2-6}{2+x-3x^2}=$$

$$\lim_{x\to\infty}\frac{x^2-6}{2+x-3x^2}=\lim_{x\to\infty}\frac{\frac{x^2}{x^2}-\frac{6}{x^2}}{\frac{2}{x^2}+\frac{x}{x^2}-\frac{3x^2}{x^2}}=\lim_{x\to\infty}\frac{1-\frac{6}{x^2}}{\frac{2}{x^2}+\frac{x}{x^2}-3}=\frac{1-0}{0+0-3}=\boxed{-\frac{1}{3}}$$

226. *Answer is B.*

$$\lim_{x\to\infty}\frac{3x^2+1}{(3-x)(3+x)}=$$

$$\lim_{x\to\infty}\frac{3x^2+1}{9-x^2}=\lim_{x\to\infty}\frac{\frac{3x^2}{x^2}+\frac{1}{x^2}}{\frac{9}{x^2}-\frac{x^2}{x^2}}=\lim_{x\to\infty}\frac{3+\frac{1}{x^2}}{\frac{9}{x^2}-1}=\frac{3+0}{0-1}=\boxed{-3}$$

227. *Answer is D.*

$$\lim_{x\to+\infty}\frac{x-\frac{1}{2x}}{2x-\frac{1}{6x}}=$$

$$\lim_{x\to+\infty}\frac{x-\frac{1}{2x}}{2x-\frac{1}{6x}}=\lim_{x\to+\infty}\frac{\frac{x}{x}-\frac{1}{2x(x)}}{\frac{2x}{x}-\frac{1}{6x(x)}}=\lim_{x\to+\infty}\frac{1-\frac{1}{2x(x)}}{2-\frac{1}{6x(x)}}=\frac{1-0}{2-0}=\boxed{\frac{1}{2}}$$

228. *Answer is C.*

$$\lim_{x\to\infty}\left[x^2\left(\frac{1}{x-2}-\frac{1}{x-3}\right)\right]=$$

$$\lim_{x\to\infty}\left[x^2\left(\frac{1}{x-2}-\frac{1}{x-3}\right)\right]=\lim_{x\to\infty}\left[x^2\left(\frac{(x-3)-(x-2)}{(x-2)(x-3)}\right)\right]=\lim_{x\to\infty}\left[x^2\left(\frac{-1}{x^2-5x+6}\right)\right]$$

$$=\lim_{x\to\infty}\frac{\frac{-x^2}{x^2}}{\frac{x^2}{x^2}-\frac{5x}{x^2}+\frac{6}{x^2}}=\lim_{x\to\infty}\frac{-1}{1-\frac{5}{x}+\frac{6}{x^2}}=\frac{-1}{1-0+0}=\boxed{-1}$$

229. *Answer is B.*

$$\lim_{x\to\infty}\frac{x^3-4x+1}{2x^3-5}=$$

$$\lim_{x\to\infty}\frac{x^3-4x+1}{2x^3-5}=\lim_{x\to\infty}\frac{\frac{x^3}{x^3}-\frac{4x}{x^3}+\frac{1}{x^3}}{\frac{2x^3}{x^3}-\frac{5}{x^3}}=\lim_{x\to\infty}\frac{1-\frac{4}{x^2}+\frac{1}{x^3}}{2-\frac{5}{x^3}}=\frac{1-0+0}{2-0}=\boxed{\frac{1}{2}}$$

230. *Answer is D.*

$$\lim_{x\to-\infty}\frac{3x}{\sqrt{3x^2-4}}=$$

For $x<0 \;\rightarrow\; x=-\sqrt{x^2}$

$$\lim_{x\to-\infty}\frac{3x}{\sqrt{3x^2-4}}=\lim_{x\to-\infty}\frac{\frac{3x}{x}}{-\sqrt{\frac{3x^2}{x^2}-\frac{4}{x^2}}}=\lim_{x\to-\infty}\frac{3}{-\sqrt{3-\frac{4}{x^2}}}=\frac{3}{-\sqrt{3-0}}=\frac{3}{-\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)=\boxed{-\sqrt{3}}$$

231. *Answer is C.*

$$\lim_{x\to\infty}\left[x^2\left(\frac{1}{x-2}-\frac{1}{x-3}\right)\right]=$$

$$\lim_{x\to\infty}\left[x^2\left(\frac{1}{x-2}-\frac{1}{x-3}\right)\right]=\lim_{x\to\infty}\left[x^2\left(\frac{(x-3)-(x-2)}{(x-2)(x-3)}\right)\right]=\lim_{x\to\infty}\left[x^2\left(\frac{-1}{x^2-5x+6}\right)\right]$$

$$=\lim_{x\to\infty}\frac{\frac{-x^2}{x^2}}{\frac{x^2}{x^2}-\frac{5x}{x^2}+\frac{6}{x^2}}=\lim_{x\to\infty}\frac{-1}{1-\frac{5}{x}+\frac{6}{x^2}}=\frac{-1}{1-0+0}=\boxed{-1}$$

232. *Answer is A.*

$$\lim_{x\to-\infty}\frac{2x+3}{\sqrt{x^2+x+1}}=$$

$$\lim_{x\to-\infty}\frac{2x+3}{\sqrt{x^2+x+1}}=\lim_{x\to-\infty}\frac{\frac{2x}{x}+\frac{3}{x}}{-\sqrt{\frac{x^2}{x^2}+\frac{x}{x^2}+\frac{1}{x^2}}}=\lim_{x\to-\infty}\frac{2+\frac{3}{x}}{-\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}}=\frac{2+0}{-\sqrt{1+0+0}}=\frac{2}{-1}=\boxed{-2}$$

233. *Answer is C.*

$$\lim_{x\to-\infty}\frac{2x-1}{1+2x}=$$

$$\lim_{x\to-\infty}\frac{2x-1}{1+2x}=\lim_{x\to-\infty}\frac{\frac{2x}{x}-\frac{1}{x}}{\frac{1}{x}+\frac{2x}{x}}=\lim_{x\to-\infty}\frac{2-\frac{1}{x}}{\frac{1}{x}+2}=\frac{2-0}{0+2}=\boxed{1}$$

234. *Answer is A.*

$$\lim_{x\to-\infty}\frac{x^2+4x-5}{x^3-1}=$$

$$\lim_{x\to-\infty}\frac{x^2+4x-5}{x^3-1}=\lim_{x\to-\infty}\frac{\frac{x^2}{x^3}+\frac{4x}{x^3}-\frac{5}{x^3}}{\frac{x^3}{x^3}-\frac{1}{x^3}}=\lim_{x\to-\infty}\frac{-\frac{1}{x}+\frac{4}{x^2}+\frac{5}{x^3}}{1+\frac{1}{x^3}}=\frac{0+0+0}{1+0}=\boxed{0}$$

235. *Answer is C.*

$$\lim_{x\to-\infty}\frac{4x^2+x-7}{x^2-5x-3}=$$

$$\lim_{x\to-\infty}\frac{4x^2+x-7}{x^2-5x-3}=\lim_{x\to-\infty}\frac{\frac{4x^2}{x^2}+\frac{x}{x^2}-\frac{7}{x^2}}{\frac{x^2}{x^2}-\frac{5x}{x^2}-\frac{3}{x^2}}=\lim_{x\to-\infty}\frac{4-\frac{1}{x}-\frac{7}{x^2}}{1+\frac{5}{x}-\frac{3}{x^2}}=\frac{4-0-0}{1+0-0}=\boxed{4}$$

236. *Answer is C.*

$$\lim_{x\to\infty}\frac{3x^2+2x+2}{4x^2+x+5}=$$

$$\lim_{x\to\infty}\frac{3x^2+2x+2}{4x^2+x+5}=\lim_{x\to\infty}\frac{\frac{3x^2}{x^2}+\frac{2x}{x^2}+\frac{2}{x^2}}{\frac{4x^2}{x^2}+\frac{x}{x^2}+\frac{5}{x^2}}=\lim_{x\to\infty}\frac{3+\frac{2}{x}+\frac{2}{x^2}}{4+\frac{1}{x}+\frac{5}{x^2}}=\frac{3+0+0}{4+0+0}=\boxed{\frac{3}{4}}$$

237. *Answer is E.*

$$\lim_{x\to\infty}\frac{3x^2-5x+4}{6x^2+7x-1}=$$

$$\lim_{x\to\infty}\frac{3x^2-5x+4}{6x^2+7x-1}=\lim_{x\to\infty}\frac{\frac{3x^2}{x^2}-\frac{5x}{x^2}+\frac{4}{x^2}}{\frac{6x^2}{x^2}+\frac{7x}{x^2}-\frac{1}{x^2}}=\lim_{x\to\infty}\frac{3-\frac{5}{x}+\frac{4}{x^2}}{6+\frac{7}{x}-\frac{1}{x^2}}=\frac{3-0+0}{6+0+0}=\boxed{\frac{1}{2}}$$

238. *Answer is C.*

$$\lim_{x\to\infty}\frac{\sqrt{3x^2+2x+1}}{x+1}=$$

$$\lim_{x\to\infty}\frac{\sqrt{3x^2+2x+1}}{x+1}=\lim_{x\to\infty}\frac{\sqrt{\frac{3x^2}{x^2}+\frac{2x}{x^2}+\frac{1}{x^2}}}{\frac{x}{x}+\frac{1}{x}}=\lim_{x\to\infty}\frac{\sqrt{3+\frac{2}{x}+\frac{1}{x^2}}}{1+\frac{1}{x}}=\frac{\sqrt{3+0+0}}{1+0}=\boxed{\sqrt{3}}$$

239. *Answer is D.*

$$\lim_{x\to\infty}\frac{5x^2-3x+1}{4x^2+2x+5}=$$

$$\lim_{x\to\infty}\frac{5x^2-3x+1}{4x^2+2x+5}=\lim_{x\to\infty}\frac{\frac{5x^2}{x^2}-\frac{3x}{x^2}+\frac{1}{x^2}}{\frac{4x^2}{x^2}+\frac{2x}{x^2}+\frac{5}{x^2}}=\lim_{x\to\infty}\frac{5-\frac{3}{x}+\frac{1}{x^2}}{4+\frac{2}{x}+\frac{5}{x^2}}=\frac{5-0+0}{4+0+0}=\boxed{\frac{5}{4}}$$

240. *Answer is A.*

$$\lim_{x\to+\infty}\frac{2x^2+3x-5}{5x-3x^2-2}=$$

$$\lim_{x\to+\infty}\frac{2x^2+3x-5}{5x-3x^2-2}=\lim_{x\to+\infty}\frac{\frac{2x^2}{x^2}+\frac{3x}{x^2}-\frac{5}{x^2}}{\frac{5x}{x^2}-\frac{3x^2}{x^2}-\frac{2}{x^2}}=\lim_{x\to+\infty}\frac{2+\frac{3}{x}-\frac{5}{x^2}}{\frac{5}{x}-3-\frac{2}{x^2}}=\frac{2+0-0}{0-3-0}=\boxed{\frac{-2}{3}}$$

241. *Answer is D.*

$$\lim_{x\to-\infty}\frac{x^3-111x^2+3x-2}{1-2x+22x^2+3x^3}=$$

$$\lim_{x\to-\infty}\frac{x^3-111x^2+3x-2}{1-2x+22x^2+3x^3}=\lim_{x\to-\infty}\frac{\frac{x^3}{x^3}-\frac{111x^2}{x^3}+\frac{3x}{x^3}-\frac{2}{x^3}}{\frac{1}{x^3}-\frac{2x}{x^3}+\frac{22x^2}{x^3}+\frac{3x^3}{x^3}}=\lim_{x\to-\infty}\frac{1-\frac{111}{x}+\frac{3}{x^2}-\frac{2}{x^3}}{\frac{1}{x^3}-\frac{2}{x^2}+\frac{22}{x}+3}=\frac{1-0+0-0}{0-0+0+3}=\boxed{\frac{1}{3}}$$

242. *Answer is C.*

$$\lim_{x\to0}\frac{\frac{3}{x^2}}{\frac{2}{x^2}+\frac{105}{x}}=$$

$$\lim_{x\to0}\frac{\frac{3}{x^2}}{\frac{2}{x^2}+\frac{105}{x}}=\lim_{x\to0}\frac{\frac{3x^2}{x^2}}{\frac{2x^2}{x^2}+\frac{105x^2}{x}}=\lim_{x\to0}\frac{3}{2+105x}=\frac{3}{2+0}=\boxed{\frac{3}{2}}$$

243. *Answer is C.*

$$\text{If}\quad \lim_{n\to\infty}\frac{6n^2}{200-4n+kn^2}=\frac{1}{2},\quad \text{then}\quad k=$$

$$\lim_{n\to\infty}\frac{6n^2}{200-4n+kn^2}=\lim_{n\to\infty}\frac{\frac{6n^2}{n^2}}{\frac{200}{n^2}-\frac{4n}{n^2}+\frac{kn^2}{n^2}}=\lim_{n\to\infty}\frac{6}{\frac{200}{n^2}-\frac{4}{n}+k}=\frac{6}{0-0+k}=\frac{1}{2}$$

$$\frac{6}{k}=\frac{1}{2}$$

$$\boxed{k=12}$$

244. *Answer is D.*

$$\lim_{x\to\infty}\sqrt[3]{\frac{8+x^2}{x(x+1)}}=$$

$$\lim_{x\to\infty}\sqrt[3]{\frac{8+x^2}{x(x+1)}}=\lim_{x\to\infty}\sqrt[3]{\frac{\frac{8}{x^2}+\frac{x^2}{x^2}}{\frac{x^2}{x^2}+\frac{x}{x^2}}}=\lim_{x\to\infty}\sqrt[3]{\frac{\frac{8}{x^2}+1}{1+\frac{1}{x}}}=\sqrt[3]{\frac{0+1}{1+0}}=\sqrt[3]{\frac{1}{1}}=\sqrt[3]{1}=\boxed{1}$$

245. *Answer is A.*

$$\lim_{x\to+\infty}\left(\frac{1}{x}-\frac{x}{x-1}\right)=$$

$$\lim_{x\to+\infty}\left(\frac{1}{x}-\frac{x}{x-1}\right)=\lim_{x\to+\infty}\frac{1-x^2}{x(x-1)}=\lim_{x\to+\infty}\frac{\frac{1}{x^2}-\frac{x^2}{x^2}}{\frac{x^2}{x^2}-\frac{x}{x^2}}=\lim_{x\to+\infty}\frac{\frac{1}{x^2}-1}{1-\frac{1}{x}}=\frac{0-1}{1-0}=\boxed{-1}$$

246. *Answer is E.*

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x=$$

Let $\quad y=(1+\frac{1}{x})^x$

$$\lim_{x\to\infty}\ln y=\lim_{x\to\infty}x\ln(1+\tfrac{1}{x})=\lim_{x\to\infty}\frac{\ln(1+\frac{1}{x})}{\frac{1}{x}}=\frac{0}{0}\quad\leftarrow \text{indeterminant form}$$

$$\lim_{x\to\infty}\ln y=\lim_{x\to\infty}\frac{\ln(1+\frac{1}{x})}{\frac{1}{x}}=\lim_{x\to\infty}\frac{\frac{-\frac{1}{x^2}}{1+\frac{1}{x}}}{\frac{-\frac{1}{x^2}}{1}}=\lim_{x\to\infty}\frac{\cancel{-\frac{1}{x^2}}}{1+\frac{1}{x}}\cdot\frac{1}{\cancel{-\frac{1}{x^2}}}=\lim_{x\to\infty}\frac{1}{1+\frac{1}{x}}=1\quad\leftarrow \text{L'hopitals rule}$$

$$\lim_{x\to\infty}\ln y=1$$

$$\lim_{x\to\infty}y=e^1=e$$

$$\boxed{\lim_{x\to\infty}(1+\tfrac{1}{x})^x=e}$$

247. *Answer is D.*

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n+2} =$$

$$\left(1+\frac{1}{n}\right)^{n+2} = \left(1+\frac{1}{n}\right)^{n}\left(1+\frac{1}{n}\right)^{2}$$

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n+2} = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n} \times \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{2}$$

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n+2} = e\times(1+0)^2 = \boxed{e}$$

248. *Answer is B.*

What is $\lim_{x\to\infty}\frac{\sqrt{9x^2+2}}{4x+3} =$

For $\lim_{x\to\infty} \rightarrow x>0 \rightarrow x=\sqrt{x^2}$

$$\lim_{x\to\infty}\frac{\frac{\sqrt{9x^2+2}}{\sqrt{x^2}}}{\frac{4x}{x}+\frac{3}{x}} = \lim_{x\to\infty}\frac{\sqrt{\frac{9x^2}{x^2}+\frac{2}{x^2}}}{4+\frac{3}{x}} = \lim_{x\to\infty}\frac{\sqrt{9+\frac{2}{x^2}}}{4+\frac{3}{x}} = \frac{\sqrt{9+0}}{4+0} = \boxed{\frac{3}{4}}$$

249. *Answer is B.*

$$\lim_{x\to\infty}\frac{\sqrt{x-2}}{x-2} =$$

********** as $x\to\infty$ $x=\sqrt{x^2}$ **********

$$\lim_{x\to\infty}\frac{\sqrt{x-2}}{x-2} = \lim_{x\to\infty}\frac{\sqrt{\frac{x}{x^2}-\frac{2}{x^2}}}{\frac{x}{x}-\frac{2}{x}} = \lim_{x\to\infty}\frac{\sqrt{\frac{1}{x}-\frac{2}{x^2}}}{1-\frac{2}{x}} = \frac{\sqrt{0-0}}{1-0} = \boxed{0}$$

250. *Answer is D.*

Which of the following are asymptotes of

$y+xy-2x=0$

I. $x=-1$ **II.** $x=1$ **III.** $y=2$

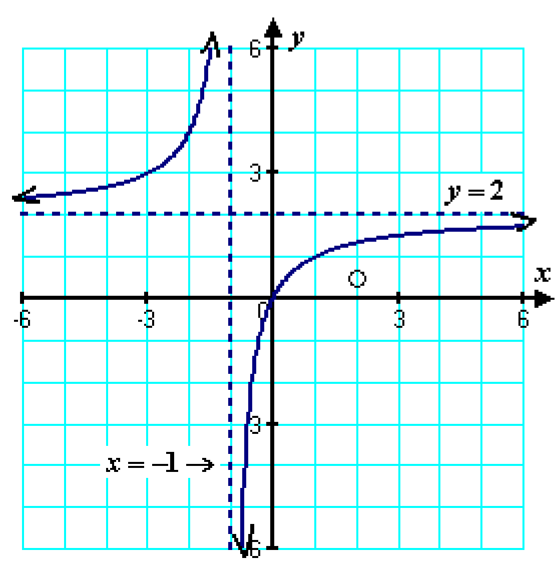
Rearrange $\rightarrow$ $y(1+x)=2x$ $\rightarrow$ $y=\frac{2x}{1+x}$

$$\lim_{x\to\infty}\frac{2x}{1+x} = \lim_{x\to\infty}\frac{\frac{2x}{x}}{\frac{1}{x}+\frac{x}{x}} = \frac{2}{0+1} = 2$$

$$\lim_{x\to-1}\frac{2x}{1+x} = \lim_{x\to-1}\frac{2(-1)}{1+(-1)} = \frac{-2}{0} = \textit{undefined}$$

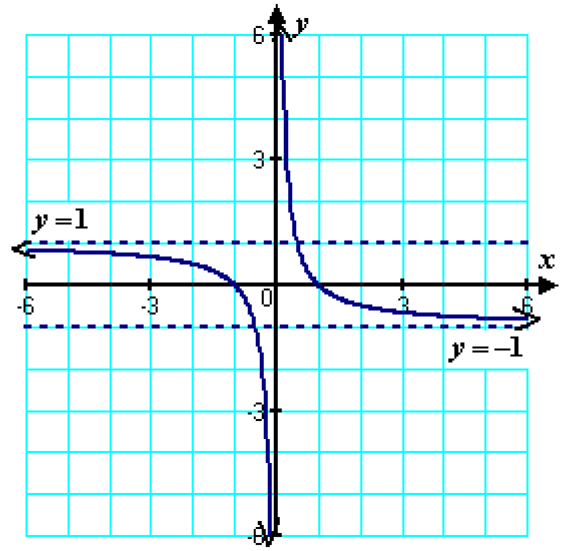
One horizontal asymptote $\boxed{y=2}$

One vertical asymptote at $\boxed{x=-1}$



251. *Answer is E.*

The horizontal asymptotes of $f(x)=\dfrac{1-|x|}{x}$ are given by



$$\lim_{x\to\infty}\frac{1-|x|}{x}=\lim_{x\to\infty}\frac{\frac{1}{x}-\frac{x}{x}}{\frac{x}{x}}=\frac{0-1}{1}=\boxed{-1}$$

as $x\to\infty$ then $x>0$ and $x=|x|$

$$\lim_{x\to-\infty}\frac{1-|x|}{x}=\lim_{x\to-\infty}\frac{-\frac{1}{x}+\frac{x}{x}}{\frac{x}{x}}=\frac{0+1}{1}=\boxed{1}$$

as $x\to-\infty$ then $x<0$ and $-x=|x|$

$\therefore$ $\boxed{y=-1,\ 1}$ are the ***horizontal*** asymptotes

$$\lim_{x\to0^+}\frac{1-|x|}{x}=\frac{1-|0|}{+0}=\infty=\boldsymbol{undefined}$$

$$\lim_{x\to0^-}\frac{1-|x|}{x}=\frac{1-|0|}{-0}=-\infty=\boldsymbol{undefined}$$

$\therefore$ $x=0$ is a ***horizontal*** asymptote

252. *Answer is B.*

The vertical asymptote and horizontal asymptote for $f(x)=\dfrac{\sqrt{x}}{x+4}$ are

Domain of function is $x\ge0$!!!!!!!!!!!!!!!!

$$\lim_{x\to\infty}\frac{\sqrt{x}}{x+4}=\lim_{x\to\infty}\frac{\sqrt{\frac{x}{x^2}}}{\frac{x}{x}+\frac{4}{x}}=\frac{\sqrt{0}}{1+0}=\boxed{0}$$

as $x\to\infty$ then $x>0$ and $x=\sqrt{x^2}$

$\therefore$ $\boxed{y=0}$ is a ***horizontal*** asymptote

$$\lim_{x\to0^+}\frac{\sqrt{x}}{x+4}=\frac{\sqrt{0}}{4}=0 \quad\leftarrow \text{endpoint } (0,\ 0)$$

and **NO** vertical asymptote

253. *Answer is E.*

Difficulty = **0.41**

For $x \geq 0$ the horizontal line $y = 2$ is an asymptote for the graph of the function f Which of the following statements ***must*** be true ?

Must know if $\lim_{x\to\infty} f(x) = L$ then L is a ***horizontal asymptote

Consider $\rightarrow \lim_{x\to\infty} \frac{\sin x}{x} + 2 = L$

$f(0) = 2$ ☒

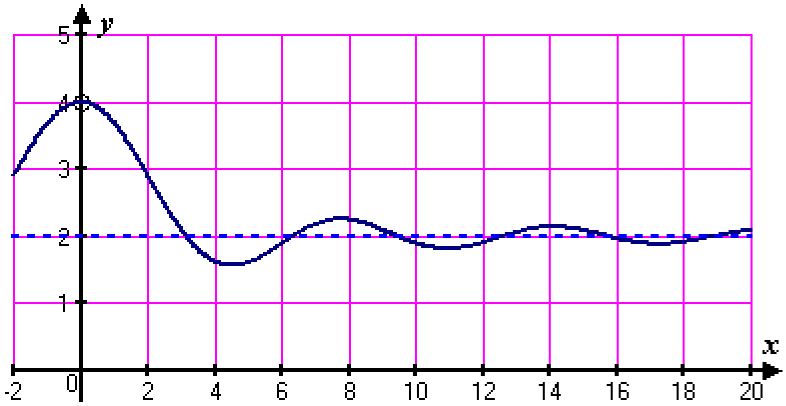
$f(x) \neq 2$ for all $x \geq 0$ ☒

$f(2)$ is undefined ☒

$\lim_{x\to 2} f(x) = \infty$ ☒

All above ***could*** be true

$\lim_{x\to\infty} f(x) = 2$ ☑ $\boxed{must}$ be true



254. *Answer is E.*

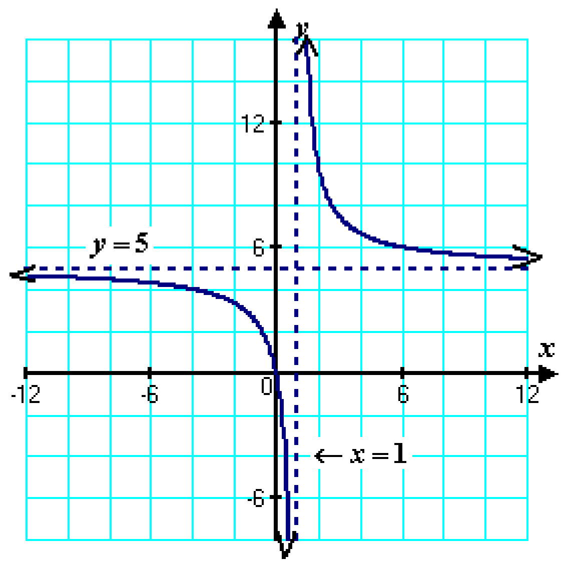
Find the equation of the horizontal asymptote of $y = \frac{5x}{x-1}$

$$\lim_{x\to\infty} \frac{5x}{x-1} = \lim_{x\to\infty} \frac{\frac{5x}{x}}{\frac{x}{x} - \frac{1}{x}} = \frac{5}{1-0} = 5$$

$$\lim_{x\to 1} \frac{5x}{x-1} = \frac{5}{0} = \textit{undefined}$$

Horizontal asymptote at $\boxed{y = 5}$

Vertical asymptote at $x = 1$



255. *Answer is C.*

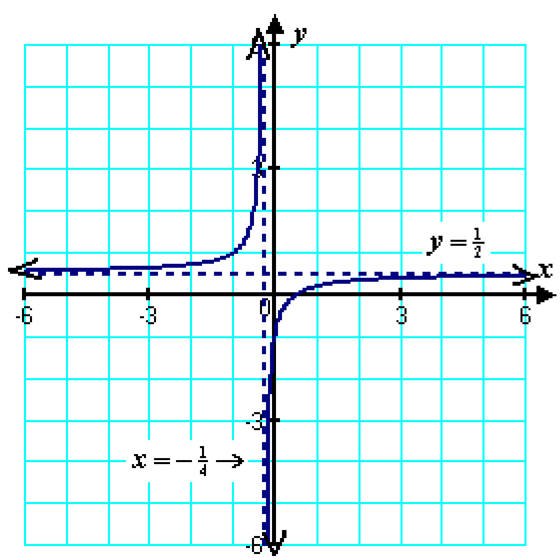
Find the equation of the horizontal asymptote of $f(x) = \frac{2x-1}{4x+1}$

$$\lim_{x\to\infty} \frac{2x-1}{4x+1} = \lim_{x\to\infty} \frac{\frac{2x}{x} - \frac{1}{x}}{\frac{4x}{x} - \frac{1}{x}} = \frac{2-0}{4-0} = \frac{1}{2}$$

$$\lim_{x\to\frac{-1}{4}} \frac{2x-1}{4x+1} = \frac{2(\frac{-1}{4})-1}{4(\frac{-1}{4})+1} = \frac{\frac{-3}{2}}{0} = \textit{undefined}$$

Horizontal asymptote at $\boxed{y = \frac{1}{2}}$

Vertical asymptotes at $\boxed{x = -\frac{1}{4}}$



256. *Answer is D.*

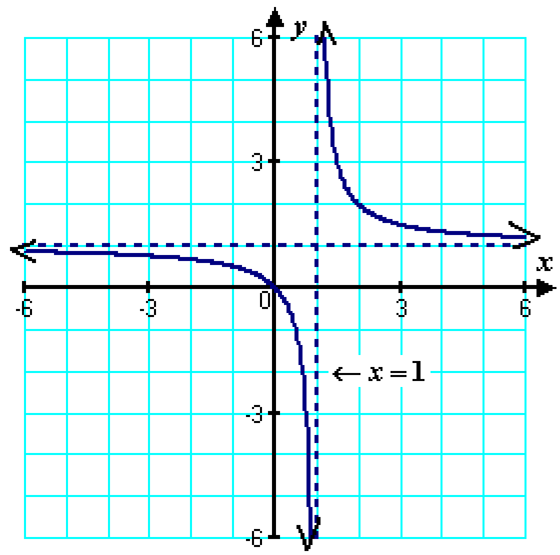
The horizontal asymptote of $f(x)=\dfrac{x}{x-1}$ is

$$\lim_{x\to\infty}\frac{x}{x-1}=\lim_{x\to\infty}\frac{\frac{x}{x}}{\frac{x}{x}-\frac{1}{x}}=\frac{1}{1-0}=1$$

$$\lim_{x\to 1}\frac{x}{x-1}=\lim_{x\to\infty}\frac{1}{1-1}=\frac{1}{0}=\textit{undefined}$$

Horizontal asymptote at $\boxed{y=1}$

Vertical asymptotes at $x=1$



257. *Answer is D.*

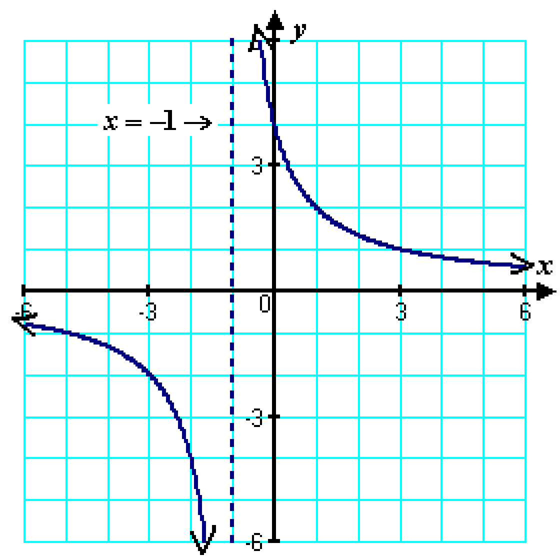
The horizontal asymptote of $f(x)=\dfrac{4}{x+1}$ is

$$\lim_{x\to\infty}\frac{4}{x+1}=\lim_{x\to\infty}\frac{\frac{4}{x}}{\frac{x}{x}+\frac{1}{x}}=\frac{0}{1+0}=0$$

$\therefore$ $\boxed{y=0}$ is a ***horizontal*** asymptote

$$\lim_{x\to -1}\frac{4}{x+1}=\frac{4}{0}=\infty=\textit{undefined}$$

$\therefore$ $x=-1$ is a ***vertical*** asymptote



258. *Answer is C.*

Find the equation of the horizontal asymptote of $y=\dfrac{x^2}{2x^2-2}$

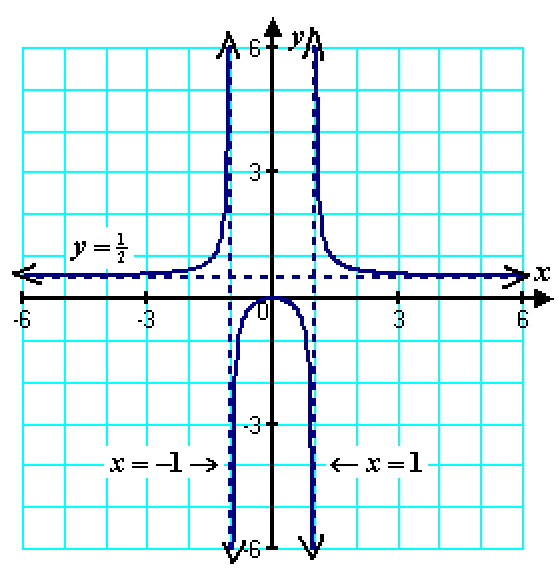
$$\lim_{x\to\infty}\frac{x^2}{2x^2-2}=\lim_{x\to\infty}\frac{\frac{x^2}{x^2}}{\frac{2x^2}{x^2}-\frac{2}{x^2}}=\frac{1}{2-0}=\frac{1}{2}$$

$$\lim_{x\to -1}\frac{x^2}{2(x-1)(x+1)}=\frac{1}{0}=\textit{undefined}$$

$$\lim_{x\to 1}\frac{x^2}{2(x-1)(x+1)}=\frac{1}{0}=\textit{undefined}$$

Horizontal asymptote at $\boxed{y=\frac{1}{2}}$

Vertical asymptotes at $x=\pm 1$



259. *Answer is E.*

$f(x)=\dfrac{(x-1)^2}{x^2-1}$ has

$$\lim_{x\to\infty}\frac{x^2-2x+1}{x^2-1}=\lim_{x\to\infty}\frac{\frac{x^2}{x^2}-\frac{2x}{x^2}+\frac{1}{x^2}}{\frac{x^2}{x^2}-\frac{1}{x^2}}=\frac{1-0+0}{1-0}=1$$

$$\lim_{x\to 1}\frac{(x-1)(x-1)}{(x-1)(x+1)}=\frac{0}{0}=\text{indefinite form}$$

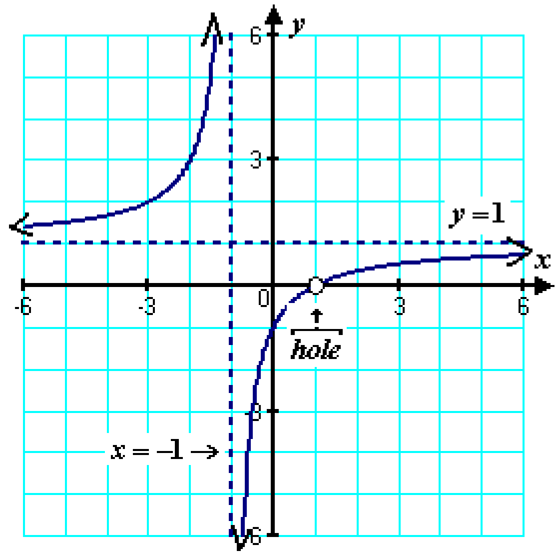
(hole when $x=1$ at point $(1,\ 0)$)

$$\lim_{x\to -1}\frac{(x-1)(x-1)}{(x-1)(x+1)}=\frac{4}{0}=\textit{undefined}$$

Horizontal asymptote at $\boxed{y=1}$

Vertical asymptotes at $x=-1$

Hole at $(1,\ 0)$



260. *Answer is A.*

How many vertical and horizontal asymptotes are there to the graph of $y=\dfrac{x^2}{x^2-1}$

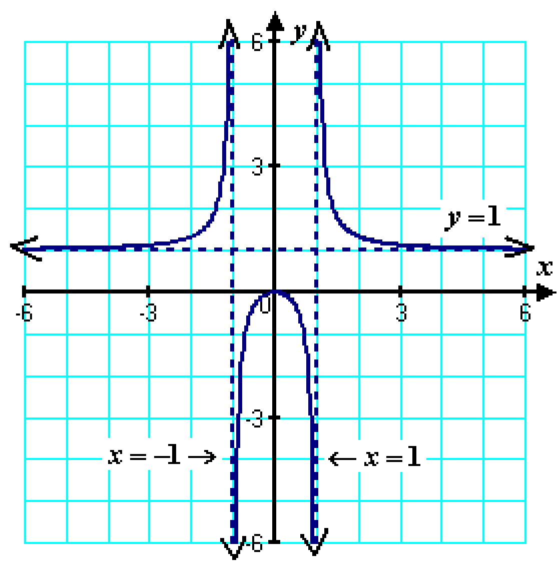
$$\lim_{x\to\infty}\frac{x^2}{x^2-1}=\lim_{x\to\infty}\frac{\frac{x^2}{x^2}}{\frac{x^2}{x^2}-\frac{1}{x^2}}=\frac{1}{1-0}=1$$

$$\lim_{x\to 1}\frac{x^2}{(x-1)(x+1)}=\frac{1}{0}=\textit{undefined}$$

$$\lim_{x\to -1}\frac{x^2}{(x-1)(x+1)}=\frac{1}{0}=\textit{undefined}$$

Horizontal asymptote at $\boxed{y=1}$

Vertical asymptotes at $\boxed{x=-1,\ 1}$



261. *Answer is D.*

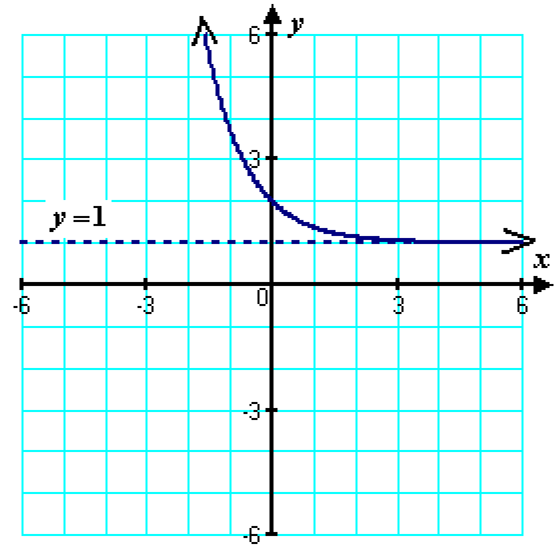
The graph of $y = e^{-x} + 1$ has a horizontal asymptote with equation

$$\lim_{x\to\infty} e^{-x} + 1 = \lim_{x\to\infty} \frac{1}{e^x} + 1 = 0 + 1 = \boxed{1}$$

Horizontal asymptote at $\boxed{y = 1}$

Should know this graph from memory !!!

(exponential decay)



262. *Answer is C.*

An asymptote for $y = \dfrac{(x+2)(x-7)}{x-5}$ is

$$\lim_{x\to\infty} \frac{x^2 - 5x - 14}{x-5} = \lim_{x\to\infty} \frac{\frac{x^2}{x} - \frac{5x}{x} - \frac{14}{x}}{\frac{x}{x} - \frac{5}{x}}$$

$$= \frac{x - 5 - 0}{1 - 0} = \infty$$

$$\lim_{x\to 5} \frac{(x+2)(x-7)}{x-5} = \frac{-14}{0} = \textit{undefined}$$

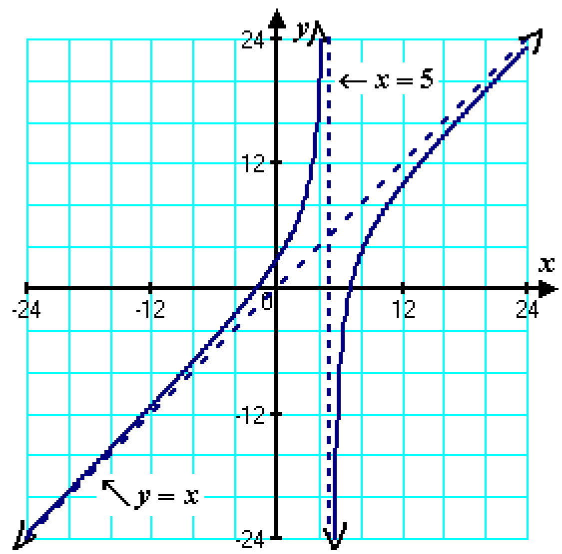
Vertical asymptotes at $\boxed{x = 5}$

No horizontal asymptote

Do the long division and get

$$y = \frac{(x+2)(x-7)}{x-5} = x - \frac{14}{x-5}$$

Slant asymptote $\boxed{y = x}$



263. *Answer is C.*

The graph of $f(x)=\dfrac{4}{x^2-1}$ has

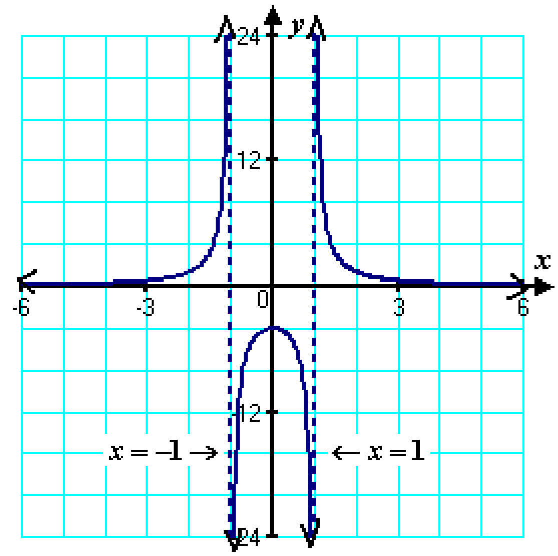
$$\lim_{x\to\infty}\frac{4}{x^2-1}=\lim_{x\to\infty}\frac{\frac{4}{x^2}}{\frac{x^2}{x^2}-\frac{1}{x^2}}=\frac{0}{1-0}=0$$

$$\lim_{x\to1}\frac{4}{(x-1)(x+1)}=\frac{4}{0}=\textit{undefined}$$

$$\lim_{x\to-1}\frac{4}{(x-1)(x+1)}=\frac{4}{0}=\textit{undefined}$$

One horizontal asymptote at $\boxed{y=0}$

Two vertical asymptotes at $\boxed{x=-1,\ 1}$



264. *Answer is A.*

Which statement below is true about the curve

$y=\dfrac{2x^2+4}{2+7x-4x^2}$

$$\lim_{x\to\infty}\frac{2x^2+4}{2+7x-4x^2}=\lim_{x\to\infty}\frac{\frac{2x^2}{x^2}+\frac{4}{x^2}}{\frac{2}{x^2}+\frac{7x}{x^2}-\frac{4x^2}{x^2}}$$

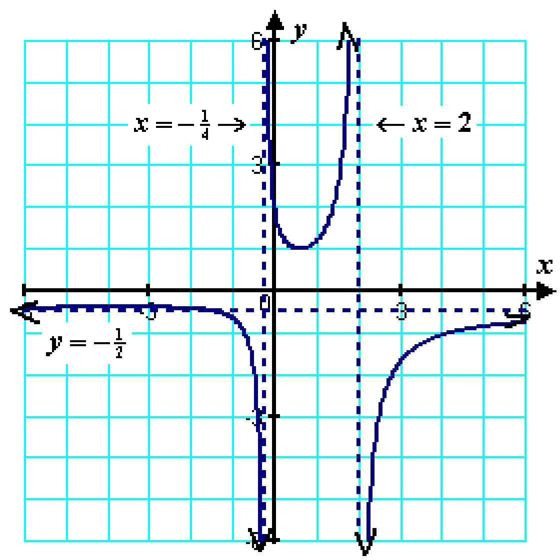
$$=\frac{2+0}{0+0-4}=-\frac{1}{2}$$

$$\lim_{x\to2}\frac{2(x^2+2)}{(2-x)(1+4x)}=\frac{12}{0}=\textit{undefined}$$

$$\lim_{x\to-\frac{1}{4}}\frac{2(x^2+2)}{(2-x)(1+4x)}=\frac{\frac{33}{8}}{0}=\textit{undefined}$$

One horizontal asymptote at $\boxed{y=-\frac{1}{2}}$

Two vertical asymptotes at $\boxed{x=2,-\frac{1}{4}}$



265. *Answer is C.*

$\boxed{\text{Difficulty} = \mathbf{0.56}}$

The graph of which of the following equations has $y=1$ as an asymptote ?

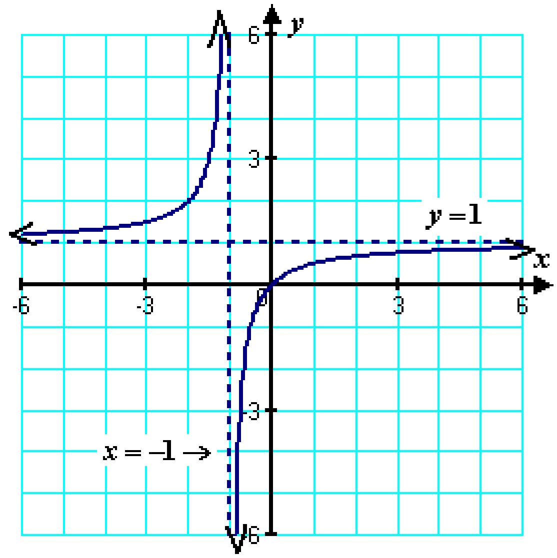
$\lim\limits_{x\to\infty} \ln x = \text{no limit}$ ☒

$\lim\limits_{x\to\infty} \sin x = \text{no limit (oscilates between } 1 \text{ and } -1)$

$\lim\limits_{x\to\infty} \dfrac{x}{x+1} = \dfrac{\frac{x}{x}}{\frac{x}{x}+\frac{1}{x}} = \dfrac{1}{1+0} = \boxed{1}$ ☑

$\lim\limits_{x\to\infty} \dfrac{x^2}{x-1} = \dfrac{\frac{x^2}{x}}{\frac{x}{x}-\frac{1}{x}} = \dfrac{x}{1-0} = \text{no limit}$ ☒

$\lim\limits_{x\to\infty} e^{-x} = \lim\limits_{x\to\infty} \dfrac{1}{e^x} = 0$ ☒



266. *Answer is C.*

The graph of $y = \dfrac{2x^2+2x+3}{4x^2-4x}$ has

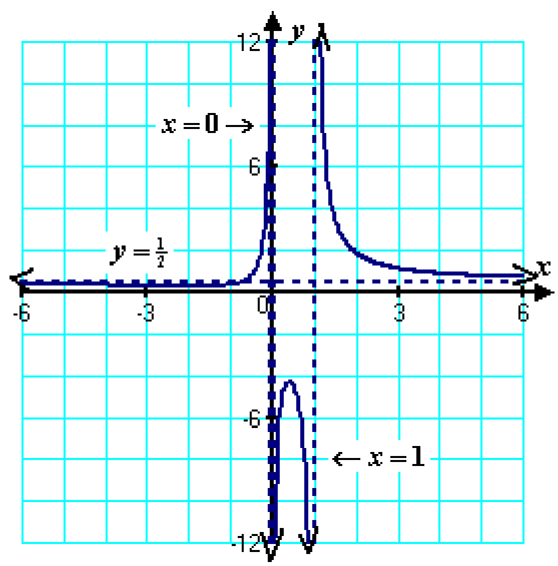
$\lim\limits_{x\to\infty} \dfrac{\frac{2x^2}{x^2}+\frac{2x}{x^2}+\frac{3}{x^2}}{\frac{4x^2}{x^2}-\frac{4x}{x^2}} = \dfrac{2+0+0}{4-0} = \boxed{\dfrac{1}{2}}$

$\lim\limits_{x\to 1} \dfrac{2x^2+2x+3}{4x(x-1)} = \dfrac{7}{0} = \textit{undefined}$

$\lim\limits_{x\to 0} \dfrac{2x^2+2x+3}{4x(x-1)} = \dfrac{3}{0} = \textit{undefined}$

Two vertical asymptotes at $\boxed{x = 0,\ 1}$

One horizontal asymptote at $\boxed{y = \frac{1}{2}}$



267. *Answer is A.*

268. *Answer is C.*

The graph of the function $f(x)=\dfrac{|x^3|}{x^3-8}$ has the following asymptotes:

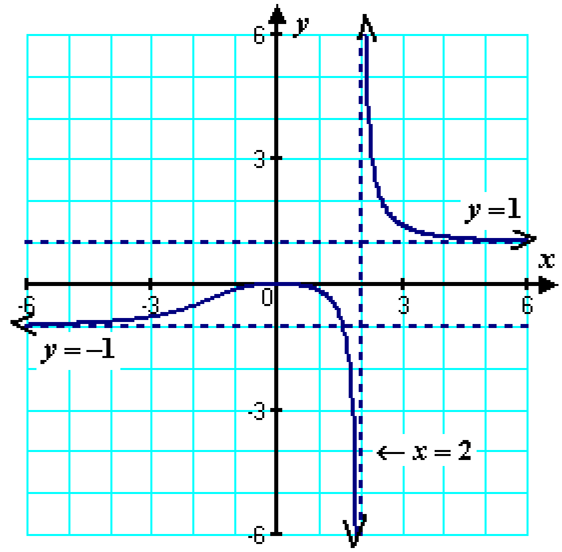
$$\lim_{x\to\infty}\frac{x^3}{x^3-8}=\lim_{x\to\infty}\frac{\frac{x^3}{x^3}}{\frac{x^3}{x^3}-\frac{8}{x^3}}=\frac{1}{1-0}=\boxed{1}$$

$$\lim_{x\to-\infty}\frac{x^3}{x^3-8}=\lim_{x\to\infty}\frac{-\frac{x^3}{x^3}}{\frac{x^3}{x^3}+\frac{8}{x^3}}=\frac{-1}{1+0}=\boxed{-1}$$

$$\lim_{x\to 2}\frac{x^3}{(x-2)(x^2+2x+4)}=\frac{8}{0}=\textit{undefined}$$

Two horizontal asymptotes at $\boxed{y=1,-1}$

One vertical asymptote at $\boxed{x=2}$



269. *Answer is E.*

Difficulty = **0.48**

If the graph of $y=\dfrac{ax+b}{x+c}$ has a horizontal asymptote $y=2$ and a vertical asymptote $x=-3$ then $a+c=$

Vertical asymptote $x=-3$

$$\lim_{x\to-3}\frac{ax+b}{x+c}=\textit{undefined}$$

$$x+c=0$$
$$-3+c=0$$
$$c=3$$

Horizontal asymptote $y=2$

$$\lim_{x\to\infty}\frac{ax+b}{x+c}=\lim_{x\to\infty}\frac{\frac{ax}{x}+\frac{b}{x}}{\frac{x}{x}+\frac{c}{x}}=\lim_{x\to\infty}\frac{a+\frac{b}{x}}{1+\frac{c}{x}}=\frac{a+0}{1+0}=a=2$$

$$\therefore\ a+c=2+3=\boxed{5}$$

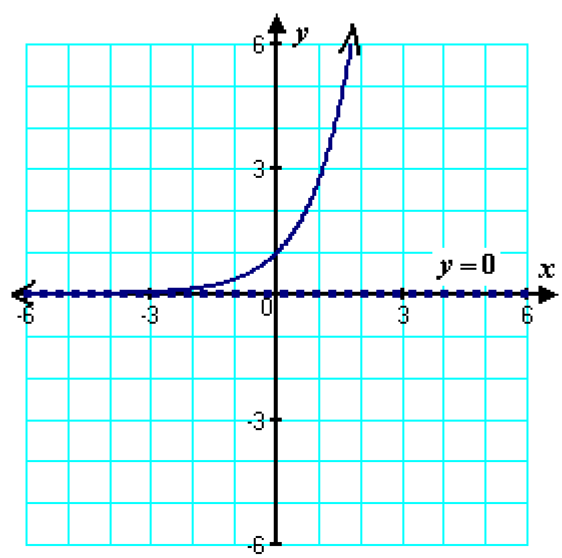
270. *Answer is A.*

If $f(x)=e^x$ which of the following lines is an asymptote to the graph of f

$$\lim_{x\to-\infty}e^x=\lim_{x\to-\infty}\frac{1}{e^{-x}}=\lim_{x\to\infty}\frac{1}{e^x}=\frac{1}{\infty}=\boxed{0}$$

One horizontal asymptotes at $\boxed{y=0}$

Basic exponential growth curve and must know from memory.



271. *Answer is D.*

The equation for the horizontal asymptote for the function $f(x)=\dfrac{(2x-5)(3x+6)(x+1)^4}{(x-9)^6}$ is

$$\lim_{x\to\infty}\frac{(2x-5)(3x+6)(x+1)^4}{(x-9)^6}=\lim_{x\to\infty}\frac{6x^6+\dots}{1x^6+\dots}=\lim_{x\to\infty}\frac{\frac{6x^6}{x^6}+\frac{\dots}{x^6}}{\frac{1x^6}{x^6}+\frac{\dots}{x^6}}=\boxed{6}$$

No need to fully expand the numerator as it is the same degree as the denominator and the limit is determined only by the coefficient of the x^6 term

272. *Answer is D.*

The graph of $f(x)=\dfrac{x^2-x-2}{2x^2-x-1}$ has a horizontal asymptote which it

$$\lim_{x\to\infty}\frac{x^2-x-2}{2x^2-x-1}=\lim_{x\to\infty}\frac{\frac{x^2}{x^2}-\frac{x}{x^2}-\frac{2}{x^2}}{\frac{2x^2}{x^2}-\frac{x}{x^2}-\frac{1}{x^2}}$$

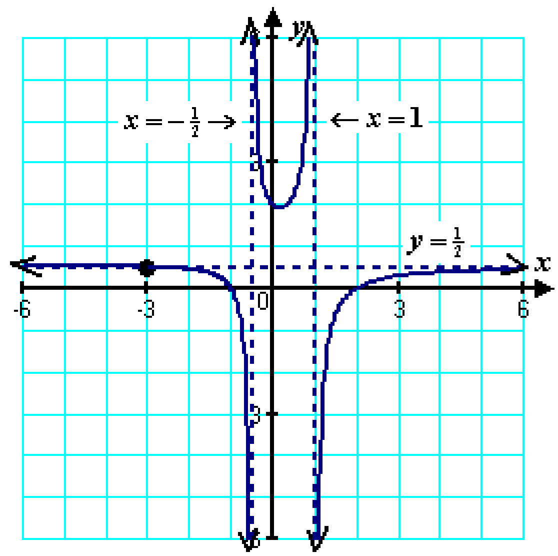
$$=\frac{1-0-0}{2-0-0}=\boxed{\frac{1}{2}}$$

$$f(x)=\frac{x^2-x-2}{2x^2-x-1}=\frac{1}{2}$$

$$\cancel{2x^2}-x-1=\cancel{2x^2}-2x-4$$

$$\boxed{x=-3}$$

$y=f(x)$ and its horizontal asymptote cross when $x=-3$



273. *Answer is C.*

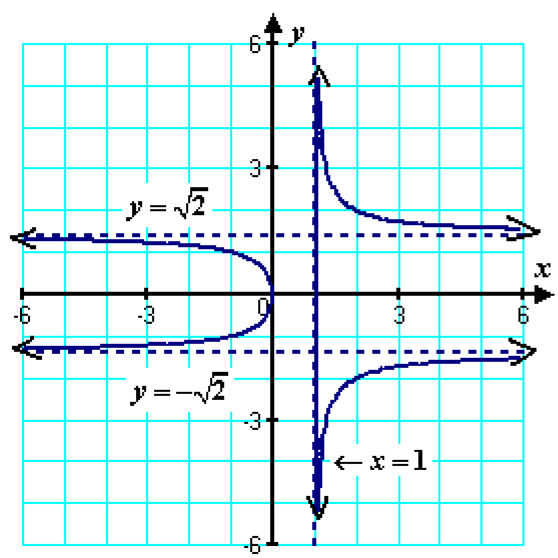
The number of horizontal asymptotes for the graph of the curve $y^2=\dfrac{2x}{x-1}$ is

$$y=\pm\sqrt{\frac{2x}{x-1}}=\frac{\pm\sqrt{2x}}{\sqrt{x-1}}$$

$$\lim_{x\to\infty}\frac{\sqrt{2x}}{\sqrt{x-1}}=\lim_{x\to\infty}\frac{\sqrt{\frac{2x}{x}}}{\sqrt{\frac{x}{x}-\frac{1}{x}}}=\frac{\sqrt{2}}{\sqrt{1-0}}=\boxed{\sqrt{2}}$$

$$\lim_{x\to-\infty}\frac{-\sqrt{2x}}{\sqrt{x-1}}=\lim_{x\to-\infty}\frac{-\sqrt{\frac{2x}{x}}}{\sqrt{\frac{x}{x}-\frac{1}{x}}}=\frac{-\sqrt{2}}{\sqrt{1-0}}=\boxed{-\sqrt{2}}$$

The graph has $\boxed{2}$ horizontal asymptotes



274. *Answer is B.*

The graph of $y=\dfrac{3x}{1+|x|}$ has the following number of vertical and horizontal asymptotes:

as $x\to\infty$ $\quad x>0$ and $x=|x|$

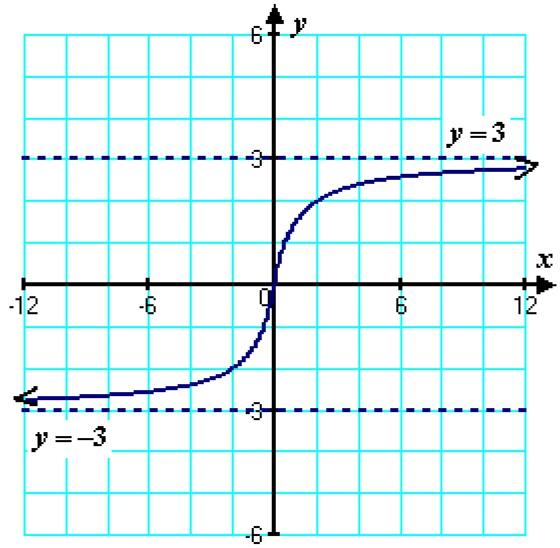
$$\lim_{x\to\infty}\frac{3x}{1+|x|}=\lim_{x\to\infty}\frac{3x}{1+x}=\lim_{x\to\infty}\frac{\frac{3x}{x}}{\frac{1}{x}+\frac{x}{x}}=\frac{3}{0+1}=\boxed{3}$$

as $x\to-\infty$ $\quad x<0$ and $-x=|x|$

$$\lim_{x\to-\infty}\frac{3x}{1+|x|}=\lim_{x\to-\infty}\frac{3x}{1-x}\lim_{x\to-\infty}\frac{\frac{3x}{x}}{\frac{1}{x}-\frac{x}{x}}=\frac{3}{0-1}=\boxed{-3}$$

The graph has $\boxed{2}$ horizontal asymptotes

NO vertical asymptotes (denominator $\neq 0$)



275. *Answer is D.*

The graph of $y=\dfrac{2x^2}{4-x^2}$ has

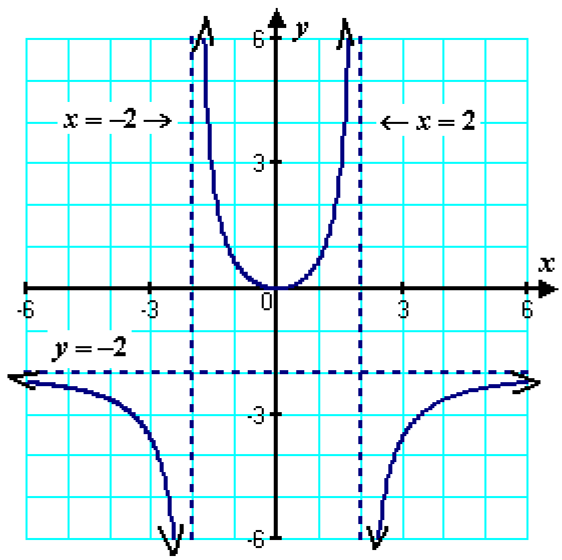
$$\lim_{x\to\infty}\frac{2x^2}{4-x^2}=\lim_{x\to\infty}\frac{\frac{2x^2}{x^2}}{\frac{4}{x^2}-\frac{x^2}{x^2}}=\frac{2}{0-1}=\boxed{-2}$$

$$\lim_{x\to2}\frac{2x^2}{(2-x)(2+x)}=\frac{8}{0}=\textit{undefined}$$

$$\lim_{x\to-2}\frac{2x^2}{(2-x)(2+x)}=\frac{8}{0}=\textit{undefined}$$

One horizontal asymptote at $\boxed{y=-2}$

Two vertical asymptotes at $\boxed{x=-2,\ 2}$



276. *Answer is B.*

The equation of the horizontal asymptote for the graph of $f(x)=\dfrac{2x^3-7x^2+8x-1}{(x-2)(4x-3)(x+1)}$ is

$$\lim_{x\to\infty}\frac{2x^3-7x^2+8x-1}{(x-2)(4x-3)(x+1)}=\lim_{x\to\infty}\frac{2x^3+\ldots}{4x^3+\ldots}=\lim_{x\to\infty}\frac{\frac{2x^3}{x^3}+\frac{\ldots}{x^3}}{\frac{4x^3}{x^3}+\frac{\ldots}{x^3}}=\frac{2}{4}=\boxed{\frac{1}{2}}$$

No need to fully expand the numerator as it is the same degree as the denominator and the limit is determined only by the coefficients of the x^3 terms

277. *Answer is E.*

If $y = 7$ is a horizontal asymptote of a rational function f, then which of the following must be true ?

$\lim\limits_{x \to 7} f(x) = \infty$ ☒ $\leftarrow$ vertical asymptote at $x = 7$ (unbounded behavior)

$\lim\limits_{x \to -\infty} f(x) = -7$ ☒ $\leftarrow$ horizontal asymptote of $y = -7$ as $x \to -\infty$

$\lim\limits_{x \to 0} f(x) = 7$ ☒ $\leftarrow$ either $f(0) = 7$ or a hole at $(0,\ 7)$

$\lim\limits_{x \to 7} f(x) = 0$ ☒ $\leftarrow$ either $f(7) = 0$ or a hole at $(7,\ 0)$

$\boxed{\lim\limits_{x \to \infty} f(x) = 7}$ ☑ $\leftarrow$ horizontal asymptote of $y = 7$ as $x \to \infty$

278. *Answer is E.*

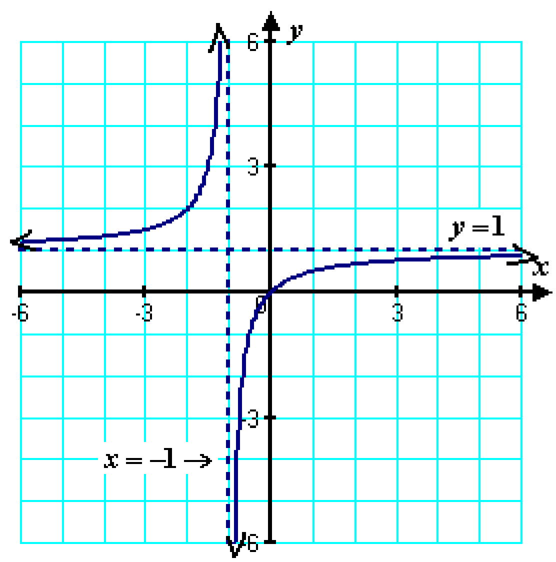
The graph of $f(x) = \dfrac{x}{x+1}$ has

$$\lim_{x \to \infty} \frac{x}{x+1} = \lim_{x \to \infty} \frac{\frac{x}{x}}{\frac{x}{x} + \frac{1}{x}} = \frac{1}{1+0} = \boxed{1}$$

$$\lim_{x \to -1} \frac{x}{x+1} = \frac{-1}{-1+1} = \frac{-1}{0} = \textit{undefined}$$

One horizontal asymptote at $\boxed{y = 1}$

One vertical asymptote at $\boxed{x = -1}$



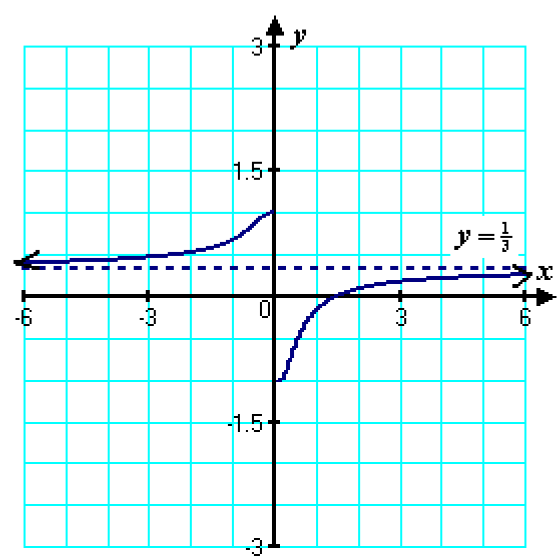
279. *Answer is C.*

The equation of the horizontal asymptote for the graph of $y = \dfrac{2 - e^{\frac{1}{x}}}{2 + e^{\frac{1}{x}}}$ is

$$\lim_{x \to \infty} \frac{2 - e^{\frac{1}{x}}}{2 + e^{\frac{1}{x}}} = \frac{2 - e^{0}}{2 + e^{0}} = \frac{2-1}{2+1} = \boxed{\frac{1}{3}}$$

One horizontal asymptote at $\boxed{y = 1}$

Domain $x \in \mathbb{R}$ and $x \neq 0$



280. *Answer is E.*

The graph of $y = \dfrac{\sin x}{x}$ has

$$\lim_{x\to\infty} \frac{\sin x}{x} = \frac{\textit{oscilates} \text{ between } 1 \text{ and } -1}{\infty} = \boxed{0}$$

$$\lim_{x\to 0} \frac{\sin x}{x} = \frac{\sin 0}{0} = \frac{0}{0} \quad \leftarrow \text{indeterminate form}$$

$$\lim_{x\to 0} \frac{\sin x}{x} = \lim_{x\to 0} \frac{\cos x}{1} = \frac{\cos 0}{1} = 1 \quad \text{(hole at (} 0,\ 1\text{))}$$

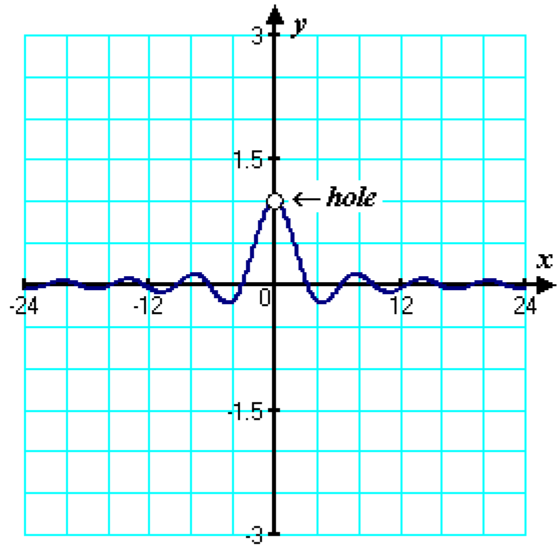
One horizontal asymptote at $\boxed{y = 1}$

Domain $x \in \mathbb{R}$ and $x \neq 0$

I. a vertical asymptote at $x = 0$ ☒

II. a horizontal asymptote at $y = 0$ ☑

III. an infinite number of zeros ☑



281. *Answer is B.*

The graph of $f(x) = \dfrac{\sin x}{\lvert x \rvert}$ has

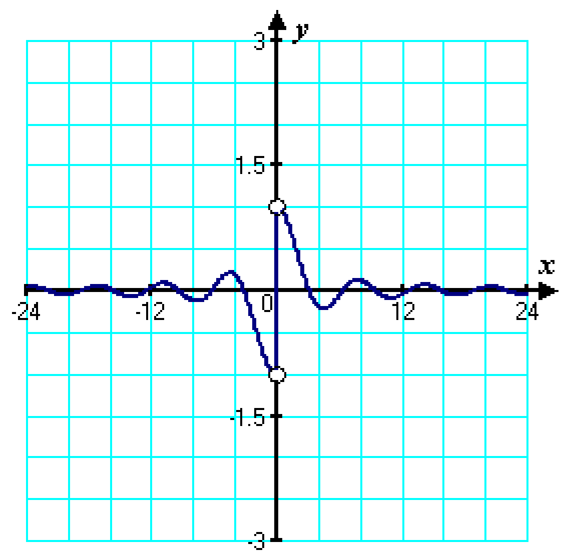
$$\lim_{x\to\infty} \frac{\sin x}{\lvert x \rvert} = \frac{\textit{oscilates} \text{ between } 1 \text{ and } -1}{\infty} = \boxed{0}$$

$$\lim_{x\to 0^+} \frac{\sin x}{\lvert x \rvert} = \lim_{x\to 0^+} \frac{\sin x}{x} = 1 \quad \leftarrow \text{squeeze theorem}$$

$$\lim_{x\to 0^-} \frac{\sin x}{\lvert x \rvert} = \lim_{x\to 0^-} \frac{\sin x}{\lvert x \rvert} = -1$$

$$\lim_{x\to 0} \frac{\sin x}{\lvert x \rvert} = \textit{undefined}, \text{ jump discontinuity}$$

Horizontal asymptote $y = 0$ and **NO** vertical asymptotes, jump discontinuity $x = 0$



282. *Answer is E.*

Which of the following best describes the behavior of the function $f(x)=\dfrac{x^2-2x}{x^2-4}$ at the values not in its domain ?

$$\lim_{x\to\infty}\frac{x^2-2x}{x^2-4}=\lim_{x\to\infty}\frac{\frac{x^2}{x^2}-\frac{2x}{x^2}}{\frac{x^2}{x^2}-\frac{4}{x^2}}=\frac{1-0}{1-0}=\boxed{1}$$

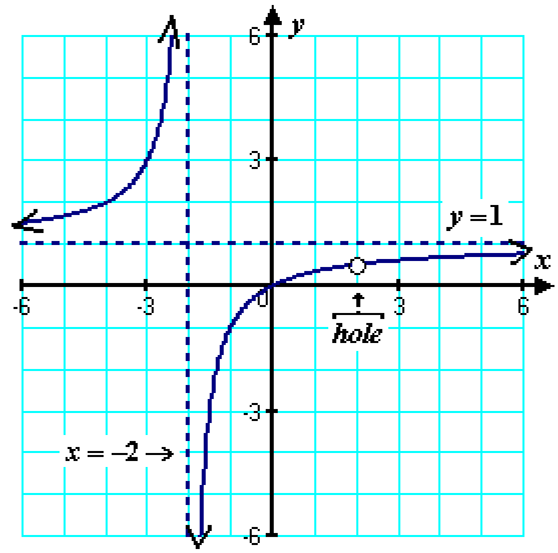
$$\lim_{x\to 2}\frac{x\cancel{(x-2)}}{\cancel{(x-2)}(x+2)}=\frac{0}{0}\leftarrow \text{indeterminate}$$

$$\lim_{x\to 2}\frac{x}{x+2}=\frac{2}{2+2}=\frac{1}{2}\quad\leftarrow \text{hole at } (2,\tfrac{1}{2})$$

$$\lim_{x\to -2}\frac{x}{x+2}=\frac{-2}{-2+2}=\frac{-2}{0}=\textit{undefined}$$

Hole/discontinuity at $(2,\frac{1}{2})$

Vertical asymptote/discontinuity at $x=-2$



283. *Answer is D.*

The graph of which of the following equations has $y=1$ as an asymptote ?

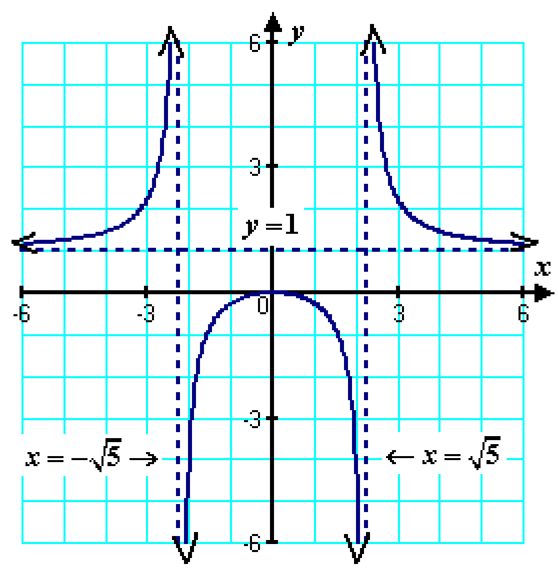
$\lim_{x\to\infty}\cos x=$ no horizontal asymptote

$\lim_{x\to-\infty}e^x=0$ asymptote $y=0$

$$\lim_{x\to\infty}\frac{x^3}{x^2+1}=\lim_{x\to-\infty}\frac{\frac{x^3}{x^2}}{\frac{x^2}{x^2}+\frac{1}{x^2}}=\frac{x}{1+0}=\infty$$

$$\boxed{\lim_{x\to\infty}\frac{x^2}{x^2-5}}=\lim_{x\to\infty}\frac{\frac{x^2}{x^2}}{\frac{x^2}{x^2}-\frac{5}{x^2}}=\frac{1}{1-0}=\boxed{1}$$

$\lim_{x\to\infty}-\ln x=$ no horizontal asymptote

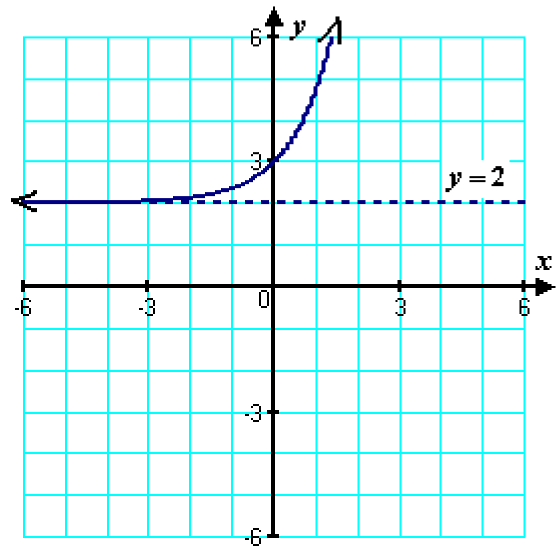


284. *Answer is C.*

If $f(x)=e^x+2$ which of the following lines is an asymptote to the graph of f

$$\lim_{x\to-\infty} e^x+2=\lim_{x\to-\infty}\frac{1}{e^{-x}}+2=\frac{1}{e^{\infty}}+2=\boxed{2}$$

Should know this exponential growth curve from memory.



285. *Answer is D.*

If the graph of $y=\dfrac{ax+b}{x+c}$ has a horizontal asymptote $y=-2$, a vertical asymptote $x=4$, and an x-intercept of 1.5, then $a-b+c=$

Horizontal asymptote $\Rightarrow \lim_{x\to\infty}\dfrac{ax+b}{x+c}=\lim_{x\to\infty}\dfrac{\frac{ax}{x}+\frac{b}{x}}{\frac{x}{x}+\frac{c}{x}}=\dfrac{a+0}{1+0}=a=-2$

Vertical asymptote $\Rightarrow \lim_{x\to4}\dfrac{-2x+b}{x+c}=\infty \quad\Rightarrow\quad 4+c=0 \quad\Rightarrow\quad c=-4$

An x-intercept of 1.5 $\Rightarrow 0=\dfrac{-2(1.5)+b}{(1.5)-4} \quad\Rightarrow\quad -2(1.5)+b=0 \quad\Rightarrow\quad b=3$

Then $a-b+c=-2-3-4=\boxed{-9}$

286. *Answer is E.*

The function $f(x)=\dfrac{4x^2-3}{2x^2+1}$ is

$$\lim_{x\to\infty}\frac{4x^2-3}{2x^2+1}=\lim_{x\to\infty}\frac{\frac{4x^2}{x^2}-\frac{3}{x^2}}{\frac{2x^2}{x^2}+\frac{1}{x^2}}=\frac{4-0}{2+0}=\boxed{2}$$

$$\lim_{x\to0}\frac{4x^2-3}{2x^2+1}=\frac{4(0)^2-3}{2(0)^2+1}=\boxed{-3}$$

Sketch graph: symmetry wrt y-axis

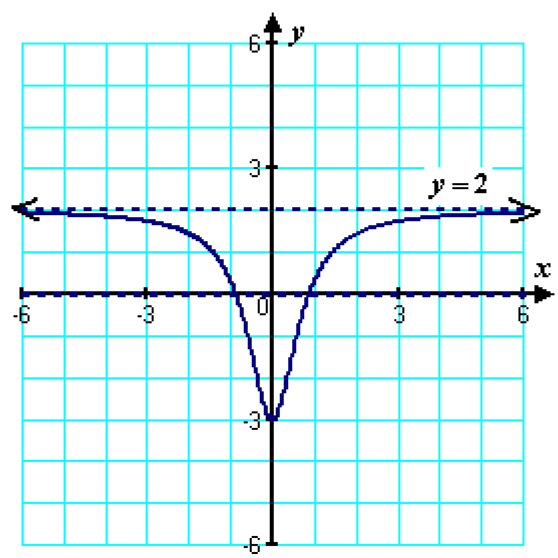
$4x^2-3=0 \;\to\;$ zero's $x=\pm\frac{\sqrt{3}}{2}$

I. unbounded ☒

II. bounded below by $y=-3$ ☑

III. bounded above by $y=2$ ☑

IV. bounded below by $y=2$ ☒



287. *Answer is A.*

Let $f(x)=\dfrac{2}{x^2+4}$

Which of the following are true ?

$$\lim_{x\to\infty}\frac{2}{x^2+4}=\lim_{x\to\infty}\frac{\frac{2}{x^2}}{\frac{x^2}{x^2}+\frac{4}{x^2}}=\frac{0}{1+0}=\boxed{0}$$

$$\lim_{x\to -2}\frac{2}{x^2+4}=\frac{2}{(-2)^2+4}=\frac{2}{8}=\frac{1}{4}$$

I. $f(x)$ has an absolute maximum at $x=0$

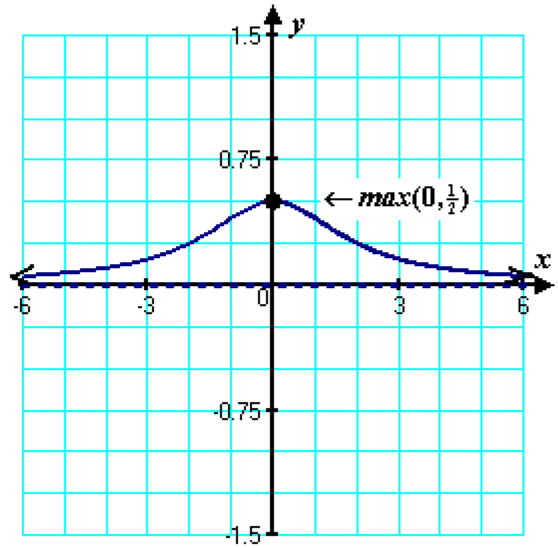
☑ denominator increases $x\neq 0$

II. $f(x)$ has a vertical asymptote at $x=-2$

☒ $f(-2)=\frac{1}{4}$

III. $f(x)$ has a horizontal asymptote at $y=\dfrac{1}{2}$

☒ $\lim\limits_{x\to\infty} f(x)=0$



288. *Answer is A.*

The equation of the horizontal asymptote of

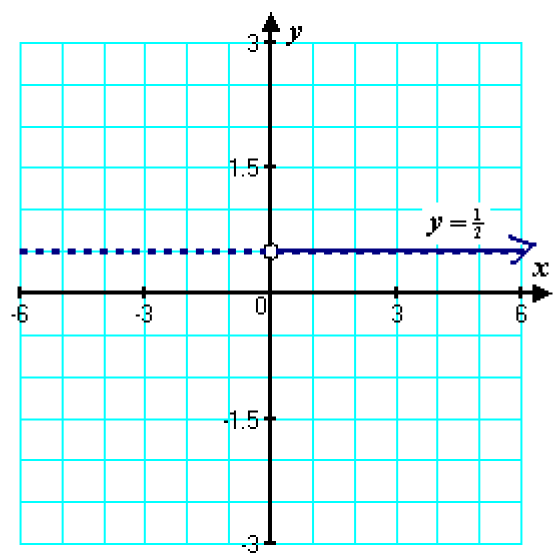
$f(x)=\dfrac{|x|}{|x|+x}$ is:

$$\lim_{x\to\infty}\frac{|x|}{|x|+x}=\lim_{x\to\infty}\frac{\frac{x}{x}}{\frac{x}{x}+\frac{x}{x}}=\frac{1}{1+1}=\boxed{\frac{1}{2}}$$

$$\lim_{x\to 0}\frac{|x|}{|x|+x}=\frac{0}{0+0}=\frac{0}{0}\quad\leftarrow \text{indeterminant}$$

$$\lim_{x\to 0^+}\frac{1}{1+1}=\frac{1}{2}$$

Note: domain of function $x>0$



289. *Answer is E.*

If $y = \dfrac{2(x-1)^2}{x^2}$, then which of the following must be true ?

I. the range is $y \ge 0$ **II.** the y-intercept is 1 **III.** the horizontal asymptote is $y = 2$

$\lim\limits_{x\to\infty} \dfrac{2x^2-4x+2}{x^2} = \lim\limits_{x\to\infty} \dfrac{\frac{2x^2}{x^2}-\frac{4x}{x^2}+\frac{2}{x^2}}{\frac{x^2}{x^2}} = \dfrac{2-0+0}{1} = \boxed{2}$ ☑ ← horizontal asymptote

$y = \dfrac{2(x-1)^2}{x^2}$ $\quad 2(x-1)^2 \ge 0$, $x^2 \ge 0$ ← range $y \ge 0$

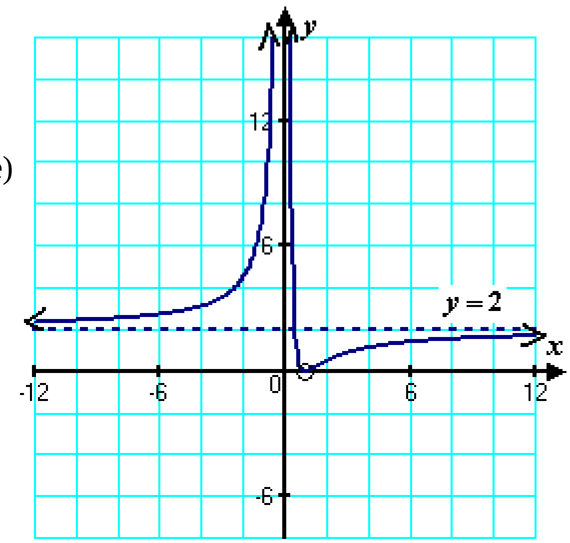
$\lim\limits_{x\to 0} \dfrac{2(x-1)^2}{x^2} = \dfrac{2}{0} = \textbf{\textit{undefined}}$ (vertical asymptote)

I. the range is $y \ge 0$ ☑ ← $y \ge 0$

II. the y-intercept is 1 ☒ ← $f(0) \ne 1$

III. the horizontal asymptote is $y = 2$

☑ ← $\lim\limits_{x\to\infty} f(x) = 2$



290. *Answer is C.*

Which of the following is true about the function f if $f(x) = \dfrac{(x-1)^2}{2x^2-5x+3}$

I. f is continuous at $x = 1$

II. The graph of f has a vertical asymptote at $x = 1$

III. The graph of f has a horizontal asymptote at $y = \dfrac{1}{2}$

$\lim\limits_{x\to\infty} \dfrac{x^2-2x+1}{2x^2-5x+3} = \lim\limits_{x\to\infty} \dfrac{\frac{x^2}{x^2}-\frac{2x}{x^2}+\frac{1}{x^2}}{\frac{2x^2}{x^2}-\frac{5x}{x^2}+\frac{3}{x^2}} = \dfrac{1-0+0}{2-0+0} = \boxed{\dfrac{1}{2}}$ ← horizontal asymptote

$\lim\limits_{x\to 1} \dfrac{(x-1)\cancel{(x-1)}}{(2x-3)\cancel{(x-1)}} = \dfrac{0}{0}$ ← indeterminate form

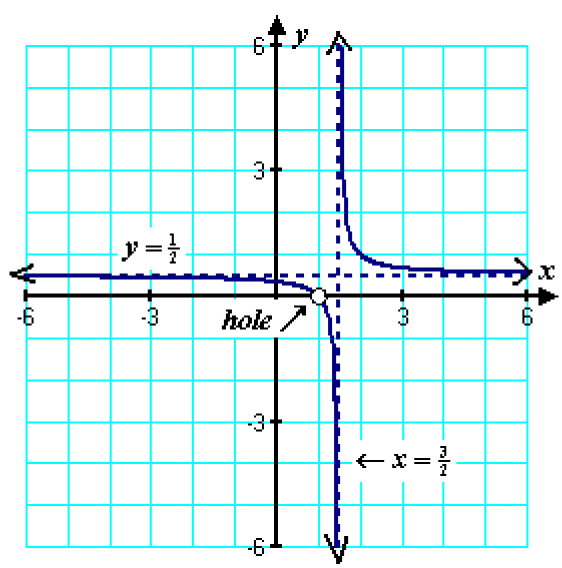
$\lim\limits_{x\to 1} \dfrac{x-1}{2x-3} = \dfrac{0}{-1} = 0$ ← (hole at $(1,0)$)

I. f is continuous at $x = 1$

☒ ← (hole at $(1,0)$)

II. The graph of f has a vertical asymptote at $x = 1$ ☒ ← $\lim\limits_{x\to 1} f(x) \ne \infty$

III. The graph of f has a horizontal asymptote at $y = \frac{1}{2}$ ☑ ← $\lim\limits_{x\to\infty} f(x) = \frac{1}{2}$



291.

$\boxed{\text{Limits}}$ → needed to handle ***holes***, ***asymptotes***, ***sharp points***, ***endpoints***,

→ **definition of derivative**

$\boxed{6a}$ Trig → basic $\quad \lim_{x\to 0}\frac{\sin x}{x}=1 \qquad \lim_{x\to 0}\frac{1-\cos x}{x}=0$ ← squeeze theorem proof

$\boxed{6b}$ Trig → advanced questions based on basics

292. *Answer is B.*

$$\lim_{t\to 0}\frac{\sin t}{t}=$$

$$\lim_{t\to 0}\frac{\sin t}{t}=\frac{\sin 0}{0}=\frac{0}{0}$$ ← indeterminant form

$$\lim_{t\to 0}\frac{\sin t}{t}=\boxed{1}$$ ← (geometry squeeze theorem) must know this limit !!!

293. *Answer is C.*

$$\lim_{x\to 0}\frac{\sin 7x}{7x}=$$

$$\lim_{x\to 0}\frac{\sin 7x}{7x}=\lim_{7x\to 0}\frac{\sin 7x}{7x}=\boxed{1}$$ ← must know from memory (squeeze theorem)

294. *Answer is D.*

$$\lim_{x\to 0}\frac{\sin 7x}{x}=$$

$$\lim_{x\to 0}\frac{\sin 7x}{x}=\lim_{7x\to 0}\frac{7}{1}\left(\frac{\sin 7x}{7x}\right)=7(1)=\boxed{7}$$ ← must know from memory (squeeze theorem)

295. *Answer is B.*

$$\lim_{x\to 0}\frac{\sin x}{7x}=$$

$$\lim_{x\to 0}\frac{\sin x}{7x}=\lim_{7x\to 0}\frac{1}{7}\left(\frac{\sin x}{x}\right)=\frac{1}{7}(1)=\boxed{\frac{1}{7}}$$ ← must know from memory (squeeze theorem)

296. *Answer is C.*

$$\lim_{x\to \pi}\frac{\sin x}{x}=$$

$$\lim_{x\to \pi}\frac{\sin x}{x}=\frac{\sin \pi}{\pi}=\frac{0}{\pi}=\boxed{0}$$ ← direct substitution

297. *Answer is C.*

$$\lim_{x\to\frac{\pi}{2}} \frac{\sin x}{x} =$$

$$\lim_{x\to\frac{\pi}{2}} \frac{\sin x}{x} = \frac{\sin\frac{\pi}{2}}{\frac{\pi}{2}} = \frac{1}{\frac{\pi}{2}} = \boxed{\frac{2}{\pi}} \quad \leftarrow \text{direct substitution}$$

298. *Answer is D.*

$$\lim_{x\to 0} \frac{\sin 2x}{x\cos x} =$$

$$\lim_{x\to 0} \frac{\sin 2x}{x\cos x} = \lim_{x\to 0}\left(\frac{2}{1}\right)\left(\frac{\sin 2x}{2x}\right)\left(\frac{1}{\cos x}\right) = \left(\lim_{x\to 0}\frac{2}{1}\right)\left(\lim_{x\to 0}\frac{\sin 2x}{2x}\right)\left(\lim_{x\to 0}\frac{1}{\cos x}\right)$$

$$= (2)\left(\lim_{2x\to 0}\frac{\sin 2x}{2x}\right)\left(\frac{1}{\cos 0}\right) = (2)(1)\left(\frac{1}{1}\right) = \boxed{2}$$

299. *Answer is C.*

$$\lim_{x\to\frac{\pi}{3}} \frac{1-\cos x}{x} =$$

$$\lim_{x\to\frac{\pi}{3}} \frac{1-\cos x}{x} = \frac{1-\cos\frac{\pi}{3}}{\frac{\pi}{3}} = \frac{1-\frac{1}{2}}{\frac{\pi}{3}} = \frac{\frac{1}{2}}{\frac{\pi}{3}} = \boxed{\frac{3}{2\pi}} \quad \leftarrow \text{direct substitution}$$

300. *Answer is A.*

$$\lim_{x\to 0} \frac{\sin 3x}{7x} =$$

$$\lim_{x\to 0} \frac{\sin 3x}{7x} = \lim_{3x\to 0} \frac{3}{7}\left(\frac{\sin 3x}{3x}\right) = \frac{3}{7}(1) = \boxed{\frac{3}{7}} \quad \leftarrow \text{direct substitution}$$

301. *Answer is B.*

$$\lim_{\theta\to 0} \frac{\sin\theta}{\sec\theta} =$$

$$\lim_{\theta\to 0} \frac{\sin\theta}{\sec\theta} = \lim_{\theta\to 0} \frac{\sin\theta}{\frac{1}{\cos\theta}} = \lim_{\theta\to 0} \sin\theta\cos\theta = (0)(1) = \boxed{0} \quad \leftarrow \text{rearrange and direct substitution}$$

302. *Answer is E.*

$$\lim_{x\to\frac{\pi}{4}} (\sin^2 x - 1) =$$

$$\lim_{x\to\frac{\pi}{4}} (\sin^2 x - 1) = \left(\sin\frac{\pi}{4}\right)^2 - 1 = \left(\frac{1}{\sqrt{2}}\right)^2 - 1 = \frac{1}{2} - 1 = \boxed{-\frac{1}{2}}$$

303. *Answer is C.*

$$\lim_{x\to 0}\sin\frac{1}{x} =$$

$$\lim_{x\to 0}\sin\frac{1}{x} = \boxed{\textit{does not exist}} \qquad \leftarrow \text{oscillating behaviour}$$

304. *Answer is E.*

$$\lim_{x\to\infty} x\sin\frac{1}{x} =$$

$$\lim_{x\to\infty} x\sin\frac{1}{x} = \lim_{\frac{1}{x}\to 0}\frac{\sin\frac{1}{x}}{\frac{1}{x}} = \boxed{1} \qquad \leftarrow \text{(rearrange first) special limit you must know}$$

305. *Answer is A.*

$$\lim_{x\to\pi}\frac{\sin(\pi - x)}{\pi - x} =$$

$$\lim_{x\to\pi}\frac{\sin(\pi - x)}{\pi - x} = \lim_{(\pi - x)\to 0}\frac{\sin(\pi - x)}{\pi - x} = \boxed{1} \qquad \leftarrow \text{rearrange into special limit !!!}$$

306. *Answer is B.*

$$\lim_{x\to 0}\frac{\cos x - 1}{x} =$$

$$\lim_{x\to 0}\frac{\cos x - 1}{x} = \lim_{x\to 0}\frac{(-1)}{1}\overbrace{\left(\frac{1-\cos x}{x}\right)}^{must\ know\ =0} = (-1)(0) = \boxed{0}$$

307. *Answer is B.*

$$\lim_{x\to 0}\frac{\sin 2x}{x} =$$

$$\lim_{x\to 0}\frac{\sin 2x}{x} = \lim_{x\to 0}\left(\frac{2}{1}\right)\frac{\sin 2x}{2x} = \lim_{x\to 0}\left(\frac{2}{1}\right)\overbrace{\left(\frac{\sin 2x}{2x}\right)}^{must\ know\ =1} = \boxed{2}$$

308. *Answer is C.*

$$\lim_{x\to 0}\frac{\sin 3x}{\sin 4x} =$$

$$\lim_{x\to 0}\frac{\sin 3x}{\sin 4x} = \lim_{x\to 0}\left(\frac{12x}{12x}\right)\frac{\sin 3x}{\sin 4x} = \lim_{x\to 0}\left(\frac{3}{4}\right)\overbrace{\left(\frac{\sin 3x}{3x}\right)}^{must\ know\ =1}\overbrace{\left(\frac{4x}{\sin 4x}\right)}^{must\ know\ =1} = \boxed{\frac{3}{4}}$$

309. *Answer is E.*

$$\lim_{x\to 0}\frac{1-\cos x}{x}=$$

$$\lim_{x\to 0}\frac{1-\cos x}{x}=\boxed{0}$$ ← must know from ***squeeze*** theorem

310. *Answer is D.*

$$\lim_{x\to 0}\frac{\tan \pi x}{x}=$$

$$\lim_{x\to 0}\frac{\tan \pi x}{x}=\lim_{x\to 0}\frac{\pi}{1}\left(\frac{\sin \pi x}{\pi x}\right)\left(\frac{1}{\cos \pi x}\right)=\pi(1)(1)=\boxed{\pi}$$

311. *Answer is C.*

$$\lim_{x\to 0}\frac{\sec x-\cos x}{x^2}=$$

$$\lim_{x\to 0}\frac{\sec x-\cos x}{x^2}=\lim_{x\to 0}\frac{\frac{1}{\cos x}-\frac{\cos^2 x}{\cos x}}{x^2}=\lim_{x\to 0}\frac{\frac{1-\cos^2 x}{\cos x}}{x^2}=\lim_{x\to 0}\frac{\sin^2 x}{x^2\cos x}=\pi(1)(1)=$$

$$=\lim_{x\to 0}\left(\frac{\sin x}{x}\right)\left(\frac{\sin x}{x}\right)\left(\frac{1}{\cos x}\right)=1(1)(1)=\boxed{1}$$

312. *Answer is B.*

$$\lim_{h\to 0}\frac{\cos(\frac{\pi}{2}+h)}{h}=$$

$$\lim_{h\to 0}\frac{\cos(\frac{\pi}{2}+h)}{h}=\frac{\cos(\frac{\pi}{2}+0)}{0}=\frac{0}{0}$$ ← indeterminate form

$$\lim_{h\to 0}\frac{\cos(\frac{\pi}{2}+h)}{h}=\lim_{h\to 0}\frac{-\sin(\frac{\pi}{2}+h)}{1}=\frac{-\sin(\frac{\pi}{2}+0)}{1}=\boxed{-1}$$ ← L'hopitals rule

313. *Answer is C.*

$$\lim_{h\to 0}\frac{\sin(\frac{\pi}{2}+h)-1}{h}=$$

$$\lim_{h\to 0}\frac{\sin(\frac{\pi}{2}+h)-1}{h}=\frac{\sin(\frac{\pi}{2}+0)-1}{0}=\frac{0}{0}$$ ← indeterminate form

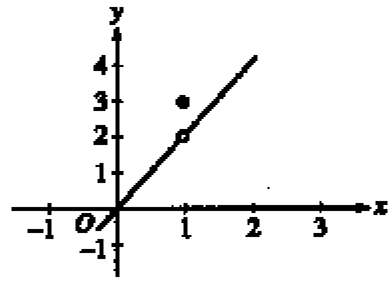
$$\lim_{h\to 0}\frac{\sin(\frac{\pi}{2}+h)-1}{h}=\lim_{h\to 0}\frac{\cos(\frac{\pi}{2}+h)}{1}=\frac{\cos(\frac{\pi}{2}+0)}{1}=\boxed{0}$$ ← L'hopitals rule

314. *Answer is A.*

Difficulty = **0.62**

The graph of the function f is show in the figure.
The value of $\lim_{x\to 1} \sin(f(x)) =$

$$\lim_{x\to 1} \sin(f(x)) = \sin(\lim_{x\to 1} f(x)) = \sin(2) = \boxed{0.9093}$$



Graph of f

315. *Answer is E.*

$$\lim_{x\to 0} \frac{1-\cos^2(2x)}{x^2} =$$

$$\lim_{x\to 0} \frac{1-\cos^2(2x)}{x^2} = \lim_{x\to 0} \frac{\sin^2(2x)}{x^2} = \lim_{x\to 0} \frac{4}{1}\left(\frac{\sin(2x)}{2x}\right)\left(\frac{\sin(2x)}{2x}\right) = 4(1)(1) = \boxed{4}$$

316. *Answer is D.*

$$\lim_{x\to 0} x\csc x =$$

$$\lim_{x\to 0} x\csc x = \lim_{x\to 0} \frac{x}{\sin x} = \boxed{1} \qquad \leftarrow \text{reciprocal of } \lim_{x\to 0} \frac{\sin x}{x} = 1$$

317. *Answer is D.*

$$\lim_{x\to \frac{\pi}{4}} \frac{\sin(x-\frac{\pi}{4})}{x-\frac{\pi}{4}} =$$

$$\lim_{x\to \frac{\pi}{4}} \frac{\sin(x-\frac{\pi}{4})}{x-\frac{\pi}{4}} = \lim_{(x-\frac{\pi}{4})\to 0} \frac{\sin(x-\frac{\pi}{4})}{x-\frac{\pi}{4}} = \boxed{1} \qquad \leftarrow \text{rearrange}$$

318. *Answer is C.*

Difficulty = **0.40**

$$\lim_{\theta\to 0} \frac{1-\cos\theta}{2\sin^2\theta} =$$

$$\lim_{\theta\to 0} \frac{1-\cos\theta}{2\sin^2\theta} = \lim_{\theta\to 0} \frac{1-\cos 0}{2\sin^2 0} = \frac{1-1}{2(0)} = \frac{0}{0} \qquad \leftarrow \text{indeterminant form}$$

$$\lim_{\theta\to 0} \frac{1-\cos\theta}{2\sin^2\theta} = \lim_{\theta\to 0} \frac{1-\cos\theta}{2(1-\cos^2\theta)} = \lim_{\theta\to 0} \frac{\cancel{1-\cos\theta}}{2\,\cancel{(1-\cos\theta)}\,(1+\cos\theta)} = \lim_{\theta\to 0} \frac{1}{2(1+\cos\theta)} = \frac{1}{2(1+1)} = \boxed{\frac{1}{4}}$$

319. *Answer is E.*

$$\lim_{x\to 0}\frac{\tan x}{x}=$$

$$\lim_{x\to 0}\frac{\tan x}{x}=\lim_{x\to 0}\left(\frac{\sin x}{x}\right)\left(\frac{1}{\cos x}\right)=(1)(1)=\boxed{1} \quad \leftarrow \text{rearrange and direct substitution}$$

320. *Answer is B.*

$$\lim_{x\to 0}\frac{\cos x-1}{\sin^2 x}=$$

$$\lim_{x\to 0}\frac{\cos x-1}{\sin^2 x}=\frac{\cos 0-1}{\sin^2 0}=\frac{0}{0} \quad \leftarrow \text{indeterminant form}$$

$$\lim_{x\to 0}\frac{\cos x-1}{\sin^2 x}=\lim_{x\to 0}\frac{-(1-\cos x)}{1-\cos^2 x}=\lim_{x\to 0}\frac{-\cancel{(1-\cos x)}}{\cancel{(1-\cos x)}(1+\cos x)}=\frac{-1}{(1+\cos 0)}=\boxed{\frac{-1}{2}}$$

321. *Answer is D.*

$$\lim_{x\to 0}\frac{\sin 3x}{\tan 2x}=$$

$$\lim_{x\to 0}\frac{\sin 3x}{\tan 2x}=\frac{\sin 0}{\tan 0}=\frac{0}{0} \quad \leftarrow \text{indeterminant form}$$

$$\lim_{x\to 0}\frac{\sin 3x}{\tan 2x}=\lim_{x\to 0}\frac{3}{2}\left(\frac{\sin 3x}{3x}\right)\left(\frac{2x}{\tan 2x}\right)=\frac{3}{2}(1)(1)=\boxed{\frac{3}{2}}$$

322. *Answer is A.*

$$\lim_{x\to 0}\frac{-\cos x+1-\sin x}{x}=$$

$$\lim_{x\to 0}\frac{-\cos x+1-\sin x}{x}=\frac{-\cos 0+1-\sin 0}{0}=\frac{0}{0} \quad \leftarrow \text{indeterminant form}$$

$$\lim_{x\to 0}\frac{1-\cos x-\sin x}{x}=\lim_{x\to 0}\frac{1-\cos x}{x}-\lim_{x\to 0}\frac{\sin x}{x}=0-1=\boxed{-1}$$

323. *Answer is D.*

$$\lim_{x\to 0}\frac{2\sin x\cos x}{2x}=$$

$$\lim_{x\to 0}\frac{2\sin x\cos x}{2x}=\frac{2\sin 0\cos 0}{2(0)}=\frac{0}{0} \quad \leftarrow \text{indeterminate form}$$

$$\lim_{x\to 0}\frac{2\sin x\cos x}{2x}=\lim_{x\to 0}\frac{\sin 2x}{2x}=\boxed{1} \quad \leftarrow \text{trig identity}$$

324. *Answer is D.*

$$\lim_{x\to 0}\frac{\sin^2 3x}{x^2}=$$

$$\lim_{x\to 0}\frac{\sin^2 3x}{x^2}=\frac{\sin^2 0}{0^2}=\frac{0}{0}\quad\leftarrow\text{indeterminate form}$$

$$\lim_{x\to 0}\frac{\sin^2 3x}{x^2}=\lim_{x\to 0}\frac{9}{1}\left(\frac{\sin 3x}{3x}\right)\left(\frac{\sin 3x}{3x}\right)=9(1)(1)=\boxed{9}$$

325. *Answer is D.*

$$\lim_{x\to 0}4\frac{\sin x\cos x-\sin x}{x^2}=$$

$$\lim_{x\to 0}4\frac{\sin x\cos x-\sin x}{x^2}=4\frac{\sin 0\cos 0-\sin 0}{0^2}=\frac{0}{0}\quad\leftarrow\text{indeterminate form}$$

$$\lim_{x\to 0}4\frac{\sin x\cos x-\sin x}{x^2}=\lim_{x\to 0}4\left(\frac{\sin x}{x}\right)\left(\frac{-(1-\cos x)}{x}\right)=4(1)(0)=\boxed{0}$$

326. *Answer is B.*

$$\lim_{x\to 0}\frac{\cos^2 x-1}{2x\sin x}=$$

$$\lim_{x\to 0}\frac{\cos^2 x-1}{2x\sin x}=\lim_{x\to 0}\frac{-(1-\cos^2 x)}{2x\sin x}=\lim_{x\to 0}\frac{-(\sin^{\cancel{2}} x)}{2x\cancel{\sin x}}=\frac{-1}{2}\lim_{x\to 0}\frac{\sin x}{x}=\frac{-1}{2}(1)=\boxed{\frac{-1}{2}}$$

327. *Answer is D.*

$$\lim_{x\to 0}\frac{\sin 2x}{x\cos x}=$$

$$\lim_{x\to 0}\frac{\sin 2x}{x\cos x}=\lim_{x\to 0}\frac{2\sin x\cancel{\cos x}}{x\cancel{\cos x}}=2\lim_{x\to 0}\frac{\sin x}{x}=2(1)=\boxed{2}$$

328. *Answer is B.*

$$\lim_{x\to 0}\frac{1-\cos x}{x^2}=$$

$$\lim_{x\to 0}\frac{1-\cos x}{x^2}=\frac{1-\cos 0}{0^2}=\frac{0}{0}\quad\leftarrow\text{indeterminant form (L'hopitals rule)}$$

$$\lim_{x\to 0}\frac{1-\cos x}{x^2}=\lim_{x\to 0}\frac{\sin x}{2x}=\frac{\sin 0}{2(0)}=\frac{0}{0}\quad\leftarrow\text{indeterminant form (L'hopitals rule again)}$$

$$\lim_{x\to 0}\frac{\sin x}{2x}=\lim_{x\to 0}\frac{\cos x}{2}=\frac{\cos 0}{2}=\boxed{\frac{1}{2}}$$

329. *Answer is C.*

$$\lim_{x\to 0}\frac{\sin 2x-2x}{x^3}=$$

$$\lim_{x\to 0}\frac{\sin 2x-2x}{x^3}=\frac{\sin 0-2(0)}{0^3}=\frac{0}{0} \quad \leftarrow \text{indeterminant form (L'hopitals rule)}$$

$$\lim_{x\to 0}\frac{\sin 2x-2x}{x^3}=\lim_{x\to 0}\frac{2\cos 2x-2}{3x^2}=\frac{2\cos 0-2}{3(0)^2}=\frac{0}{0} \quad \leftarrow \text{(L'hopitals rule again)}$$

$$\lim_{x\to 0}\frac{2\cos 2x-2}{3x^2}=\lim_{x\to 0}\frac{-4\sin 2x}{6x}=\frac{-4\sin 0}{6(0)}=\frac{0}{0} \quad \leftarrow \text{(L'hopitals rule again)}$$

$$\lim_{x\to 0}\frac{-4\sin 2x}{6x}=\lim_{x\to 0}\frac{-8\cos 2x}{6}=\frac{-8\cos 0}{6}=\boxed{\frac{-4}{3}}$$

330. *Answer is B.*

$$\lim_{x\to\frac{\pi}{4}}\frac{\tan x-1}{\sin x-\cos x}=$$

$$\lim_{x\to\frac{\pi}{4}}\frac{\tan x-1}{\sin x-\cos x}=\frac{\tan\frac{\pi}{4}-1}{\sin\frac{\pi}{4}-\cos\frac{\pi}{4}}=\frac{1-1}{\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}}=\frac{0}{0} \quad \leftarrow \text{indeterminant form (L'hopitals rule)}$$

$$\lim_{x\to\frac{\pi}{4}}\frac{\tan x-1}{\sin x-\cos x}=\lim_{x\to\frac{\pi}{4}}\frac{\sec^2 x}{\cos x+\sin x}=\frac{\sec^2\frac{\pi}{4}}{\cos\frac{\pi}{4}+\sin\frac{\pi}{4}}=\frac{2}{\frac{2}{\sqrt{2}}}=\boxed{\sqrt{2}}$$

331. *Answer is B.*

$$\lim_{x\to\frac{\pi}{2}}(\sec x-\tan x)=$$

$$\lim_{x\to\frac{\pi}{2}}(\sec x-\tan x)=\lim_{x\to\frac{\pi}{2}}\frac{1-\sin x}{\cos x}=\frac{1-\sin\frac{\pi}{2}}{\cos\frac{\pi}{2}}=\frac{0}{0} \quad \leftarrow \text{indeterminant form (L'hopitals rule)}$$

$$\lim_{x\to\frac{\pi}{2}}\frac{1-\sin x}{\cos x}=\lim_{x\to\frac{\pi}{2}}\frac{-\cos x}{-\sin x}=\lim_{x\to\frac{\pi}{2}}\frac{\cos x}{\sin x}=\frac{\cos\frac{\pi}{2}}{\sin\frac{\pi}{2}}=\frac{0}{1}=\boxed{0}$$

332. *Answer is B.*

$$\lim_{x\to\pi/2}\frac{\sin x}{x}=$$

$$\lim_{x\to\pi/2}\frac{\sin x}{x}=\frac{\sin\frac{\pi}{2}}{\frac{\pi}{2}}=\frac{1}{\frac{\pi}{2}}=\boxed{\frac{2}{\pi}}$$

333. *Answer is D.*

$$\lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h}=$$

$$\lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h}= \quad \leftarrow \text{means the derivative of } f(x)=\sin x$$

$$f'(x)=\boxed{\cos x}$$

334. *Answer is B.*

$\boxed{\text{Difficulty} = \mathbf{0.45}}$

$$\boxed{\lim_{h\to 0}\frac{\tan 3(x+h)-\tan 3x}{h}=}$$

$$\lim_{h\to 0}\frac{\tan 3(x+h)-\tan 3x}{h}= \quad \leftarrow \text{means the derivative of } f(x)=\tan 3x$$

$$f'(x)=\boxed{3\sec^2(3x)}$$

335. *Answer is C.*

$$\boxed{\lim_{x\to 0}\frac{\tan(x+h)-\tan x}{h}=}$$

$$\lim_{x\to 0}\frac{\tan(x+h)-\tan x}{h}= \quad \leftarrow \text{means the derivative of } f(x)=\tan x$$

$$f'(x)=\boxed{\sec^2 x}$$

336. *Answer is B.*

$$\boxed{\lim_{h\to 0}\frac{\cos(x+h)-\cos x}{h}=}$$

$$\lim_{h\to 0}\frac{\cos(x+h)-\cos x}{h}= \quad \leftarrow \text{means the derivative of } f(x)=\cos x$$

$$f'(x)=\boxed{-\sin x}$$

337. *Answer is D.*

$$\boxed{\lim_{h\to 0}\frac{\tan(2(x+h))-\tan(2x)}{h}=}$$

$$\lim_{h\to 0}\frac{\tan(2(x+h))-\tan(2x)}{h}= \quad \leftarrow \text{means the derivative of } f(x)=\tan(2x)$$

$$f'(x)=\boxed{2\sec^2(2x)}$$

338. *Answer is B.*

$$\boxed{\lim_{h\to 0}\frac{\tan(\frac{\pi}{6}+h)-\tan(\frac{\pi}{6})}{h}=}$$

$$\lim_{h\to 0}\frac{\tan(\frac{\pi}{6}+h)-\tan(\frac{\pi}{6})}{h}= \quad \leftarrow \text{means the derivative of } f(x)=\tan x \text{ evaluated at } x=\tfrac{\pi}{6}$$

$$f'(x)=\sec^2 x$$

$$f'(\tfrac{\pi}{6})=\sec^2(\tfrac{\pi}{6})=\boxed{\frac{4}{3}}$$

339. *Answer is B.*

$$\lim_{h\to 0}\frac{\sec(\pi+h)-\sec\pi}{h}=$$

$$\lim_{h\to 0}\frac{\sec(\pi+h)-\sec\pi}{h}= \quad \leftarrow \text{means the derivative of } f(x)=\sec x \text{ evaluated at } x=\pi$$

$$f'(x)=\sec x\tan x$$

$$f'(\pi)=\sec\pi\tan\pi=\boxed{0}$$

340. *Answer is C.*

$$\lim_{h\to 0}\frac{\sin(\pi+h)-\sin\pi}{h}=$$

$$\lim_{h\to 0}\frac{\sin(\pi+h)-\sin\pi}{h}= \quad \leftarrow \text{definition of derivative for } f(x)=\sin x \text{ when } x=\pi$$

$$f'(x)=\cos x$$

$$f'(\pi)=\cos\pi=\boxed{-1}$$

341. *Answer is A.*

$$\lim_{h\to 0}\frac{\cos(\frac{3\pi}{2}+h)-\cos(\frac{3\pi}{2})}{h}=$$

$$\lim_{h\to 0}\frac{\cos(\frac{3\pi}{2}+h)-\cos(\frac{3\pi}{2})}{h}= \quad \leftarrow \text{derivative of } f(x)=\cos x \text{ evaluated at } x=\tfrac{3\pi}{2}$$

$$f'(x)=-\sin x$$

$$f'(\tfrac{3\pi}{2})=-\sin(\tfrac{3\pi}{2})=\boxed{1}$$

342. *Answer is C.*

$$\lim_{\Delta x\to 0}\frac{\sin(\frac{\pi}{3}+\Delta x)-\sin(\frac{\pi}{3})}{\Delta x}=$$

$$\lim_{\Delta x\to 0}\frac{\sin(\frac{\pi}{3}+\Delta x)-\sin(\frac{\pi}{3})}{\Delta x}= \quad \leftarrow \text{derivative of } f(x)=\sin x \text{ evaluated at } x=\tfrac{\pi}{3}$$

$$f'(x)=\cos x$$

$$f'(\tfrac{\pi}{3})=\cos(\tfrac{\pi}{3})=\boxed{\frac{1}{2}}$$

343. *Answer is B.*

$$\lim_{h\to 0}\frac{\csc(\frac{\pi}{4}+h)-\csc(\frac{\pi}{4})}{h}=$$

$$\lim_{h\to 0}\frac{\csc(\frac{\pi}{4}+h)-\csc(\frac{\pi}{4})}{h}= \quad \leftarrow \text{ derivative of } f(x)=\csc x \text{ evaluated at } x=\tfrac{\pi}{4}$$

$$f'(x)=-\csc x\cot x$$

$$f'(\tfrac{\pi}{4})=-\csc(\tfrac{\pi}{4})\cot(\tfrac{\pi}{4})=\boxed{-\sqrt{2}}$$

344. *Answer is B.*

$$\lim_{h\to 0}\frac{\cos(\frac{\pi}{2}+h)-\cos\frac{\pi}{2}}{h}=$$

$$\lim_{h\to 0}\frac{\cos(\frac{\pi}{2}+h)-\cos\frac{\pi}{2}}{h}= \quad \leftarrow \text{ derivative of } f(x)=\cos x \text{ evaluated at } x=\tfrac{\pi}{2}$$

$$f'(x)=-\sin x$$

$$f'(\tfrac{\pi}{2})=-\sin(\tfrac{\pi}{2})=\boxed{-1}$$

345. *Answer is D.*

$$\lim_{h\to 0}\frac{\sec(\frac{\pi}{3}+h)-\sec(\frac{\pi}{3})}{h}=$$

$$\lim_{h\to 0}\frac{\sec(\frac{\pi}{3}+h)-\sec(\frac{\pi}{3})}{h}= \quad \leftarrow \text{ derivative of } f(x)=\sec x \text{ evaluated at } x=\tfrac{\pi}{3}$$

$$f'(x)=\sec x\tan x$$

$$f'(\tfrac{\pi}{3})=\sec(\tfrac{\pi}{3})\tan(\tfrac{\pi}{3})=\boxed{2\sqrt{3}}$$

346. *Answer is A.*

$$\lim_{h\to 0}\frac{\cot(\frac{5\pi}{6}+h)-\cot\frac{5\pi}{6}}{h}=$$

$$\lim_{h\to 0}\frac{\cot(\frac{5\pi}{6}+h)-\cot\frac{5\pi}{6}}{h}= \quad \leftarrow \text{ derivative of } f(x)=\cot x \text{ evaluated at } x=\tfrac{5\pi}{6}$$

$$f'(x)=-\csc^2 x$$

$$f'(\tfrac{5\pi}{6})=-\csc^2(\tfrac{5\pi}{6})=\boxed{-4}$$

347. *Answer is D.*

$$\lim_{k\to 0}\frac{\tan(\frac{\pi}{4}+k)-\tan(\frac{\pi}{4})}{k}=$$

$\lim_{k\to 0}\frac{\tan(\frac{\pi}{4}+k)-\tan(\frac{\pi}{4})}{k}=$ $\leftarrow$ derivative of $f(x)=\tan x$ evaluated at $x=\frac{\pi}{4}$

$$f'(x)=\sec^2 x$$

$$f'(\tfrac{\pi}{4})=\sec^2(\tfrac{\pi}{4})=\boxed{2}$$

348. *Answer is D.*

$$\lim_{h\to 0}\frac{\tan(\frac{\pi}{4}+h)-\tan(\frac{\pi}{4})}{h}=$$

$\lim_{h\to 0}\frac{\tan(\frac{\pi}{4}+h)-\tan(\frac{\pi}{4})}{h}=$ $\leftarrow$ derivative of $f(x)=\tan(x)$ evaluated at $x=\frac{\pi}{4}$

$$f'(x)=\sec^2(x)$$

$$f'(\tfrac{\pi}{4})=\left[\sec(\tfrac{\pi}{4})\right]^2=\left(\sqrt{2}\right)^2=\boxed{2}$$

349. *Answer is B.*

$$\lim_{h\to 0}\frac{\cos(\frac{\pi}{4}+h)-\cos\frac{\pi}{4}}{h}=$$

$\lim_{h\to 0}\frac{\cos(\frac{\pi}{4}+h)-\cos\frac{\pi}{4}}{h}=$ $\leftarrow$ derivative of $f(x)=\cos(x)$ evaluated at $x=\frac{\pi}{4}$

$$f'(x)=-\sin x$$

$$f'(\tfrac{\pi}{4})=-\sin(\tfrac{\pi}{4})=-\frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)=\boxed{\frac{-\sqrt{2}}{2}}$$

350. *Answer is A.*

$$\lim_{h\to 0}\frac{\sin(\frac{\pi}{2}+h)-\sin\frac{\pi}{2}}{h}=$$

$\lim_{h\to 0}\frac{\sin(\frac{\pi}{2}+h)-\sin\frac{\pi}{2}}{h}=$ $\leftarrow$ derivative of $f(x)=\sin(x)$ evaluated at $x=\frac{\pi}{2}$

$$f'(x)=\cos(x)$$

$$f'(\tfrac{\pi}{2})=\cos(\tfrac{\pi}{2})=\boxed{0}$$

351. *Answer is E.*

$$\lim_{h\to 0}\frac{\sin(\frac{\pi}{2}+h)-1}{h}=$$

$$\lim_{h\to 0}\frac{\sin(\frac{\pi}{2}+h)-1}{h}= \qquad \leftarrow \text{ derivative of } f(x)=\sin x \text{ evaluated at } x=\tfrac{\pi}{2}$$

$$f'(x)=\cos x$$

$$f'(\tfrac{\pi}{2})=\cos(\tfrac{\pi}{2})=\boxed{0}$$

$$f'\left(\frac{\pi}{2}\right), \text{ where } f(x)=\sin x$$

352. *Answer is C.*

If $g(x)=x+\cos x$ then $\lim_{h\to 0}\frac{g(x+h)-g(x)}{h}=$

$$\lim_{h\to 0}\frac{g(x+h)-g(h)}{h}= \qquad \leftarrow \text{ means the derivative of } g(x) \text{ at any point } x$$

$$g(x)=x+\cos x$$

$$g'(x)=\boxed{1-\sin x}$$

353. *Answer is B.*

$$\lim_{x\to\infty}\frac{3\sqrt{x^4-3x}}{2x^2+\cos x}=$$

$$\lim_{x\to\infty}\frac{3\sqrt{x^4-3x}}{2x^2+\cos x}=\lim_{x\to\infty}\frac{3\sqrt{\frac{x^4}{x^4}-\frac{3x}{x^4}}}{\frac{2x^2}{x^2}+\frac{\cos x}{x^2}}=\lim_{x\to\infty}\frac{3\sqrt{1-\frac{3}{x^3}}}{2+\frac{\cos x}{x^2}}=\frac{3\sqrt{1-0}}{2+0}=\boxed{\frac{3}{2}}$$

354. *Answer is E.*

$$\lim_{x\to 0}\frac{\sin x}{|x|}=$$

For $x>0$ $\quad \lim_{x\to 0^+}\frac{\sin x}{|x|}=\lim_{x\to 0^+}\frac{\sin x}{x}=+1$ $\quad \leftarrow$ squeeze theorem

For $x<0$ $\quad \lim_{x\to 0^-}\frac{\sin x}{|x|}=\lim_{x\to 0^-}\frac{\sin x}{|x|}=-1$

Since $\lim_{x\to 0^-}\frac{\sin x}{x}\neq\lim_{x\to 0^+}\frac{\sin x}{x} \Rightarrow \lim_{x\to 0}\frac{\sin x}{|x|}=\boxed{\text{does not exist}}$

355.

$\boxed{\textbf{Continuity}} \rightarrow$ A function $f(x)$ is said to be continuous at $x = c$ if

$\boxed{1}$ $f(c)$ is defined $\boxed{2}$ $\lim_{x \to c} f(x)$ exists $\boxed{3}$ $\lim_{x \to c} f(x) = f(c)$

If any of these conditions fails to be satisfied, the function is ***discontinuous*** at $x = c$

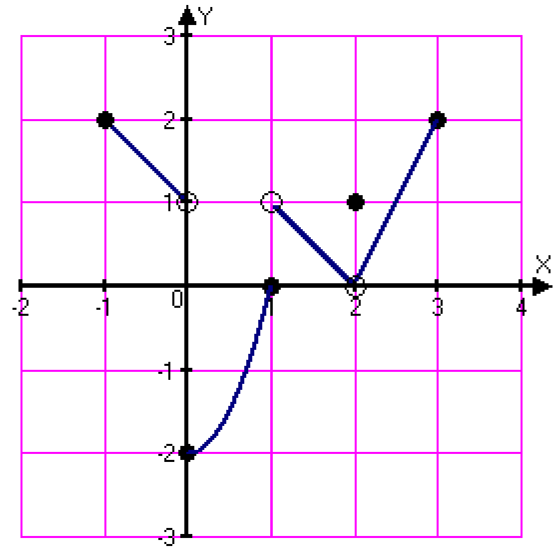
356. *Answer is A.*

The function f is shown in the graph and defined below:

$$f(x) = \begin{cases} 1 - x & -1 \le x < 0 \\ 2x^2 - 2 & 0 \le x \le 1 \\ -x + 2 & 1 < x < 2 \\ 1 & x = 2 \\ 2x - 4 & 2 < x \le 3 \end{cases}$$

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = 0$$

$$\therefore \lim_{x \to 2} f(x) = \boxed{0}$$



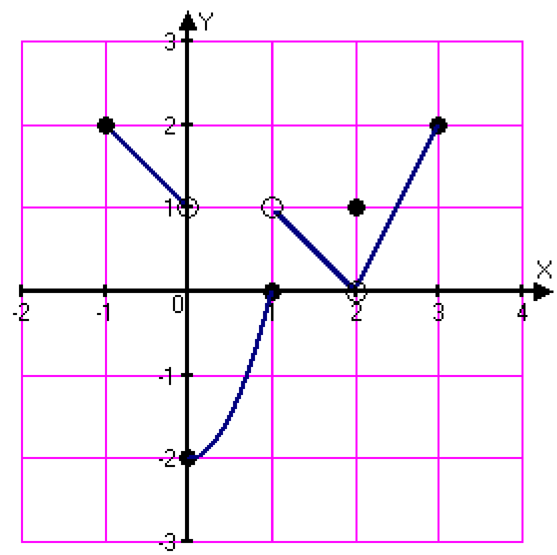
357. *Answer is E.*

The function f is shown in the graph and defined below:

$$f(x) = \begin{cases} 1 - x & -1 \le x < 0 \\ 2x^2 - 2 & 0 \le x \le 1 \\ -x + 2 & 1 < x < 2 \\ 1 & x = 2 \\ 2x - 4 & 2 < x \le 3 \end{cases}$$

The function f is defined on $[-1,\ 3]$ ☑

There is a value of f for all x values in the closed interval $[-1,\ 3]$



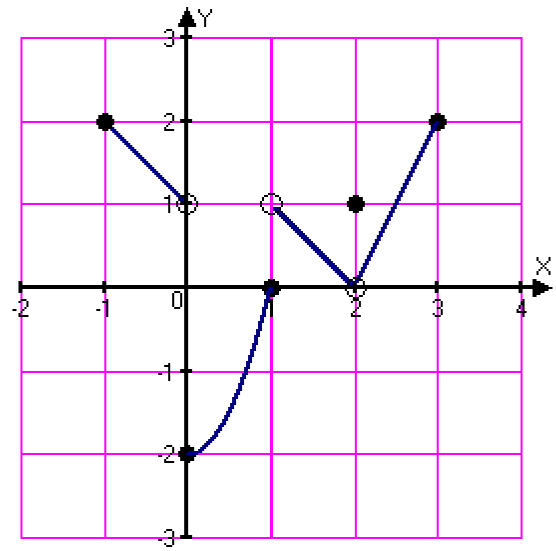
358. *Answer is C.*

The function f is shown in the graph and defined below:

$$f(x)=\begin{cases} 1-x & -1\le x<0 \\ 2x^2-2 & 0\le x\le 1 \\ -x+2 & 1<x<2 \\ 1 & x=2 \\ 2x-4 & 2<x\le 3 \end{cases}$$

The function has a ***removable*** discontinuity at $x=2$

(can be removed by redefining $f(2)=0$)



359. *Answer is B.*

The function f is shown in the graph and defined below:

$$f(x)=\begin{cases} 1-x & -1\le x<0 \\ 2x^2-2 & 0\le x\le 1 \\ -x+2 & 1<x<2 \\ 1 & x=2 \\ 2x-4 & 2<x\le 3 \end{cases}$$

On which of the following intervals is f continuous ?

$-1\le x\le 0$ ☒

jump discontinuity at $x=0$

$\boxed{0<x<1 \quad \textit{continuous}}$ ☑

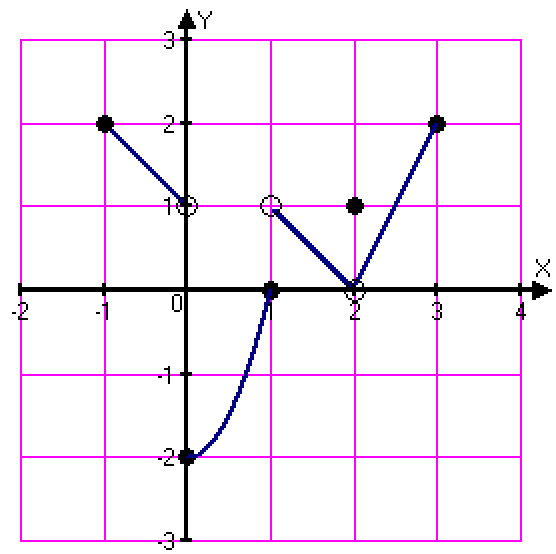
$1\le x\le 2$ ☒

jump discontinuity at $x=1$

removable discontinuity at $x=2$

$2\le x\le 3$ ☒

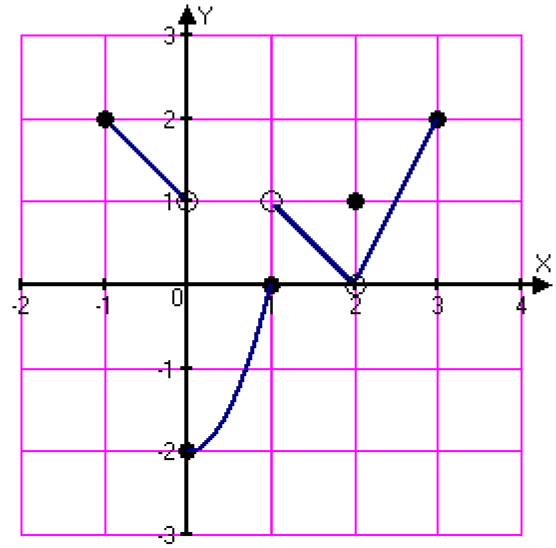
removable discontinuity at $x=2$



360. *Answer is B.*

The function f is shown in the graph and defined below:

$$f(x)=\begin{cases}1-x & -1\le x<0\\ 2x^2-2 & 0\le x\le 1\\ -x+2 & 1<x<2\\ 1 & x=2\\ 2x-4 & 2<x\le 3\end{cases}$$

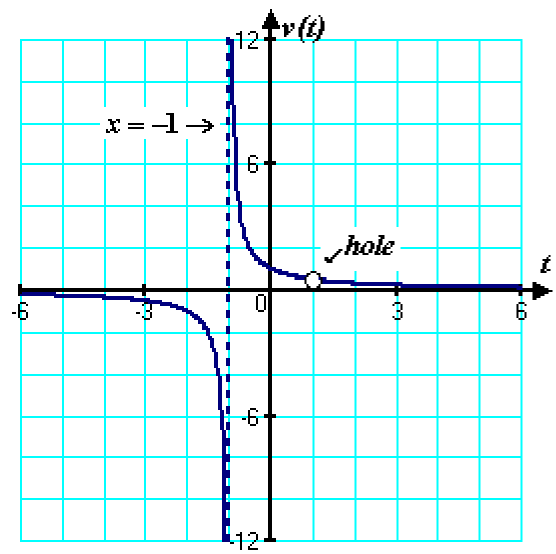


The function has a jump discontinuity at $\boxed{x=0}$ and $\boxed{x=1}$ which cannot be made continuous by redefining $f(0)$ or $f(1)$

Only $x=1$ was given as a choice to pick from.

361. *Answer is B.*

For what value(s) of x does $f(x)=\dfrac{x-1}{x^2-1}$ have a removable discontinuity ?



$$f(x)=\frac{x-1}{x^2-1}=\frac{\cancel{x-1}}{\cancel{(x-1)}(x+1)}=\frac{1}{x+1}$$

$$f(1)=\frac{1}{1+1}=\frac{1}{2}$$

Hole at $(1,\ \tfrac{1}{2})$ that can be filled by assigning

$f(1)=\dfrac{1}{2}$ (a removable discontinuity at $x=1$)

362. *Answer is A.*

363. *Answer is C.*

364. *Answer is E.*

365. *Answer is D.*

If f is continuous on $[\ 4,\ 7\ ]$, ***how many*** of the following statements ***must*** be true ?

I. f has a maximum value on $[\ 4,\ 7\ ]$ **III.** $f(7) > f(4)$

II. f has a minimum value on $[\ 4,\ 7\ ]$ **IV.** $\lim\limits_{x\to 6} f(x) = f(6)$

If f is continuous on $[\ 4,\ 7\ ]$ $\leftarrow$ ***closed interval*** includes end points

I. f has a maximum value on $[\ 4,\ 7\ ]$ ☑

II. f has a minimum value on $[\ 4,\ 7\ ]$ ☑

III. $f(7) > f(4)$ ☒ ***may*** be but need not be

IV. $\lim\limits_{x\to 6} f(x) = f(6)$ ☑ continuous

366. *Answer is C.*

If $f(x) = \begin{cases} 2x^2+3 & \text{if } x \ge 1 \\ g(x) & \text{if } x < 1 \end{cases}$, then f will be continuous at $x=1$ if $g(x) =$

$f(x) = 2x^2+3$

$f(1) = 2(1)^2 + 3 = 5$ $\Rightarrow$

	$\boxed{g(1)=5}$	
$g(x) = x$	$g(1) = 1$	☒
$g(x) = \cos(x+4)$	$g(1) = \cos 5$	☒
$g(x) = \boxed{6-x}$	$g(1) = 5$	☑
$g(x) = x^2+2$	$g(1) = 3$	☒
$g(x) = 2x^2-3$	$g(1) = -2$	☒

367. *Answer is B.*

Given that $f(x) = |x-3| + 2$, which one of the following statements is ***false*** ?

f is continuous at $x=3$ ***point*** $(\ 3,\ 2)$ ☑

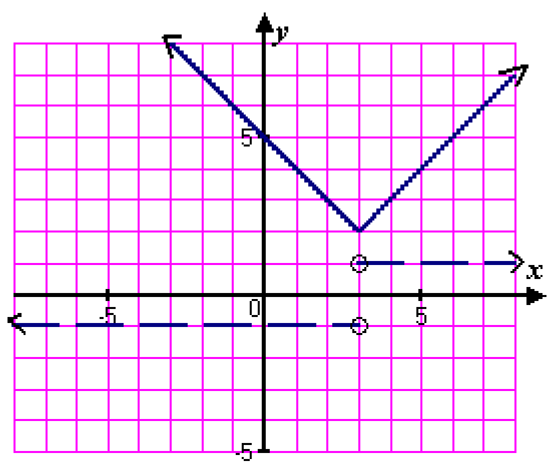
f is differentiable at $x=3$ ☒

$\boxed{\lim\limits_{x\to 3^-} f'(x) = -1} \ne \boxed{\lim\limits_{x\to 3^+} f'(x) = 1}$

$f'(5) = 1$ ☑

$f'(0) = -1$ ☑

$f(2) = f(4) = 3$ ☑



Dashed line y' has a discontinuity at $x = 3$

368. *Answer is B.*

369. *Answer is A.*

Which of the following is a point of discontinuity for $f(x)=\dfrac{x^2-4}{x^2+2x-3}$

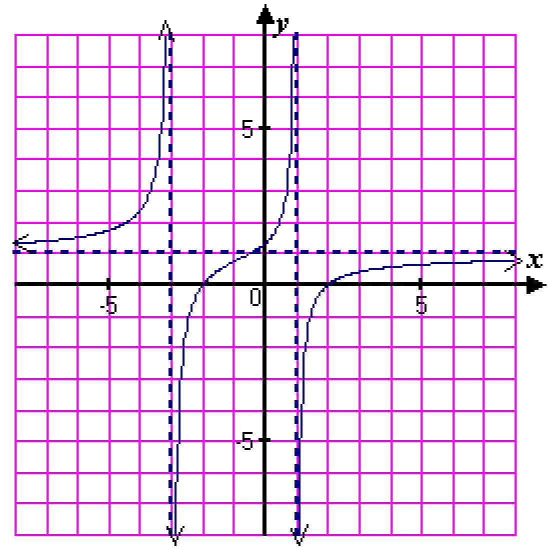
$$f(x)=\frac{x^2-4}{x^2+2x-3}=\frac{(x-2)(x+2)}{(x+3)(x-1)}$$

Horizontal asymptote $y=1$

$$\lim_{x\to\infty}\frac{x^2-4}{x^2+2x-3}=\frac{\frac{x^2}{x^2}-\frac{4}{x^2}}{\frac{x^2}{x^2}+\frac{2x}{x^2}-\frac{3}{x^2}}=\frac{1-0}{1+0-0}=1$$

Vertical asymptotes $x=1,-3$

(vertical asymptotes are nonremovable discontinuities)



370. *Answer is B.*

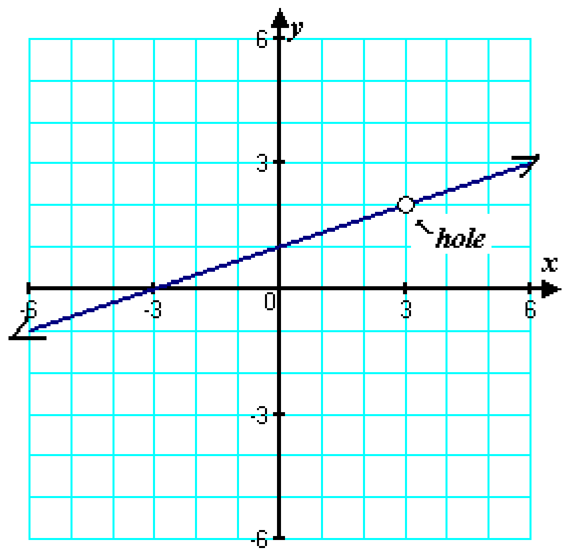
371. *Answer is C.*

The graph of $y=\dfrac{x^2-9}{3x-9}$ has

$$y=\frac{x^2-9}{3x-9}=\frac{(x+3)\cancel{(x-3)}}{3\cancel{(x-3)}}=\frac{(x+3)}{3}$$

$$y(3)=\frac{3+3}{3}=2$$

Hole at $(3,\ 2)$ in the straight line graph



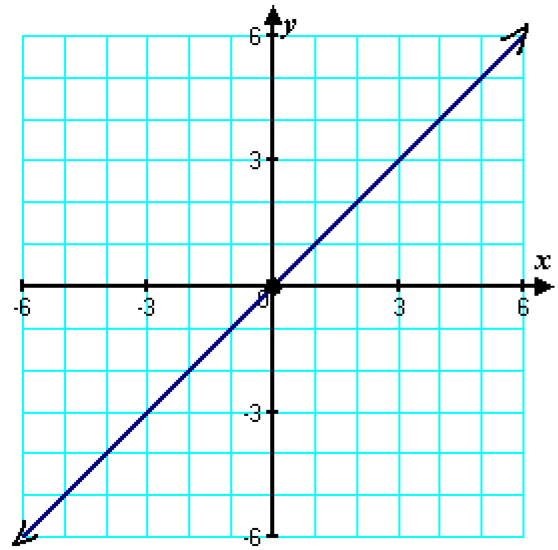
372. *Answer is A.*

The function $f(x)=\begin{cases} \dfrac{x^2}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

$f(x)=\dfrac{x^2}{x}=x$ when $x \neq 0$

Was a hole at $(0,\ 0)$ but was filled by assigning $f(0)=0$

Now continuous everywhere



373. *Answer is B.*

Suppose $f(x)=\begin{cases} f(x)=\dfrac{3x(x-1)}{x^2-3x+2} & \text{for } x \neq 1,2 \\ f(1)=-3 \\ f(2)=4 \end{cases}$ then $f(x)$ is continuous

$f(x)=\dfrac{3x(x-1)}{x^2-3x+2}=\dfrac{3x\cancel{(x-1)}}{\cancel{(x-1)}(x-2)}=\dfrac{3x}{x-2}$

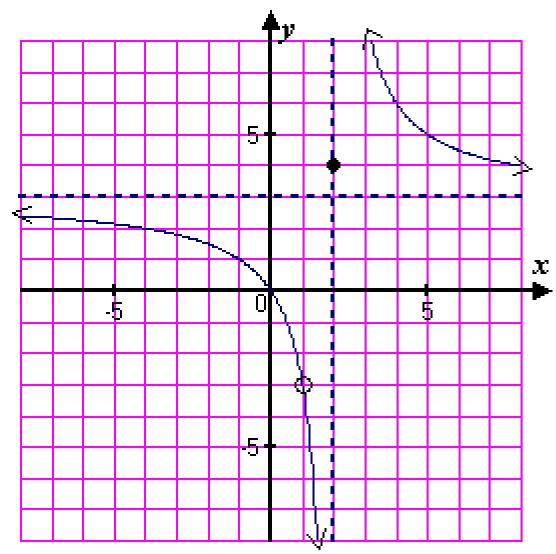
Horizontal asymptote at $x=3$

$\lim\limits_{x\to\infty}\dfrac{3x}{x-2}=\dfrac{\frac{3x}{x}}{\frac{x}{x}-\frac{2}{x}}=\dfrac{3}{3-0}=3$

Vertical asymptote at $x=2$

Hole at $(1,-3)$ filled by $f(1)=-3$

$f(2)=4$ fills the ***domain*** but there is still a nonremovable jump discontinuity at $x=2$



374. *Answer is D.*

375. *Answer is D.*

Let $f(x)=\begin{cases}\dfrac{x^2+x}{x} & \text{if } x\neq 0\\ 1 & \text{if } x=0\end{cases}$

Which of the following statements, **I**, **II**, and **III**, are true ?

I. $f(0)$ exists ☑

II. $\lim\limits_{x\to 0} f(x)$ exists ☑

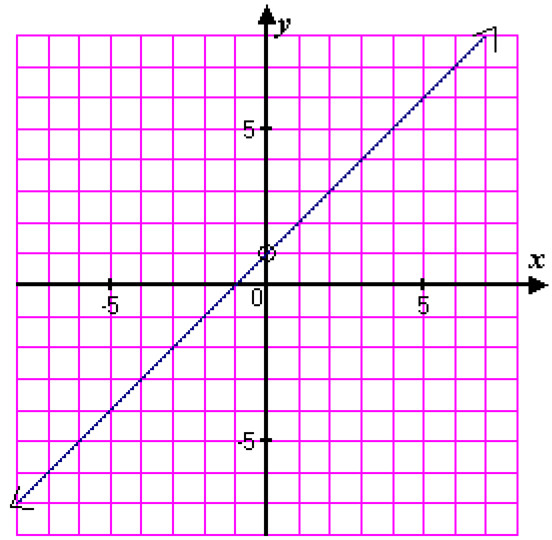
III. f is continuous at $x=0$ ☑

If $x\neq 0$ then

$$f(x)=\frac{x^2+x}{x}=\frac{x(x+1)}{x}=x+1$$

The graph of $f(x)$ is the line $y=x+1$ with a ***hole*** at $x=0$

Since $\boxed{f(0)=1}$ fills the hole $f(x)$ ***continuous***



376. *Answer is B.*

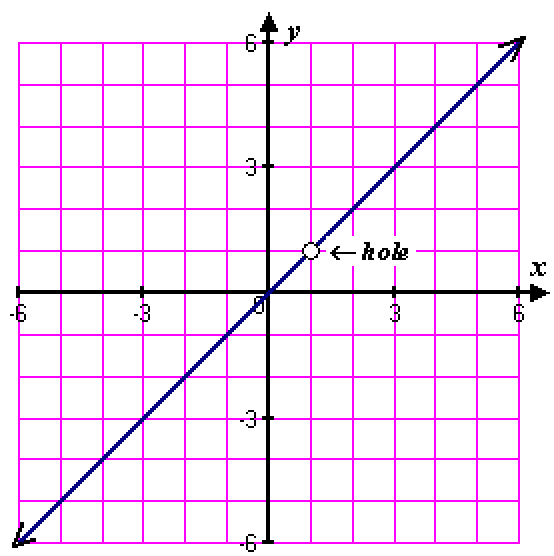
A function $f(x)$ equals $\dfrac{x^2-x}{x-1}$ for all x except $x=1$. In order that the function be continuous at $x=1$, the value of $f(1)$ must be

If $x\neq 1$ then

$$f(x)=\frac{x^2-x}{x-1}=\frac{x(x-1)}{x-1}=x$$

The graph of $f(x)$ is the line $y=x$ with a ***hole*** at $x=1$

If $\boxed{f(1)=1}$ then the function is ***continuous***



377. *Answer is D.*

If $f(x)=\begin{cases} x^2 & \text{for } x \le 1 \\ 2x-1 & \text{for } x > 1 \end{cases}$, then

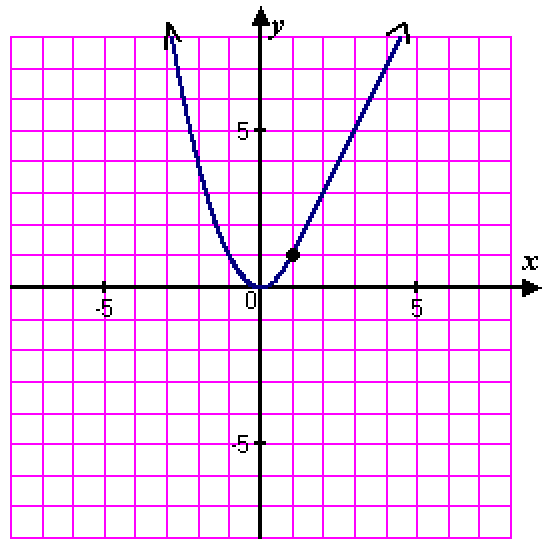
if $x \le 1$ then $f(x)=x^2$ and $f'(x)=2x$

$\rightarrow f(1)=1^2=1$ and $f'(1)=2(1)=2$

if $x>1$ then $f(x)=2x-1$ and $f'(x)=2$

$\rightarrow f(1)=2(1)-1=1$ and $f'(1)=2$

The piecewise function is continuous at $x=1$ and the derivative is also continuous

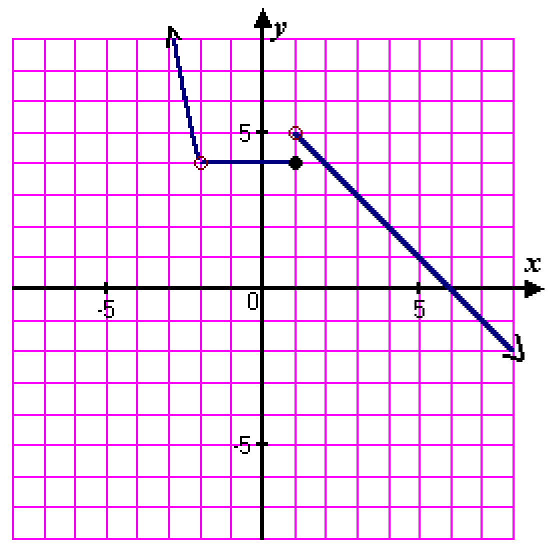


378. *Answer is C.*

Suppose $f(x)=\begin{cases} x^2 & \text{if } x<-2 \\ 4 & \text{if } -2<x\le 1 \\ 6-x & \text{if } x>1 \end{cases}$

Which statement is true ?

There is a hole at $x=-2$ and a jump discontinuity at $x=1$



379. *Answer is D.*

380. *Answer is B.*

Determine a value of k such that $f(x)$ is continuous, where $f(x)=\begin{cases} 3kx-5 & \text{for } x>2 \\ 4x-5k & \text{for } x\le 2 \end{cases}$

$$\text{Continuous at } x=2$$

$$3kx-5=4x-5k$$

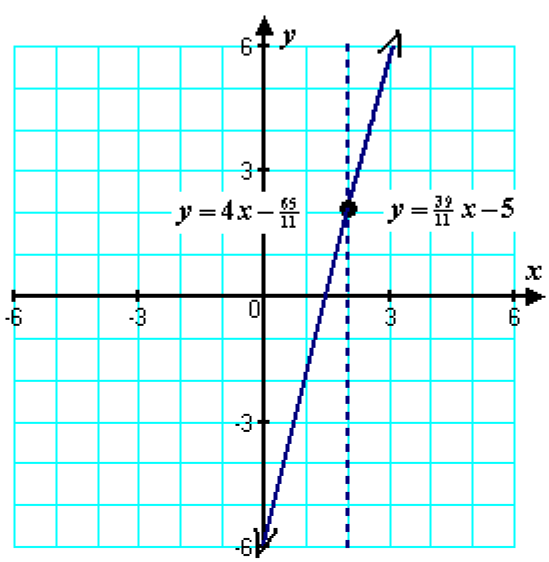
$$3k(2)-5=4(2)-5k$$

$$6k-5=8-5k$$

$$11k=13$$

$$\boxed{k=\tfrac{13}{11}}$$

Assigning $f(2)=\frac{13}{11}$ will make $f(x)$ continuous



381. *Answer is D.*

Find the value of k such that

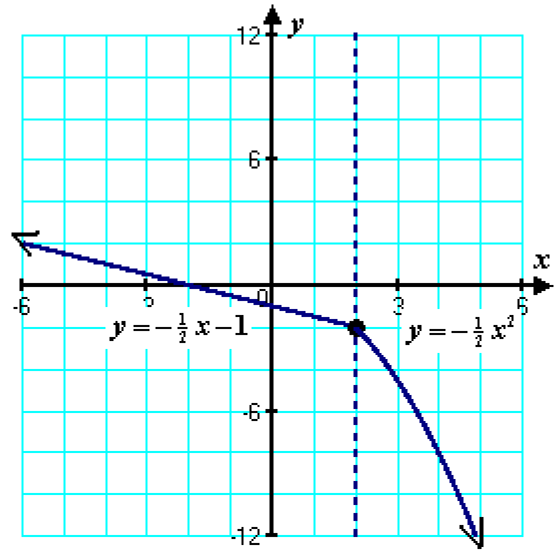
$$f(x)=\begin{cases} kx-1 & \text{for } x<2 \\ kx^2 & \text{for } x\ge 2 \end{cases}$$

is continuous for all real numbers.

Continuous at $x=2$

$$kx-1=kx^2$$
$$k(2)-1=k(2)^2$$
$$2k-1=4k$$
$$-1=2k$$
$$\boxed{-\tfrac{1}{2}=k}$$

Assigning $f(2)=-\frac{1}{2}$ will make $f(x)$ continuous at $x=2$



382. *Answer is E.*

383. *Answer is C.*

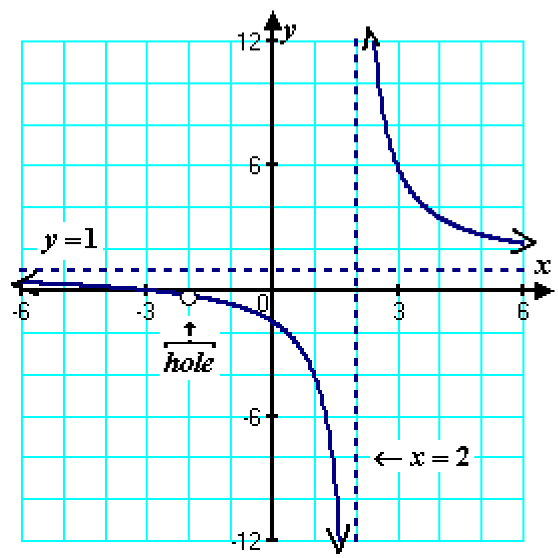
The function $f(x)=\dfrac{x^2+5x+6}{x^2-4}$ has

$$f(x)=\frac{(x+3)\cancel{(x+2)}}{(x-2)\cancel{(x+2)}}=\frac{x+3}{x-2}$$

$$f(-2)=\frac{-2+3}{-2-2}=\frac{-1}{4}$$

Hole at $(-2,-\frac{1}{4})$ removable

Vertical asymptote at $x=2$ nonremovable



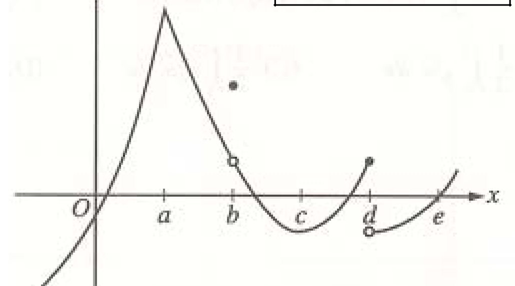
384. *Answer is A.*

The graph of a function f is shown. At which value of x is f continuous, but not differentiable ?

Sharp point a is continuous but has no derivative !

$$\lim_{x\to a^-} f'(x)\ne \lim_{x\to a^+} f'(x)$$

Difficulty $= \mathbf{0.75}$



Graph of f

385. *Answer is B.*

If $f(x)=\begin{cases}\dfrac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}, & \text{for } x\neq 2\\ k, & \text{for } x=2\end{cases}$ and if f is continuous at $x=2$, then $k=$

$$g(x)=\frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}$$

$$g(x)=\left(\frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}\right)\left(\frac{\sqrt{2x+5}+\sqrt{x+7}}{\sqrt{2x+5}+\sqrt{x+7}}\right)$$

$$g(x)=\frac{2x+5-(x+7)}{x-2\left(\sqrt{2x+5}+\sqrt{x+7}\right)}=\frac{\cancel{x-2}}{\cancel{x-2}\left(\sqrt{2x+5}+\sqrt{x+7}\right)}=\frac{1}{\sqrt{2x+5}+\sqrt{x+7}}$$

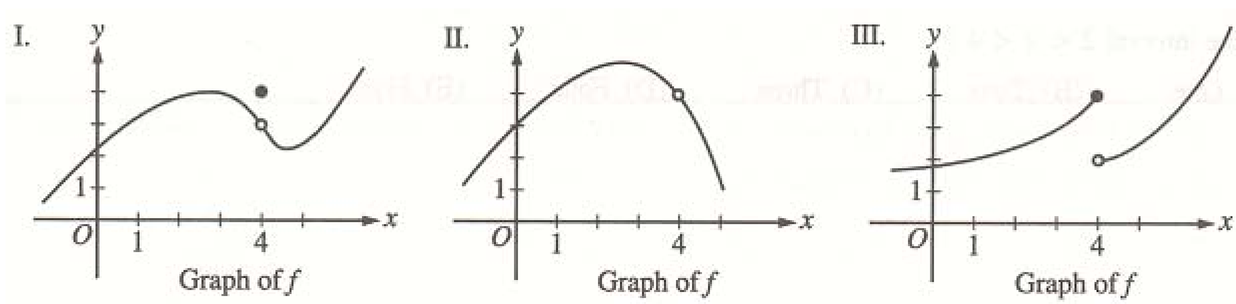
$$g(2)=\frac{1}{\sqrt{2(2)+5}+\sqrt{2+7}}=\frac{1}{\sqrt{9}+\sqrt{9}}=\frac{1}{3+3}=\boxed{\frac{1}{6}}$$

If $f(x)$ is continuous at $x=2$, then $k=\dfrac{1}{6}$

386. *Answer is D.*

Difficulty = **0.55**

For which of the following does $\lim\limits_{x\to 4} f(x)$ exist ?



I. $\lim\limits_{x\to 4} f(x)=3$ ☑ **II.** $\lim\limits_{x\to 4} f(x)=4$ ☑ **III.** $\lim\limits_{x\to 4} f(x)=$ ***does not exist*** ☒

387. *Answer is C.*

388. *Answer is E.*

Difficulty = **0.75**

The graph of the function f is shown in the diagram. Which of the following statements ***must*** be false ?

$f(a)$ exists ☑

$f(x)$ is defined for $0 < x < a$ ☑

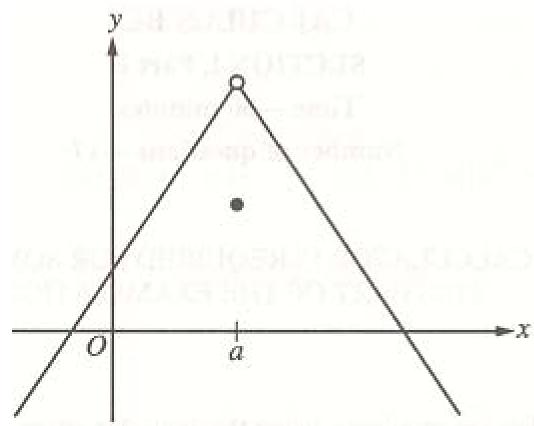
f is ***not*** continuous at $x = a$ ☑

$$\lim_{x \to a} f(x) \neq f(a)$$

$\lim_{x \to a} f(x)$ exists ☑

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$$

$\lim_{x \to a} f'(x)$ exists $\leftarrow$ ***false*** * * *
(derivative has a jump discontinuity)
slope $f(x)$ when $x < a$ is positive whereas
slope $f(x)$ when $x > a$ is negative



Graph of f

389. *Answer is D.*

390. *Answer is E.*

Difficulty = **0.53**

At $x = 3$ the function given by $f(x) = \begin{cases} x^2, & \text{for } x < 3 \\ 6x - 9, & \text{for } x \geq 3 \end{cases}$ is

Continuous at $x = 3$

$$x^2 = 6x - 9$$

$$3^2 = 6(3) - 9$$

$$9 = 9 \ ☑$$

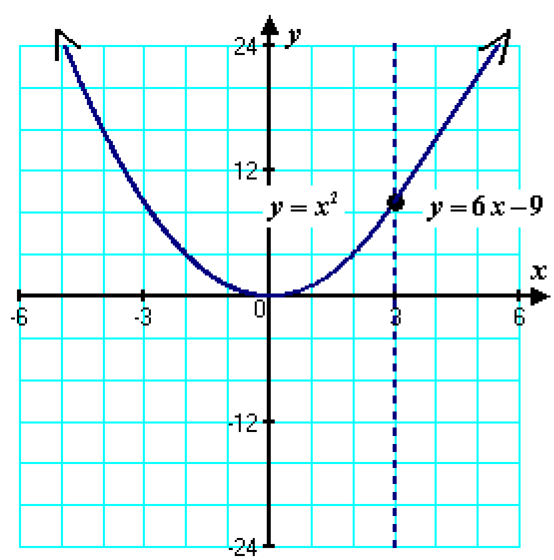
Differentiable at $x = 3$

$$2x = 6$$

$$2(3) = 6$$

$$6 = 6 \ ☑$$

$f(x) =$ is continuous ***and*** differentiable at $x = 3$

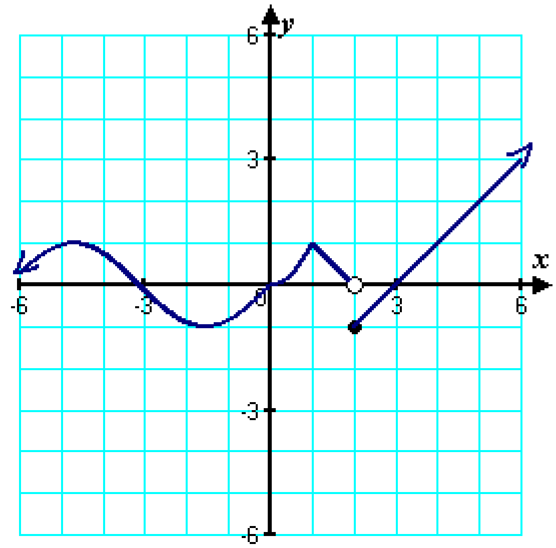


391. *Answer is C.*

Let f be the function defined by the following

$$f(x)=\begin{cases}\sin x & \text{for } x<0\\ x^2 & \text{for } 0\le x<1\\ 2-x & \text{for } 1\le x<2\\ x-3 & \text{for } x\ge 2\end{cases}$$

For what values of x is f **NOT** continuous ?



$\lim\limits_{x\to 2^-} = 0 \qquad \lim\limits_{x\to 2^+} = -1$

$\lim\limits_{x\to 2^-} \neq \lim\limits_{x\to 2^+}$

$\lim\limits_{x\to 2} = \textit{undefined}$ (jump discontinuity)

392. *Answer is C.*

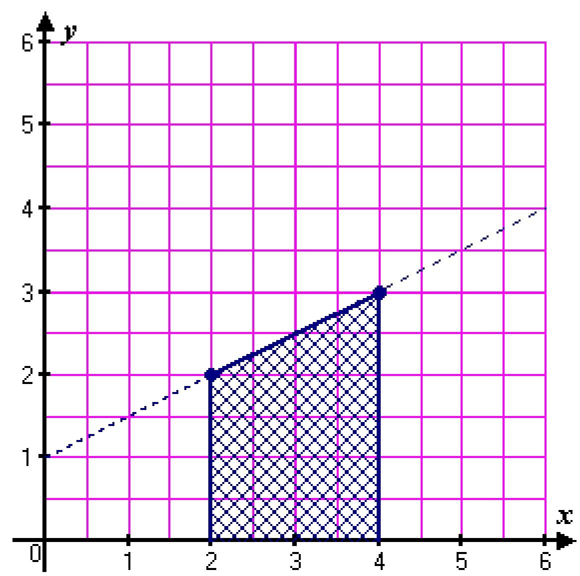
Difficulty = **0.40**

393. *Answer is B.*

Difficulty = **0.52**

If f is ***continuous*** for $a\le x\le b$ and ***differentiable*** for $a<x<b$, which of the following ***could be*** false ?

Sketch a graph that fits all conditions but one (any linear function with a non-zero slope)



394. *Answer is E.*

Difficulty = **0.52**

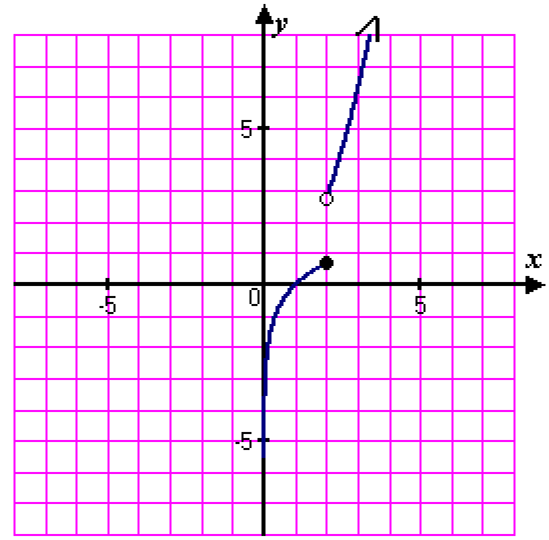
If $f(x)=\begin{cases}\ln x & \text{for } 0<x\le 2\\ x^2\ln 2 & \text{for } 2<x\le 4\end{cases}$, then $\lim\limits_{x\to 2} f(x)=$

$\lim\limits_{x\to 2^-}\ln x=\ln 2$

$\lim\limits_{x\to 2^+}x^2\ln 2=4\ln 2$

$\lim\limits_{x\to 2} f(x)=$ ***does not exist***

(Limit from the ***left*** and the ***right*** are differrent)



395. *Answer is D.*

Difficulty = **0.74**

Let f be the function given by $f(x)=|x|$
Which of the following statements about f are true ? **I.** f is continuous at $x=0$
II. f is differentiable at $x=0$
III. f has an absolute minimum at $x=0$

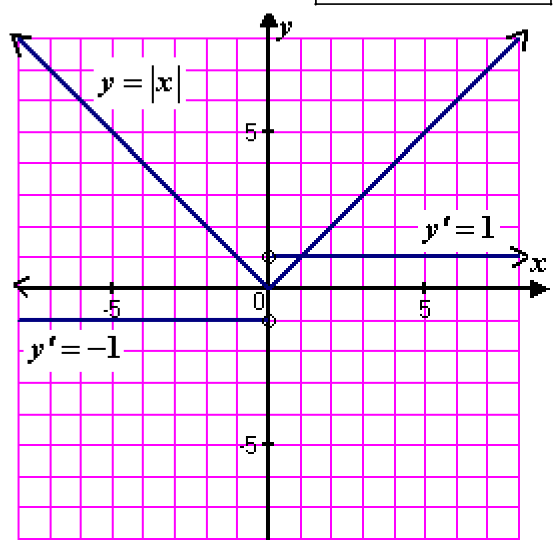
I. f is continuous at $x=0$ $f(0)=|0|=0$ ☑

II. f is differentiable at $x=0$ ☒

$\left(\lim\limits_{x\to 0^-} y'=-1 \;\neq\; \lim\limits_{x\to 0+} y'=1\right)$

III. f has an absolute minimum at $x=0$ ☑

See the graph of $f(x)=|x|$



396. *Answer is A.*

Difficulty = **0.61**

The graph of a function f is shown in the diagram.
Which of the following statements about f is false ?

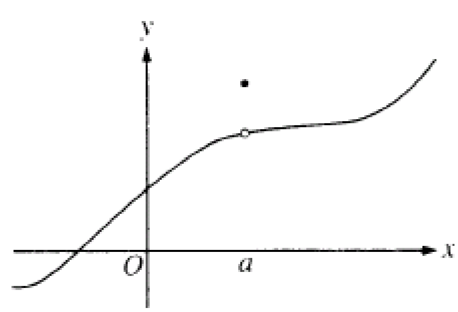
f is continuous at $x=a$ ☒ ← hole

f has a relative maximum at $x=a$ ☒

$x=a$ is in the domain of f ☑

$\lim\limits_{x\to a^+} f(x)$ is equal to $\lim\limits_{x\to a^-} f(x)$ ☑

$\lim\limits_{x\to a} f(x)$ exists ☑



397. *Answer is D.*

Let f be defined as follows, where $a \neq 0$

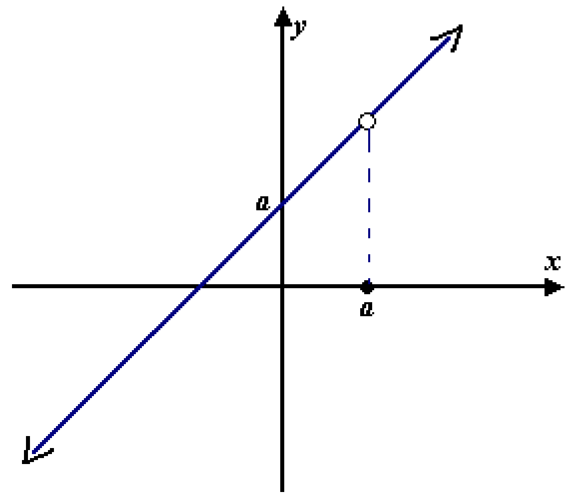
$$f(x) = \begin{cases} \dfrac{x^2 - a^2}{x - a} & \text{for } a \neq 0 \\ 0 & \text{for } x = a \end{cases}$$

Which of the following are true about f

I. $\lim\limits_{x \to a} f(x)$ exists

II. $f(a)$ exists

III. $f(x)$ is continuous at $x = a$



$$f(x) = \frac{(x+a)\cancel{(x-a)}}{\cancel{x-a}} = x + a$$

Hole at $(a,\ 2a)$ and $f(a) = 0$

I. $\lim\limits_{x \to a} f(x) = 2a$ ☑

II. $f(a) = 0$ ☑

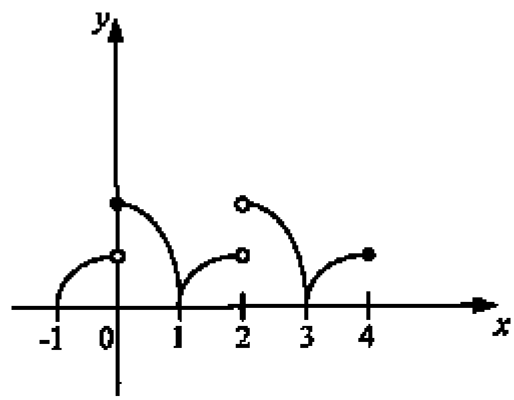
III. $f(x)$ is continuous at $x = a$ ☒

$$\lim_{x \to a} f(x) \neq f(a)$$

398. *Answer is D.*

The function shown is ***defined*** on the closed interval $-1 \leq x \leq 4$ for

$$f(2) = \textit{undefined}$$



399. *Answer is C.*

Let $f(x)=\begin{cases}\dfrac{x^3+8}{x+2}, & \text{if } x\neq -2\\ 4, & \text{if } x=-2\end{cases}$

Which of the following four statements are true ?

$$f(x)=\frac{\cancel{(x+2)}(x^2-2x+4)}{\cancel{x+2}}=x^2-2x+4$$

Hole at $(-2,\ 12)$

I. $f(x)$ is defined at $x=-2$ ☑

$f(-2)=4$

II. $f(x)$ is continuous at $x=-2$ ☒

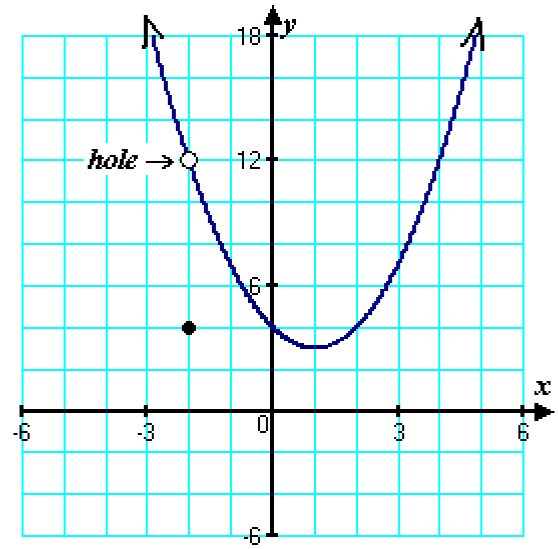
$\lim\limits_{x\to -2} f(x)\neq f(-2)$

III. $\lim\limits_{x\to -2} f(x)$ exists ☑

$\lim\limits_{x\to -2} f(x)=12$

IV. $f(x)$ is differentiable at $x=-2$ ☒

$f(x)$ is **NOT** continuous at $x=-2$



400. *Answer is C.*

For what value of k is the function

$y=\begin{cases}x+k & \text{if } x<2\\ x^2+4 & \text{if } x\geq 2\end{cases}$

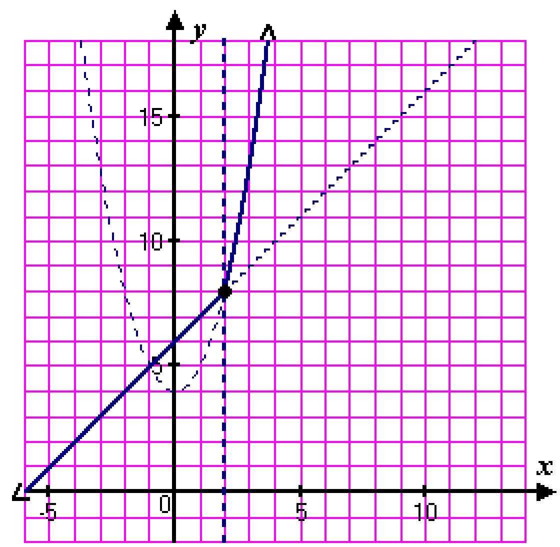
continuous at $x=2$

Continuous at $x=2$

$x+k=x^2+4$

$2+k=2^2+4$

$k=8-2=\boxed{6}$



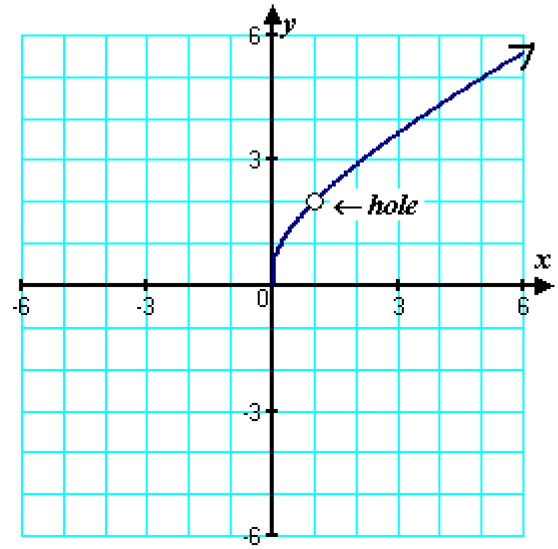
401. *Answer is E.*

The function $f(x)=\dfrac{2x-2}{\ln x}$ is defined for all $x>0$ except $x=1$. The value that must be assigned to $f(1)$ to make $f(x)$ continuous at $x=1$ is

$$\lim_{x\to1}\frac{2x-2}{\ln x}=\frac{0}{0} \quad \leftarrow \text{indeterminant form}$$

$$\lim_{x\to1}\frac{2}{\frac{1}{x}}=\frac{2}{1}=2 \quad \leftarrow \text{L'hopitals rule}$$

$f(1)=2$ will make $f(x)$ continuous at $x=1$



402. *Answer is C.*

If the function defined by

$$f(x)=\begin{cases} x^3 & \text{if } x\le 2 \\ 2x+k & \text{if } x>2 \end{cases}$$

is continuous at $x=2$, then $k=$

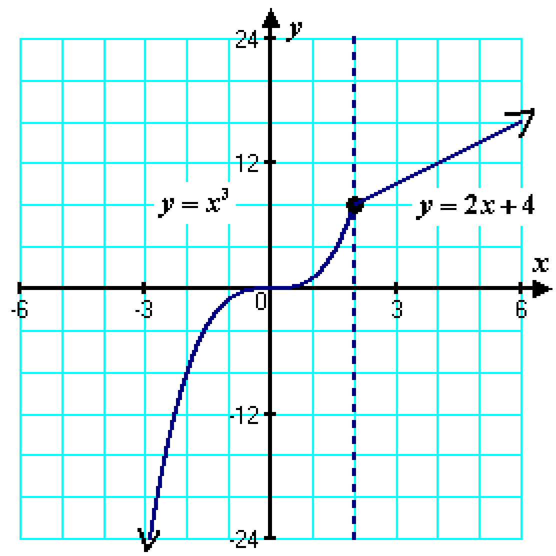
Continuous at $x=2$

$$x^3=2x+k$$

$$2^3=2(2)+k$$

$$8=4+k$$

$$\boxed{4=k}$$



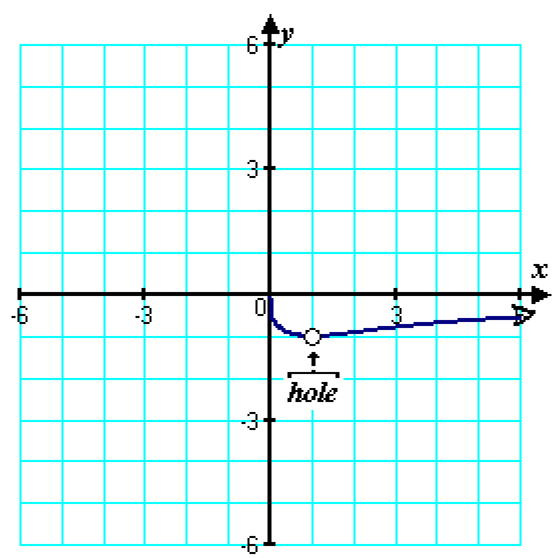
403. *Answer is A.*

The function $f(x)=\dfrac{e^{-x+1}-1}{\ln x}$ is defined for all $x>0$ except $x=1$ The value that must be assigned to $f(1)$ to make $f(x)$ continuous at $x=1$ is

$$\lim_{x\to1}\frac{e^{-x+1}-1}{\ln x}=\frac{e^{-1+1}-1}{\ln 1}=\frac{0}{0} \quad \leftarrow \text{indeterminant}$$

$$\lim_{x\to1}\frac{-e^{-x+1}}{\frac{1}{x}}=\frac{-e^{-1+1}}{\frac{1}{1}}=-1 \quad \leftarrow \text{L'hopitals rule}$$

Assigning $f(1)=-1$ makes $f(x)$ continuous at $x=1$



404. *Answer is E.*

The function $f(x)=\dfrac{x^3-8}{x-2}$ is not defined at $x=2$ Which of the following expressions for $f(2)$ can be used to make $f(x)$ continuous at $x=2$

$$f(x)=\frac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{x-2}}=x^2+2x+4$$

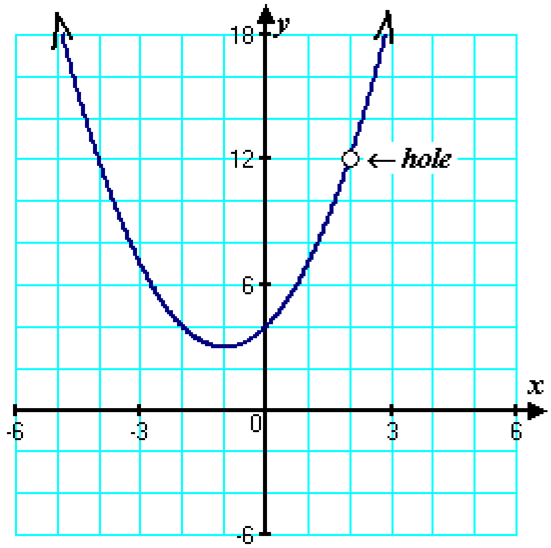
$$f(2)=x^2+2x+4=12$$

Assigning $f(2)=12$ makes $f(x)$ continuous at $x=2$

I. $f(2)=12$ ☑

II. $f(2)=\lim\limits_{x\to 2} f(x)$ ☑

III. $f(2)=\lim\limits_{x\to 2^-} f(x)$ ☑



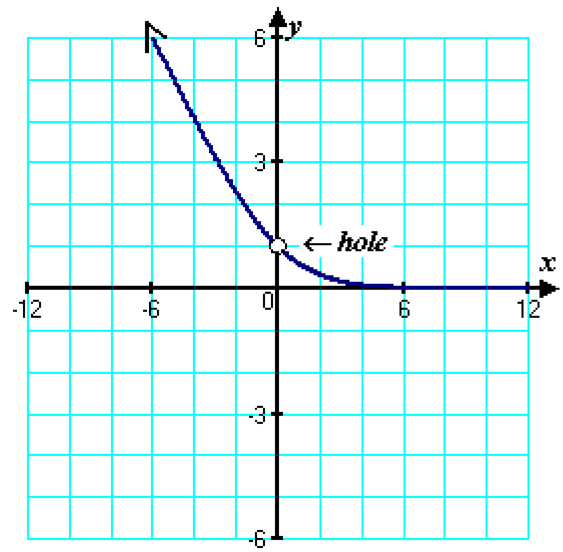
405. *Answer is D.*

What value should be assigned to $f(x)=\dfrac{x}{e^x-1}$ at $x=0$ to make $f(x)$ continuous at $x=0$

$$\lim_{x\to 0}\frac{x}{e^x-1}=\frac{0}{e^0-1}=\frac{0}{0} \leftarrow \text{indeterminant}$$

$$\lim_{x\to 0}\frac{1}{e^x}=\frac{1}{1}=1 \leftarrow \text{L'hopitals rule}$$

$f(0)=1$ to make $f(x)$ continuous at $x=0$



406. *Answer is C.*

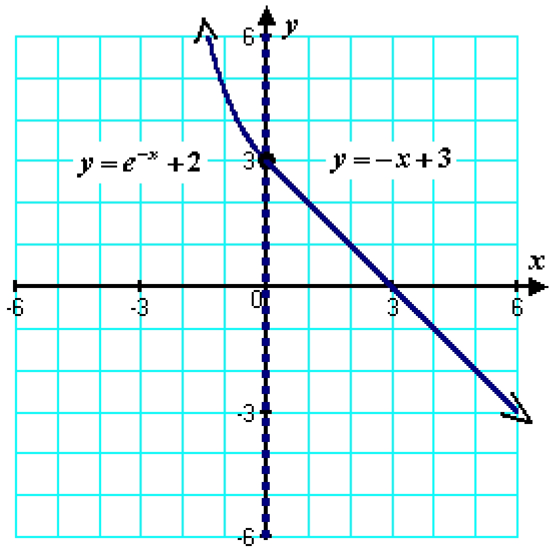
If $f(x) = \begin{cases} e^{-x} + 2 & \text{for } x < 0 \\ ax + b & \text{for } x \geq 0 \end{cases}$ is ***differentiable*** at $x = 0$ then $a + b =$

Continuous at $x = 0$

$$e^{-x} + 2 = ax + b$$
$$e^{0} + 2 = a(0) + b$$
$$3 = b$$

Differentiable at $x = 0$

$$-e^{-x} = a$$
$$-e^{0} = a$$
$$-1 = a$$
$$a + b = -1 + 3 = \boxed{2}$$



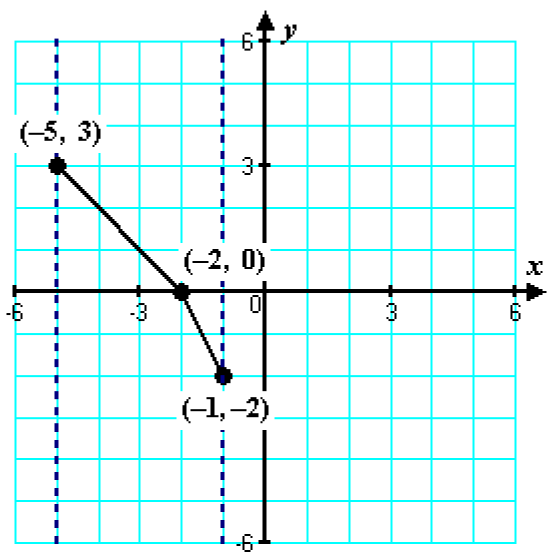
407. *Answer is B.*

Suppose that f is a ***continuous*** function defined for all real numbers x with $f(-5) = 3$ and $f(-1) = -2$ If $f(x) = 0$ for one and only one value of x, then which of the following could be x

$$f(-5) = \underbrace{\boxed{3}}_{positive} \qquad \underbrace{f(-2) = 0}_{zero} \qquad \underbrace{\boxed{-2}}_{negative} = f(-1)$$

Function is ***continuous*** and to go from a positive $+3$ to a negative -2 it must have passed through $f(x) = 0$ somewhere in interval $(-5, -1)$

$$-5 < \boxed{-2} < -1 \qquad -7 \quad \overbrace{\boxed{-2}}^{\uparrow} \quad 0 \quad 1 \quad 2$$



408. *Answer is C.*

If $f(x)=\begin{cases} x^2+2 & \text{for } x\le 1 \\ 2x+1 & \text{for } x>1 \end{cases}$ then $f'(1)=$

Continuous at $x=1$

$$x^2+2=2x+1 \quad \text{at } x=1$$

$$1^2+2=2(1)+1$$

$$3=3 \ ☑$$

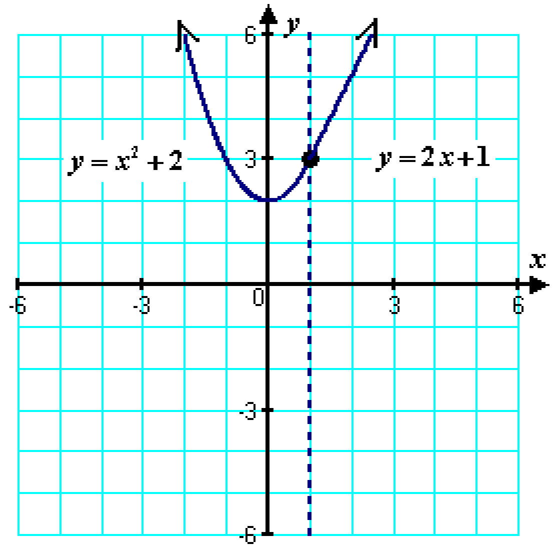
Differentiable at $x=1$

$$2x=2$$

$$2(1)=2$$

$$2=2 \ ☑$$

$$\boxed{f'(1)=2}$$



409. *Answer is C.*

If f is a continuous function defined by

$f(x)=\begin{cases} x^2+bx & \text{for } x\le 5 \\ 5\sin(\frac{\pi}{2}x) & \text{for } x>5 \end{cases}$ then $b=$

Continuous at $x=5$

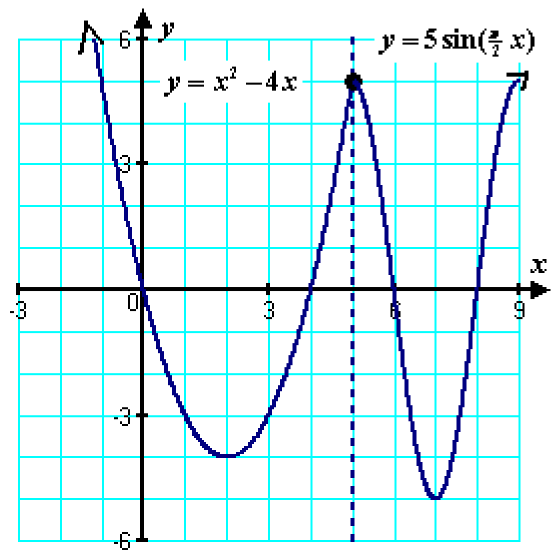
$$x^2+bx=5\sin(\tfrac{\pi}{2}x)$$

$$(5)^2+b(5)=5\sin(\tfrac{5\pi}{2})$$

$$25+5b=5$$

$$5b=-20$$

$$\boxed{b=-4}$$



410. *Answer is B.*

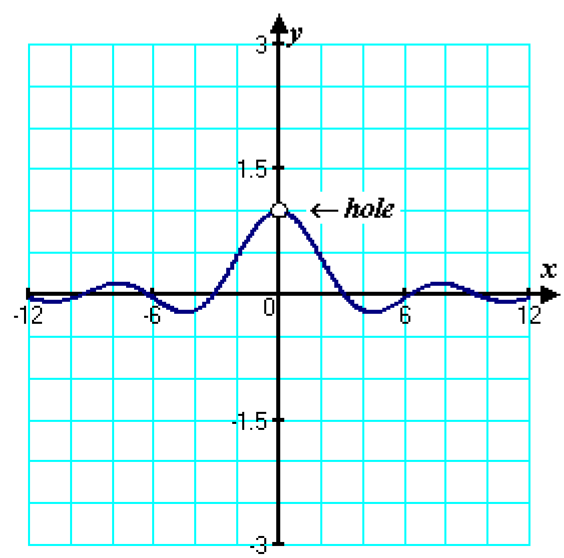
Consider the function $f(x)=\begin{cases} \frac{\sin x}{x} & \text{for } x\ne 0 \\ k & \text{for } x=0 \end{cases}$

In order for $f(x)$ to be continuous at $x=0$, the value of k must be

$$\lim_{x\to 0}\frac{\sin x}{x}=1 \quad \leftarrow \text{squeeze theorem}$$

Assigning $f(x)=1$ will make $f(x)$ continuous

at $x=1 \quad \rightarrow \quad \boxed{1=k}$



411. *Answer is E.*

412. *Answer is D.*

> Consider the function f defined on $\frac{\pi}{2} \le x \le \frac{3\pi}{2}$ by $f(x) = \frac{\tan x}{\sin x}$ for all $x \ne \pi$ If f is continuous at $x = \pi$ then $f(\pi) =$

Continuous at $x = \pi$

$$\lim_{x\to\pi} \frac{\tan x}{\sin x} = \lim_{x\to\pi} \frac{\tan \pi}{\sin \pi} = \frac{0}{0} \leftarrow \text{indeterminant form}$$

$$\lim_{x\to\pi} \frac{\frac{\sin x}{\cos x}}{\frac{\sin x}{1}} = \lim_{x\to\pi} \frac{\sin x}{\cos x}\left(\frac{1}{\sin x}\right) = \lim_{x\to\pi} \frac{1}{\cos x} = \lim_{x\to\pi} \frac{1}{\cos \pi} = \frac{1}{-1} = \boxed{-1}$$

$$f(\pi) = -1$$

413. *Answer is D.*

> If the function f is continuous for all positive real numbers and if $f(x) = \frac{\ln x^2 - x\ln x}{x-2}$ when $x \ne 2$ then $f(2) =$

Continuous at $x = 2$

$$\lim_{x\to 2} \frac{\ln x^2 - x\ln x}{x-2} = \lim_{x\to 2} \frac{\ln 4 - 2\ln 2}{2-2} = \lim_{x\to 2} \frac{\ln 4 - \ln 2^2}{2-2} = \frac{0}{0} \leftarrow \text{indeterminant form}$$

$$\lim_{x\to 2} \frac{\frac{2}{x} - \left[x\left(\frac{1}{x}\right) + \ln x\right]}{1} = \frac{\frac{2}{2} - 2\left(\frac{1}{2}\right) - \ln 2}{1} = \frac{1 - 1 - \ln 2}{1} = \boxed{-\ln 2}$$

$$f(2) = 0$$

414. *Answer is C.*

415. *Answer is A.*

> Let f be defined by $f(x) = \begin{cases} \frac{x^2-2x+1}{x-1} & \text{for } x \ne 1 \\ k & \text{for } x = 1 \end{cases}$
> Determine the value of k for which f is continuous for all real x

Continuous at $x = 1$

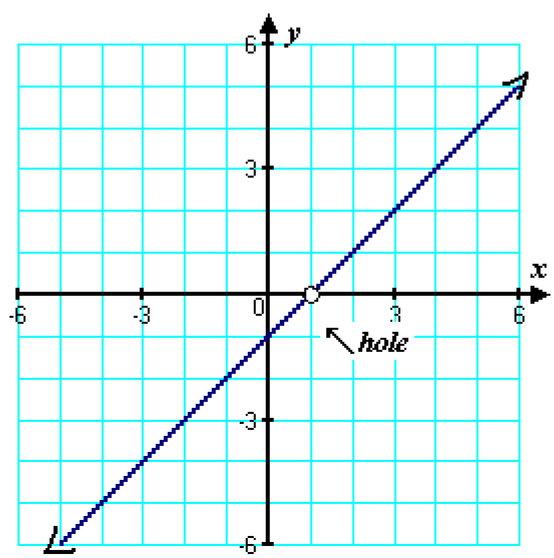
$$\lim_{x\to 1} \frac{\cancel{(x-1)}(x-1)}{\cancel{x-1}} = \frac{0}{0} \leftarrow \text{indeterminant form}$$

$$\lim_{x\to 1} x - 1 = 0$$

$$f(1) = 0$$

If $k = 0$ then it fills the hole at $x = 1$

for $f(x) = \frac{x^2-2x+1}{x-1}$



416. *Answer is C.*

417. *Answer is B.*

If f is continuous at $x = 2$, and if

$$f(x) = \begin{cases} \dfrac{\sqrt{x+2} - \sqrt{2x}}{x-2} & \text{for } x \neq 2 \\ k & \text{for } x = 2 \end{cases} \quad \text{then } k =$$

Continuous at $x = 2$

$$\lim_{x \to 2} \frac{\sqrt{x+2} - \sqrt{2x}}{x-2} = \frac{0}{0} \quad \leftarrow \text{indeterminant form}$$

$$\lim_{x \to 2} \frac{\sqrt{x+2} - \sqrt{2x}}{x-2} \left(\frac{\sqrt{x+2} + \sqrt{2x}}{\sqrt{x+2} + \sqrt{2x}} \right) = \lim_{x \to 2} \frac{x+2-2x}{(x-2)(\sqrt{x+2} + \sqrt{2x})}$$

$$= \lim_{x \to 2} \frac{-\cancel{(x-2)}}{\cancel{(x-2)}(\sqrt{x+2} + \sqrt{2x})} = \frac{-1}{\sqrt{x+2} + \sqrt{2x}} = \frac{-1}{4}$$

$$k = \boxed{\frac{-1}{4}}$$

418. *Answer is D.*

Which of the following values for k makes the function $f(x) = \begin{cases} \ln(x+k) & \text{for } 0 < x < 3 \\ \cos(kx) & \text{for } x \leq 0 \end{cases}$ continuous at $x = 0$

Continuous at $x = 0$

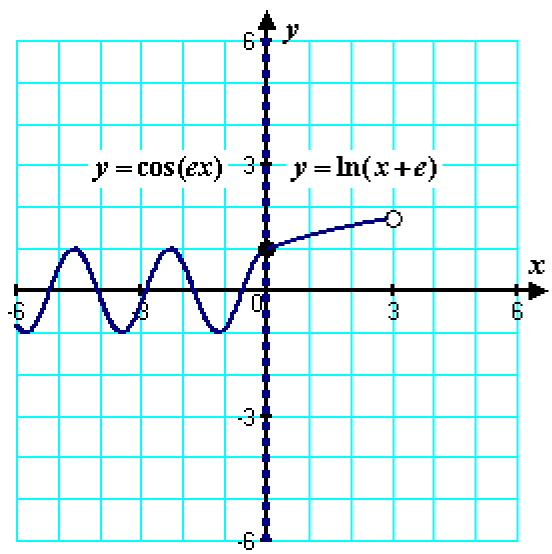
$$\ln(x+k) = \cos(kx)$$

$$\ln(0+k) = \cos(k(0))$$

$$\ln(k) = \cos(0)$$

$$\ln(k) = 1$$

$$\boxed{k = e}$$

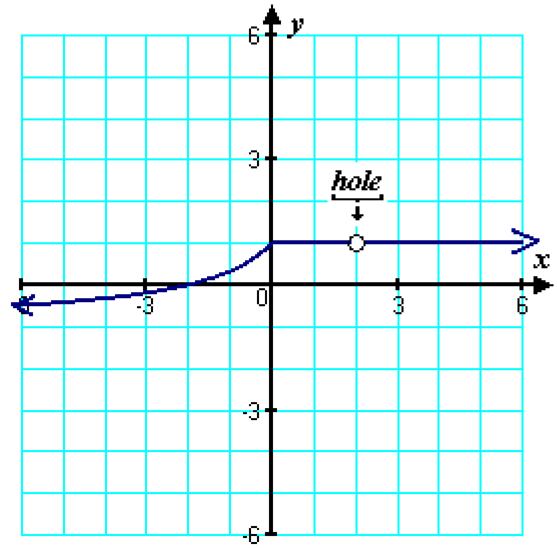


419. *Answer is C.*

420. *Answer is B.*

421. *Answer is D.*

If $f(x)=\begin{cases}\dfrac{|x|-2}{x-2} & \text{for } x\neq 2\\ k & \text{for } x=2\end{cases}$ then the value of k for which $f(x)$ is continuous for all real values of x is $k=$



Continuous at $x=2$

$$\lim_{x\to 2}\frac{|x|-2}{x-2}=\lim_{x\to 2}\frac{x-2}{x-2}=\frac{0}{0} \leftarrow \text{indeterminant}$$

$$\lim_{x\to 2}\frac{1}{1}=1$$

$$\boxed{k=1}$$

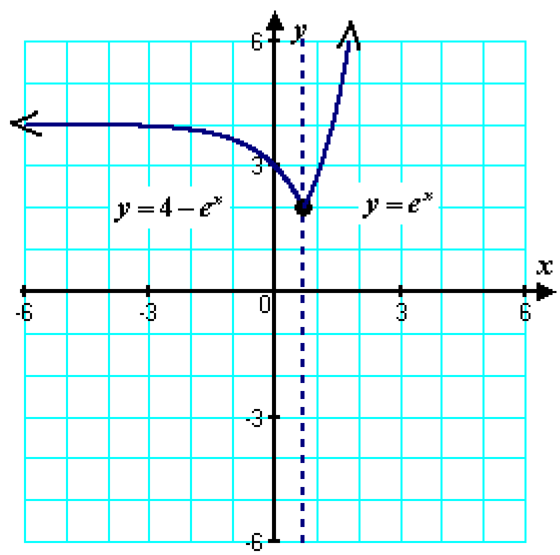
If $k=1$ then is fills the hole in $f(x)=\dfrac{|x|-2}{x-2}$

and makes it continuous

422. *Answer is D.*

What is the $\lim\limits_{x\to\ln 2} g(x)$ given

$$g(x)=\begin{cases}e^x & \text{if } x>\ln 2\\ 4-e^x & \text{if } x\leq \ln 2\end{cases}$$



Continuous at $x=\ln 2$

$$\lim_{x\to\ln 2^-} 4-e^x=\lim_{x\to\ln 2^+} e^x$$

$$\lim_{x\to\ln 2^-} 4-e^{\ln 2}=\lim_{x\to\ln 2^+} e^{\ln 2}$$

$$4-e^{\ln 2}=e^{\ln 2}$$

$$4-2=2$$

$$\boxed{2=2}$$

$$\lim_{x\to\ln 2} g(x)=2$$

423. *Answer is A.*

424. *Answer is D.*

If $\lim\limits_{x\to a} f(x)=\mathrm{L}$ where L is a real number, which of the following ***must*** be true ?

I. $f(a)=\mathrm{L}$ **II.** $\lim\limits_{x\to a^-} f(x)=\mathrm{L}$ **III.** $\lim\limits_{x\to a^+} f(x)=\mathrm{L}$

$\lim\limits_{x\to a} f(x)=\mathrm{L}$ exists if and only if $\lim\limits_{x\to a^-} f(x)=\mathrm{L}$ ☑ ***and*** $\lim\limits_{x\to a^+} f(x)=\mathrm{L}$ ☑

$f(a)=\mathrm{L}$ $\underbrace{\textit{may}\text{ be true but need not be}}_{\textit{must}\text{ be true } ☒}$

425. *Answer is E.*

426. *Answer is A.*

427. *Answer is A.*

428. *Answer is C.*

429. *Answer is E.*

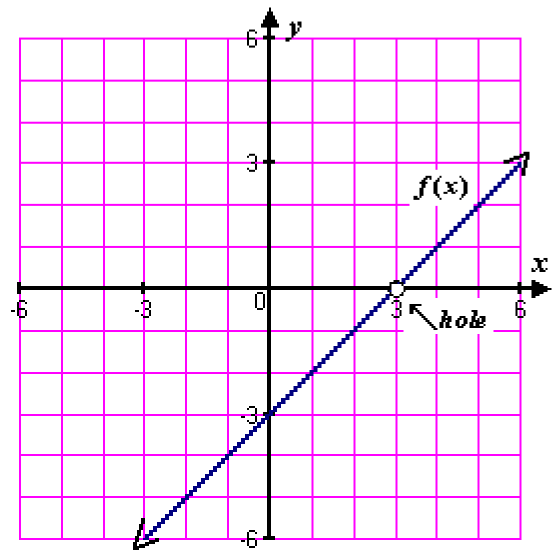
A function $f(x)$ is equal to $\dfrac{x^2-6x+9}{x-3}$ for all $x>0$ except $x=3$. In order for the function to be continuous at $x=3$, what must the value of $f(3)$ be ?

If $x \neq 3$ then

$$f(x)=\frac{x^2-6x+9}{x-3}=\frac{(x-3)(x-3)}{x-3}=x-3$$

The graph of $f(x)$ is the line $y=x-3$ with a ***hole*** at $x=3$

If $\boxed{f(3)=0}$ then the function is ***continuous***



430. *Answer is B.*

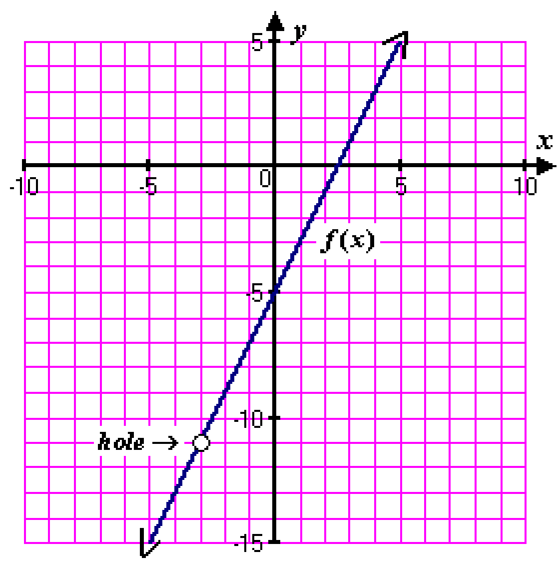
If the function f is continuous for all real numbers and if $f(x)=\dfrac{2x^2+x-15}{x+3}$ when $x \neq -3$, then $f(-3)=$

If $x \neq -3$ then

$$f(x)=\frac{2x^2+x-15}{x+3}=\frac{(2x-5)(x+3)}{x+3}=2x-5$$

The graph of $f(x)$ is the line $y=2x-5$ with a ***hole*** at $x=-3$

If $\boxed{f(-3)=-11}$ then the function is ***continuous***



431. *Answer is E.*

A function $f(x)$ is equal to $\dfrac{x^2-4}{x-2}$ for all $x>0$ except $x=2$. In order for the function to be continuous at $x=2$, what must the value of $f(2)=$

$$f(x)=\frac{x^2-4}{x-2}=\frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}}=x+2$$

$$f(2)=2+2=4$$

To be continuous at $x=2$ the value of

$f(2)=2+2=\boxed{4}$ must be ***assigned***

432. *Answer is A.*

A function $f(x)$ is equal to $\dfrac{\sin 2x}{x}$ for all x except $x=0$. In order for the function to be continuous at $x=0$, what must the value of $f(0)$ be ?

$$\lim_{x\to 0}\frac{\sin 2x}{x}=\lim_{2x\to 0}\frac{2\sin 2x}{2x}=2$$

Hole in $f(x)$ at $(0,\ 2)$ which can be filled by assigning $f(0)=2$

433. *Answer is A.*

If the function f is continuous for all real numbers and if $f(x)=\dfrac{x^2-7x+12}{x-4}$ when $x\neq 4$, then $f(4)=$

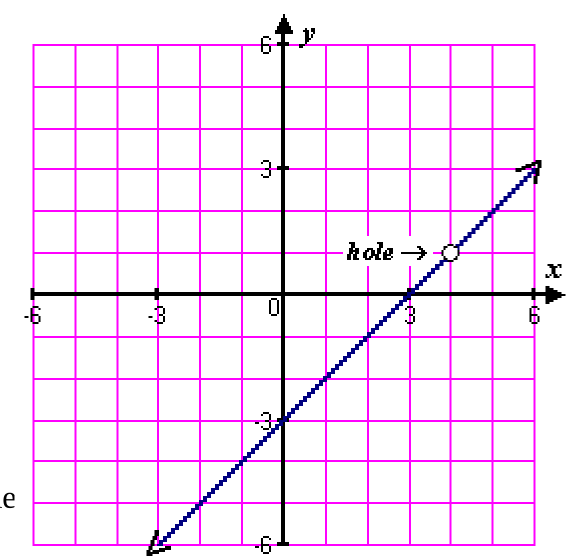
If $x\neq 4$ then

$$f(x)=\frac{x^2-7x+12}{x-4}=\frac{\cancel{(x-4)}(x-3)}{\cancel{x-4}}=x-3$$

The graph of f is the line $y=x-3$

with a hole at $x=4$

To make $f(x)$ ***continuous*** we need to fill the hole by defining $\boxed{f(4)=1}$ $\leftarrow$ add the point $(4,\ 1)$



434. *Answer is A.*

If $f(x)=\begin{cases} x^2+5 & \text{if } x<2 \\ 7x-5 & \text{if } x\geq 2 \end{cases}$,

for all real numbers x,

which of the following must be true ?

Continuous at $x=2$

$$x^2+5=7x-5$$

$$(2)^2+5=7(2)-5$$

$$9=9 \ ☑$$

Differentiable at $x=2$

$$2x=7$$

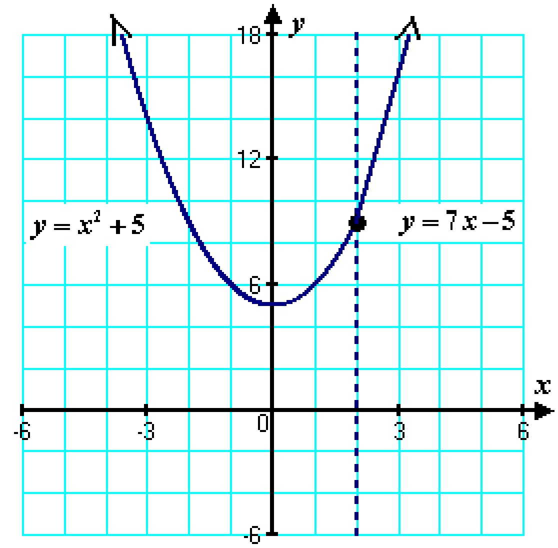
$$2(2)=7$$

$$4\neq 7 \ ☒$$

I. $f(x)$ is continuous everywhere ☑

II. $f(x)$ is differentiable everywhere ☒

III. $f(x)$ has a local minimum at $x=2$ ☒



435. Let $f(x)=\begin{cases} x+2a, & \text{if } x<1 \\ ax^2+7x-4, & \text{if } x\geq 1 \end{cases}$ If a is such that $f(x)$ is continuous at $x=1$, is $f(x)$ also differentiable at $x=1$ Justify your answer.

Continuous at $x=1$

$$x+2a=ax^2+7x-4$$

$$(1)+2a=a(1)^2+7(1)-4$$

$$a=2$$

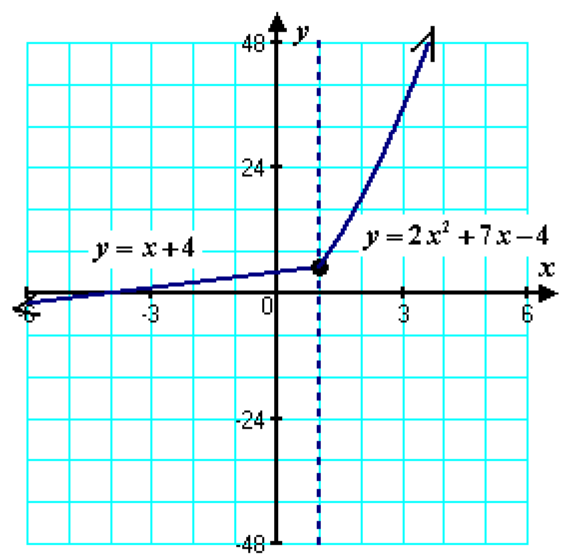
Differentiable at $x=1$

$$1=4x+7$$

$$1=4(1)+7$$

$$1\neq 11$$

$\therefore$ ***not*** differentiable at $x=1$



436. $f(x)=\begin{cases} bx^3+7, & \text{if } x<3 \\ ax^2+3, & \text{if } x\geq 3 \end{cases}$ Find the values of a and b such that $f(x)$ is ***differentiable*** at $x=3$

continuous

$$bx^3+7=ax^2+3 \text{ when } x=3$$
$$b(3)^3+7=a(3)^2+3$$
$$27b+7=9a+3$$
$$4=9a-27b$$

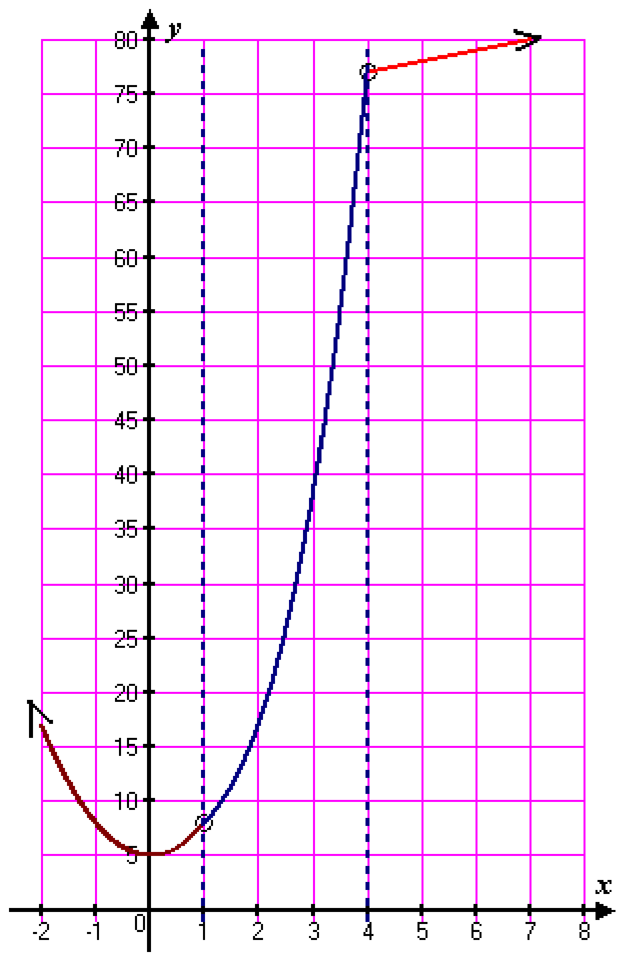
$\leftarrow$ ***differentiable***

$$bx^3+7=ax^2+3 \text{ when } x=3$$
$$3bx^2=2ax \quad \leftarrow \text{differentiate}$$
$$3b(3)^2=2a(3)$$
$$0=6a-27b$$

$$\boxed{\begin{matrix}4=9a-27b\\0=6a-27b\end{matrix}} \rightarrow \boxed{\begin{matrix}4=9a-27b\\0=-6a+27b\end{matrix}} \rightarrow \boxed{\frac{4}{3}=a} \rightarrow \boxed{\begin{matrix}27b=6a\\27b=8\\b=\frac{8}{27}\end{matrix}}$$

437. If $f(x)=\begin{cases} 3x^2+5 & \text{if} \quad x<1 \\ x^3+2x+5 & \text{if} \quad 1\leq x\leq 4 \\ x+c & \text{if} \quad x>4 \end{cases}$ for what value of c is $f(x)$ continuous at $x=4$

$$\overbrace{\text{continuous}}$$
$$x^3+2x+5=x+c \quad \leftarrow \text{at } x=4$$
$$4^3+2(4)+5=4+c$$
$$64+8+5=4+c$$
$$77=4+c$$
$$\boxed{73=c}$$



438. For what value of c is the function defined by

$$f(x)=\begin{cases} -x & \text{if} \quad x<-1 \\ -x^2+x+c & \text{if} \quad -1\le x\le 2 \\ 2x-3 & \text{if} \quad x>2 \end{cases}$$

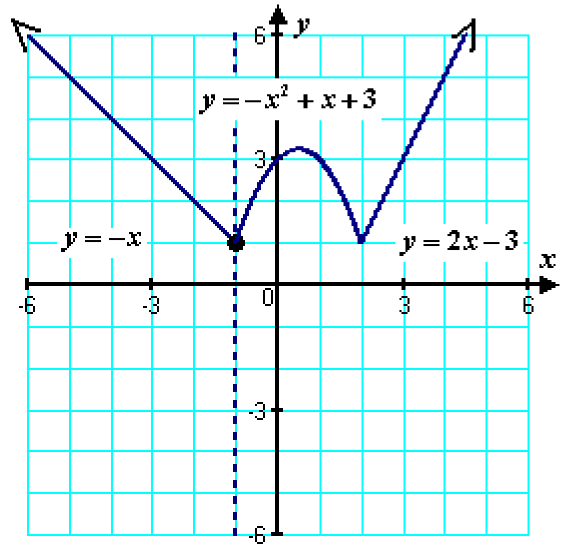
continuous at $x=-1$

Continuous at $x=-1$

$$-x=-x^2+x+c$$

$$-(-1)=-(-1)^2+(-1)+c$$

$$\boxed{3=c}$$



439. $f(x)=\begin{cases} cx^2+5 & \text{if } x<1 \\ ax+b & \text{if } x\ge 1 \end{cases}$ The slope of the line tangent to the graph of $f(x)$ is -3 at $x=-1$ Find the values of a, b and c such that $f'(x)$ is defined for all real numbers.

From tangent line at $x=-1$

$$f(x)=cx^2+5$$

$$f'(x)=2cx$$

$$f'(-1)=2c(-1)=-3$$

$$\boxed{c=\tfrac{3}{2}}$$

Differentiable $x\in\Re$

$$\tfrac{3}{2}x^2+5=ax+b \quad \leftarrow \text{at } x=1$$

$$3x=a$$

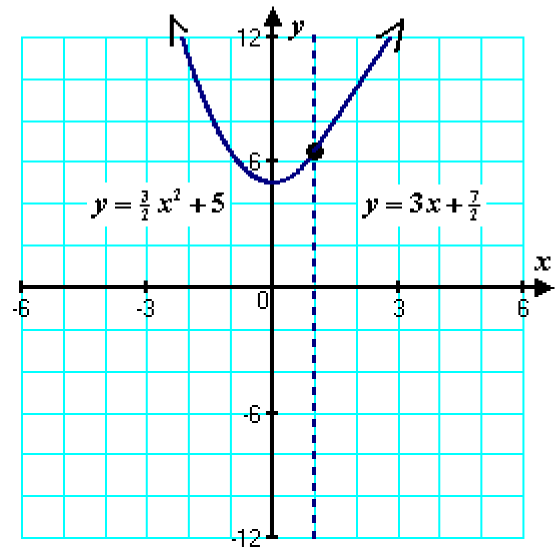
$$\boxed{3=a}$$

Continuous $x\in\Re$

$$\tfrac{3}{2}x^2+5=3x+b$$

$$\tfrac{3}{2}+5=3+b \quad \leftarrow \text{at } x=1$$

$$\boxed{\tfrac{7}{2}=b}$$



440. Given $f(x)=\begin{cases} ax^2 & \text{if } x\le 2 \\ bx-6 & \text{if } x>2 \end{cases}$ find the value of a that will make $f(x)$ differentiable at $x=2$

Continuous at $x=2$

$$ax^2 = bx-6$$

$$a(2)^2 = b(2)-6$$

$$4a = 2b-6$$

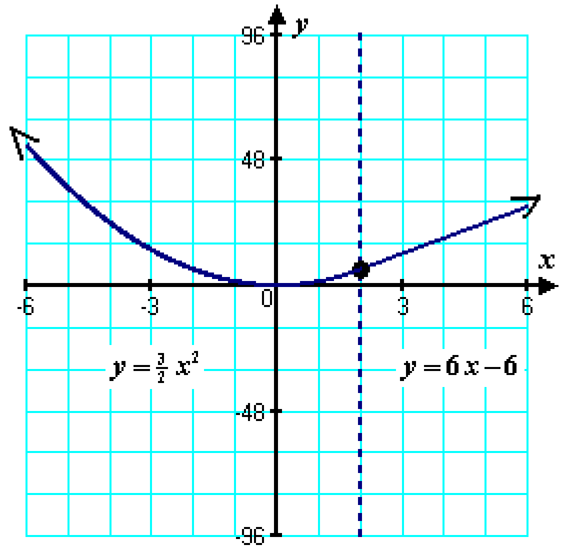
Differentiable at $x=2$

$$2ax = b$$

$$2a(2) = b$$

$$4a = b$$

$$\boxed{a=\tfrac{3}{2} \text{ and } b=6}$$



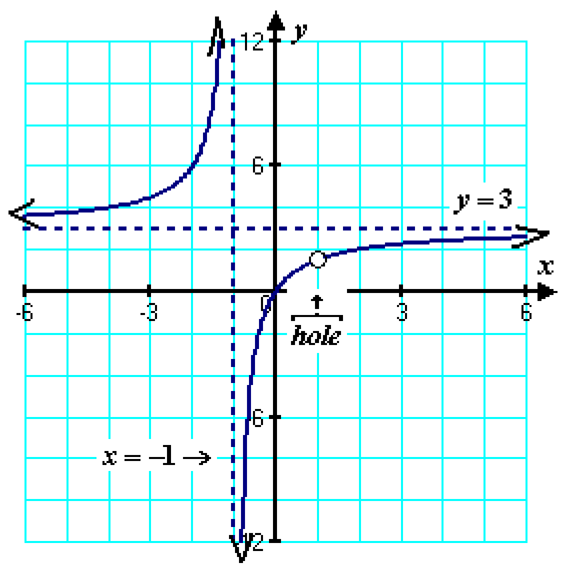
441. The function $f(x)=\dfrac{3x^2-3x}{x^2-1}$ is not defined at $x=\pm1$ What value should be assigned to $f(1)$ to make $f(x)$ continuous at that point ?

$$f(x)=\frac{3x^2-2x}{x^2-1}=\frac{3x\,\cancel{(x-1)}}{\cancel{(x-1)}(x+1)}=\frac{3x}{(x+1)}$$

$$f(1)=\frac{3(1)}{(1+1)}=\frac{3}{2}$$

Hole at $(1, \tfrac{3}{2})$ which can be filled by assigning

$$f(1)=\boxed{\frac{3}{2}}$$



442. Given $f(x)=\begin{cases} ax^2+bx+1 & \text{if } x<\frac{1}{2} \\ x^3+2 & \text{if } x\ge\frac{1}{2} \end{cases}$ find the values of a and b that will make $f(x)$ differentiable at $x=\frac{1}{2}$

Continuous at $x=\frac{1}{2}$

$$ax^2+bx+1=x^3+2$$

$$a(\tfrac{1}{2})^2+b(\tfrac{1}{2})+1=(\tfrac{1}{2})^3+2$$

$$\tfrac{a}{4}+\tfrac{b}{2}+1=\tfrac{1}{8}+2$$

$$2a+4b+8=1+16$$

$$2a+4b=9$$

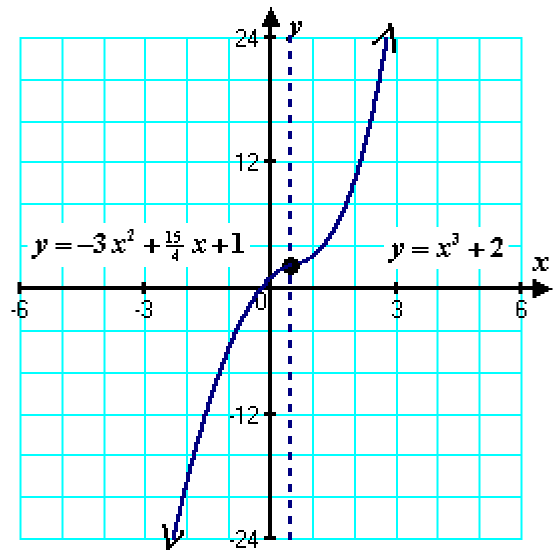
Differentiable at $x=\frac{1}{2}$

$$2ax+b=3x^2$$

$$2a(\tfrac{1}{2})+b=3(\tfrac{1}{2})^2$$

$$a+b=\tfrac{3}{4}$$

$$\boxed{a=-3 \text{ and } b=\tfrac{15}{4}}$$



443. The function $f(x)=\begin{cases} ax^3 & \text{if } x\le 2 \\ 2x+k & \text{if } x>2 \end{cases}$ is differentiable at $x=2$ Find a and k

Continuous at $x=2$

$$ax^3=2x+k$$

$$a(2)^3=2(2)+k$$

$$8a=4+k$$

$$8(\tfrac{1}{6})=4+k$$

$$\boxed{-\tfrac{8}{3}=k}$$

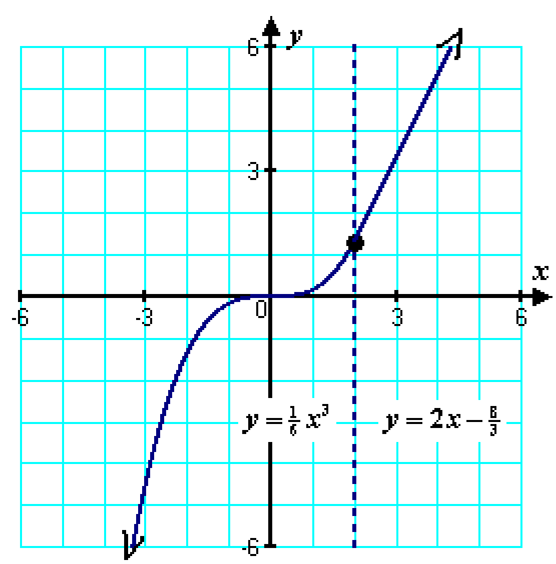
Differentiable at $x=2$

$$3ax^2=2$$

$$3a(2)^2=2$$

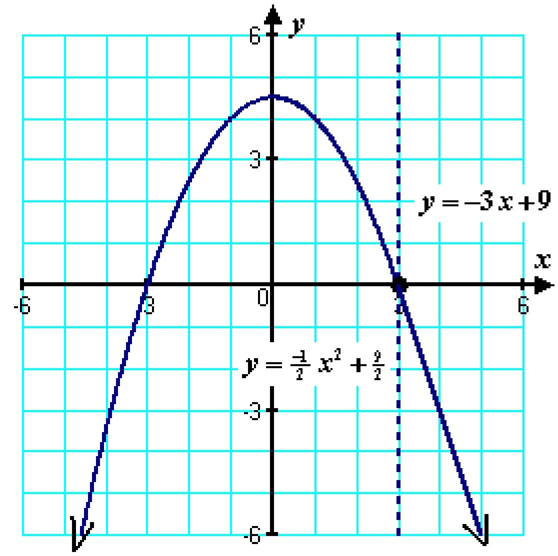
$$12a=2$$

$$\boxed{a=\tfrac{1}{6}}$$



445.

$f(x)=\begin{cases} cx^2+\frac{9}{2} & \text{if } x<3 \\ ax+b & \text{if } x\geq 3 \end{cases}$ The slope of the line tangent to the graph of $f(x)$ at $x=-1$ is 1. Find the values of a, b and c such that $f'(x)$ is defined over the domain of f



Continuous at $x=3$

$$cx^2+\tfrac{9}{2}=ax+b$$
$$c(3)^2+\tfrac{9}{2}=a(3)+b$$
$$9c+\tfrac{9}{2}=3a+b$$
$$9(\tfrac{-1}{2})+\tfrac{9}{2}=3(-3)+b$$
$$\boxed{9=b}$$

Differentiable at $x=-1$

$$f(x)=cx^2+\tfrac{9}{2}$$
$$f'(x)=2cx$$
$$f'(-1)=2c(-1)=1$$
$$\boxed{c=\tfrac{-1}{2}}$$

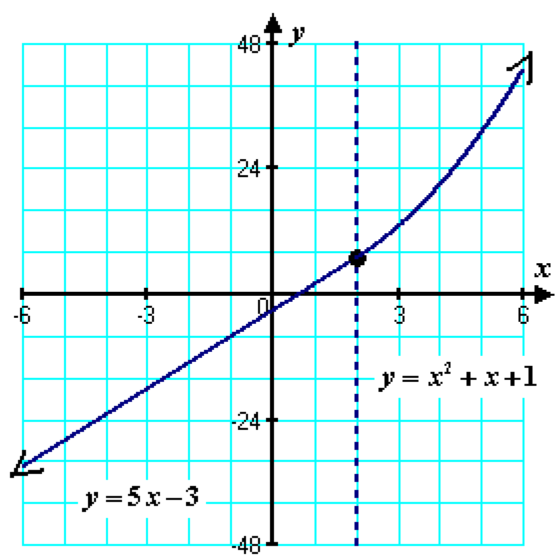
Differentiable at $x=3$

$$2cx=a$$
$$2(\tfrac{-1}{2})(3)=a$$
$$\boxed{-3=a}$$

446.

Let f be defined by $f(x)=\begin{cases} ax+b & \text{if } x<2 \\ x^2+x+1 & \text{if } x\geq 2 \end{cases}$

Find the values of a and b such that f is differentiable at $x=2$



Continuous at $x=2$

$$ax+b=x^2+x+1$$
$$a(2)+b=(2)^2+(2)+1$$
$$2a+b=7$$

Differentiable at $x=2$

$$a=2x+1$$
$$a=2(2)+1$$
$$a=5$$
$$\boxed{a=5 \text{ and } b=-3}$$

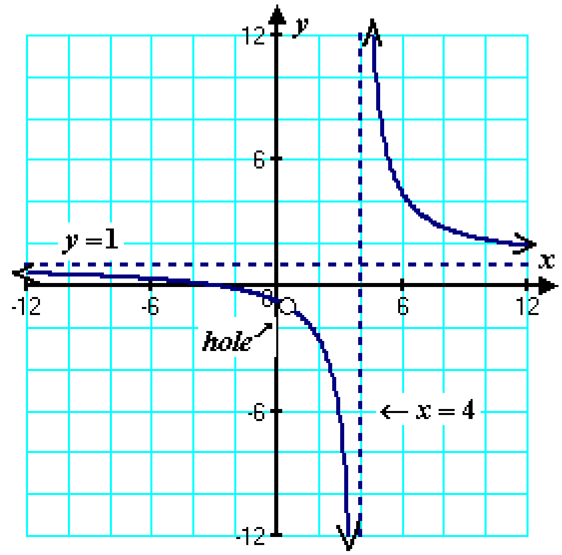
447. What value must be assigned to $f(\frac{1}{2})$ if $f(x)=\dfrac{2x^2+5x-3}{2x^2-9x+4}$ is to be continuous at $x=\frac{1}{2}$

$$f(x)=\frac{\cancel{(2x-1)}(x+3)}{\cancel{(2x-1)}(x-4)}=\frac{x+3}{x-4} \quad x\neq \tfrac{1}{2}$$

hole at $(\frac{1}{2},-1)$ and vertical asymptote at $x=4$

Hole at $(\frac{1}{2},-1)$ can be filled by making $f(\frac{1}{2})=-1$

and the function is now continuous at $x=\frac{1}{2}$



448. A function is defined for $-2<x<2$ by

$$f(x)=\begin{cases} 5x^2+ax+b & \text{if } -2<x\le 0 \\ (2x+4)^{\frac{5}{2}} & \text{if } 0<x<2 \end{cases}$$

Find the values of a and b if $f(x)$ is differentiable at $x=0$

Continuous at $x=0$

$$5x^2+ax+b=(2x+4)^{\frac{5}{2}}$$

$$5(0)^2+a(0)+b=(2(0)+4)^{\frac{5}{2}}$$

$$b=32$$

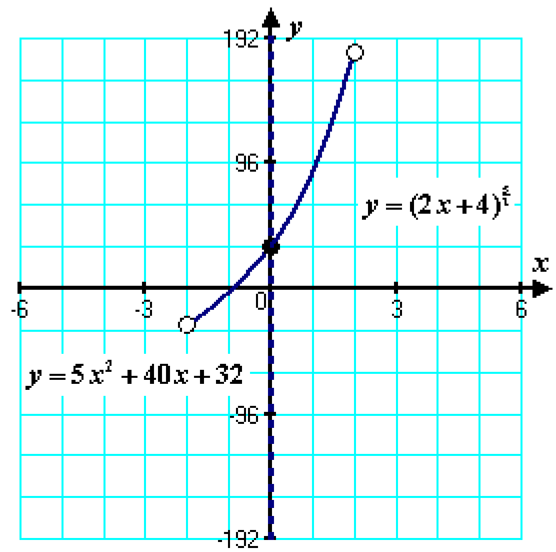
Differentiable at $x=0$

$$10x+a=\tfrac{5}{2}(2x+4)^{\frac{3}{2}}(2)$$

$$10(0)+a=\tfrac{5}{\cancel{2}}(2(0)+4)^{\frac{3}{2}}(\cancel{2})$$

$$a=40$$

$$\boxed{a=40 \text{ and } b=32}$$



449. $f(x)=\dfrac{1-e^{2x}}{1-e^{x}}$ What value must be assigned to $f(x)$ at $x=0$ to make $f(x)$ continuous at $x=0$

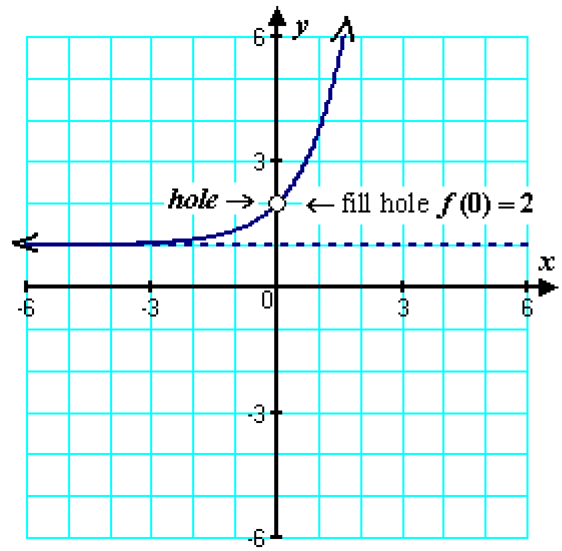
$$\lim_{x\to 0}\frac{1-e^{2x}}{1-e^{x}}=\frac{1-e^{2(0)}}{1-e^{0}}=\frac{0}{0} \leftarrow \text{indefinite form}$$

$$\lim_{x\to 0}\frac{-2e^{2x}}{-e^{x}}=\frac{-2e^{2(0)}}{-e^{0}}=\frac{-2}{-1}=2$$

also $f(x)=\dfrac{1-e^{2x}}{1-e^{x}}=\dfrac{\cancel{(1-e^{x})}(1+e^{x})}{\cancel{1-e^{x}}}=1+e^{x}$

when $x\neq 2$ so $f(0)=1+e^{0}=2$

$f(x)=\dfrac{1-e^{2x}}{1-e^{x}}$ has a hole at $(0,\ 2)$ that can be filled by making $f(0)=2$ and the function is now continuous at $x=0$



450. Given $f(x)=\begin{cases} ax+b & \text{if } x<\frac{1}{2} \\ 3x^{2} & \text{if } x\geq\frac{1}{2}\end{cases}$ find the values of a and b for which this function is differentiable at $x=\dfrac{1}{2}$

Continuous at $x=\frac{1}{2}$

$$ax+b=3x^{2}$$

$$a(\tfrac{1}{2})+b=3(\tfrac{1}{2})^{2}$$

$$\tfrac{a}{2}+b=\tfrac{3}{4}$$

$$2a+4b=3$$

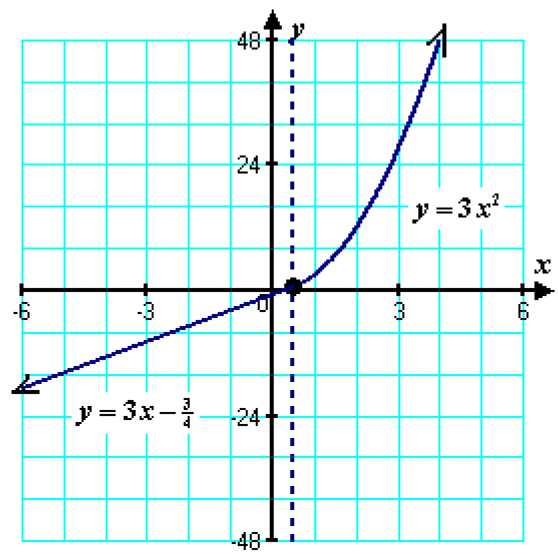
Differentiable at $x=\frac{1}{2}$

$$a=6x$$

$$a=6(\tfrac{1}{2})$$

$$a=3$$

$$\boxed{a=3 \text{ and } b=-\tfrac{3}{4}}$$



451. *Answer is C.*

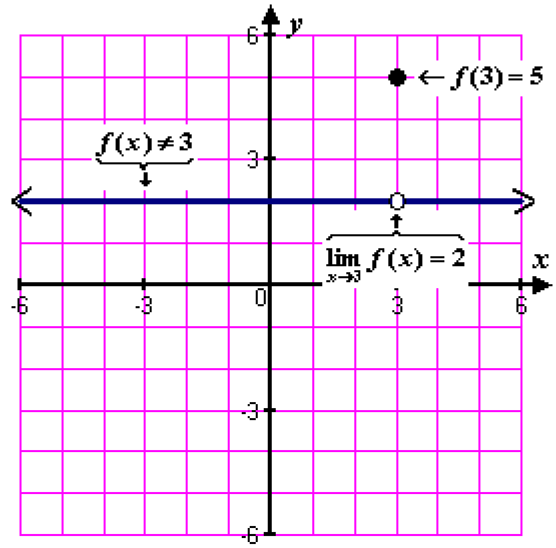
If $f(x)=\begin{cases}\dfrac{2x-6}{x-3} & x\neq 3\\ 5 & x=3\end{cases}$, then $\lim\limits_{x\to 3} f(x)=$

Piecewise function $\to$ domain $x\varepsilon\mathbb{R}$

$$\frac{2x-6}{x-3}=\frac{2(x-3)}{x-3}=2 \text{ when } x\neq 3$$

$$f(3)=5$$

$$\lim_{x\to 3} f(x)=\boxed{2}$$



452. *Answer is B.*

If $f(x)=\begin{cases}x+1 & x\leq 1\\ 3+ax^2 & x>1\end{cases}$

then $f(x)$ is continuous for all x if $a=$

Ends must meet at $x=1$

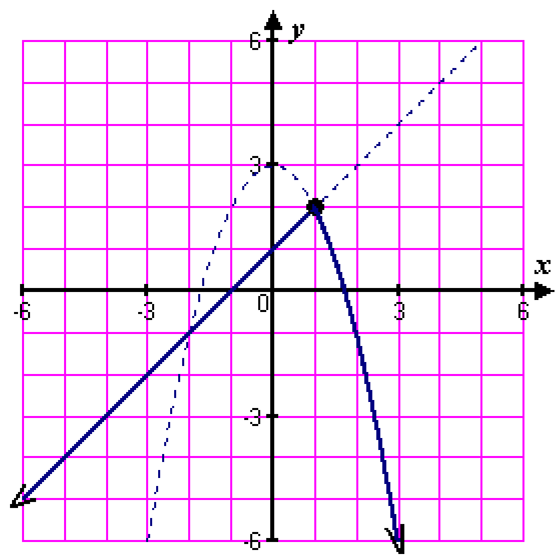
$$x+1=3+ax^2$$

$$1+1=3+a(1)^2$$

$$2=3+a$$

$$\boxed{-1=a}$$

If $a=-1$ then ends meet at $x=1$ and function is continuous for all x



453. *Answer is C.*

If $f(x)=\dfrac{1}{10-\sqrt{x^2+64}}$ is not continuous at c, then $c=$

$$\frac{1}{10-\sqrt{x^2+64}}=\frac{1}{0}=\textit{undefined}$$

$$10-\sqrt{x^2+64}=0$$

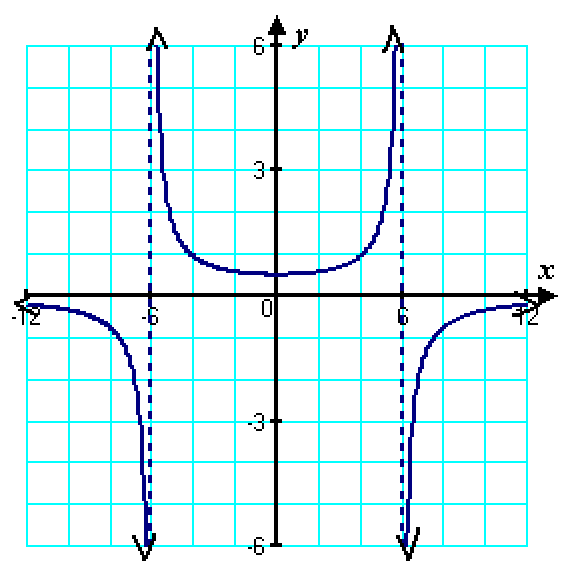
$$10=\sqrt{x^2+64}$$

$$100=x^2+64$$

$$36=x^2$$

$$\boxed{\pm 6=x}$$

$f(x)$ has vertical asymptotes at $x=\pm 6$

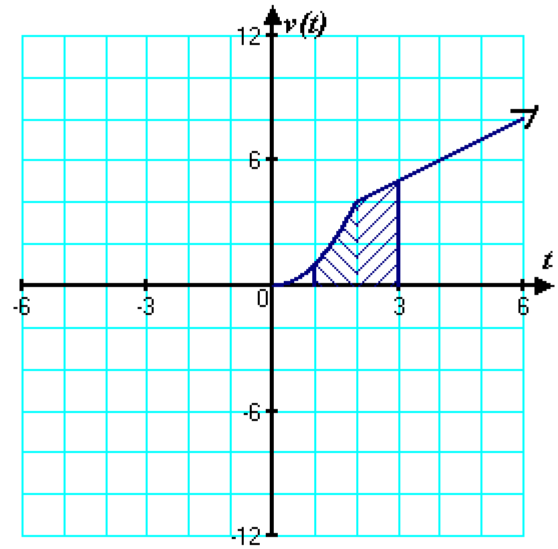


454. *Answer is B.*

455. *Answer is E.*

A particle moves along a straight line. Its velocity is $V(t)=\begin{cases} t^2 & \text{for } 0\le t\le 2 \\ t+2 & \text{for } t\ge 2\end{cases}$

The distance travelled by the particle in the interval $1\le t\le 3$ is



$$\text{Distance} = \int_1^3 V(t)\,dt = \int_1^2 t^2\,dt + \int_2^3 (t+2)\,dt$$

$$= \left[\frac{t^3}{3}\right]_1^2 + \left[\frac{t^2}{2}+2t\right]_2^3$$

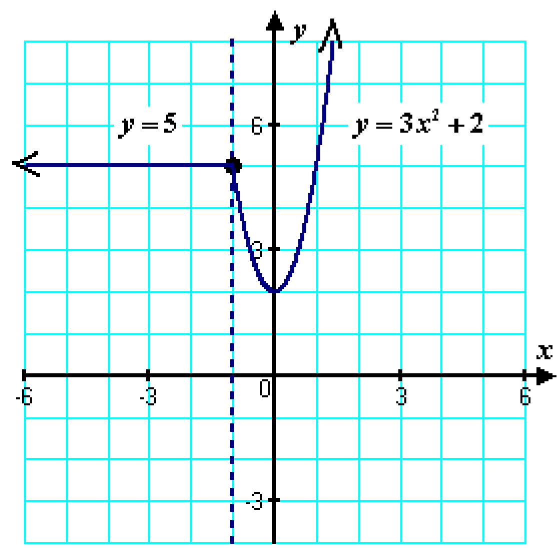
$$= \left[\frac{8}{3}-\frac{1}{3}\right] + \left[\frac{9}{2}+6-(2+4)\right]$$

$$= 6\tfrac{5}{6} \approx \boxed{6.83}$$

456. *Answer is E.*

For what value of c is $f(x)$ continuous ?

$$f(x)=\begin{cases} -cx+5, & x<-1 \\ 3x^2+2, & x\ge -1\end{cases}$$



Continuous at $x=-1$

$$-cx+5=3x^2+2$$

$$-c(-1)+5=3(-1)^2+2$$

$$c+5=5$$

$$\boxed{c=0}$$

457. *Answer is E.*

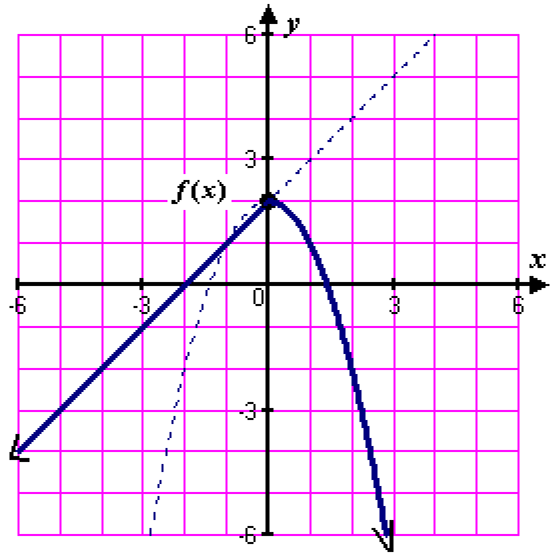
Use $f(x)=\begin{cases}2-x^2 & \text{for } x\ge 0\\ 2+x & \text{for } x<0\end{cases}$

and find $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

Domain $f(x)$ is $x\varepsilon\mathbb{R}$

$\begin{cases}\text{for } x\ge 0 & y=-x^2+2\\ \text{for } x<0 & y=x+2\end{cases}$

see graph of piecewise function



Now consider the graphs of **derivative**

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$f(x)=x+2$ $\quad$ $f(x)=-x^2+2$

$f'(x)=1$ $\quad$ $\leftarrow f'(x)=-2x$

$\underbrace{f'(x)=1}_{\downarrow}$ $\quad$ $f'(0)=0$

Derivative of $f(x)$ at $x=0$ (from the left)

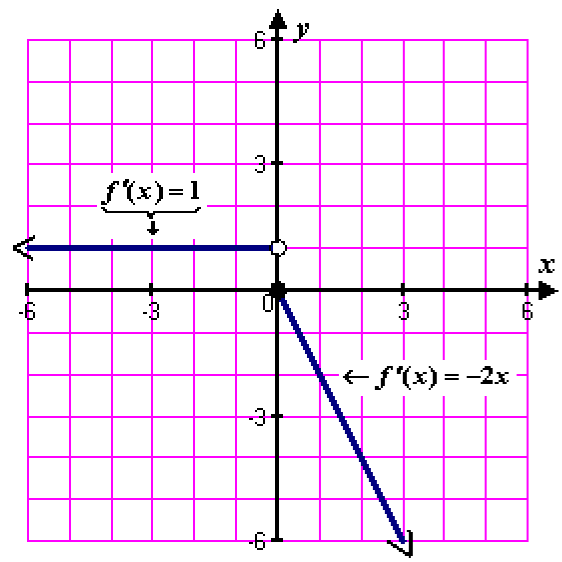
$$f'(0)=\lim_{h\to 0^-}\frac{f(x+h)-f(x)}{h}=1$$

Derivative of $f(x)$ at $x=0$ (from the right)

$$f'(0)=\lim_{h\to 0^+}\frac{f(x+h)-f(x)}{h}=0$$

Derivative of $f(x)$ at $x=0$

$$f'(0)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\textbf{\textit{undefined}}$$ (different left/right) See derivative graphs above



458. *Answer is C.*

459. *Answer is E.*

460. *Answer is E.*

461. *Answer is C.*

462. *Answer is C.*

Let $f(x) = \begin{cases} \dfrac{x^2-1}{x-1} & \text{if } x \neq 1 \\ 4 & \text{if } x = 1 \end{cases}$

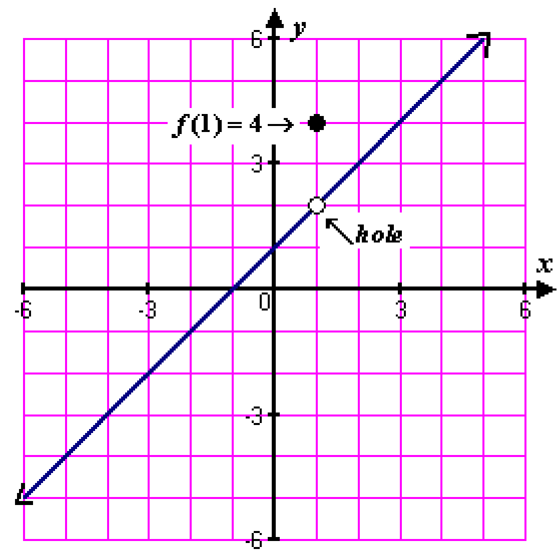
Which of the following statements,

I, II and III are true ?

I. $\lim\limits_{x\to 1} f(x)$ exists

II. $f(1)$ exists

III. f is continuous at $x = 1$



$\textit{if } x \neq 1 \quad \dfrac{x^2-1}{x-1} = \dfrac{(x-1)(x+1)}{x-1} = x+1$

So the graph of $f(x)$ is $y = x+1$ with a ***hole***

at $x = 1$ and the point $f(1) = 4 \rightarrow (1,\ 4)$

I. $\lim\limits_{x\to 1} f(x)$ exists ☑ $\lim\limits_{x\to 1} f(x) = 2$

II. $f(1)$ exists ☑ $f(1) = 4$

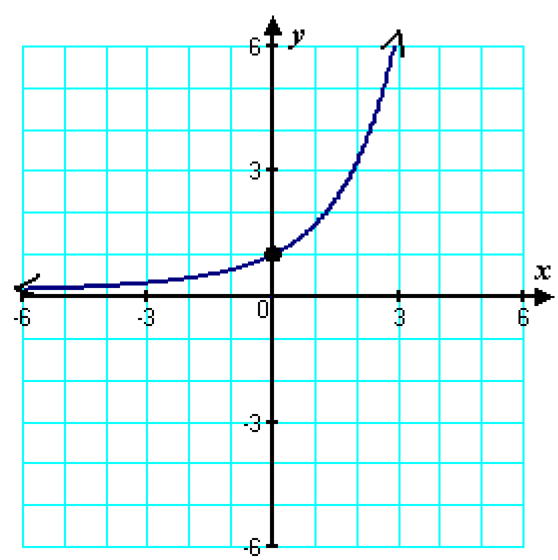
III. f is continuous at $x = 1$ ☒

$\lim\limits_{x\to 1} f(x) \neq f(1)$

463. *Answer is C.*

464. *Answer is A.*

Let $f(x) = \begin{cases} \dfrac{e^x-1}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$ Which of the following statements is true ?



$\lim\limits_{x\to 0} \dfrac{e^x-1}{x} = \dfrac{e^0-1}{0} = \dfrac{0}{0} \leftarrow$ indeterminant

$\lim\limits_{x\to 0} \dfrac{e^x}{1} = \dfrac{e^0}{1} = 1 \leftarrow$ L'hopitals rule

hole at $(0,\ 1)$ ***filled*** by piecewise function

---

f is continuous at $x = 0$ ☑

$\lim\limits_{x\to 0^+} f(x) = 0$ ☒

$\lim\limits_{x\to 0^+} f(x) > 1$ ☒

$\lim\limits_{x\to 0^-} f(x) < 1$ ☒

$\lim\limits_{x\to 0^-} f(x)$ does not exist ☒

465. *Answer is D.*

The function f is given by

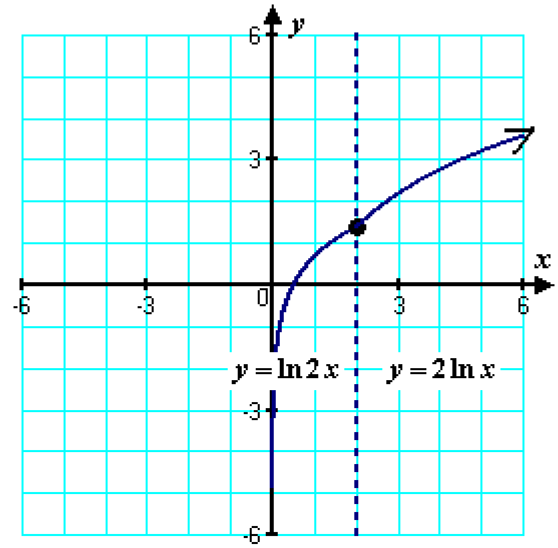
$$f(x)=\begin{cases}\ln 2x, & \text{for } 0<x<2\\ 2\ln x, & \text{for } x\ge 2\end{cases}$$

The limit $\lim_{x\to 2} f(x)=$

$$\lim_{x\to 2^-}\ln 2x=\ln 2(2)=\ln 4=\ln 2^2=2\ln 2$$

$$\lim_{x\to 2^+} 2\ln x=2\ln 2$$

$$\therefore\ \lim_{x\to 2} f(x)=\boxed{2\ln 2}$$



466. *Answer is E.*

The function f is given by

$$f(x)=\begin{cases}\dfrac{x^3}{|x|}, & x\ne 0\\ 0, & x=0\end{cases}$$

Which of the following statements are true ?

for $x>0$ $y=\dfrac{x^3}{|x|}=\dfrac{x^3}{x}=x^2$

for $x<0$ $y=\dfrac{x^3}{|x|}=\dfrac{x^3}{-x}=-x^2$

for $x=0$ $y=0$

I. f is continuous at the point $x=0$ ☑

$f(0)=0$ $\lim_{x\to 0} f(x)=0$ $\lim_{x\to 0} f(x)=f(0)$

II. f is differentiable at the point $x=0$ ☑

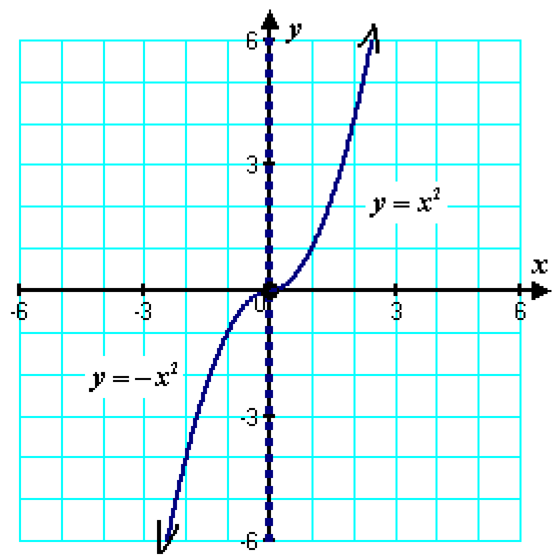
$y=-x^2$ $y=x^2$

$y'=-2x$ $y'=2x$

$y'(0)=0$ $y'(0)=0$

III. $x=0$ is a point of inflection for a graph of f

At $x=0$ concavity changes



467. *Answer is E.*

468. *Answer is D.*

469. *Answer is E.*

If f is the function given by

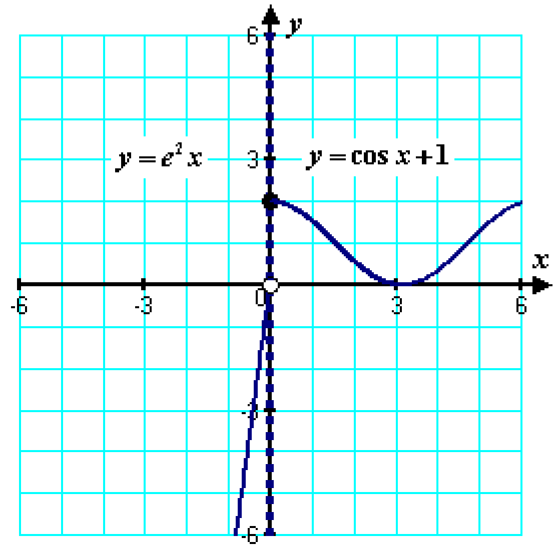
$$f(x)=\begin{cases} e^2x, & x<0 \\ \cos x+1, & x\geq 0 \end{cases}$$

then $\lim_{x\to 0} f(x)=$

$$\lim_{x\to 0^-} e^2x=0 \qquad \lim_{x\to 0^+} \cos x+1=2$$

$$\lim_{x\to 0^-} e^2x \neq \lim_{x\to 0^+} \cos x+1$$

$$\therefore \lim_{x\to 0} f(x)=\text{does } \textbf{\textit{not}} \text{ exist}$$



470. *Answer is B.*

Determine c so that $f(x)$ is continuous everywhere when $f(x)=\begin{cases} x-2, & x\leq 5 \\ cx-3, & x>5 \end{cases}$

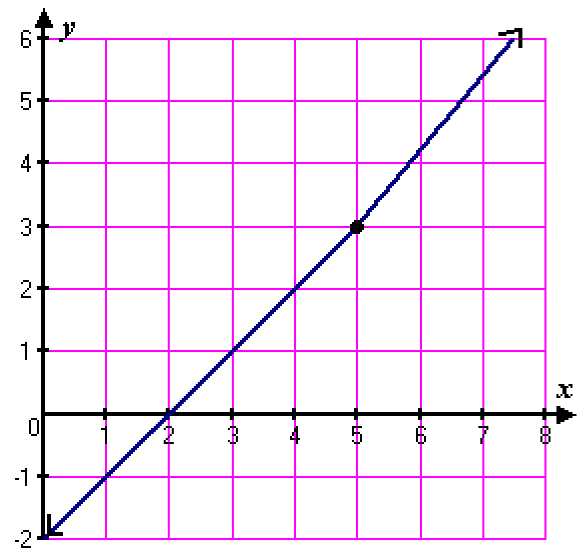
Functions must meet at $x=5$ (continuous)

$$\overbrace{x-2}^{\text{left of 5}} \overbrace{=}^{\substack{\text{continuous}\\ \text{at } x=5}} \overbrace{cx-3}^{\text{right of 5}}$$

$$5-2=c5-3$$

$$3+3=5c$$

$$\boxed{\tfrac{6}{5}=c}$$



471. *Answer is D.*

Difficulty = **0.45**

Let f be the function given by

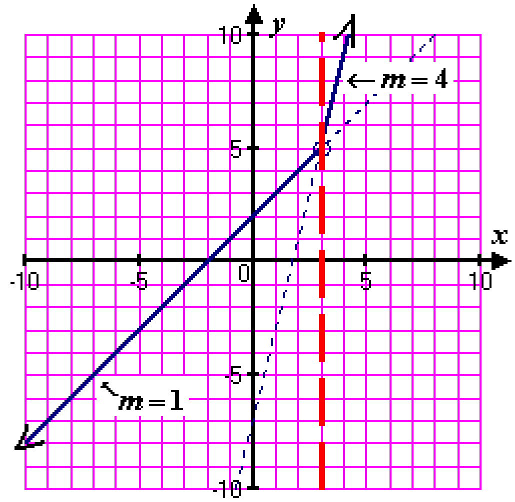
$$f(x)=\begin{cases} x+2 & \text{if } x\le 3 \\ 4x-7 & \text{if } x>3 \end{cases}$$

Which of the following statements are true about f

I. $\lim\limits_{x\to 3} f(x)$ exists

II. f is continuous at $x=3$

III. f is differentiable at $x=3$



I. $\lim\limits_{x\to 3} f(x)=5$ ☑ exists $\quad \lim\limits_{x\to 3^-} f(x)=\lim\limits_{x\to 3^+} f(x)=5$

II. f is continuous at $x=3$ ☑ $\quad \lim\limits_{x\to 3} f(x)=5=f(3)$

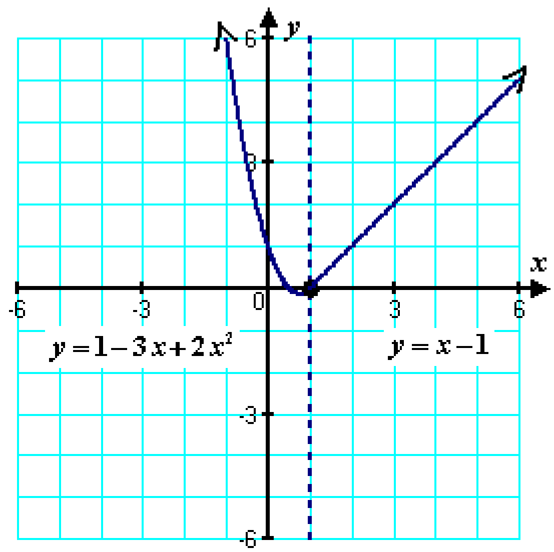
III. f is differentiable at $x=3$ ☒ $\lim\limits_{x\to 3^-} f'(x)\Rightarrow 1\neq 4\Leftarrow \lim\limits_{x\to 3^+} f'(x)$

472. *Answer is B.*

Let m and b be real numbers and let the function f be defined by

$$f(x)=\begin{cases} 1+3bx+2x^2 & \text{for } x\le 1 \\ mx+b & \text{for } x>1 \end{cases}$$

If f is both continuous and differentiable at $x=1$ then



Continuous at $x=1$

$$1+3bx+2x^2=mx+b$$
$$1+3b(1)+2(1)^2=m(1)+b$$
$$3=m-2b$$

Differentiable at $x=1$

$$1+3bx+2x^2=mx+b$$
$$3b+4x=m$$
$$3b+4(1)=m$$
$$4=m-3b$$

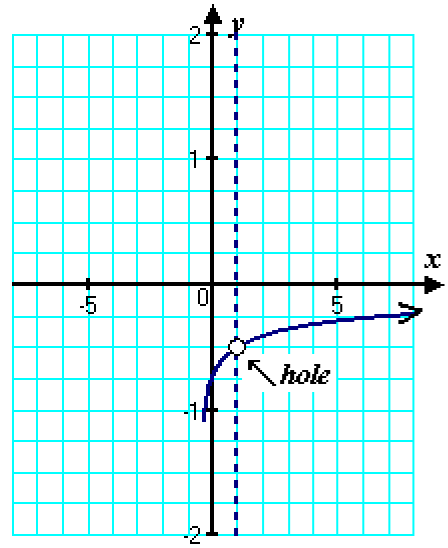
Now solve system of equations $\rightarrow$ $\boxed{\begin{aligned}3&=m-2b\\4&=m-3b\end{aligned}}\Rightarrow\boxed{\begin{aligned}3&=m-2b\\-4&=-m+3b\\\hline &\boxed{-1=b}\end{aligned}}\Rightarrow\boxed{\begin{aligned}3&=m-2b\\3&=m-2(-1)\\&\boxed{1=m}\end{aligned}}$

473. *Answer is D.*

The function f is continuous at $x=1$

If $f(x)=\begin{cases} \dfrac{\sqrt{x+3}-\sqrt{3x+1}}{x-1} & \text{for } x\neq 1 \\ k & \text{for } x=1 \end{cases}$

then $k=$



$$\lim_{x\to 1}\frac{\sqrt{x+3}-\sqrt{3x+1}}{x-1}=\frac{\sqrt{1+3}-\sqrt{3(1)+1}}{1-1}=\frac{0}{0}$$

$$\lim_{x\to 1}\frac{\sqrt{x+3}-\sqrt{3x+1}}{x-1}\left(\frac{\sqrt{x+3}+\sqrt{3x+1}}{\sqrt{x+3}+\sqrt{3x+1}}\right)$$

$$=\lim_{x\to 1}\frac{x+3-(3x+1)}{(x-1)\left(\sqrt{x+3}+\sqrt{3x+1}\right)}$$

$$=\lim_{x\to 1}\frac{2-2x}{(x-1)\left(\sqrt{x+3}+\sqrt{3x+1}\right)}$$

$$=\lim_{x\to 1}\frac{-2\,\cancel{(x-1)}}{\cancel{(x-1)}\left(\sqrt{x+3}+\sqrt{3x+1}\right)}$$

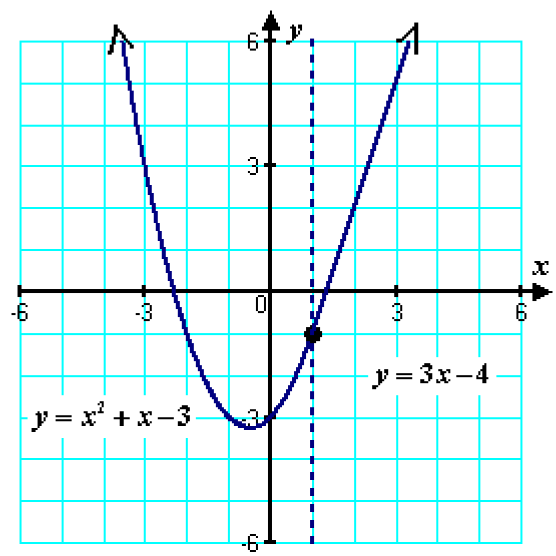
$$=\lim_{x\to 1}\frac{-2}{\sqrt{1+3}+\sqrt{3+1}}=\frac{-2}{2+2}=\boxed{-\frac{1}{2}}$$

The function f is continuous at $x=1$ if $k=-\dfrac{1}{2}$

474. *Answer is D.*

The function f is defined on all the reals such that $f(x)=\begin{cases} x^2+kx-3 & \text{for } x\le 1 \\ 3x+b & \text{for } x>1 \end{cases}$

For which of the following values of k and b will the function be both continuous and differentiable on its entire domain ?



Continuous at $x=1$

$$x^2+kx-3=3x+b$$
$$1^2+k-3=3+b$$
$$-5=b-k$$

Differentiable at $x=1$

$$2x+k=3$$
$$2+k=3$$
$$k=1$$

$$\boxed{b=-4 \text{ and } k=1}$$

475. *Answer is D.*

The function $f(x)=\begin{cases} 4-x^2 & \text{for } x\le 1 \\ mx+b & \text{for } x>1 \end{cases}$ is continuous and differentiable for all real numbers. The values of m and b are

Continuous at $x=1$

$$4-x^2=mx+b$$

$$4-(1)^2=m(1)+b$$

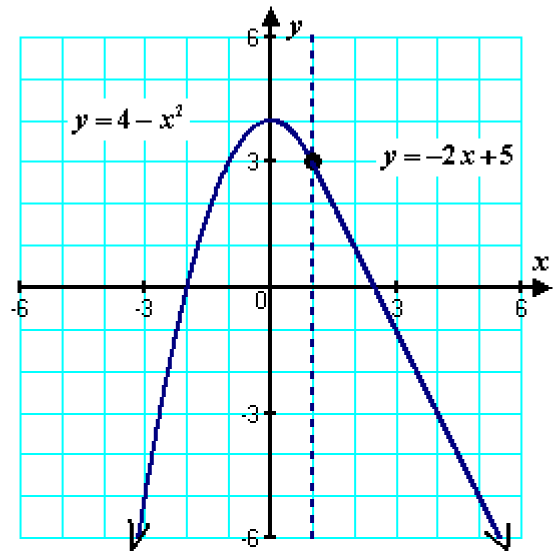
$$3=m+b$$

Differentiable at $x=1$

$$-2x=m$$

$$-2=m$$

$$\boxed{m=-2 \text{ and } b=5}$$



476. *Answer is D.*

The function f is defined for all real numbers by $f(x)=\begin{cases} e^{-x}+3 & \text{for } x>0 \\ ax+b & \text{for } x\le 0 \end{cases}$

If f is differentiable at $x=0$, then $a+b=$

Continuous at $x=0$

$$ax+b=e^{-x}+3$$

$$a(0)+b=e^{-0}+3$$

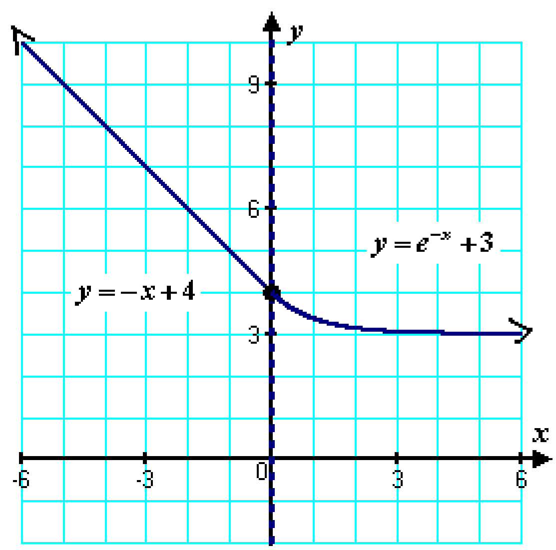
$$b=4$$

Differentiable at $x=0$

$$a=-e^{-x}$$

$$a=-e^{-0}=-1$$

$$\boxed{a+b=3}$$



477. *Answer is C.*

Find the values of a and b that assure that

$$f(x) = \begin{cases} \ln(3-x) & \text{if } x < 2 \\ a - bx & \text{if } x \geq 2 \end{cases} \quad \text{is}$$

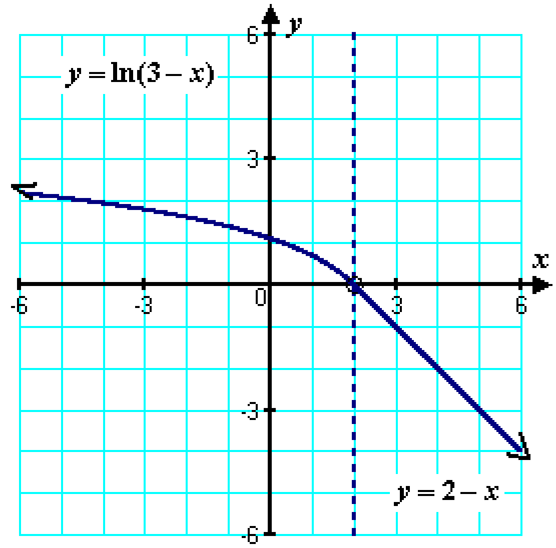
differentiable at $x = 2$

Continuous at $x = 2$

$$\ln(3-x) = a - bx$$
$$\ln(3-2) = a - b(2)$$
$$0 = a - 2b$$
$$2b = a$$

Differentiable at $x = 2$

$$\frac{-1}{3-x} = -b$$
$$\frac{1}{3-2} = b$$
$$\boxed{1 = b} \ \ \textit{and} \ \ \boxed{2 = a}$$



478.

$\boxed{\text{Trig derivatives}} \rightarrow$

Trig limits $\rightarrow \displaystyle\lim_{x\to 0}\frac{\sin x}{x}=1$

$\rightarrow \displaystyle\lim_{x\to 0}\frac{1-\cos x}{x}=0$

(squeeze theorem proofs pg 80)

y	y'
$\sin x$	$\cos x$
$\sec x$	$\sec x\tan x$
$\tan x$	$\sec^2 x$
$\cos x$	$-\sin x$
$\csc x$	$-\csc x\cot x$
$\cot x$	$-\csc^2 x$

479. *Answer is E.*

Find the derivative of $y=\sin(x^2)$

$$y'=\cos(x^2)(2x)$$

$$y'=\boxed{2x\cos(x^2)}$$

480. *Answer is A.*

If $y=\ln(\tan x)$ then $y'=$

$$y'=\frac{\sec^2 x}{\tan x}=\frac{\frac{1}{\cos^2 x}}{\frac{\sin x}{\cos x}}=\frac{1}{\cos^{\cancel{2}} x}\times\frac{\cancel{\cos x}}{\sin x}=\frac{2}{2\sin x\cos x}=\boxed{\frac{2}{\sin 2x}}$$

481. *Answer is E.*

Find the derivative of $y=e^x\sin x$

$$y'=e^x\cos x+\sin x e^x=\boxed{e^x(\sin x+\cos x)}$$

482. *Answer is B.*

The derivative of $y=\tan(x^2)$ is

$$y'=\sec^2(x^2)(2x)=\boxed{2x\sec^2(x^2)}$$

483. *Answer is E.*

If $y=\cancel{\ln e}^{\tan^2 x}$ then $y'\left(\frac{\pi}{4}\right)=$

$$y=(\tan x)^2$$

$$y'=2(\tan x)^1\sec^2 x$$

$$y'(\tfrac{\pi}{4})=2\tan(\tfrac{\pi}{4})\sec^2(\tfrac{\pi}{4})=2(1)\left(\sqrt{2}\right)^2=\boxed{4}$$

484. *Answer is D.*

Given $f(x)=x\cos x$ the second derivative of $f(x)=$

$$f'(x)=x(-\sin x)+\cos x=\cos x-x\sin x$$

$$f''(x)=-\sin x-\left[x\cos x+\sin x\right]=\boxed{-x\cos x-2\sin x}$$

485. *Answer is D.*

Find the derivative of $\cos^2 x$

$$y=(\cos x)^2$$

$$y=2(\cos x)^1(-\sin x)=\boxed{-2\sin x\cos x}$$

486. *Answer is E.*

Find the value of the derivative of $e^x\sin x$ at $x=\pi$

$$y'=e^x\cos x+e^x\sin x$$

$$y'(\pi)=e^\pi\cos\pi+\cancel{e^\pi\sin\pi}=\boxed{-e^\pi}$$

487. *Answer is D.*

The y-intercept of the line tangent to $y=x\sin x$ at $x=\pi$ is

$$y'=x\cos x+\sin x$$

$$y'(\pi)=\pi\cos\pi+\cancel{\sin\pi}=-\pi \quad \leftarrow \text{slope}$$

$$y(\pi)=\pi\sin\pi=0 \quad \leftarrow \text{point } (\pi,\ 0)$$

$$\text{Tangent line}=\frac{-\pi}{1}=\frac{y-0}{x-\pi}$$

$$y=-\pi x+\pi^2$$

$$y\text{-intercept of the line tangent}\rightarrow\boxed{\pi^2}$$

488. *Answer is D.*

If $f(x)=\ln(\sin x)$ then $f'\left(\dfrac{\pi}{4}\right)=$

$$f'(x)=\frac{\cos x}{\sin x}=\cot x$$

$$f'(\tfrac{\pi}{4})=\cot\tfrac{\pi}{4}=\boxed{1}$$

489. *Answer is A.*

If $y=\sin 2x-x$ then $y'\left(\dfrac{\pi}{2}\right)=$

$$y'=2\cos 2x-1$$

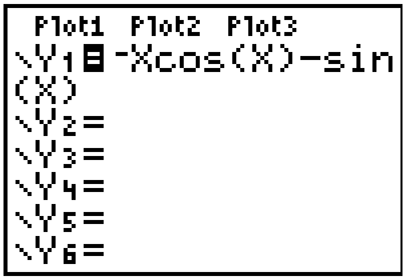
$$y'(\tfrac{\pi}{2})=2\cos 2(\tfrac{\pi}{2})-1=-2-1=\boxed{-3}$$

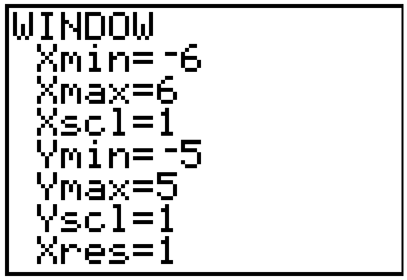
490. *Answer is D.*

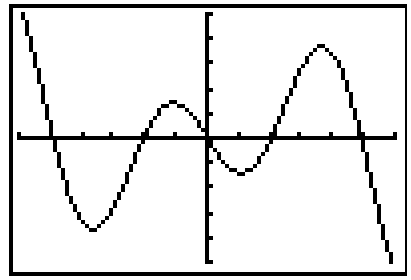
The number of critical points of the function $f(x) = -x\sin x$ on $[-6,\ 6]$ is

$$f'(x) = -\left[x\cos x + \sin x\right] = -x\cos x - \sin x = 0$$

Use graphing calculator to find the ***number*** of critical points







491. *Answer is D.*

If $y = 3\left[\tan\left(\dfrac{x}{3}\right)\right]^2$ then $y'(\pi) =$

$$y' = \cancel{3}(2)\left[\tan\tfrac{x}{3}\right]^1 \sec^2\tfrac{x}{3}\left(\tfrac{1}{\cancel{3}}\right) = 2\tan\tfrac{x}{3}\sec^2\tfrac{x}{3}$$

$$y'(\pi) = 2\tan\tfrac{\pi}{3}\sec^2\tfrac{\pi}{3} = 2(\sqrt{3})(2)^2 = \boxed{8\sqrt{3}}$$

492. *Answer is A.*

If $f(x) = 2\sin^2 5x$ then $f'(x) =$

$$f(x) = 2\left[\sin 5x\right]^2$$

$$f'(x) = 2(2)\left[\sin 5x\right]^1(\cos 5x)(5)$$

$$f'(x) = 10(2\sin 5x\cos 5x)$$

$$f'(x) = \boxed{10\sin 10x}$$

493. *Answer is D.*

If $y = e^{\sin x}$, then $\dfrac{dy}{dx} =$

$$y' = e^{\sin x}\cos x$$

494. *Answer is D.*

If $g(x) = x^2\cos x$ then $g'(x) =$

$$g(x) = x^2\cos x$$

$$g'(x) = x^2(-\sin x) + (\cos x)(2x)$$

$$g'(x) = \boxed{2x\cos x - x^2\sin x}$$

495. *Answer is C.*

If $f(x)=\cos 3x$, find all values in the interval $(0,\ \pi)$ for which $f^{(3)}(x)=0$

$$f'(x)=-3\sin 3x$$
$$f''(x)=-9\cos 3x$$
$$f^{(3)}(x)=27\sin 3x=0$$
$$\sin 3x=0$$
$$3x=0,\pi,2\pi,3\pi,\ \ldots$$
$$\boxed{x=\tfrac{\pi}{3},\tfrac{2\pi}{3}}$$

496. *Answer is D.*

The normal line to $y=2\sin x$ at $x=\dfrac{\pi}{3}$ intersects the x-axis at $(x_1,0)$ What is the value of x_1

Slope of tangent at $\left(\frac{\pi}{3},\sqrt{3}\right)$

$$y=2\sin x$$
$$y'=2\cos x$$
$$y'(\tfrac{\pi}{3})=2\cos\tfrac{\pi}{3}=2(\tfrac{1}{2})=1$$

Slope of normal $=-1$

Equation of normal at $\left(\frac{\pi}{3},\sqrt{3}\right)$

$$\text{Slope}=\frac{rise}{run}=\frac{-1}{1}=\frac{y-\sqrt{3}}{x-\frac{\pi}{3}}$$
$$y-\sqrt{3}=\tfrac{\pi}{3}-x$$
$$y=-x+\tfrac{\pi}{3}+\sqrt{3}$$

x_1-intercept $(y=0)$

$$0=-x+\tfrac{\pi}{3}+\sqrt{3}$$
$$x=\tfrac{\pi}{3}+\sqrt{3}$$
$$x_1=\boxed{\frac{\pi+3\sqrt{3}}{3}}$$

497. *Answer is B.*

If $f(x)=\sin x+\cos x+e^x$, then $f'(x)=$

$$f'(x)=\cos x-\sin x+e^x$$

498. *Answer is A.*

If $y=\sin u$, $u=3w$, and $w=e^{2x}$, then $\dfrac{dy}{dx}=$

$$u=3w$$
$$u=3e^{2x}$$

$$y=\sin u$$
$$y=\sin 3e^{2x}$$
$$\frac{dy}{dx}=\cos 3e^{2x}(6e^{2x})=\boxed{6e^{2x}\cos 3e^{2x}}$$

499. *Answer is C.*

In the interval $[\ 0,\ \pi\ ]$, where are the inflection points for the function $y=\sin^2 x-\cos^2 x$

$$y=\sin^2 x-\cos^2 x=-\left[\cos^2 x-\sin^2 x\right]=-\cos 2x$$

$$y'=2\sin(2x)$$

$$y''=4\cos(2x)=0$$

$\cos(2x)=0 \quad \leftarrow$ Sketch graph of $\cos 2x$ in the interval $[\ 0,\ \pi\ ]$

$$\boxed{x=\tfrac{\pi}{4},\tfrac{3\pi}{4}}$$

500. *Answer is A.*

Find y', if $y=\ln(\sec x+\tan x)$

$$y'=\frac{\sec x\tan x+\sec^2 x}{\sec x+\tan x}=\frac{\sec x\,\cancel{(\tan x+\sec x)}}{\cancel{\sec x+\tan x}}=\boxed{\sec x}$$

501. *Answer is D.*

Find $\dfrac{dy}{dx}$ if $y=x^2\sin\dfrac{1}{x}$ $\quad (x\neq 0)$

$$y=x^2\sin\frac{1}{x}$$

$$\frac{dy}{dx}=\cancel{x^2}\left(\cos\frac{1}{x}\right)\left(-\frac{1}{\cancel{x^2}}\right)+\left(\sin\frac{1}{x}\right)(2x)$$

$$\frac{dy}{dx}=\boxed{2x\sin\frac{1}{x}-\cos\frac{1}{x}}$$

502. *Answer is A.*

Find $\dfrac{dy}{dx}$ if $y=\dfrac{1}{2\sin 2x}$

$$y=\frac{1}{2\sin 2x}=\frac{1}{2}\csc 2x$$

$$y'=\frac{1}{\cancel{2}}\left[-\csc 2x\cot 2x(\cancel{2})\right]=\boxed{-\csc 2x\cot 2x}$$

503. *Answer is A.*

Find $\dfrac{dy}{dx}$, if $y=e^{-x}\cos 2x$

$$y'=e^{-x}(-\sin 2x)(2)-e^{-x}\cos 2x=-e^{-x}(2\sin 2x)-e^{-x}\cos 2x$$

$$y'=\boxed{-e^{-x}(2\sin 2x+\cos 2x)}$$

504. *Answer is D.*

Find $\dfrac{dy}{dx}$, if $y = \sec^2\sqrt{x} = \left[\sec x^{\frac{1}{2}}\right]^2$

$$y' = \cancel{2}\left[\sec x^{\frac{1}{2}}\right]^1 \sec x^{\frac{1}{2}} \tan x^{\frac{1}{2}} \left(\tfrac{1}{\cancel{2}} x^{\frac{-1}{2}}\right)$$

$$y' = \boxed{\dfrac{\sec^2\sqrt{x}\tan\sqrt{x}}{\sqrt{x}}}$$

505. *Answer is B.*

If $y = a\sin ct + b\cos ct$, where a, b, and c are constants, then $\dfrac{d^2y}{dt^2}$ is

$$y' = a\cos ct(c) + b(-\sin ct)(c) = ac\cos ct - bc\sin ct$$

$$y'' = ac(-\sin ct)(c) - bc\cos ct(c) = -ac^2\sin ct - bc^2\cos ct$$

$$y'' = -c^2\left[a\sin ct + b\cos ct\right] = \boxed{-c^2 y}$$

506. *Answer is E.*

The equation of the tangent to the curve $y = x\sin x$ at the point $\left(\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$ is

$$y = x\sin x$$

$$y' = x\cos x + \sin x$$

$$y'(\tfrac{\pi}{2}) = (\tfrac{\pi}{2})\cos(\tfrac{\pi}{2}) + \sin(\tfrac{\pi}{2})$$

$$y'(\tfrac{\pi}{2}) = (\tfrac{\pi}{2})(0) + 1 = 1$$

Slope of tangent at $\left(\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$ is 1

Equation of tangent at the point $\left(\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = 1 = \frac{y - \frac{\pi}{2}}{x - \frac{\pi}{2}}$$

$$y - \tfrac{\pi}{2} = x - \tfrac{\pi}{2}$$

$$\boxed{y = x}$$

507. *Answer is E.*

If $x \neq 0$, then the slope of $x\sin\dfrac{1}{x}$ equals zero whenever

$$y = x\sin x^{-1}$$

$$y' = x\cos x^{-1}(-x^{-2}) + \sin x^{-1} = 0$$

$$-\cancel{x}\cos\tfrac{1}{x}\left(\tfrac{1}{x^{\cancel{2}}}\right) + \sin\tfrac{1}{x} = 0$$

$$\sin\tfrac{1}{x} = \tfrac{1}{x}\cos\tfrac{1}{x}$$

$$\frac{\sin\frac{1}{x}}{\cos\frac{1}{x}} = \frac{1}{x}$$

$$\boxed{\tan\tfrac{1}{x} = \tfrac{1}{x}}$$

508. *Answer is E.*

If $f(x) = \cos x \sin 3x$ then $f'\left(\frac{\pi}{6}\right) =$

$$f'(x) = \cos x(\cos 3x)(3) + \sin 3x(-\sin x)$$
$$f'(x) = 3\cos x \cos 3x - \sin x \sin 3x$$
$$f'(\tfrac{\pi}{6}) = 3\cos\tfrac{\pi}{6}\cos\tfrac{\pi}{2} - \sin\tfrac{\pi}{6}\sin\tfrac{\pi}{2}$$
$$f'(\tfrac{\pi}{6}) = 3\left(\tfrac{\sqrt{3}}{2}\right)(0) - \left(\tfrac{1}{2}\right)(1) = \boxed{-\frac{1}{2}}$$

509. *Answer is D.*

If $y = \sin^3(1-2x)$ then $\frac{dy}{dx} =$

$$y = \left[\sin(1-2x)\right]^3$$
$$y' = 3\left[\sin(1-2x)\right]^2 \cos(1-2x)(-2)$$
$$y' = \boxed{-6\cos(1-2x)\sin^2(1-2x)}$$

510. *Answer is D.*

$\frac{d}{dx}\left(\sin(\cos x)\right) =$

$$\frac{d}{dx}\left(\sin(\cos x)\right) = \cos(\cos x)\overbrace{(-\sin x)}^{chain} = \boxed{-\left(\cos(\cos x)\right)\sin x}$$

511. *Answer is E.* $\boxed{\text{Difficulty} = \mathbf{0.77}}$

If $y = x^2 \sin 2x$ then $\frac{dy}{dx} =$

$$y' = x^2(2\cos 2x) + 2x\sin 2x$$
$$y' = 2x\sin 2x + 2x^2\cos 2x = \boxed{2x(\sin 2x + x\cos 2x)}$$

512. *Answer is C.* $\boxed{\text{Difficulty} = \mathbf{0.79}}$

A particle moves along the x-axis so that at any time $t \geq 0$, its velocity is given by $v(t) = 3 + 4.1\cos(0.9t)$ What is the acceleration of the particle at time $t = 4$

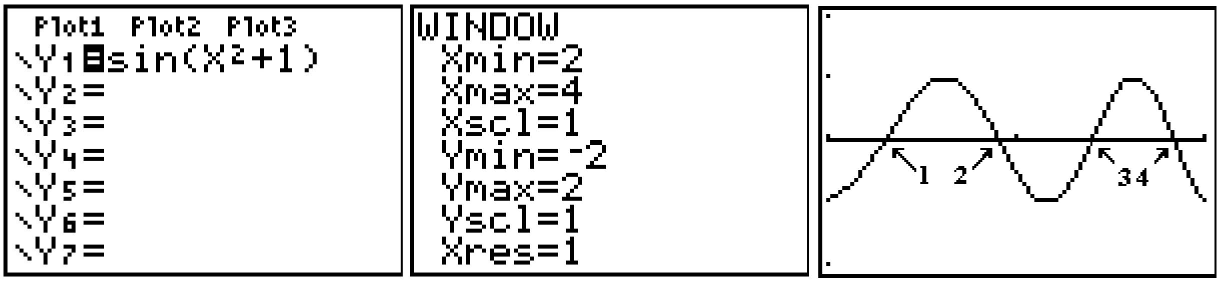
$$v(t) = 3 + 4.1\cos(0.9t)$$
$$v'(t) = -4.1(.9)\sin(0.9t)$$
$$a(4) = -4.1(.9)\sin(0.9(4)) = \boxed{1.6329}$$

513. *Answer is D.*

Difficulty $= \mathbf{0.54}$

Let f be the function with derivative given by $f'(x) = \sin(x^2+1)$ How many relative extrema does f have on the interval $2 < x < 4$

function $f(x)$ has a relative extrema whenever its derivative $f'(x) = \sin(x^2+1)$ changes sign.



514. *Answer is E.*

Difficulty $= \mathbf{0.96}$

If $y = \sin(3x)$ then $\dfrac{dy}{dx} =$

$y' = 3\cos(3x)$

515. *Answer is A.*

If $f(x) = x + \sin x$ then $f'(x) =$

$f'(x) = 1 + \cos x$

516. *Answer is A.*

If $y = \cos^2 3x$ then $\dfrac{dy}{dx} =$

$y = (\cos 3x)^2$

$y' = 2(\cos 3x)^1(-\sin 3x)(3)$

$\boxed{y' = -6\sin 3x \cos 3x}$

517. *Answer is C.*

If $y = \cos^2 x - \sin^2 x$ then $y' =$

$y = \cos(2x)$

$y' = -\sin(2x)(2) = \boxed{-2\sin(2x)}$

518. *Answer is B.*

Difficulty $= \mathbf{0.76}$

If $f(x) = \sin x$, then $f'\left(\dfrac{\pi}{3}\right) =$

$f'(x) = \cos x$

$f'\left(\dfrac{\pi}{3}\right) = \cos\dfrac{\pi}{3} = \boxed{\dfrac{1}{2}}$

519. *Answer is E.*

Difficulty = **0.73**

If $y = 2\cos\left(\frac{x}{2}\right)$ then $\frac{d^2y}{dx^2} =$

$$y' = \cancel{2}\left(-\sin\left(\frac{x}{2}\right)\right)\left(\frac{1}{\cancel{2}}\right) = -\sin\left(\frac{x}{2}\right)$$

$$y'' = -\cos\left(\frac{x}{2}\right)\left(\frac{1}{2}\right) = \boxed{-\frac{1}{2}\cos\left(\frac{x}{2}\right)}$$

520. *Answer is E.*

Difficulty = **0.70**

If $y = \tan x - \cot x$ then $\frac{dy}{dx} =$

$$y' = \sec^2 x - (-\csc^2 x)$$

$$y'' = \boxed{\sec^2 x + \csc^2 x}$$

521. *Answer is D.*

Difficulty = **0.76**

If $f(x) = (x-1)^2 \sin x$ then $f'(0) =$

$$f'(x) = (x-1)^2 \cos x + 2(x-1)\sin x$$

$$f'(0) = (0-1)^2 \cos 0 + 2(0-1)\sin 0 = 1(1) - 2(0) = \boxed{1}$$

522. *Answer is B.*

Difficulty = **0.71**

An equation of the line tangent to the graph of $y = x + \cos x$ at the point $(0, 1)$ is

$$y = x + \cos x$$

$$y' = 1 - \sin x$$

$$y'(0) = 1 - \sin 0 = 1 - 0 = 1$$

Slope of tangent at point $(0, 1)$ is $m = 1$

Equation of the line tangent at the point $(0, 1)$

$$\text{Slope} = \frac{rise}{run} = \frac{1}{1} = \frac{y-1}{x-0}$$

$$y - 1 = x$$

$$\boxed{y = x + 1}$$

523. *Answer is E.*

Difficulty = **0.56**

If $f(x) = \tan 2x$, then $f'\left(\frac{\pi}{6}\right) =$

$$f'(x) = \sec^2 2x(2) = 2\sec^2 2x$$

$$f'\left(\frac{\pi}{6}\right) = 2\sec^2 2\left(\frac{\pi}{6}\right) = 2\sec^2\left(\frac{\pi}{3}\right) = 2(2)^2 = \boxed{8}$$

524. *Answer is B.*

If $f(x)=\sin^2(3-x)$ then $f'(0)=$

$$f(x)=\sin^2(3-x)=\left[\sin(3-x)\right]^2$$
$$f'(x)=2\left[\sin(3-x)\right]^1\cos(3-x)(-1)$$
$$f'(x)=-2\sin(3-x)\cos(3-x)$$
$$f'(0)=\boxed{-2\sin 3\cos 3}$$

525. *Answer is A.*

If $y=3\sin x+4\cos x$ then $y''-y=$

$$y'=3\cos x-4\sin x$$
$$y''=-3\sin x-4\cos x$$
$$y''-y=-3\sin x-4\cos x-(3\sin x+4\cos x)$$
$$y''-y=-3\sin x-4\cos x-3\sin x-4\cos x=\boxed{-6\sin x-8\cos x}$$

526. *Answer is C.*

If $y=\sin x+e^{-x}$ then $y+y''=$

$$y'=\cos x-e^{-x}$$
$$y''=-\sin x+e^{-x}$$
$$y+y''=\sin x+e^{-x}+(-\sin x+e^{-x}x)$$
$$y+y''=\cancel{\sin x}+e^{-x}\cancel{-\sin x}+e^{-x}x=\boxed{2e^{-x}}$$

527. *Answer is E.*

If $f(x)=\sqrt{4\sin x+2}$ then $f'(0)=$

$$f(x)=(4\sin x+2)^{\frac{1}{2}}$$
$$f'(x)=\tfrac{1}{2}(4\sin x+2)^{-\frac{1}{2}}(4\cos x)$$
$$f'(x)=\frac{2\cos x}{\sqrt{4\sin x+2}}$$
$$f'(0)=\frac{2\cos 0}{\sqrt{4\sin 0+2}}=\frac{2}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)=\boxed{\sqrt{2}}$$

528. *Answer is B.*

The equation of the tangent line to the graph of $y = \cos x + \tan(2x)$ at the point $(0, 1)$ is

$$y = \cos x + \tan(2x)$$
$$y' = -\sin x + 2\sec^2(2x)$$
$$y'(0) = -\sin 0 + 2\sec^2(0)$$
$$y'(0) = 0 + 2(1)^2 = 2$$

Slope of tangent at point $(0, 1)$

Equation of the tangent

$$\frac{2}{1} = \frac{y-1}{x-0}$$
$$y - 1 = 2x$$
$$\boxed{y = 2x + 1}$$

529. *Answer is A.*

If $f(x) = (x-1)^2 \cos x$ then $f'(0) =$

$$f'(x) = -(x-1)^2 \sin x + 2\cos x(x-1)$$
$$f'(0) = -(0-1)^2 \sin 0 + 2\cos 0(0-1)$$
$$f'(0) = -(-1)^2(0) + 2(1)(-1) = \boxed{-2}$$

530. *Answer is C.*

If $f(x) = \dfrac{\sin^2 x}{1-\cos x}$ then $f'(x) =$

$$f(x) = \frac{(1-\cos^2 x)}{1-\cos x} = \frac{\cancel{(1-\cos x)}(1+\cos x)}{\cancel{1-\cos x}} = 1 + \cos x$$
$$f'(x) = \boxed{-\sin x}$$

531. *Answer is B.*

For $x \neq 0$ the slope of the tangent to $y = x\cos x$ equals zero whenever

$$y' = \left[x(-\sin x) + \cos x\right]$$
$$y' = -x\sin x + \cos x = 0$$
$$\frac{\cos x}{\cos x} = \frac{x\sin x}{\cos x}$$
$$1 = x\tan x$$
$$\boxed{\tfrac{1}{x} = \tan x}$$

532. *Answer is E.*

If $y = \cos^2 x - \sin^2 x$ then $y' =$

$$y = \left[\cos x\right]^2 - \left[\sin x\right]^2$$
$$y' = 2\left[\cos x\right]^1(-\sin x) - 2\left[\sin x\right]^1(\cos x)$$
$$y' = \boxed{-4\sin x\cos x}$$

533. *Answer is E.*

If $y = \cos^2(2x)$ then $\dfrac{dy}{dx} =$

$$y = \left[\cos(2x)\right]^2$$
$$y' = 2\left[\cos(2x)\right]^1(-\sin(2x))(2)$$
$$y' = \boxed{-4\sin(2x)\cos(2x)}$$

534. *Answer is B.*

A particle moves along the x-axis and its position for time $t \ge 0$ is $x(t) = \cos(2t) + \sec t$
When $t = \pi$ the acceleration of the particle is

$$x(t) = \cos(2t) + \sec t$$
$$v(t) = -\sin(2t)(2) + \sec t \tan t$$
$$v(t) = -2\sin(2t) + \sec t \tan t$$
$$a(t) = -2\cos(2t)(2) + \left[\sec t \sec^2 t + \tan t(\sec t \tan t)\right]$$
$$a(t) = -4\cos(2t) + \left[\sec^3 t + \sec t \tan^2 t\right]$$
$$a(\pi) = -4\cos(2\pi) + \sec^3 \pi + \sec \pi \tan^2 \pi$$
$$a(\pi) = -4(1) + (-1) + (-1)(0)^2 = \boxed{-5}$$

535. *Answer is A.*

If $g(x) = \tan^2(e^x)$ then $g'(x) =$

$$g(x) = \left[\tan(e^x)\right]^2$$
$$g'(x) = 2\left[\tan(e^x)\right]^1 \sec^2(e^x)(e^x)$$
$$g'(x) = \boxed{2e^x \tan(e^x)\sec^2(e^x)}$$

536.

Mean Value Theorem → for derivatives

If $f(x)$ is a function that is continuous on $[a,b]$ and differentiable on (a,b) then there is at least one number $c \in (a,b)$ such that

$$\underbrace{\boxed{\frac{f(b)-f(a)}{b-a}}}_{\text{average rate of change}} = \underbrace{\boxed{f'(c)}}_{\text{instantaneous rate of change}}$$

537. *Answer is C.*

How many values of c satisfy the conclusion of the Mean Value Theorem for $f(x)=x^3+1$ on the interval $[-1,\ 1]$

$f(x)=x^3+1$

$f(-1)=(-1)^3+1=0 \rightarrow$ point $(-1,\ 0)$

$f(1)=(1)^3+1=2 \quad \rightarrow$ point $(1,\ 2)$

Slope of secant $=\dfrac{f(b)-f(a)}{b-a}=\dfrac{2-0}{1-(-1)}=1$

$$f(x)=x^3+1$$
$$f'(x)=3x^2=1$$
$$x^2=\frac{1}{3}$$
$$\boxed{x=\pm\sqrt{\tfrac{1}{3}}}$$

Two values of c in interval $[-1,\ 1]$

538. *Answer is A.*

The value of c guaranteed by the Mean Value Theorem for $f(x)=\dfrac{2}{x-1}$ on the interval $[\ 3,\ 5]$

$f(x)=\frac{2}{x-1}$

$f(3)=\frac{2}{3-1}=1 \quad \rightarrow$ point $(\ 3,\ 1)$

$f(5)=\frac{2}{5-1}=\frac{1}{2} \quad \rightarrow$ point $(\ 5,\ \frac{1}{2})$

Slope of secant $=\dfrac{f(b)-f(a)}{b-a}=\dfrac{\frac{1}{2}-1}{5-3}=-\frac{1}{4}$

$$f(x)=\tfrac{2}{x-1}=2(x-1)^{-1}$$
$$f'(x)=2(-1)(x-1)^{-2}=\tfrac{-2}{(x-1)^2}=\tfrac{-1}{4}$$
$$(x-1)^2=8$$
$$x-1=\pm 2\sqrt{2}$$
$$\boxed{x=1+2\sqrt{2}}$$

One values of c in interval $[\ 3,\ 5]$

539. *Answer is D.*

Given that $f(x) = x^4 - 3$, find $c \in (0, 2)$ such that $\dfrac{f(2) - f(0)}{2 - 0} = f'(c)$

$f(x) = x^4 - 3$

$f(0) = -3 \quad \rightarrow$ point $(0, -3)$

$f(2) = 2^4 - 3 = 13 \quad \rightarrow$ point $(2, 13)$

Slope of secant $= \dfrac{f(b) - f(a)}{b - a} = \dfrac{13 - (-3)}{2 - 0} = 8$

$$f(x) = x^4 - 3$$

$$f'(x) = 4x^3 = 8$$

$$x^3 = 2$$

$$\boxed{x = \sqrt[3]{2}}$$

One values of c in interval $[0, 2]$

540. *Answer is E.*

Given that $f(x) = x^3$, find c such that $f(3) - f(1) = (3 - 1) f'(c)$

$f(x) = x^3$

$f(1) = (1)^3 = 1 \quad \rightarrow$ point $(1, 1)$

$f(3) = (3)^3 = 27 \quad \rightarrow$ point $(3, 27)$

Slope of secant $= \dfrac{f(3) - f(1)}{3 - 1} = \dfrac{27 - 1}{3 - 1} = 13$

$$f(x) = x^3$$

$$f'(x) = 3x^2 = 13$$

$$x^2 = \frac{13}{3}$$

$$\boxed{x = \sqrt{\tfrac{13}{3}}}$$

One value of c in interval $[1, 3]$

541. *Answer is A.*

> If $f(x) = 2x^3 - 6x$, at what point on the interval $0 \le x \le \sqrt{3}$, if any, is the tangent to the curve parallel to the secant line ?

$f(x) = 2x^3 - 6x$

$f(0) = 0 \quad \rightarrow$ point $(0,\ 0)$

$f(\sqrt{3}) = 2(\sqrt{3})^3 - 6(\sqrt{3}) = 0 \quad \rightarrow$ point $(\sqrt{3},\ 0)$

Slope of secant $= \dfrac{f(\sqrt{3}) - f(0)}{\sqrt{3} - 0} = \dfrac{0-0}{3-1} = 0$

$$f(x) = 2x^3 - 6x$$
$$f'(x) = 6x^2 - 6 = 0$$
$$6(x^2 - 1) = 0$$
$$6(x+1)(x-1) = 0$$

$x = -1$	$\boxed{x = 1}$

One value of c in interval $\left[\ 0,\ \sqrt{3}\right]$

542. *Answer is B.*

> Let f be the function given by $f(x) = x^3 - 3x^2$ What are all values of c that satisfy the conclusion of the Mean Value Theorem of differential calculus on the closed interval $[\ 0,\ 3]$

$f(x) = x^3 - 3x^2$

$f(0) = 0 \quad \rightarrow$ point $(0,\ 0)$

$f(3) = 3^3 - 3(3)^2 = 0 \quad \rightarrow$ point $(3,\ 0)$

Slope of secant $= \dfrac{f(3) - f(0)}{3 - 0} = \dfrac{0-0}{3-0} = 0$

$$f(x) = x^3 - 3x^2$$
$$f'(x) = 3x^2 - 6x = 0$$
$$3x(x-2) = 0$$

$x = 0$	$\boxed{x = 2}$

One value of c in interval $[\ 0,\ 3]$ that satisfies MVT of differential calculus (cannot include ***endpoints***)

543. *Answer is D.*

Find the point on the graph of $y = \sqrt{x}$ between **(1, 1)** and **(9, 3)** at which the tangent to the graph has the same slope as the line through **(1, 1)** and **(9, 3)**

$f(x) = \sqrt{x}$

$f(1) = 1 \quad \rightarrow$ point **(1, 1)**

$f(9) = \sqrt{9} = 3 \quad \rightarrow$ point **(9, 3)**

Slope of secant $= \dfrac{f(9) - f(1)}{9-1} = \dfrac{3-1}{9-1} = \dfrac{1}{4}$

$$f(x) = \sqrt{x} = x^{\frac{1}{2}}$$

$$f'(x) = \tfrac{1}{2} x^{-\frac{1}{2}} = \tfrac{1}{2\sqrt{x}} = \tfrac{1}{4}$$

$$2\sqrt{x} = 4$$

$$\boxed{x = 4}$$

One value of c in interval $[\,1,\ 9\,]$

or the point **(4, 4)**

544. *Answer is D.*

If c is the number that satisfies the conclusion of the Mean Value Theorem for $f(x) = x^3 - 2x^2$ on the interval $0 \le x \le 2$ then $c =$

$f(x) = x^3 - 2x^2$

$f(0) = 0 \quad \rightarrow$ point **(0, 0)**

$f(2) = 2^3 - 2(2)^2 = 0 \quad \rightarrow$ point **(2, 0)**

Slope of secant $= \dfrac{f(2) - f(0)}{2-0} = \dfrac{0-0}{2-0} = 0$

$$f(x) = x^3 - 2x^2$$

$$f'(x) = 3x^2 - 4x = 0$$

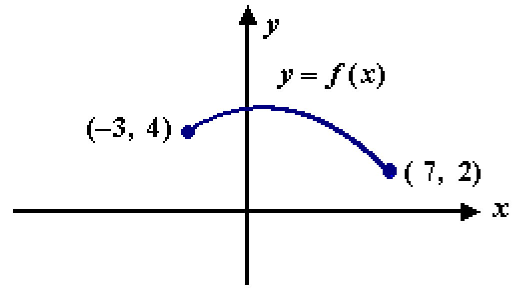
$$x(3x-4) = 0$$

$$x = 0 \quad \Big| \quad \boxed{x = \tfrac{4}{3}}$$

One value of c in interval $[\,0,\ 2\,]$

545. *Answer is D.*

The graph of $y = f(x)$ on the closed interval $\left[-3,\ 7\ \right]$ is shown. If f is continuous on $\left[-3,\ 7\ \right]$ and differentiable on $(-3,\ 7)$, then there exist a c, $-3 < c < 7$ such that



$$f(x) = f(x)$$
$$f(-3) = 4 \quad \rightarrow \text{ point } (-3,\ 4)$$
$$f(7) = 2 \quad \rightarrow \text{ point } (\ 7,\ 2)$$
$$\text{Slope of secant} = \frac{f(7) - f(-3)}{7-(-3)} = \frac{2-4}{7+3} = -\frac{1}{5}$$
$$f(x) = f(x)$$
$$f'(x) = -\tfrac{1}{5} \text{ at some value } x = c$$
$$f'(c) = -\tfrac{1}{5}$$

One value of c in interval $\left[-3,\ 7\right]$

546. *Answer is B*

Let f be the function given by $f(x) = x^3$
What are all values of c that satisfy the conclusion of the **M**ean **V**alue **T**heorem on the closed interval $\left[\ -1,\ 2\ \right]$

$$f(x) = x^3$$
$$f(-1) = -1 \quad \rightarrow \text{ point } (-1, -1)$$
$$f(2) = 8 \quad \rightarrow \text{ point } (\ 2,\ 8)$$
$$\text{Slope of secant} = \frac{f(2) - f(-1)}{2-(-1)} = \frac{8-(-1)}{2+1} = 3$$
$$f(x) = x^3$$
$$f'(x) = 3x^2 = 3$$
$$x^2 = 1$$
$$x = -1 \quad \Big| \quad \boxed{x = 1}$$

One value of c in interval $\left[-1,\ 2\right]$ (must exclude endpoint value of $x = -1$)

547. *Answer is B.*

At time $t \geq 0$ the position of a particle moving along the x-axis is given by $x(t) = \dfrac{t^3}{3} + 2t + 2$
For what value of t in the interval $[\ 0,\ 3]$ will the instantaneous velocity of the particle equal the average velocity of the particle from time $t = 0$ to time $t = 3$

$x(t) = \frac{t^3}{3} + 2t + 2$

$x(0) = 2 \quad \rightarrow$ point $(\ 0,\ 2)$

$x(3) = 9 + 6 + 2 = 17 \quad \rightarrow$ point $(\ 3,\ 17)$

Slope of secant $= \dfrac{x(3) - x(0)}{3 - 0} = \dfrac{17 - 2}{3} = 5$

$x(t) = \frac{t^3}{3} + 2t + 2$

$x'(t) = t^2 + 2 = 5$

$t^2 = 3$

$x = -\sqrt{3} \quad \Big| \quad \boxed{x = \sqrt{3}}$

One value of c in interval $[\ 0,\ 3]$

548. *Answer is D.*

Let $f(x)$ be a function with a continuous derivative on the interval $(\ 1,\ 3)$ such that $f(1) = 2$ and $f(3) = -4$ Which of the following must be true for some a in $(\ 1,\ 3)$

$f(x) = f(x)$

$f(1) = 2 \quad \rightarrow$ point $(\ 1,\ 2)$

$f(3) = -4 \quad \rightarrow$ point $(\ 3, -4)$

Slope of secant $= \dfrac{f(3) - f(1)}{3 - 1} = \dfrac{-4 - 2}{2} = -3$

$f(x) = f(x)$

$f'(x) = -3$ at some point $x = a$

$f'(a) = -3$

One value of c in interval $[\ 1,\ 3]$

549. *Answer is A.*

There is a point between $\mathbf{P(1,0)}$ and $\mathbf{Q}(e,1)$ on the graph of $y = \ln x$ such that the tangent to the graph at that point is parallel to the line through points **P** and **Q**. The x-coordinate of this point is

$$f(x) = \ln x$$

$$f(1) = 0 \quad \rightarrow \text{ point } (1,\ 0)$$

$$f(e) = 1 \quad \rightarrow \text{ point } (e,\ 1)$$

$$\text{Slope of secant} = \frac{f(e) - f(1)}{e-1} = \frac{1-0}{e-1} = \frac{1}{e-1}$$

$$f(x) = \ln x$$

$$f'(x) = \tfrac{1}{x} = \tfrac{1}{e-1}$$

$$\boxed{x = e-1}$$

One value of c in interval $[1,\ e]$

550. *Answer is B.*

Let f be a continuous function on the interval $[-1,\ 3]$ If $f(-1) = 9$ and $f(3) = 1$, then the Mean Value Theorem guarantees that

$$f(x) = f(x)$$

$$f(-1) = 9 \quad \rightarrow \text{ point } (-1,\ 9)$$

$$f(3) = 1 \quad \rightarrow \text{ point } (3,\ 1)$$

$$\text{Slope of secant} = \frac{f(3) - f(-1)}{3-(-1)} = \frac{1-9}{4} = -2$$

$$f(x) = f(x)$$

$$f'(x) = -2 \quad \text{at some point } x = c$$

$$\boxed{f'(c) = -2}$$

One value of c in interval $[-1,\ 3]$

551. *Answer is D.*

If $f'(x)$ exists for all x and $f(1)=10$ and $f(8)=-4$ then, for at least one value of c in the open interval **(1, 8)**, which of the following must be true ?

$$f(x)=f(x)$$

$f(1)=10 \quad \rightarrow$ point **(1, 10)**

$f(8)=-4 \quad \rightarrow$ point **(8,–4)**

$$\text{Slope of secant} = \frac{f(8)-f(1)}{8-1}=\frac{-4-10}{7}=-2$$

$$f(x)=f(x)$$

$f'(x)=-2$ at some point $x=c$

$$\boxed{f'(c)=-2}$$

One value of c in interval $\left[\ 1,\ 8\ \right]$

552. *Answer is E.*

$f(x)$ is a differentiable function with $f(1)=-3$ and $f(5)=4$ Which of the following must be true ?

$$f(x)=f(x)$$

$f(1)=-3 \quad \rightarrow$ point **(1,–3)**

$f(5)=4 \quad \rightarrow$ point **(5, 4)**

$$\text{Slope of secant} = \frac{f(5)-f(1)}{5-1}=\frac{4-(-3)}{4}=\frac{7}{4}$$

$$f(x)=f(x)$$

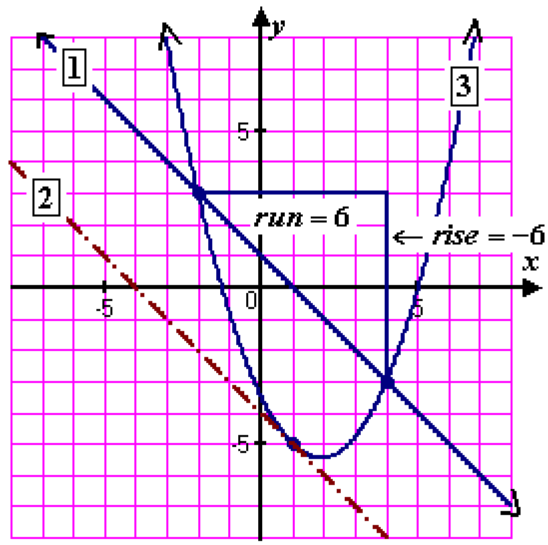
$f'(x)=\frac{7}{4}$ at some point $x=k$

$$\boxed{f'(k)=\frac{7}{4}}$$

One value of k in interval $\left[\ 1,\ 5\ \right]$

553. *Answer is B.*

Let f be a continuous function on the interval $[-2,\ 4]$ If $f(-2)=3$ and $f(4)=-3,$ then the Mean Value Theorem guarantees that



$f(x)=f(x)$

$f(-2)=3 \quad \rightarrow$ point $(-2,\ 3)$

$f(4)=-3 \quad \rightarrow$ point $(\ 4,-3)$

Slope of secant $=\dfrac{f(4)-f(-2)}{4-(-2)}=\dfrac{-3-3}{6}=-1$

$f(x)=f(x)$

$f'(x)=-1$ at some point $x=c$

$\boxed{f'(c)=-1}$

One value of c in interval $[\ -2,\ 4\ ]$

554. *Answer is B.*

There is a point between $\mathbf{P(-1,\ 1)}$ and $\mathbf{Q(\ 7,\ 3)}$ on the graph of $y=\sqrt{x+2}$ such that the line tangent to the graph at that point is parallel to the line through **P** and **Q** The coordinates of this point are

$f(x)=\sqrt{x+2}$

$f(-1)=1 \quad \rightarrow$ point $(-1,\ 1)$

$f(7)=3 \quad \rightarrow$ point $(\ 7,\ 3)$

Slope of secant $=\dfrac{f(7)-f(-1)}{7-(-1)}=\dfrac{3-1}{8}=\dfrac{1}{4}$

$f(x)=\sqrt{x+2}=(x+2)^{\frac{1}{2}}$

$f'(x)=\frac{1}{2}(x+2)^{-\frac{1}{2}}=\frac{1}{2\sqrt{x+2}}=\frac{1}{4}$

$2\sqrt{x+2}=4$

$\sqrt{x+2}=2$

$x+2=4$

$\boxed{x=2}$

One value of c in interval $[\ -1,\ 7\ ]$ or at the point $(\ 2,\ 2)$

555. *Answer is D.*

If f is a differentiable function where $f(0)=-1$ and $f(4)=3$ then which of the following ***must*** be true ?

I. there exists a c in $\left[\,0,\ 4\right]$ where $f(c)=0$

(from sketch) ***must*** be true ☑

II. there exists a c in $\left[\,0,\ 4\right]$ where $f'(c)=0$

(from sketch) ***could*** be true ☒

III. there exists a c in $\left[\,0,\ 4\right]$ where $f'(c)=1$

(from MVT) ***must*** be true ☑

$f(x)=f(x)$

$f(0)=-1 \quad \rightarrow$ point $(\,0,-1)$

$f(4)=3 \quad \rightarrow$ point $(\,4,\ 3)$

Slope of secant $=\dfrac{f(4)-f(0)}{4-0}=\dfrac{3-(-1)}{4}=1$

$f(x)=f(x)$

$f'(x)=1$ at some point $x=c$

$\boxed{f'(c)=1}$

One value of c in interval $\left[\,0,\ 4\,\right]$

556. *Answer is D.*

Let f be a continuous function on the interval $[-1,\ 9]$ If $f(-1)=2$ and $f(9)=7$, then which of the following are ***necessarily*** true ?
I. $f'(c)=\frac{1}{2}$ for some c between -1 and 9
must be true MVT ☑
II. $f(c)>0$ for all c between -1 and 9
could be true ☒
III. $f(c)=5$ for some c between -1 and 9
must be true sketch ☑

$f(x)=f(x)$

$f(-1)=2 \quad \rightarrow$ point $(-1,\ 2)$

$f(9)=7 \quad \rightarrow$ point $(\ 9,\ 7)$

Slope of secant $=\dfrac{f(9)-f(-1)}{9-(-1)}=\dfrac{7-2}{10}=\dfrac{1}{2}$

$f(x)=f(x)$

$f'(x)=\frac{1}{2}$ at some point $x=c$

$\boxed{f'(c)=\frac{1}{2}}$

One value of c in interval $[\ -1,\ 9\ ]$

557. *Answer is B.*

The point $(\ c, f(c))$ on the curve $f(x)=\sqrt{x}$ between $x=a=0$ and $x=b=4$ that satisfies $f'(x)=\dfrac{f(b)-f(a)}{b-a}$ is

$f(x)=\sqrt{x}$

$f(0)=0 \quad \rightarrow$ point $(\ 0,\ 0)$

$f(4)=2 \quad \rightarrow$ point $(\ 4,\ 2)$

Slope of secant $=\dfrac{f(4)-f(0)}{4-0}=\dfrac{2-0}{4}=\dfrac{1}{2}$

$f(x)=\sqrt{x}=x^{\frac{1}{2}}$

$f'(x)=\frac{1}{2}x^{-\frac{1}{2}}=\frac{1}{2\sqrt{x}}=\frac{1}{2}$

$2\sqrt{x}=2$

$\sqrt{x}=1$

$\boxed{x=1}$

One value of c in interval $[\ 0,\ 4\ ]$ at point $(\ 1,\ 1)$

558. *Answer is C.*

Let $f(x)$ be a differentiable function defined only on the interval $-2 \le x \le 10$ The table gives the value of $f(x)$ and its derivative $f'(x)$ at several points of the domain. The line tangent to the graph of $f(x)$ and parallel to the segment between the endpoints intersects the y-axis at the point

$f(x) = f(x)$

$f(-2) = 26 \quad \rightarrow$ point $(0,\ 26)$

$f(10) = 2 \quad \rightarrow$ point $(4,\ 2)$

Slope of secant $= \dfrac{f(4) - f(0)}{10 - (-2)} = \dfrac{2 - 26}{12} = -2$

$f(x) = f(x)$

$f'(4) = -2 \quad \leftarrow$ from table

$f(4) = 23 \quad \leftarrow$ point $(4,\ 23)$ table

Slope $= \dfrac{rise}{run} = \dfrac{-2}{1} = \dfrac{y - 23}{x - 4}$

$y - 23 = -2x + 8$

$\boxed{y = -2x + 31} \leftarrow$ tangent

One value of c in interval $[\ -2,\ 10\ ]$

y-intercept of tangent to f at point

$(4,\ 23)$ is $(0,\ 31)$

559. *Answer is E.*

A function f is continuous for $0 \le x \le 5$ and differentiable for $0 < x < 5$. Given that $f(0) = -2$ and $f(5) = 3$, which of the following statements ***must*** be true ?

I. $f'(c) = 1$ for some c such that $0 < c < 5$

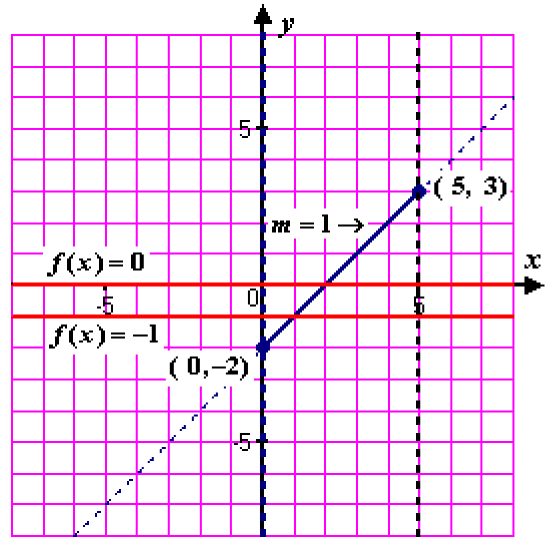
must be true MVT ☑

II. $f(c) = 0$ for some c such that $0 < c < 5$

sketch graph ***must*** be true ☑

III. $f(c) = -1$ for some c such that $0 < c < 5$

sketch graph ***must*** be true ☑



$f(x) = f(x)$

$f(0) = -2 \quad \rightarrow$ point $(0, -2)$

$f(5) = 3 \quad \rightarrow$ point $(5, 3)$

Slope of secant $= \dfrac{f(5) - f(0)}{5 - 0} = \dfrac{3 - (-2)}{5} = 1$

$f(x) = f(x)$

$f'(x) = 1$ for some $x = c$

$\boxed{f'(c) = 1}$

One value of c in interval $[\ 0,\ 5\ ]$

560. *Answer is C.*

Consider the function $f(x) = \sqrt{x - 2}$ On what intervals are the hypotheses of the Mean Value Theorem satisfied ?

Domain of function $\rightarrow$ $x \ge 2$

$\therefore$ end points of interval must both be ≥ 2

$[\ 0,\ 2]$ ☒

$[\ 1,\ 5]$ ☒

$[\ 2,\ 7]$ ☑

none of these ☒

561. *Answer is D.*

Consider the following graph of $f(x) = x\sin x$ on the domain $[-4,\ 4]$ How many values of c in $(-4,\ 4)$ appear to satisfy the Mean Value Theorem equation ?

$f(x) = x\sin x = (-x)\sin(-x)$

(note symmetry with respect to y-axis)

$f(-4) \approx -3 \quad \rightarrow$ point $(-4, -3)$

$f(4) \approx -3 \quad \rightarrow$ point $(4, -3)$

$$\text{Slope of secant} = \frac{f(4) - f(-4)}{4 - (-4)} = \frac{-3 - (-3)}{8} = 0$$

$f(x) = x\sin x$

$f'(-2) = 0 \quad \leftarrow$ from graph

$f'(0) = 0 \quad \leftarrow$ from graph

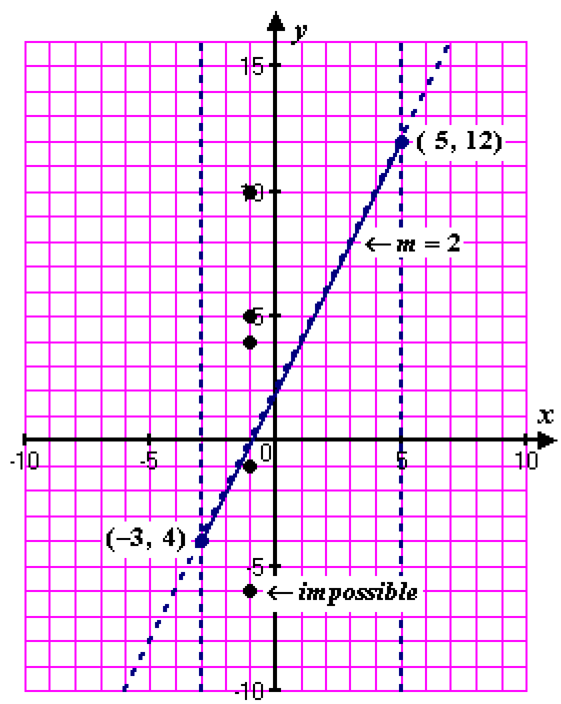
$f'(2) = 0 \quad \leftarrow$ from graph

$\boxed{x = -2,\ 0,\ 2}$

$\boxed{\textbf{Three}}$ values of c in interval $[\ -4,\ 4\ ]$

562. *Answer is A.*

The function $f(x)$ is continuous on the closed interval $[-3,\ 5]$ and differentiable on the open interval $(-3,\ 5)$. If $f'(x) > 0$ over the interval and if $f(-3) = -4$ and $f(5) = 12$, then $f(-1)$ cannot equal



$f(x) = f(x)$

$f(-3) = -4 \quad \rightarrow$ point $(-3, -4)$

$f(5) = 12 \quad \rightarrow$ point $(5,\ 12)$

$$\text{Slope of secant} = \frac{f(5) - f(-3)}{5 - (-3)} = \frac{12 - (-4)}{8} = 2$$

$f'(x) > 0$ means $f(x)$ always increasing

$f(-1) = -1,\ 4,\ 5,\ 10$ ☑

$\boxed{f(-1) \neq -6} \rightarrow f(x)$ is ***increasing*** over $[-3,\ 5]$

563. *Answer is E.*

Difficulty = **0.80**

The function f is continuous and differentiable on the closed interval $[\,0,\ 4]$ The table gives selected values of f on this interval. Which of the following statements ***must*** be true ?

Sketch graph of points in table !!!

The minimum value of f on $[\,0,\ 4]$ is $\mathbf{2}$

could be true ☒

The maximum value of f on $[\,0,\ 4]$ is $\mathbf{4}$

could be true ☒

$f(x) > 0$ for $0 < x < 4$

could be true ☒

$f'(x) < 0$ for $2 < x < 4$

could be true ☒

There exists c with $0 < c < 4$ for which $f'(c) = 0$

must be true MVT ☑

$f(0) = 2 \quad \rightarrow$ point $(\,0,\ 2)$

$f(4) = 2 \quad \rightarrow$ point $(\,4,\ 2)$

Slope of secant $= \dfrac{f(4) - f(0)}{4 - 0} = \dfrac{2 - 2}{4} = 0$

$f'(x) = 0$ for at least one $x = c$

Mean Value Theorem

At least one value of c in interval $[\,0,\ 4\,]$

564.

$\boxed{\textbf{Mean Value Theorem}}$ $\rightarrow$ for integrals (text page 283)

If $f(x)$ is a function that is continuous on $[a,b]$, there exists a number $c \in [a,b]$

such that $\displaystyle\int_a^b f(x)dx = (b-a)f(c)$ ***or*** $\displaystyle\frac{1}{(b-a)}\int_a^b f(x)dx = \underbrace{\boxed{f(c)}}_{\substack{\text{mean value}\\ \text{of } f(x) \text{ on}\\ [a,b]}}$

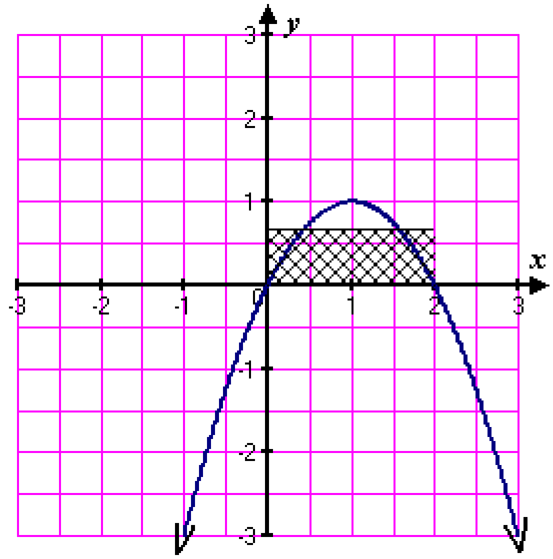
565. *Answer is D.*

Find the average value of $f(x) = 2x - x^2$ on the interval $[\ 0,\ 2\ ]$

$$\frac{1}{(b-a)}\int_a^b f(x)dx = f(c) \leftarrow \text{average } f(x)$$

$$\frac{1}{2-0}\int_0^2 (2x - x^2)dx = \frac{1}{2}\left[x^2 - \frac{x^3}{3}\right]_0^2$$

$$= \frac{1}{2}\left[4 - \frac{8}{3}\right] = \boxed{\frac{2}{3}}$$



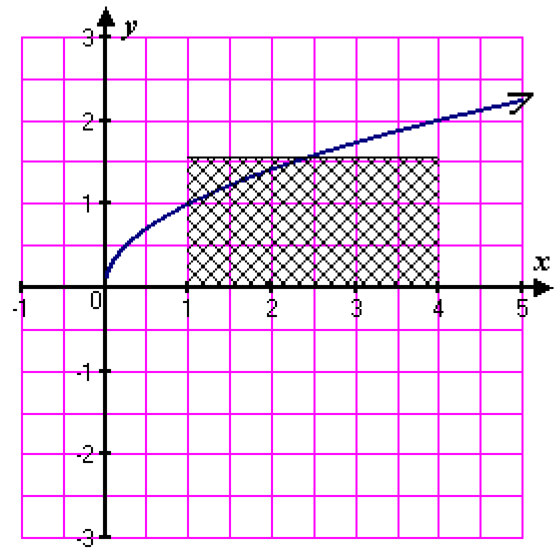
566. *Answer is C.*

Find the average value of $f(x) = \sqrt{x}$ on the interval $[\ 1,\ 4\ ]$

$$\frac{1}{(b-a)}\int_a^b f(x)dx = f(c) \leftarrow \text{average } f(x)$$

$$\frac{1}{4-1}\int_1^4 (x^{\frac{1}{2}})dx = \frac{1}{3}\left[\frac{2x^{\frac{3}{2}}}{3}\right]_1^4 = \frac{2}{9}\left[x^{\frac{3}{2}}\right]_1^4$$

$$= \frac{2}{9}[8-1] = \boxed{\frac{14}{9}}$$



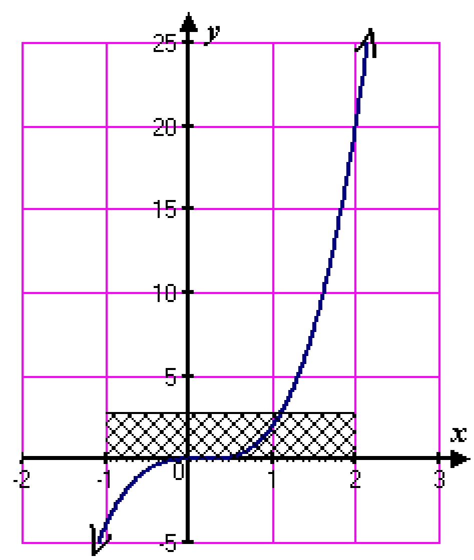
567. *Answer is A.*

What is the average value of $y = 3t^3 - t^2$ over the interval $-1 \le t \le 2$

$$\frac{1}{(b-a)}\int_a^b f(x)dx = f(c) \leftarrow \text{average } f(x)$$

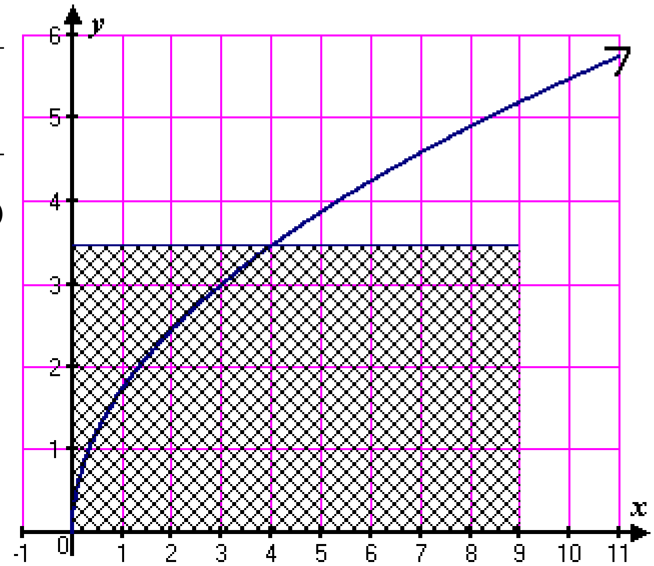
$$\frac{1}{2-(-1)}\int_{-1}^2 3t^3 - t^2 dt = \frac{1}{3}\left[\frac{3t^4}{4} - \frac{t^3}{3}\right]_{-1}^2$$

$$= \frac{1}{3}\left[\left(\frac{3(16)}{4} - \frac{8}{3}\right) - \left(\frac{3}{4} - \frac{-1}{3}\right)\right] = \boxed{\frac{11}{4}}$$



568. *Answer is B.*

The average value of $\sqrt{3x}$ on the closed interval $[\ 0,\ 9\ ]$ is



$$\frac{1}{(b-a)}\int_a^b f(x)dx = f(c) \leftarrow \text{average } f(x)$$

$$\frac{1}{9-0}\int_0^9 \tfrac{1}{3}(3x)^{\frac{1}{2}}(3)\,dx = \frac{1}{27}\left[\frac{2(3x)^{\frac{3}{2}}}{3}\right]_0^9$$

$$= \frac{2}{81}\left[27(3\sqrt{3})\right]$$

$$= \boxed{2\sqrt{3}}$$

569. *Answer is A.*

What is the average (mean) value of $3t^3 - t^2$ over the interval $-1 \le t \le 2$

$$\frac{1}{(b-a)}\int_a^b f(x)dx = f(c) \leftarrow \text{average } f(x)$$

$$\frac{1}{2-(-1)}\int_{-1}^2 3t^3 - t^2\ dx = \frac{1}{3}\left[\frac{3t^4}{4} - \frac{t^3}{3}\right]_{-1}^2$$

$$= \frac{1}{3}\left[\left(\frac{48}{4} - \frac{8}{3}\right) - \left(\frac{3}{4} - \frac{-1}{3}\right)\right]$$

$$= \frac{1}{3}\left[\left(\frac{112}{12}\right) - \left(\frac{13}{12}\right)\right] = \frac{1}{3}\left(\frac{99}{12}\right) = \boxed{\frac{11}{4}}$$

570. *Answer is C.*

The average value of $\sqrt{x}$ over the interval $0 \le x \le 2$ is

$$\frac{1}{(b-a)}\int_a^b f(x)dx = f(c) \leftarrow \text{average } f(x)$$

$$\frac{1}{(2-0)}\int_0^2 \sqrt{x}\ dx = \frac{1}{2}\left[\frac{2x^{\frac{3}{2}}}{3}\right]_0^2 = \frac{1}{\cancel{2}}\left[\frac{\cancel{2}\sqrt{8}}{3}\right] = \boxed{\frac{2\sqrt{2}}{3}}$$

571. *Answer is A.*

The average value of $f(x) = x^2\sqrt{x^3+1}$ on the closed interval $[\ 0,\ 2]$ is

$$\frac{1}{(b-a)}\int_a^b f(x)dx = f(c) \leftarrow \text{average } f(x)$$

$$\frac{1}{(2-0)}\int_0^2 x^2\sqrt{x^3+1}\ dx = \frac{1}{2}\left(\frac{1}{3}\right)\int_0^2 (x^3+1)^{\frac{1}{2}}(3x^2)\ dx$$

$$= \frac{1}{6}\int_0^2 (x^3+1)^{\frac{1}{2}}(3x^2)\ dx$$

$$= \frac{1}{6}\left[\frac{2(x^3+1)^{\frac{3}{2}}}{3}\right]_0^2$$

$$= \frac{1}{9}\left[(x^3+1)^{\frac{3}{2}}\right]_0^2 = \frac{1}{9}\left[(9)^{\frac{3}{2}} - (1)^{\frac{3}{2}}\right]$$

$$= \frac{1}{9}[27-1] = \boxed{\frac{26}{9}}$$

572. *Answer is C.*

Difficulty = **0.60**

What is the average value of y for the part of the curve $y = 3x - x^2$ which is in the first quadrant ?

$$\frac{1}{(b-a)}\int_a^b f(x)dx = f(c) \leftarrow \text{average } f(x)$$

In the first quadrant ? $y = 3x - x^2 = 0$

$x(3-x) = 0$

$[\ 0,\ 3]$ interval $\leftarrow$ $x = 0 \mid 3 = x$

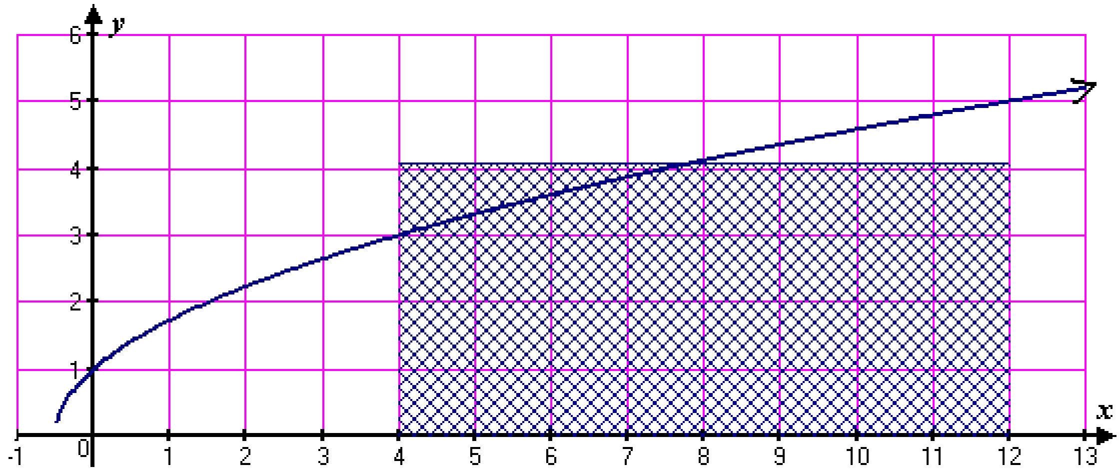
$$\frac{1}{(3-0)}\int_0^3 3x - x^2\ dx = \frac{1}{3}\left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^3$$

$$= \frac{1}{3}\left(\frac{27}{2} - \frac{27}{3}\right) = \frac{1}{3}\left[\frac{27}{6}\right] = \boxed{\frac{3}{2}}$$

573. *Answer is B.*

The average value of the function $y=\sqrt{2x+1}$ from $x=4$ to $x=12$ is

$$\frac{1}{12-4}\int_4^{12}\tfrac{1}{2}(2x+1)^{\frac{1}{2}}(2)\,dx=\frac{1}{16}\left[\frac{2(2x+1)^{\frac{3}{2}}}{3}\right]_4^{12}=\frac{1}{24}\left[(2x+1)^{\frac{3}{2}}\right]_4^{12}=\frac{1}{24}[125-27]=\boxed{\frac{49}{12}}$$



574. *Answer is C.*

The average value of the function $y=3x^2$ over the interval $1\le x\le 3$ is

$$\frac{1}{(b-a)}\int_a^b f(x)dx=f(c) \leftarrow \text{average } f(x)$$

$$\frac{1}{3-1}\int_1^3 3x^2dx=\frac{1}{2}\left[x^3\right]_1^3$$

$$=\frac{1}{2}[27-1]=\boxed{13}$$

575. *Answer is D.*

The average value of the function $f(x)=3x^2-4$ from $x=2$ to $x=4$ is

$$\frac{1}{(b-a)}\int_a^b f(x)dx=f(c) \leftarrow \text{average } f(x)$$

$$\frac{1}{4-2}\int_2^4 3x^2-4dx=\frac{1}{2}\left[x^3-4x\right]_2^4$$

$$=\frac{1}{2}[(64-16)-(8-8)]$$

$$=\frac{1}{2}[48]=\boxed{24}$$

576. *Answer is C.*

The average value of the function $f(x) = 4x^3 - 2x$ over the interval $2 \le x \le 3$ is

$$\frac{1}{(b-a)}\int_a^b f(x)dx = f(c) \leftarrow \text{average } f(x)$$

$$\frac{1}{3-2}\int_2^3 4x^3 - 2x\,dx = \left[x^4 - x^2\right]_2^3$$

$$= \left[(81-9)-(16-4)\right]$$

$$= \left[72-12\right] = \boxed{60}$$

577. *Answer is C.*

What is the average (mean) value of $2t^3 - 3t^2 + 4$ over the interval $-1 \le t \le 1$

$$\frac{1}{(b-a)}\int_a^b f(x)dx = f(c) \leftarrow \text{average } f(x)$$

$$\frac{1}{1-(-1)}\int_{-1}^{1}(2t^3 - 3t^2 + 4)\,dx = \frac{1}{2}\left[\frac{t^4}{2} - t^3 + 4t\right]_{-1}^{1}$$

$$= \frac{1}{2}\left[\left(\frac{1}{2} - 1 + 4\right) - \left(\frac{1}{2} + 1 - 4\right)\right]$$

$$= \frac{1}{2}\left[\frac{1}{2} - 1 + 4 - \frac{1}{2} - 1 + 4\right] = \frac{1}{2}[6] = \boxed{3}$$

578. *Answer is E.*

The average (mean) value of $\frac{1}{x}$ over the interval $1 \le x \le e$ is

$$\frac{1}{(b-a)}\int_a^b f(x)dx = f(c) \leftarrow \text{average } f(x)$$

$$\frac{1}{e-1}\int_1^e \frac{1}{x}dx = \frac{1}{e-1}\left[\ln x\right]_1^e = \frac{1}{e-1}\left[\ln e - \ln 1\right]$$

$$= \boxed{\frac{1}{e-1}}$$

579. *Answer is A.*

The average value of $f(x)=e^{2x}+1$ on the interval $0\le x\le \frac{1}{2}$ is

$$\frac{1}{(b-a)}\int_a^b f(x)dx=f(c)\leftarrow \text{average } f(x)$$

$$\frac{1}{\frac{1}{2}-0}\int_0^{\frac{1}{2}}(e^{2x}+1)\,dx=2\int_0^{\frac{1}{2}}(\tfrac{1}{2}e^{2x}(2)+1)dx=2\Big[\tfrac{1}{2}e^{2x}+x\Big]_0^{\frac{1}{2}}$$

$$=2\Big[(\tfrac{1}{2}e+\tfrac{1}{2})-(\tfrac{1}{2}+0)\Big]=\boxed{e}$$

580. *Answer is C.*

If $f(x)=\sqrt{x-1}$ then the average value of f over the interval $1\le x\le 5$ is

$$\frac{1}{(b-a)}\int_a^b f(x)dx=f(c)\leftarrow \text{average } f(x)$$

$$\frac{1}{5-1}\int_1^5 (x-1)^{\frac{1}{2}}dx=\frac{1}{4}\left[\frac{2(x-1)^{\frac{3}{2}}}{3}\right]_1^5$$

$$=\frac{1}{6}\Big[(x-1)^{\frac{3}{2}}\Big]_1^5$$

$$=\frac{1}{6}\big[8-0\big]=\boxed{\frac{4}{3}}$$

581. *Answer is D.*

The average value of $(3x+1)^2$ on the interval -1 to 1 is

$$\frac{1}{(b-a)}\int_a^b f(x)dx=f(c)\leftarrow \text{average } f(x)$$

$$\frac{1}{1-(-1)}\int_{-1}^1(3x+1)^2dx=\frac{1}{2}\left(\frac{1}{3}\right)\int_{-1}^1(3x+1)^2(3)dx$$

$$=\frac{1}{6}\left[\frac{(3x+1)^3}{3}\right]_{-1}^1$$

$$=\frac{1}{18}\big[64-(-8)\big]=\boxed{4}$$

582. *Answer is D.*

What is the average value of $f(x)=e^{2x}$ over the interval $[\,1,\ 4\,]$

$$\frac{1}{(b-a)}\int_a^b f(x)dx = f(c) \leftarrow \text{average } f(x)$$

$$\frac{1}{4-1}\int_1^4 e^{2x}\ dx = \frac{1}{3}\left(\frac{1}{2}\right)\int_1^4 e^{2x}(2)\ dx$$

$$= \frac{1}{6}\left[e^{2x}\right]_1^4 = \boxed{\frac{e^8-e^2}{6}}$$

583. *Answer is D.*

The average value of $y=(2x+5)^3$ over the interval $[\,1,\ 4\,]$ is

$$\frac{1}{(b-a)}\int_a^b f(x)dx = f(c) \leftarrow \text{average } f(x)$$

$$\frac{1}{4-1}\int_1^4 (2x+5)^3\ dx = \frac{1}{3}\left(\frac{1}{2}\right)\int_1^4 (2x+5)^3(2)\ dx$$

$$= \frac{1}{6}\left[\frac{(2x+5)^4}{4}\right]_1^4 = \frac{1}{24}\left[(2x+5)^4\right]_1^4$$

$$= \frac{1}{24}(13^4-7^4) = \boxed{1090}$$

584. *Answer is B.*

The average value of the function $f(x)=(x-1)^2$ on the interval from $x=1$ to $x=5$ is

$$\frac{1}{(b-a)}\int_a^b f(x)dx = f(c) \leftarrow \text{average } f(x)$$

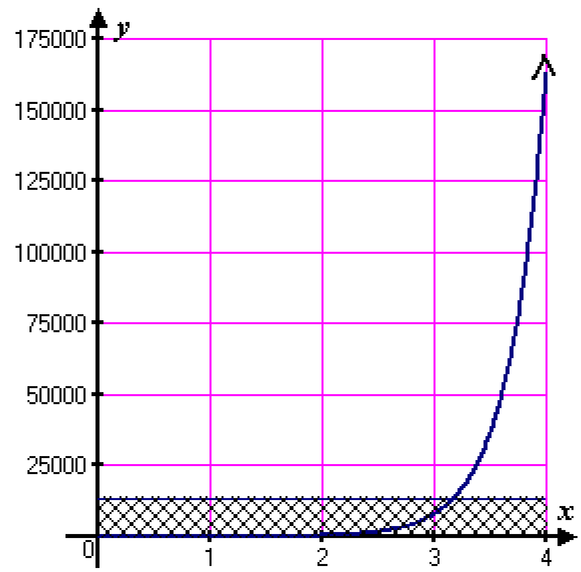
$$\frac{1}{5-1}\int_1^5 (x-1)^2\ dx = \frac{1}{4}\left[\frac{(x-1)^3}{3}\right]_1^5$$

$$= \frac{1}{12}(4^3-0^3) = \boxed{\frac{16}{3}}$$

585. *Answer is A.*

The average value of e^{3x} on the interval $[\,0,\ 4\,]$ is

$$\frac{1}{4-0}\int_0^9 \tfrac{1}{3}e^{3x}(3)\,dx = \frac{1}{12}\Big[e^{3x}\Big]_0^4$$
$$= \frac{1}{12}\Big[e^{12}-1\Big]$$
$$= \boxed{\frac{e^{12}-1}{12}}$$



586. *Answer is D.*

The average value of $\dfrac{1}{x}$ on the closed interval $[\,1,\ 3\,]$ is

$$\frac{1}{(b-a)}\int_a^b f(x)dx = f(c) \leftarrow \text{average } f(x)$$
$$\frac{1}{3-1}\int_1^3 \frac{1}{x}dx = \frac{1}{2}\Big[\ln x\Big]_1^3 = \frac{1}{2}\Big[\ln 3 - \ln 1\Big]$$
$$= \boxed{\frac{\ln 3}{2}}$$

587. *Answer is C.*

If $\displaystyle\int_a^b f(x)\,dx = 8$, $a = 2$, f is continuous, and the average value of f on $[\,a,\ b\,]$ is 4, then $b =$

$$\frac{1}{(b-a)}\int_a^b f(x)dx = f(c) \leftarrow \text{average } f(x)$$
$$\frac{1}{b-2}\int_2^b f(x)dx = 4$$
$$\frac{1}{b-2}(8) = 4$$
$$8 = 4(b-2)$$
$$2 = b-2$$
$$\boxed{4 = b}$$

588. *Answer is E.*

The average value of a continuous function $f(x)$ on the closed interval $[\,3,\ 7\,]$ is 12
What is the value of $\int_3^7 f(x)\,dx$

$$\frac{1}{(b-a)}\int_a^b f(x)dx = f(c) \leftarrow \text{average } f(x)$$

$$\frac{1}{7-4}\int_3^7 f(x)dx = \boxed{12}$$

$$\frac{1}{4}\int_3^7 f(x)dx = \boxed{12}$$

$$\int_3^7 f(x)dx = \boxed{48}$$

589. *Answer is E.*

The average value of $f(x)=x^3$ over the interval $a \le x \le b$ is

$$\frac{1}{(b-a)}\int_a^b f(x)dx = f(c) \leftarrow \text{average } f(x)$$

$$\frac{1}{b-a}\int_a^b x^3dx = \frac{1}{b-a}\left[\frac{x^4}{4}\right]_a^b = \boxed{\frac{b^4-a^4}{4(b-a)}}$$

590. *Answer is B.*

If the average value of $y = x^2$ over the interval $[\ 1,\ b\ ]$ is $\frac{13}{3}$ then the value of b could be

$$\frac{1}{(b-a)}\int_a^b f(x)dx = f(c) \leftarrow \text{average } f(x)$$

$$\frac{1}{b-1}\int_1^b x^2dx = \frac{1}{b-1}\left[\frac{x^3}{3}\right]_1^b = \frac{13}{3}$$

$$\frac{1}{b-1}\left[\frac{b^3}{3} - \frac{1}{3}\right] = \frac{13}{3}$$

$$\frac{1}{b-1}\left[\frac{b^3-1}{3}\right] = \frac{13}{3}$$

$$\frac{\cancel{(b-1)}(b^2+b+1)}{\cancel{3}\,\cancel{(b-1)}} = \frac{13}{\cancel{3}}$$

$$b^2 + b + 1 = 13$$

$$b^2 + b - 12 = 0$$

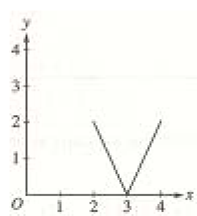
$$(b+4)(b-3) = 0$$

$$b = -4 \quad \Big| \quad \boxed{b = 3}$$

591. *Answer is C.*

Difficulty $= \mathbf{0.46}$

On the closed interval $[\ 2,\ 4]$ which of the following could be the graph of a function f with the property that $\frac{1}{4-2}\int_2^4 f(t)\,dt = 1$



Average value of the function on interval $[\ 2,\ 4]$

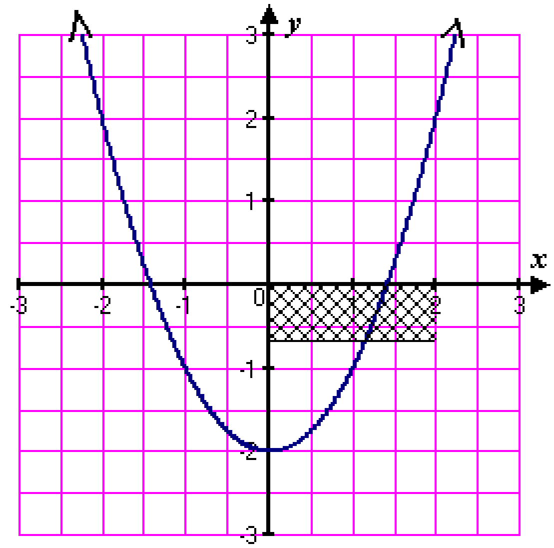
$\frac{1}{4-2}\int_2^4 f(t)\,dt = 1$ therefore the area under the curve $\int_2^4 f(t)\,dt = 2$

592. *Answer is B.*

Find the average value of $f(x)=x^2-2$ on the interval $[\,0,\ 2\,]$

$$\frac{1}{2-0}\int_0^2 (x^2-2)dx=\frac{1}{2}\left[\frac{x^3}{3}-2x\right]_0^2$$

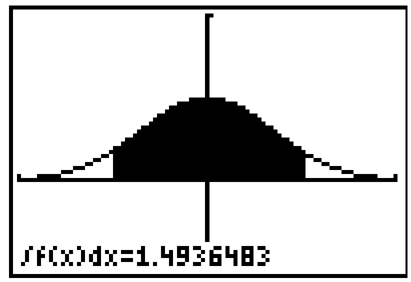
$$=\frac{1}{2}\left[\frac{8}{3}-4\right]=\boxed{-\frac{2}{3}}$$



593. *Answer is C.*

The average value of the function $f(x)=e^{-x^2}$ on the closed interval $[-1,\ 1]$ is

$$\text{Average value}=\frac{1}{2}\int_{-1}^{1} e^{-x^2}dx=\boxed{0.746824}$$

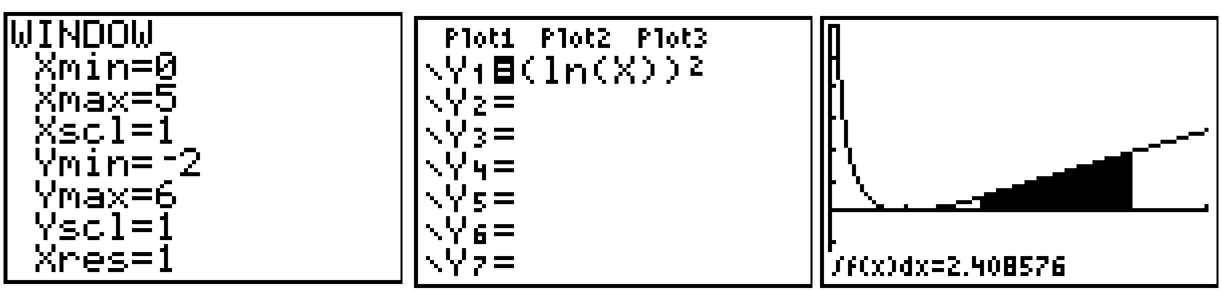


594. *Answer is B.*

The average value of the function $f(x)=\ln^2 x$ on the interval $[\,2,\ 4]$ is

$$\frac{1}{(b-a)}\int_a^b f(x)dx=f(c)$$

$$\frac{1}{4-2}\int_2^4 \ln^2 x\ dx=\boxed{1.2042}$$



595. *Answer is B.*

The average value of the function $y=e^x$ on the interval from $x=-2$ to $x=2$ is

$$\frac{1}{2-(-2)}\int_{-2}^{2} e^x dx=\frac{1}{4}\left[e^x\right]_{-2}^{2}=\frac{1}{4}\left[e^2-e^{-2}\right]_{-2}^{2}\approx 1.8134$$

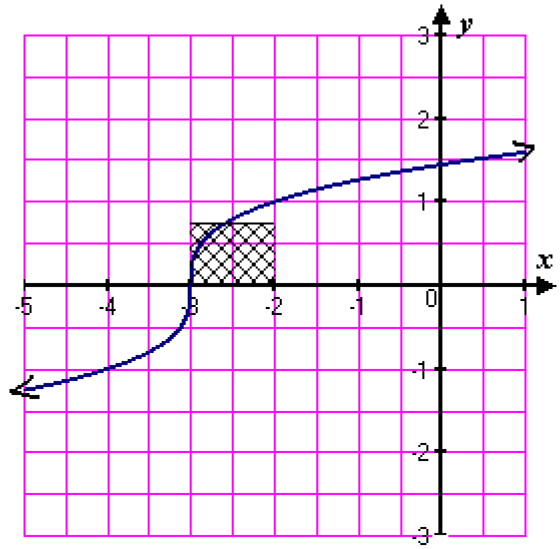
596. *Answer is B.*

597. *Answer is B.*

Find the average value of $f(x)=\sqrt[3]{x+3}$ on the interval $[-3,-2]$

$$\frac{1}{-2-(-3)}\int_{-3}^{-2}(x+3)^{\frac{1}{3}}\,dx=\frac{3}{4}\left[(x+3)^{\frac{4}{3}}\right]_{-3}^{-2}$$

$$=\frac{3}{4}[1-0]=\boxed{\frac{3}{4}}$$



598. *Answer is B.*

What is the average value of the function $f(x)=\dfrac{x}{x^2+1}$ on the interval $[\,0,\ 2]$

$$\frac{1}{(b-a)}\int_a^b f(x)dx=f(c)\ \leftarrow \text{average } f(x)$$

$$\frac{1}{2-0}\int_0^2\frac{x}{x^2+1}dx=\frac{1}{2}\left(\frac{1}{2}\right)\int_0^2\frac{2x}{x^2+1}dx$$

$$=\frac{1}{4}\left[\ln(x^2+1)\right]_0^2$$

$$=\frac{\ln 5-\ln 1}{4}\approx\boxed{0.40234...}$$

599. *Answer is D.*

If $f(x)=2+\lvert x\rvert$ find the average value of the function f on the interval $-1\le x\le 3$

$$\frac{1}{3-(-1)}\int_{-1}^{3}2+\lvert x\rvert\,dx=\frac{1}{4}\left[\overbrace{13}^{geometric}\right]_{-1}^{3}=\boxed{\frac{13}{4}}$$

600. *Answer is C.*

If the position of a particle on the x-axis at time t is $-5t^2$, then the average velocity of the particle for $0 \le t \le 3$ is

$$x(t) = -5t^2 \qquad v(t) = -10t$$

$$\text{Average velocity} = \frac{1}{3-0}\int_0^3 -10t\,dt = \frac{1}{3}\left[-5t^2\right]_0^3 = \frac{1}{3}\left[-45\right] = \boxed{-15}$$

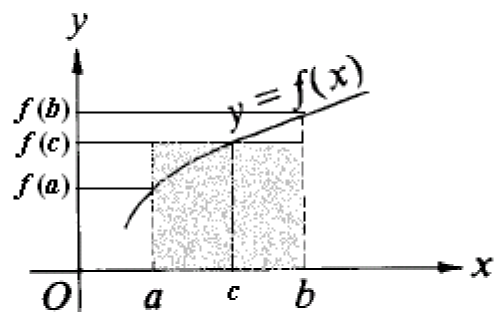
601. *Answer is E.*

If f is the continuous, strictly increasing function on the interval $a \le x \le b$ as shown, which of the following must be true ?

I. $\int_a^b f(x)\,dx < f(b)(b-a)$

II. $\int_a^b f(x)\,dx > f(a)(b-a)$

III. $\int_a^b f(x)\,dx = f(c)(b-a)$ for some number c such that $a < c < b$



Look at choices carefully and rearrange !!!
(question about MVT for integration/average value)

I. $\frac{1}{(b-a)}\int_a^b f(x)\,dx < f(b)$ ☑

II. $\frac{1}{(b-a)}\int_a^b f(x)\,dx > f(a)$ ☑

III. $\frac{1}{(b-a)}\int_a^b f(x)\,dx = f(c)$ ☑

Mark $f(b)$ and $f(a)$ and approximate $f(c)$ on the graph and notice all statements are TRUE

602. *Answer is B.*

If $f(x)$ is a continuous and even function and $\int_0^4 f(x)\,dx = -5$ and $\int_4^6 f(x)\,dx = 2$ then the average value of $f(x)$ over the interval from $x=-6$ to $x=4$ is

$$\frac{1}{4-(-6)}\int_{-6}^4 f(x)dx = \frac{-3+(-5)}{10} = \boxed{-0.8}$$

603. *Answer is C.*

$\int e^{\sin x}\cos x\;dx =$

$$\int e^{\sin x}(\cos x)\;dx = \boxed{e^{\sin x} + C}$$

604. *Answer is B.*

$$\int (\sec^2 x + \sec x \tan x)\ dx =$$

$$\int (\sec^2 x + \sec x \tan x)\ dx = \boxed{\tan x + \sec x + C}$$

605. *Answer is E.*

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \tan x \sin x \cot x \csc x\ dx =$$

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \left(\frac{\cancel{\tan x}}{1}\right)\left(\frac{\cancel{\sin x}}{1}\right)\left(\frac{1}{\cancel{\tan x}}\right)\left(\frac{1}{\cancel{\sin x}}\right) dx = \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} 1\ dx = [x]_{\frac{\pi}{3}}^{\frac{\pi}{4}} = \left[\frac{\pi}{4} - \frac{\pi}{3}\right] = \boxed{-\frac{\pi}{12}}$$

606. *Answer is C.*

$$\int \sin 7x\ dx =$$

$$\int \sin 7x\ dx = -\frac{1}{7}\int (-\sin 7x)(7)\ dx = \boxed{-\frac{1}{7}\cos 7x + C}$$

607. *Answer is E.*

$$\int_0^{\frac{\pi}{4}} \cos^2 x \sin x \sec x\, dx =$$

$$\int_0^{\frac{\pi}{4}} \cos^{\cancel{2}} x \sin x \left(\frac{1}{\cancel{\cos x}}\right) dx = \int_0^{\frac{\pi}{4}} [\sin x]^1 (\cos x)\, dx = \left[\frac{\sin^2 x}{2}\right]_0^{\frac{\pi}{4}} = \left[\frac{\left(\frac{1}{\sqrt{2}}\right)^2}{2} - \frac{(0)^2}{2}\right] = \boxed{\frac{1}{4}}$$

608. *Answer is C.*

$$\int \cos(4x+7)\ dx =$$

$$\int \cos(4x+7)\ dx = \frac{1}{4}\int \cos(4x+7)(4)\ dx = \boxed{\frac{1}{4}\sin(4x+7) + C}$$

609. *Answer is D.*

$$\int \cos 3x\, dx =$$

$$\int \cos 3x\, dx = \frac{1}{3}\int \cos 3x(3)\, dx = \boxed{\frac{1}{3}\sin 3x + C}$$

610. *Answer is A.*

$$\int t\cos(2t)^2\,dt =$$

$$\int t\cos(2t)^2\,dt = \frac{1}{8}\int \cos(4t^2)(8t)\,dt = \boxed{\frac{1}{8}\sin(4t^2) + C}$$

611. *Answer is E.*

$$\int \sin 2\theta\,d\theta =$$

$$\int \sin 2\theta\,d\theta = -\frac{1}{2}\int -\sin 2\theta(2)\,d\theta = \boxed{-\frac{1}{2}\cos 2\theta + C}$$

612. *Answer is D.*

$$\int \frac{du}{\cos^2 3u} =$$

$$\int \frac{du}{\cos^2 3u} = \frac{1}{3}\int \sec^2 3u(3)du = \boxed{\frac{1}{3}\tan 3u + C}$$

613. *Answer is E.*

$$\int \tan\theta\,d\theta =$$

$$\int \tan\theta\,d\theta = \int \frac{\sin\theta}{\cos\theta}\,d\theta = -\int \frac{-\sin\theta}{\cos\theta}\,d\theta = \boxed{-\ln\left|\cos\theta\right| + C}$$

614. *Answer is C.*

$$\int \frac{dx}{\sin^2 2x} =$$

$$\int \frac{dx}{\sin^2 2x} = -\frac{1}{2}\int -\csc^2 2x(2)dx = \boxed{-\frac{1}{2}\cot 2x + C}$$

615. *Answer is B.*

$$\int \cot 2u\,du =$$

$$\int \cot 2u\,du = \int \frac{\cos 2u}{\sin 2u}\,du = \frac{1}{2}\int \frac{\cos 2u(2)}{\sin 2u}\,du = \boxed{\frac{1}{2}\ln\left|\sin 2u\right| + C}$$

616. *Answer is B.*

$$\int \cos\theta\, e^{\sin\theta}\,d\theta =$$

$$\int e^{\sin\theta}\cos\theta\,d\theta = \boxed{e^{\sin\theta} + C}$$

617. *Answer is C.*

$$\int e^{2\theta} \sin e^{2\theta}\, d\theta =$$

$$\int e^{2\theta} \sin e^{2\theta}\, d\theta = -\frac{1}{2}\int -\sin e^{2\theta}(2e^{2\theta})\, d\theta = \boxed{-\frac{1}{2}\cos e^{2\theta} + C}$$

618. *Answer is B.*

$$\int_0^{\frac{\pi}{4}} \sin 2\theta\, d\theta =$$

$$\int_0^{\frac{\pi}{4}} \sin 2\theta\, d\theta = -\frac{1}{2}\int_0^{\frac{\pi}{4}} -\sin 2\theta(2)\, d\theta = -\frac{1}{2}\left[\cos 2\theta\right]_0^{\frac{\pi}{4}} = -\frac{1}{2}\left[\cos\frac{\pi}{2} - \cos 0\right] = -\frac{1}{2}\left[0-1\right] = \boxed{\frac{1}{2}}$$

619. *Answer is D.*

$$\int_0^{\pi} \cos^2\theta \sin\theta\, d\theta =$$

$$-\int_0^{\pi} \left[\cos\theta\right]^2 (-\sin\theta)\, d\theta = -\frac{1}{3}\left[\cos^3\theta\right]_0^{\pi} = -\frac{1}{3}\left[\cos^3\pi - \cos^3 0\right] = -\frac{1}{3}\left[(-1)^3 - (1)^3\right] = \boxed{\frac{2}{3}}$$

620. *Answer is E.*

$$\int_0^{\frac{\pi}{6}} \frac{\cos\theta}{1+2\sin\theta}\, d\theta =$$

$$\frac{1}{2}\int_0^{\frac{\pi}{6}} \frac{2\cos\theta}{1+2\sin\theta}\, d\theta = \frac{1}{2}\left[\ln\left|1+2\sin\theta\right|\right]_0^{\frac{\pi}{6}} = \frac{1}{2}\left[\ln\left|1+2\sin\frac{\pi}{6}\right| - \ln\left|1+2\sin 0\right|\right]$$

$$= \frac{1}{2}\left[\ln 2 - \ln 1\right] = \frac{1}{2}\left[\ln 2 - 0\right] = \boxed{\ln\sqrt{2}}$$

621. *Answer is C.*

$$\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} \frac{\cos 2x}{\sin^2 2x}\, dx =$$

$$\frac{1}{2}\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} \left[\sin 2x\right]^{-2} (\cos 2x)(2)\, dx = -\frac{1}{2}\left[\sin 2x\right]^{-1} = -\frac{1}{2}\left[\frac{1}{\sin 2x}\right]_{\frac{\pi}{12}}^{\frac{\pi}{4}} = -\frac{1}{2}\left[\frac{1}{\sin\frac{\pi}{2}} - \frac{1}{\sin\frac{\pi}{6}}\right]$$

$$= -\frac{1}{2}\left[\frac{1}{1} - \frac{1}{\frac{1}{2}}\right] = \boxed{\frac{1}{2}}$$

622. *Answer is A.*

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^3\theta\cos\theta\ d\theta =$$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} [\sin\theta]^3(\cos\theta)\ d\theta = \left[\frac{\sin^4\theta}{4}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} = \frac{1}{4}\left[1^4 - \left(\frac{1}{\sqrt{2}}\right)^4\right] = \frac{1}{4}(1-\tfrac{1}{4}) = \boxed{\frac{3}{16}}$$

623. *Answer is E.*

$$\int x\cos x^2 dx =$$

$$\int x\cos x^2 dx = \frac{1}{2}\int \cos x^2(2x)dx = \boxed{\frac{1}{2}\sin x^2 + C}$$

624. *Answer is B.*

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cot x\ dx =$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos x}{\sin x}\ dx = \Big[\ln|\sin x|\Big]_{\frac{\pi}{6}}^{\frac{\pi}{2}} = \left[\ln\left|\sin\frac{\pi}{2}\right| - \ln\left|\sin\frac{\pi}{6}\right|\right] = \left[\ln 1 - \ln\frac{1}{2}\right] = \left[0 - \ln 2^{-1}\right] = \boxed{\ln 2}$$

625. *Answer is A.*

$$\int \cos^2 x\sin x\ dx =$$

$$\int \cos^2 x\sin x\ dx = -\int [\cos x]^2(-\sin x)\ dx = \boxed{-\frac{\cos^3 x}{3} + C}$$

626. *Answer is D.*

Difficulty = **0.58**

$$\int_0^{\frac{\pi}{4}} \sin x\, dx =$$

$$\int_0^{\frac{\pi}{4}} \sin x\, dx = -\int_0^{\frac{\pi}{4}} -\sin x\, dx = -\big[\cos x\big]_0^{\frac{\pi}{4}} = -\big[\cos\tfrac{\pi}{4} - \cos 0\big] = -\left[\tfrac{1}{\sqrt{2}} - 1\right] = \boxed{1 - \tfrac{\sqrt{2}}{2}}$$

627. *Answer is B.*

Difficulty = **0.65**

$$\int x^2\cos(x^3)\, dx =$$

$$\int x^2\cos(x^3)\, dx = \frac{1}{3}\int \cos(x^3)(3x^2)\, dx = \boxed{\frac{1}{3}\sin(x^3) + C}$$

628. *Answer is A.*

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos x}{\sin x}\, dx =$$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos x}{\sin x}\, dx = \Big[\ln|\sin x|\Big]_{\frac{\pi}{4}}^{\frac{\pi}{2}} = \Big[\ln|\sin \tfrac{\pi}{2}| - \ln|\sin \tfrac{\pi}{4}|\Big] = \Big[0 - \ln\tfrac{1}{\sqrt{2}}\Big] = -\ln\tfrac{1}{\sqrt{2}} = \ln\left(\tfrac{1}{\sqrt{2}}\right)^{-1} = \boxed{\ln\sqrt{2}}$$

629. *Answer is D.*

$$\int \sin(2x+3)\, dx =$$

$$\int \sin(2x+3)\, dx = -\frac{1}{2}\int -\sin(2x+3)(2)\, dx = \boxed{-\frac{1}{2}\cos(2x+3)+C}$$

630. *Answer is B.*

$$\int \tan(2x)\, dx =$$

$$\int \tan(2x)\, dx = -\frac{1}{2}\int \frac{-\sin(2x)(2)}{\cos(2x)}\, dx = \boxed{-\frac{1}{2}\ln|\cos 2x| + C}$$

631. *Answer is D.*

$$\int_0^{\frac{\pi}{3}} \sin(3x)\; dx =$$

$$-\frac{1}{3}\int_0^{\frac{\pi}{3}} -\sin(3x)(3)\; dx = -\frac{1}{3}\Big[\cos(3x)\Big]_0^{\frac{\pi}{3}} = -\frac{1}{3}\Big[\cos\pi - \cos 0\Big] = -\frac{1}{3}\Big[-1-1\Big] = \boxed{\frac{2}{3}}$$

632. *Answer is C.*

$$\int \sin(2x+3)\, dx =$$

$$\int \sin(2x+3)\, dx = -\frac{1}{2}\int -\sin(2x+3)(2)\, dx = \boxed{-\frac{1}{2}\Big[\cos(2x+3)\Big]+C}$$

633.

Volume Disc $\underbrace{V = \pi\int_a^b \Big[R(x)\Big]^2 dx}_{\textbf{\textit{horizontal}}\text{ axis of revolution}}$ $\underbrace{V = \pi\int_c^d \Big[R(y)\Big]^2 dy}_{\textbf{\textit{vertical}}\text{ axis of revolution}}$

Volume Washer $V = \pi\int_a^b \underbrace{\Big[R(x)\Big]^2}_{outer} - \underbrace{\Big[r(x)\Big]^2}_{inner} dx = \pi\int_a^b \underbrace{\Big[R(x)\Big]^2}_{disc} dx - \underbrace{\pi\int_a^b \Big[r(x)\Big]^2}_{hole} dx$

Volume Cross Section $V = \int_a^b \Big[triangle, square, semicircle\Big] dx$

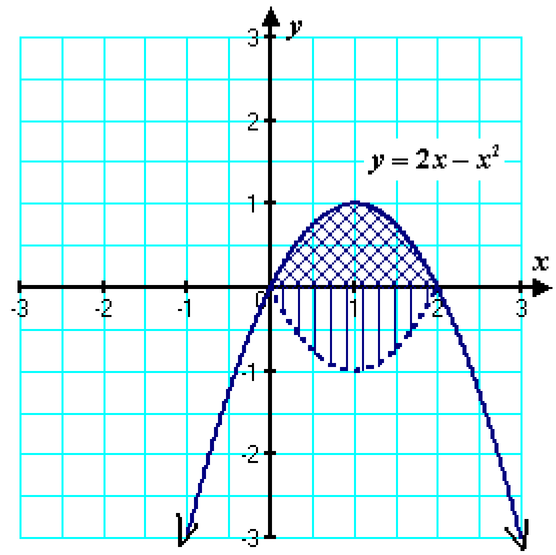
634. *Answer is C.*

An integral for the volume obtained by revolving, around the x-axis, the region bounded by $y = 2x - x^2$ and the x-axis is

$$y = 2x - x^2 = 0$$
$$x(2 - x) = 0$$
$$x = 0 \quad | \quad 2 = x$$

x-intercepts $0, 2$

$$\text{Volume} = \pi \int_0^2 (2x - x^2)^2 \, dx$$

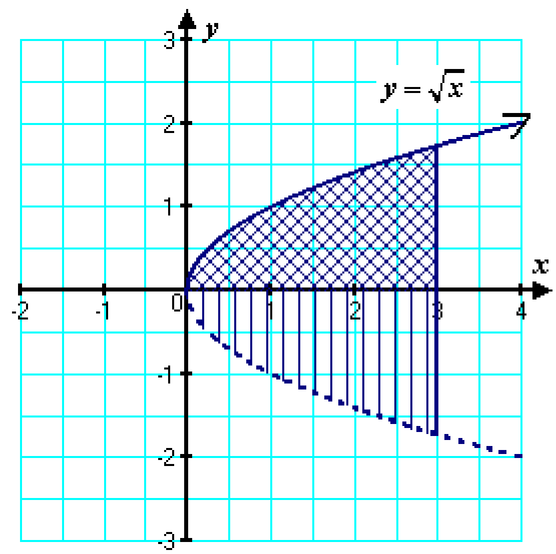


635. *Answer is C.*

Difficulty $= \mathbf{0.58}$

The region enclosed by the x-axis, the line $x = 3$, and the curve $y = \sqrt{x}$ is rotated about the x-axis. What is the volume of the solid generated ?

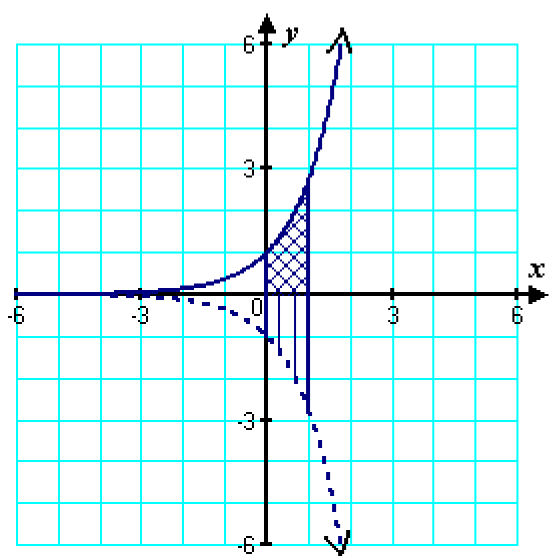
$$\text{Volume} = \pi \int_0^3 \left(\sqrt{x}\right)^2 dx = \pi \int_0^3 x \, dx$$
$$= \pi \left[\frac{x^2}{2}\right]_0^3 = \boxed{\frac{9\pi}{2}}$$



636. *Answer is C.*

What is the volume of the solid generated by revolving the area bounded by $y = e^x$, $x = 0$ and $x = 1$ about the x-axis.

$$\pi \int_0^1 (e^x)^2 dx = \frac{\pi}{2} \int_0^1 e^{2x}(2)dx$$
$$= \frac{\pi}{2}\left[e^{2x}\right]_0^1$$
$$= \frac{\pi}{2}\left[e^2 - e^0\right]$$
$$= \frac{\pi}{2}\left[e^2 - 1\right] = \boxed{\frac{\pi}{2}(e^2 - 1)}$$



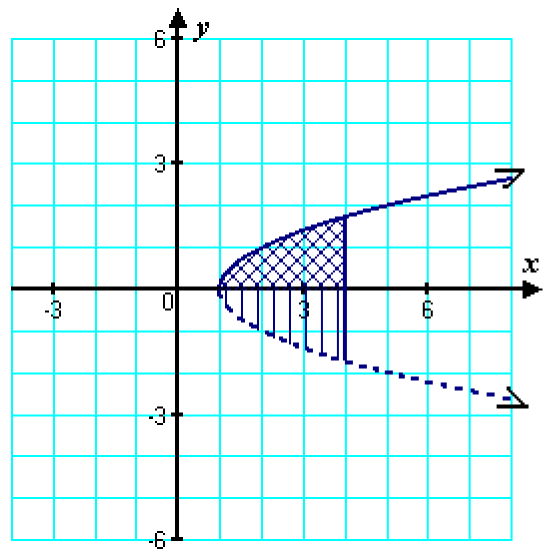
637. *Answer is D.*

If the region enclosed by the graphs of $y=\sqrt{x-1}$, $x=4$ and the x-axis is revolved about the x-axis, the volume of the solid generated is

$$\text{Volume} = \pi\int_1^4 \left(\sqrt{x-1}\right)^2 dx$$

$$\text{Volume} = \pi\int_1^4 (x-1)\,dx = \pi\left[\frac{x^2}{2}-x\right]_1^4$$

$$= \pi\left[(8-4)-(\tfrac{1}{2}-1)\right] = \boxed{\frac{9\pi}{2}}$$

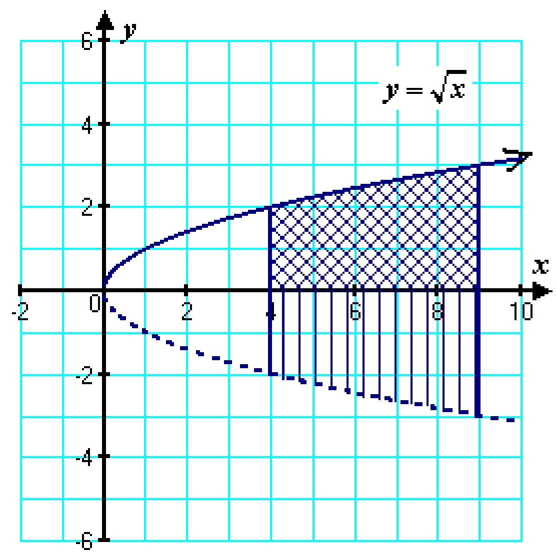


638. *Answer is C.*

What is the volume of the solid obtained when the region bounded by $x=4$, $x=9$, $y=0$, and $y=\sqrt{x}$ is rotated about the x-axis ?

$$\text{Volume} = \pi\int_4^9 \left(\sqrt{x}\right)^2 dx = \pi\int_4^9 x\,dx$$

$$= \pi\left[\frac{x^2}{2}\right]_4^9 = \pi\left(\frac{81}{2}-\frac{16}{2}\right) = \boxed{\frac{65\pi}{2}}$$



639. *Answer is B.*

Find the volume of the solid formed by rotating the graph of $x^2+4y^2=4$ about the x-axis.

$$4y^2 = 4-x^2$$

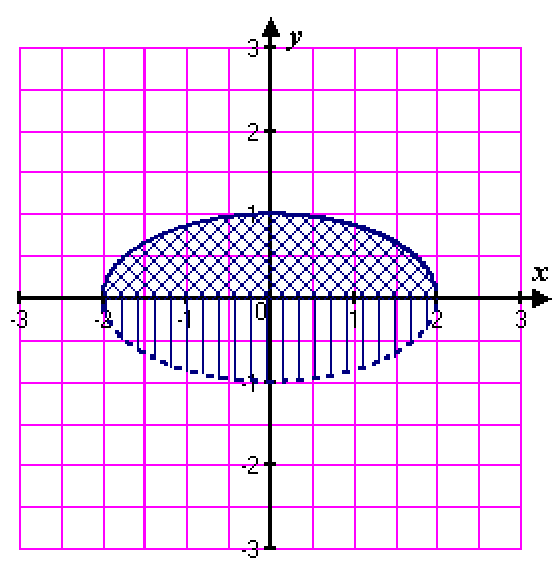
$$y^2 = 1-\tfrac{x^2}{4}$$

$$y = \sqrt{1-\tfrac{x^2}{4}}$$

$$\text{Volume} = \pi\int_{-2}^{2}\left(\sqrt{1-\tfrac{x^2}{4}}\right)^2 dx$$

$$= 2\pi\int_0^2 1-\tfrac{x^2}{4}\,dx$$

$$= 2\pi\left[x-\frac{x^3}{12}\right]_0^2 = 2\pi\left[2-\tfrac{8}{12}\right] = \boxed{\frac{8\pi}{3}}$$

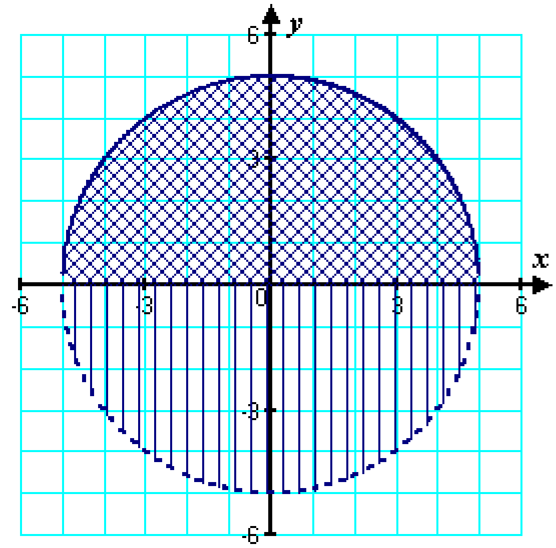


640. *Answer is D.*

Which definite integral represents the volume of a sphere with radius **5**

Circle at (**0**, **0**) with radius **5**

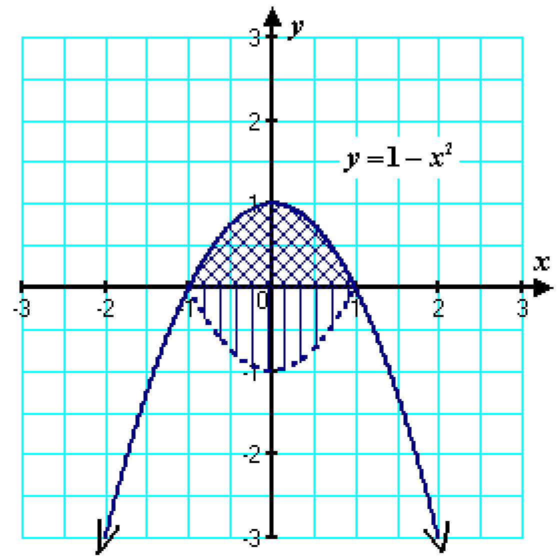
$$x^2 + y^2 = 25$$
$$y^2 = 25 - x^2$$
$$y = \sqrt{25 - x^2}$$
$$\text{Volume} = \pi \int_{-5}^{5} \left(\sqrt{25 - x^2}\right)^2 dx$$
$$= 2\pi \int_{0}^{5} (25 - x^2)\, dx$$



641. *Answer is B.*

What is the volume of the solid obtined by rotating the region bounded by $y = 1 - x^2$ and $y = 0$ about the x-axis ?

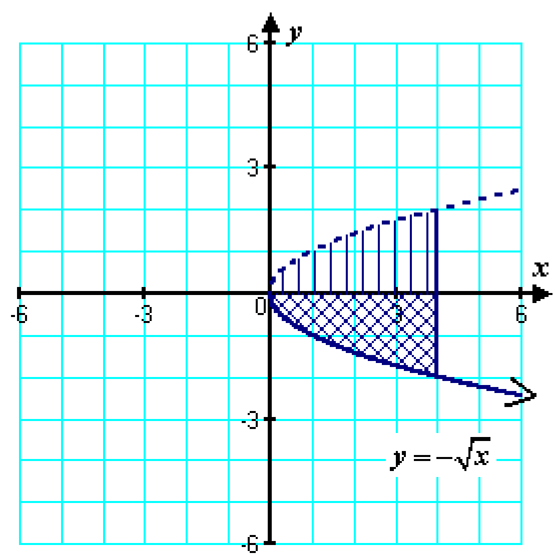
$$\text{Volume} = \pi \int_{-1}^{1} (1 - x^2)^2\, dx$$
$$= 2\pi \int_{0}^{1} (1 - 2x^2 + x^4)\, dx$$
$$= 2\pi \left[x - 2\frac{x^3}{3} + \frac{x^5}{5} \right]_0^1 = 2\pi \left[1 - \frac{2}{3} + \frac{1}{5} \right]$$
$$= 2\pi \left[\frac{15 - 10 + 3}{15} \right] = \boxed{\frac{16\pi}{15}}$$



642. *Answer is C.*

Which of the following integrals represents the volume of the solid obtained by rotating the region bounded by the graph of $y = -\sqrt{x}$, the x-axis and the line $x = 4$ about the x-axis ?

$$\text{Volume} = \pi \int_{0}^{4} \left(-\sqrt{x}\right)^2 dx = \pi \int_{0}^{4} x\, dx$$
$$= \pi \left[\frac{x^2}{2} \right]_0^4 = \boxed{8\pi}$$

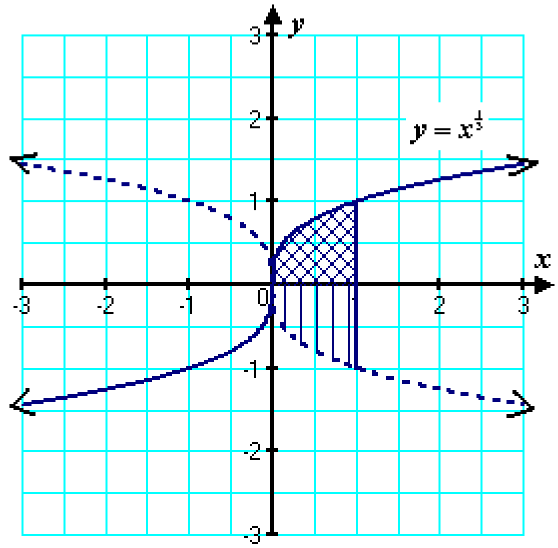


643. *Answer is A.*

The region bounded by $y = x^{\frac{1}{3}}$, $x = 0$, $x = 1$, and the x-axis is revolved about the x-axis. In terms of cubic units, what is the volume of the solid generated ?

$$\text{Volume} = \pi\int_0^1 \left(x^{\frac{1}{3}}\right)^2 dx = \pi\int_0^1 x^{\frac{2}{3}}\,dx$$

$$= \pi\left[\frac{3x^{\frac{5}{3}}}{5}\right]_0^1 = \boxed{\frac{3\pi}{5}}$$



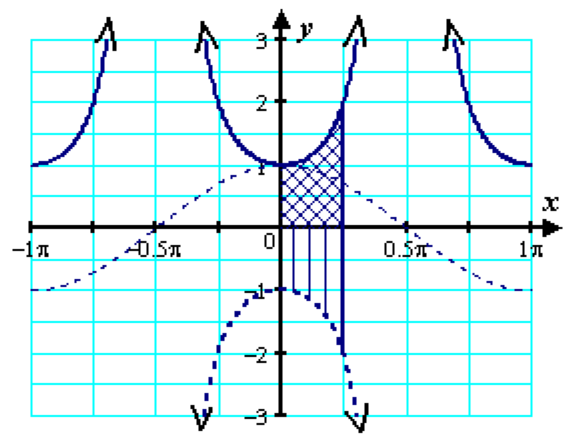
644. *Answer is C.*

The region in the first quadrant bounded by the graph of $y = \sec x$, $x = \dfrac{\pi}{4}$ and the axes is rotated about the x-axis. What is the volume of the solid generated ?

$$\text{Volume} = \pi\int_0^{\frac{\pi}{4}} \sec^2 x\,dx = \pi\left[\tan x\right]_0^{\frac{\pi}{4}}$$

$$= \pi\left[\tan\tfrac{\pi}{4} - \tan 0\right]$$

$$= \boxed{\pi}$$



645. *Answer is B.*

Difficulty = **0.24**

The volume of the solid obtained by revolving the region enclosed by the ellipse $x^2 + 9y^2 = 9$ about the x-axis is

$$x^2 + 9y^2 = 9$$

$$9y^2 = 9 - x^2$$

$$y^2 = 1 - \tfrac{x^2}{9}$$

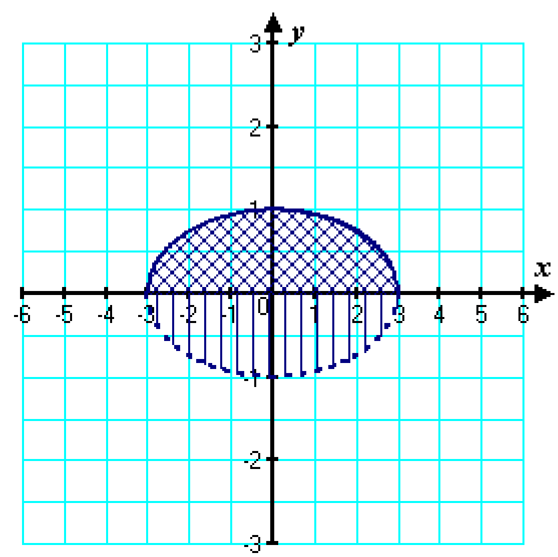
$$y = \sqrt{1 - \tfrac{x^2}{9}}$$

$$\text{Volume} = \pi\int_{-3}^{3} \left(\sqrt{1 - \tfrac{x^2}{9}}\right)^2 dx$$

$$= 2\pi\int_0^3 1 - \tfrac{1}{9}x^2\,dx$$

$$= 2\pi\left[x - \tfrac{1}{9}\left(\tfrac{x^3}{3}\right)\right]_0^3$$

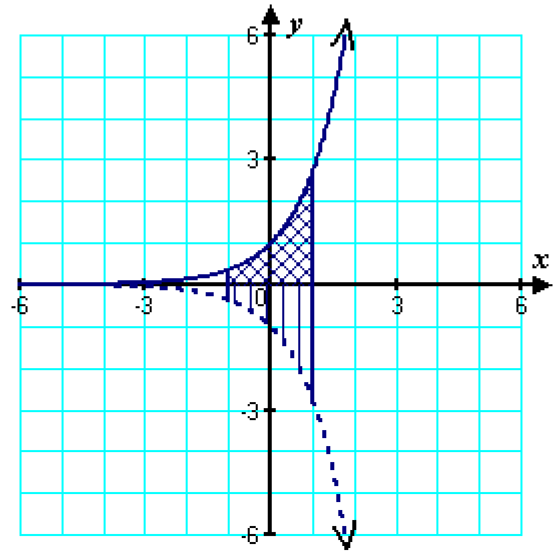
$$= 2\pi\left[3 - \tfrac{1}{9}\left(\tfrac{3^3}{3}\right)\right] = \boxed{4\pi}$$



646. *Answer is C.*

The area bounded by $y = e^x$, $x = -1$, $x = 1$ and the x-axis is revolved about the x-axis. The volume thus generated is

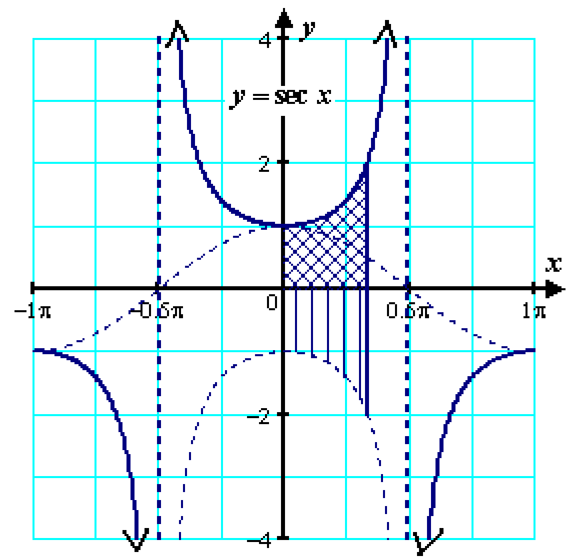
$$\pi\int_{-1}^{1}(e^x)^2\,dx = \frac{\pi}{2}\int_{-1}^{1}e^{2x}(2)dx$$
$$= \frac{\pi}{2}\left[e^{2x}\right]_{-1}^{1}$$
$$= \frac{\pi}{2}\left[e^2 - e^{-2}\right] = \boxed{\frac{\pi}{2}\left(e^2 - \frac{1}{e^2}\right)}$$



647. *Answer is C.*

What is the volume of the solid generated by rotating about the x-axis the region enclosed by the curve $y = \sec x$ and the lines $x = 0$, $y = 0$ and $x = \frac{\pi}{3}$

$$\text{Volume} = \pi\int_{0}^{\frac{\pi}{3}}\sec^2 x\,dx = \pi\left[\tan x\right]_{0}^{\frac{\pi}{3}}$$
$$= \pi\left[\tan\tfrac{\pi}{3} - \tan 0\right] = \boxed{\pi\sqrt{3}}$$



648. *Answer is C.*

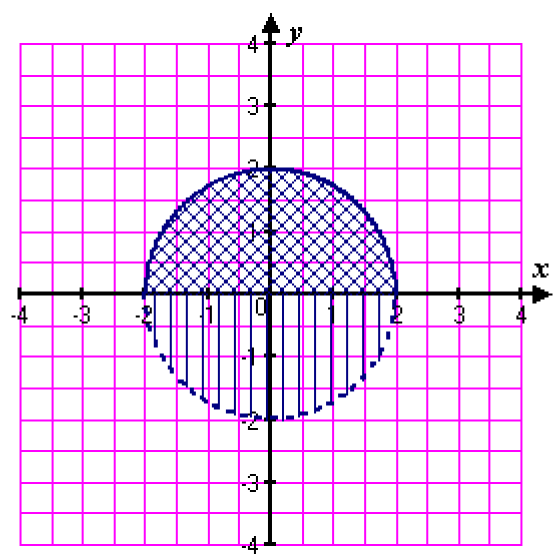
Which definite integral represents the ***volume*** of a sphere with radius **2**

Circle at $(0, 0)$ radius $= 2$

$$x^2 + y^2 = 4$$
$$y^2 = 4 - x^2$$

Radius $\quad y = \sqrt{4 - x^2} \leftarrow$ use dx

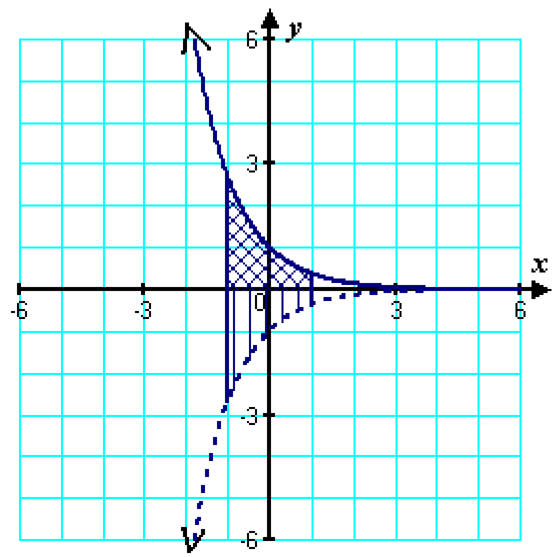
$$\text{Volume} = \pi\int_{-2}^{2}\left(\sqrt{4 - x^2}\right)^2 dx$$
$$= 2\pi\int_{0}^{2}(4 - x^2)\,dx$$



649. *Answer is A.*

The volume of a solid generated by revolving the area bounded by $x=-1,\ x=1$ and $y=e^{-x}$ about the x-axis is

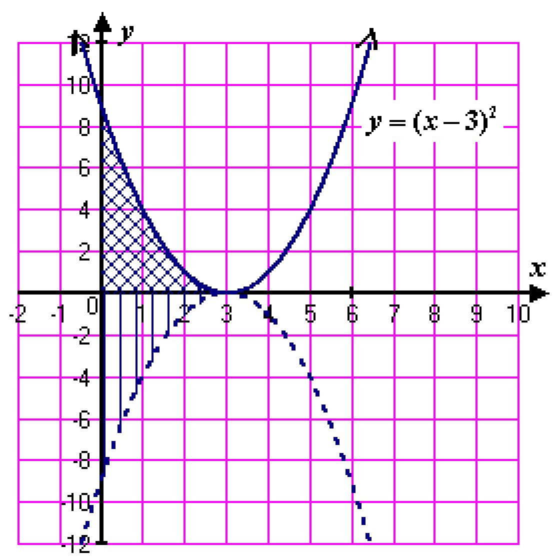
$$\pi\int_{-1}^{1}(e^{-x})^2\,dx=\frac{-\pi}{2}\int_{-1}^{1}e^{-2x}(-2)dx$$
$$=\frac{-\pi}{2}\left[e^{-2x}\right]_{-1}^{1}$$
$$=\frac{-\pi}{2}\left[e^{-2}-e^{2}\right]$$
$$=\frac{\pi e^2}{2}\left[\tfrac{e^2}{e^2}-\tfrac{e^{-2}}{e^2}\right]=\boxed{\frac{\pi e^2}{2}(1-\tfrac{1}{e^4})}$$



650. *Answer is B.*

The volume of the solid formed by revolving the region bounded by the graph of $y=(x-3)^2$ and the coordinate axes about the x-axis is given by which of the following integrals ?

$$\text{Volume}=\pi\int_0^3\left[(x-3)^2\right]^2dx$$
$$=\boxed{\pi\int_0^3(x-3)^4dx}$$



651. *Answer is D.*

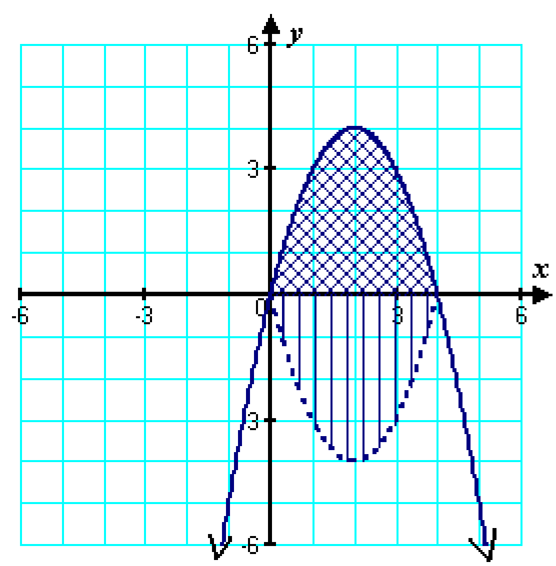
What is the volume of the solid generated by revolving the region bounded by the x-axis and the graph of $y=4x-x^2$ about the x-axis ?

Zeros of function $y=4x-x^2=0$

$$x(4-x)=0$$

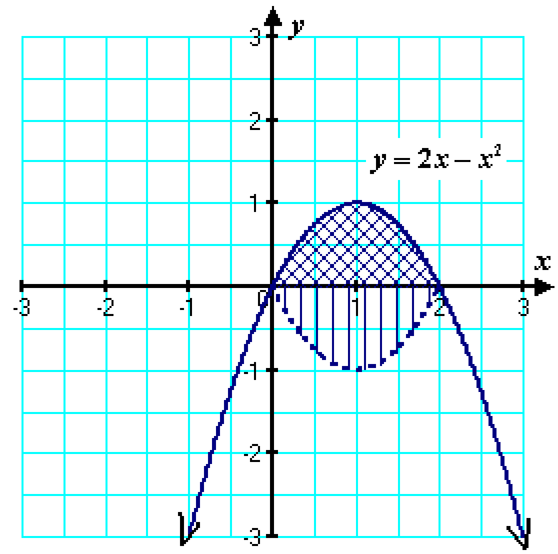
$x=0$	$4=x$

$$\text{Volume}=\pi\int_0^4(4x-x^2)^2dx$$
$$=\pi\int_0^4(16x^2-8x^3+x^4)dx$$
$$=\pi\left[\frac{16x^3}{3}-2x^4+\frac{x^5}{5}\right]_0^4=\boxed{\frac{512\pi}{15}}$$



652. *Answer is A.*

Let **R** be the region in the first quadrant bounded by the x-axis and the curve $y = 2x - x^2$ The volume produced when **R** is revolved about the x-axis is



Limits of integration $2x - x^2 = 0$

$$x(2-x) = 0$$

$$x = 0 \quad | \quad x = 2$$

$$\text{Volume} = \pi\int_0^2 (2x - x^2)^2\,dx$$

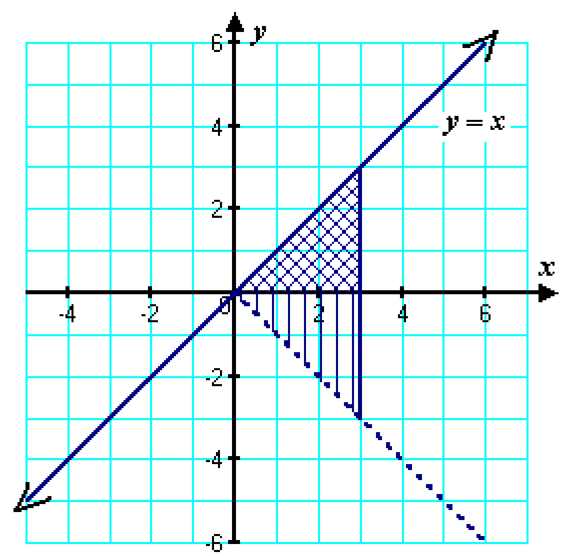
$$= \pi\int_0^2 (4x^2 - 4x^3 + x^4)\,dx$$

$$= \pi\left[\frac{4x^3}{3} - x^4 + \frac{x^5}{5}\right]_0^2$$

$$= \pi\left[\frac{32}{3} - 16 + \frac{32}{5}\right] = \boxed{\frac{16\pi}{15}}$$

653. *Answer is A.*

The region in the first quadrant between the x-axis from $x = 0$ to $x = 3$, and the graph $y = x$, is rotated about the x-axis. The volume of the resulting solid of revolution is given by



$$\text{Volume} = \pi\int_0^3 (x)^2\,dx = \boxed{\int_0^3 \pi x^2\,dx}$$

654. *Answer is B.*

> Find the volume of the solid formed by revolving the region bounded by $y = x^3$, $y = 1$, and $x = 2$ about the x-axis.

$$\text{Volume} = \pi\int_1^2 (R)^2 - (r)^2\, dx =$$

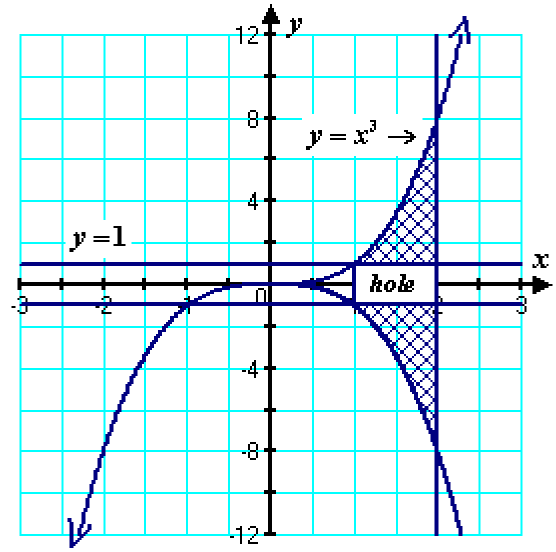
$$\pi\int_1^2 (x^3)^2 - (1)^2\, dx = \pi\int_1^2 x^6 - 1 dx$$

$$= \pi\left[\tfrac{x^7}{7} - x\right]_1^2$$

$$= \pi\left[(\tfrac{2^7}{7} - 2) - (\tfrac{1^7}{7} - 1)\right]$$

$$= \pi\left[\tfrac{128}{7} - 2 - \tfrac{1}{7} + 1\right]$$

$$= \pi\left[\tfrac{127}{7} - 1\right] = \boxed{\tfrac{120}{7}\pi}$$



655. *Answer is E.*

> Find the volume of the solid formed by revolving the region bounded by the graphs of $y = -x^2 + 4$ and $y = 0$ about the x-axis.

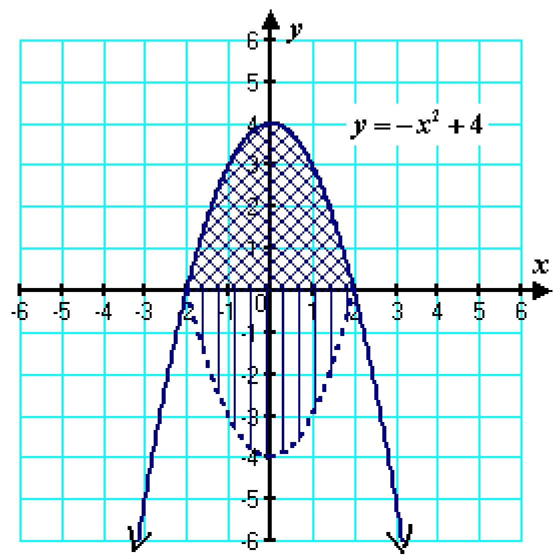
$$\text{Volume} = \pi\int_{-2}^2 (4 - x^2)^2\, dx$$

$$= 2\pi\int_0^2 (16 - 8x + x^4) dx$$

$$= 2\pi\left[16x - 4x^2 + \tfrac{x^2}{5}\right]_0^2$$

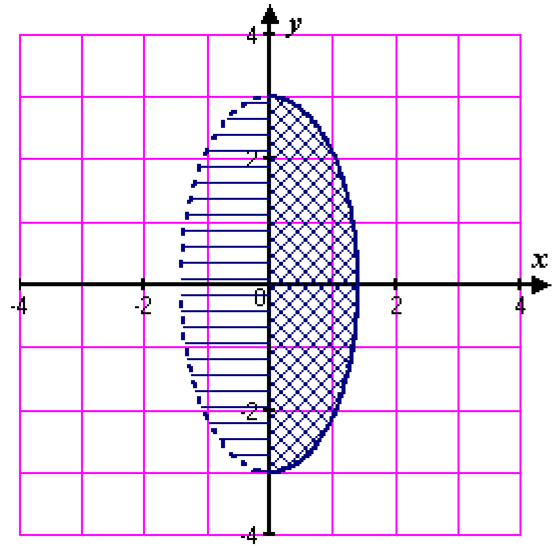
$$= 2\pi\left[32 - 16 + \tfrac{4}{5}\right]$$

$$= 2\pi\left[16 + \tfrac{4}{5}\right] = \boxed{\tfrac{168}{5}\pi}$$



656. *Answer is C.*

The ellipse $\frac{x^2}{2}+\frac{y^2}{9}=1$ is revolved around the y-axis. The number of cubic units in the resulting solid is



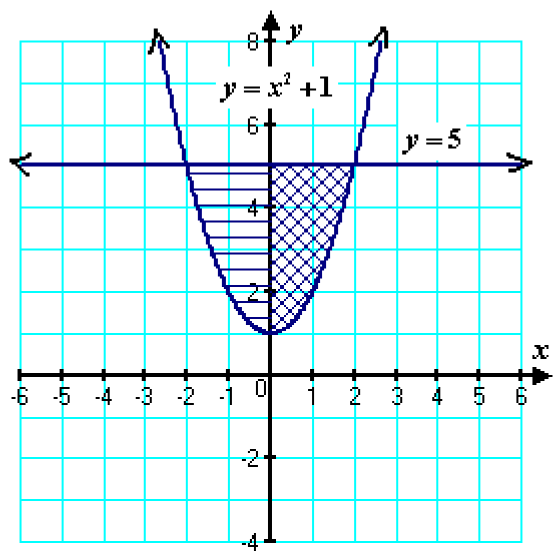
$$9x^2+2y^2=18$$
$$9x^2=18-2y^2$$
$$x^2=2-\tfrac{2}{9}y^2$$
$$x=\sqrt{2-\tfrac{2}{9}y^2}$$
$$f(y)=\sqrt{2-\tfrac{2}{9}y^2} \leftarrow y\text{-axis}$$

y-intercepts are $y=\pm 3$

$$\text{Volume}=\pi\int_{-3}^{3}\left(\sqrt{2-\tfrac{2}{9}y^2}\right)^2 dy \leftarrow y\text{-axis}$$
$$=2\pi\int_0^3 (2-\tfrac{2}{9}y^2)\,dy=2\pi\left[2y-\frac{2y^3}{27}\right]_0^3=\boxed{8\pi}$$

657. *Answer is B.*

The region **R** in the first quadrant is enclosed by the lines $x=0$ and $y=5$ and the graph of $y=x^2+1$ The volume of the solid generated when **R** is revolved about the y-axis is



$$y=x^2+1$$
$$y-1=x^2$$
$$\sqrt{y-1}=x$$
$$\text{Volume}=\pi\int_1^5 (\sqrt{y-1})^2\,dy \leftarrow y\text{-axis}$$
$$=\pi\int_1^5 y-1\,dy=\pi\left[\tfrac{y^2}{2}-y\right]_1^5$$
$$=\pi\left[(\tfrac{25}{2}-5)-(\tfrac{1}{2}-1)\right]=\boxed{8\pi}$$

658. *Answer is A.*

Difficulty = **0.30**

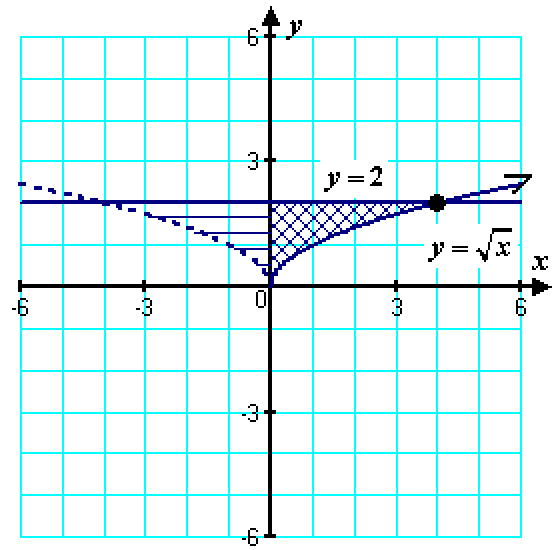
If the region enclosed by the y-axis, the line $y = 2$ and the curve $y = \sqrt{x}$ is revolved about the y-axis, the volume of the solid generated is

$$y^2 = x$$

$$\text{Volume} = \pi \int_0^2 (y^2)^2\, dy \quad \leftarrow y\text{-axis}$$

$$= \pi \int_0^2 y^4 dy$$

$$= \pi \left[\frac{y^5}{5} \right]_0^2 = \boxed{\frac{32\pi}{5}}$$



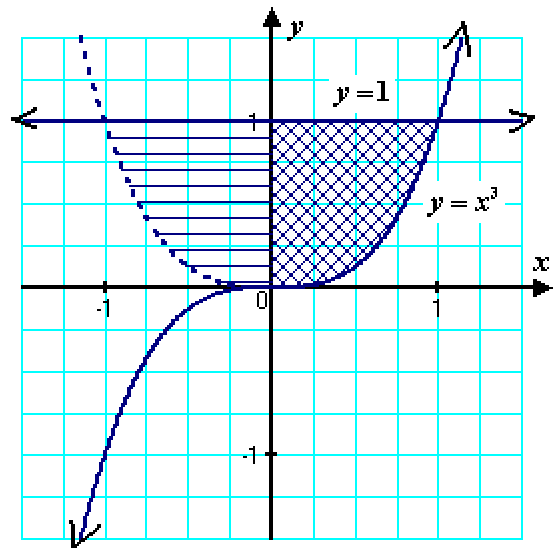
659. *Answer is C.*

The volume of the solid generated by revolving about the y-axis the region bounded by the graph of $y = x^3$, the line $y = 1$ and the y-axis is

$$\sqrt[3]{y} = x$$

$$\text{Volume} = \pi \int_0^1 \left(\sqrt[3]{y}\right)^2 dy = \pi \int_0^1 y^{\frac{2}{3}}\, dy \quad \leftarrow y\text{-axis}$$

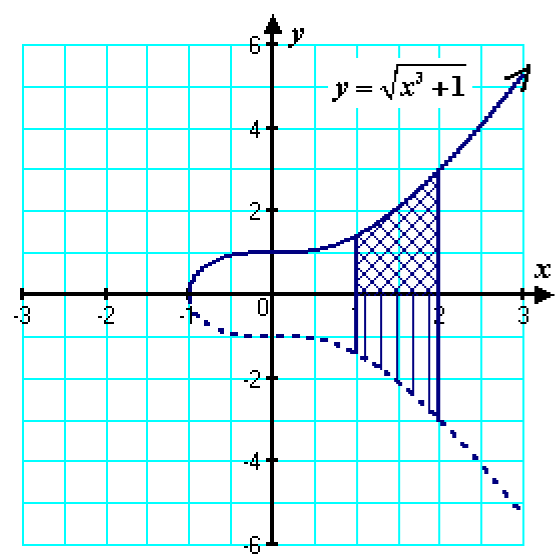
$$= \pi \left[\frac{3y^{\frac{5}{3}}}{5} \right]_0^1 = \boxed{\frac{3\pi}{5}}$$



660. *Answer is A.*

What is the volume of the solid obtained by rotating the region under the graph $y = \sqrt{x^3 + 1}$ between $x = 1$ and $x = 2$, around the x-axis ?

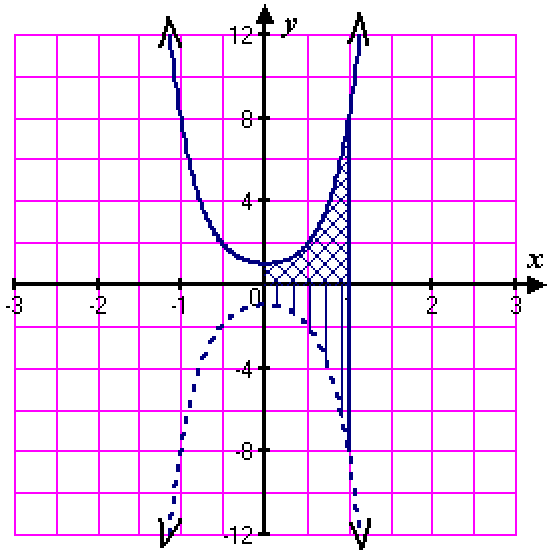
$$\text{Volume} = \pi \int_1^2 \left(\sqrt{x^3 + 1}\right)^2 dx = \pi \int_1^2 x^3 + 1\, dx$$



661. *Answer is D.*

A solid is generated when the region in the first quadrant enclosed by the graph of $y=(x^2+1)^3$, the line $x=1$, the x-axis, and the y-axis is revolved about the x-axis. Its volume is found by evaluating which of the following integrals ?

$$\text{Volume} = \pi\int_0^1 \left((x^2+1)^3\right)^2 dx = \pi\int_0^1 (x^2+1)^6\, dx$$



662. Let **R** be the region in the first quadrant that is enclosed by the graph of $y=\tan x$, the x-axis, and the line $x=\frac{\pi}{3}$

a) Find the area of **R**

b) Find the volume of the solid formed by revolving **R** about the x-axis.

$$a)\quad \text{Area} = \int_0^{\frac{\pi}{3}} \tan x dx = -\int_0^{\frac{\pi}{3}} \frac{-\sin x}{\cos x} dx$$

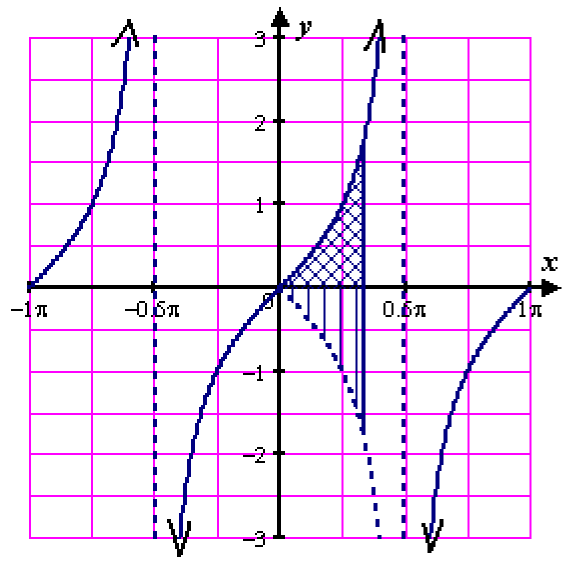
$$= -\Big[\ln|\cos x|\Big]_0^{\frac{\pi}{3}}$$

$$= -\Big[\ln|\cos \tfrac{\pi}{3}| - \ln|\cos 0|\Big]$$

$$= -\Big[\ln\tfrac{1}{2} - \ln 1\Big] = \boxed{\ln 2}$$

$$b)\quad \text{Volume} = \pi\int_0^{\frac{\pi}{3}} \tan^2 x\, dx = \pi\int_0^{\frac{\pi}{3}} \sec^2 x - 1\, dx$$

$$= \pi\Big[\tan x - x\Big]_0^{\frac{\pi}{3}} = \boxed{\pi\left[\sqrt{3}-\tfrac{\pi}{3}\right]}$$



663. *Answer is C.*

The volume generated by revolving $y = x^3$ around the y-axis is $(-1 \le x \le 1)$

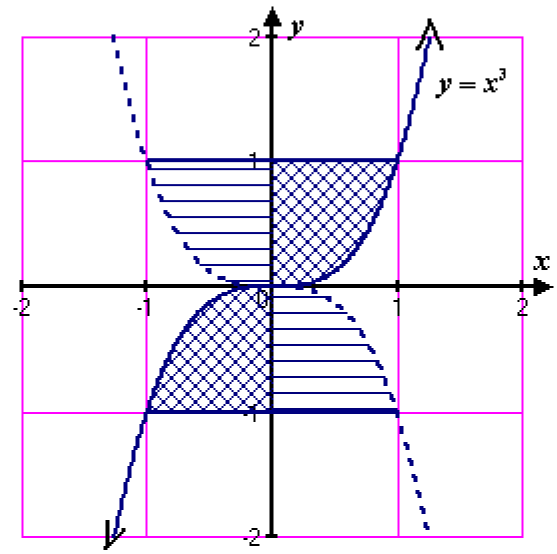
$$y = x^3$$

$$\sqrt[3]{y} = x \quad \leftarrow f(y)$$

$$\text{Volume} = 2\pi \int_0^1 \left(y^{\frac{1}{3}}\right)^2 dy \quad \leftarrow y\text{-axis}$$

$$= 2\pi \int_0^1 y^{\frac{2}{3}} dy$$

$$= 2\pi \left[\frac{3y^{\frac{5}{3}}}{5}\right]_0^1 = \boxed{\frac{6\pi}{5}}$$



664. Let **R** be the region bounded by the x-axis, the graph of $y=\sqrt{x}$, and the line $x=4$

a) Find the area of the region **R**

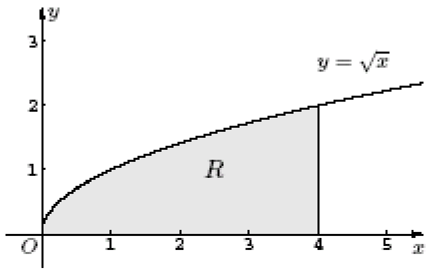
b) Find the value of h such that the vertical line $x=h$ divides the region **R** into two regions of equal area.

c) Find the volume of the solid generated when **R** is revolved about the x-axis.

d) The vertical line $x=k$ divides the region **R** into two regions such that when these two regions are revolved about the x-axis, they generate solids with equal volumes. Find the value of k

---

(a)



$$A=\int_0^4 \sqrt{x}\,dx=\frac{2}{3}x^{3/2}\Big|_0^4=\frac{16}{3} \quad \text{or} \quad 5.333$$

$2\begin{cases}1: & A=\int_0^4 \sqrt{x}\,dx\\ 1: & \text{answer}\end{cases}$

(b) $\displaystyle\int_0^h \sqrt{x}\,dx=\frac{8}{3}$ —or— $\displaystyle\int_0^h \sqrt{x}\,dx=\int_h^4 \sqrt{x}\,dx$

$\displaystyle\frac{2}{3}h^{3/2}=\frac{8}{3}$ —or— $\displaystyle\frac{2}{3}h^{3/2}=\frac{16}{3}-\frac{2}{3}h^{3/2}$

$h=\sqrt[3]{16}$ or 2.520 or 2.519

$2\begin{cases}1: & \text{equation in } h\\ 1: & \text{answer}\end{cases}$

(c) $\displaystyle V=\pi\int_0^4 (\sqrt{x})^2\,dx=\pi\frac{x^2}{2}\Big|_0^4=8\pi$

or 25.133 or 25.132

$3\begin{cases}1: & \text{limits and constant}\\ 1: & \text{integrand}\\ 1: & \text{answer}\end{cases}$

(d) $\displaystyle\pi\int_0^k (\sqrt{x})^2\,dx=4\pi$ —or— $\displaystyle\pi\int_0^k (\sqrt{x})^2\,dx=\pi\int_k^4 (\sqrt{x})^2\,dx$

$\displaystyle\pi\frac{k^2}{2}=4\pi$ —or— $\displaystyle\pi\frac{k^2}{2}=8\pi-\pi\frac{k^2}{2}$

$k=\sqrt{8}$ or 2.828

$2\begin{cases}1: & \text{equation in } k\\ 1: & \text{answer}\end{cases}$

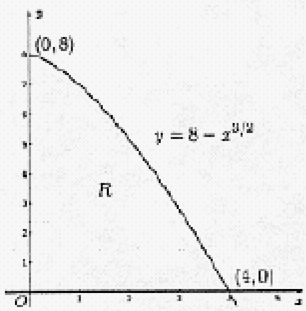
665. Let **R** be the region in the first quadrant bounded by the graph of $y = 8 - x^{\frac{3}{2}}$ the x-axis and and y-axis.

***a*)** Find the area of the region **R**

***b*)** Find the volume of the solid generated when **R** is revolved about the x-axis.

***c*)** The vertical line $x = k$ divides the region **R** into two regions such that when these two regions are revolved about the x-axis, they generate solids with equal volumes. Find the value of k

(a)



$$A = \int_0^4 \left(8 - x^{3/2}\right) dx$$

$$= 8x - \frac{2}{5}x^{5/2}\Big|_0^4 = 32 - \frac{64}{5} = \frac{96}{5} = 19.2$$

$3 \begin{cases} 2: \text{ integral} \\ \quad 1: \text{ integrand} \\ \quad 1: \text{ limits} \\ 1: \text{ answer} \end{cases}$

(b) $$V = \pi \int_0^4 \left(8 - x^{3/2}\right)^2 dx$$

$$= \frac{576\pi}{5} = 115.2\pi \approx 361.911$$

$3 \begin{cases} 2: \text{ integral} \\ \quad 1: \text{ integrand} \\ \quad 1: \text{ limits and constant} \\ 1: \text{ answer} \end{cases}$

(c) $$\pi \int_0^k \left(8 - x^{3/2}\right)^2 dx = \frac{115.2\pi}{2}$$

$$\left[\begin{array}{l} \text{or} \\ \pi \int_0^k \left(8 - x^{3/2}\right)^2 dx = \pi \int_k^4 \left(8 - x^{3/2}\right)^2 dx \end{array}\right]$$

$$\int_0^k \left(8 - x^{3/2}\right)^2 dx = 57.6$$

$$\int_0^k \left(64 - 16x^{3/2} + x^3\right) dx = 57.6$$

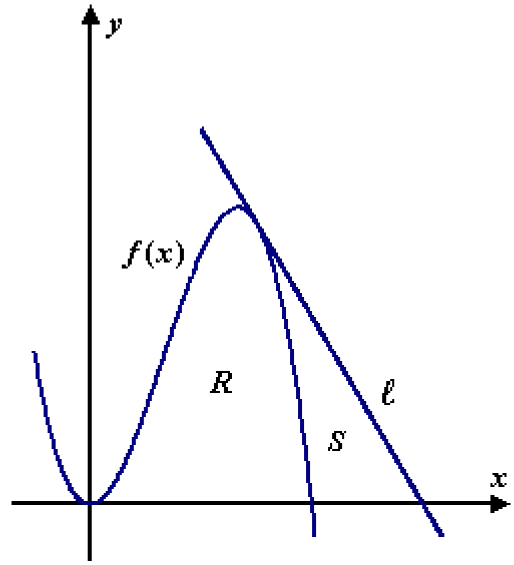
$$64k - \frac{32}{5}k^{5/2} + \frac{k^4}{4} = 57.6$$

$k \approx 0.995$ or 0.994

$3 \begin{cases} 1: \text{ integral with } k \text{ in limits} \\ 1: \text{ equates volumes} \\ 1: \text{ answer} \end{cases}$

Note: 0/1 for answer in each part if no setup points earned

666. Let f be the function given by $f(x) = 4x^2 - x^3$, and let ℓ be the line $y = 18 - 3x$, where ℓ is tangent to the graph of f. Let **R** be the region bounded by the graph of f and the x-axis, and let **S** be the region bounded by the graph of f, the line ℓ, and the x-axis, as shown on the right.



a) Show that ℓ is tangent to the graph of $y = f(x)$ at the point $x = 3$

b) Find the area of **S**

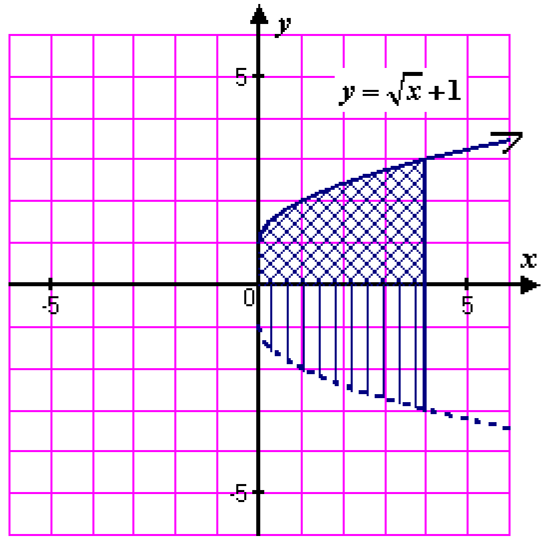
c) Find the volume of the solid generated when R is revolved about the x-axis

---

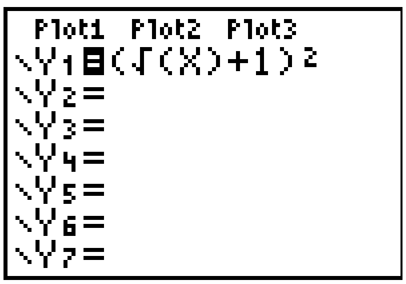
(a) $f'(x) = 8x - 3x^2$; $f'(3) = 24 - 27 = -3$ $f(3) = 36 - 27 = 9$ Tangent line at $x = 3$ is $y = -3(x-3) + 9 = -3x + 18$, which is the equation of line ℓ.	2 : { 1 : finds $f'(3)$ and $f(3)$ 1 : { finds equation of tangent line or shows (3,9) is on both the graph of f and line ℓ
(b) $f(x) = 0$ at $x = 4$ The line intersects the x-axis at $x = 6$. Area $= \frac{1}{2}(3)(9) - \int_3^4 (4x^2 - x^3)\,dx$ $= 7.916$ or 7.917 OR Area $= \int_3^4 ((18 - 3x) - (4x^2 - x^3))dx + \frac{1}{2}(2)(18 - 12)$ $= 7.916$ or 7.917	4 : { 2 : integral for non-triangular region 1 : limits 1 : integrand 1 : area of triangular region 1 : answer
(c) Volume $= \pi \int_0^4 (4x^2 - x^3)^2\,dx$ $= 156.038\pi$ or 490.208	3 : { 1 : limits and constant 1 : integrand 1 : answer

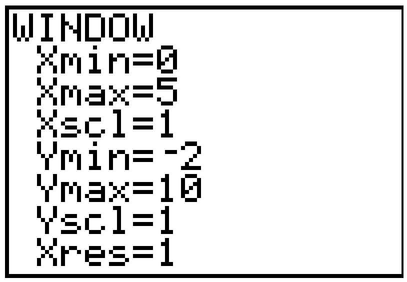
667. *Answer is E.*

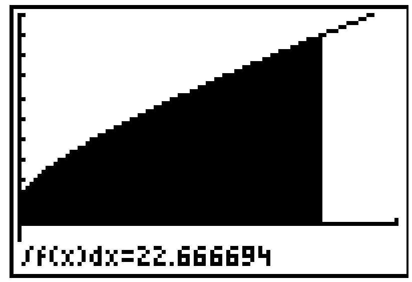
Find the volume of the solid formed by rotating about the x-axis the region enclosed by the graph of $y=\sqrt{x}+1$, the x-axis, the y-axis and the line $x=4$



$$\text{Volume} = \pi\int_0^4 (\sqrt{x}+1)^2\,dx =$$

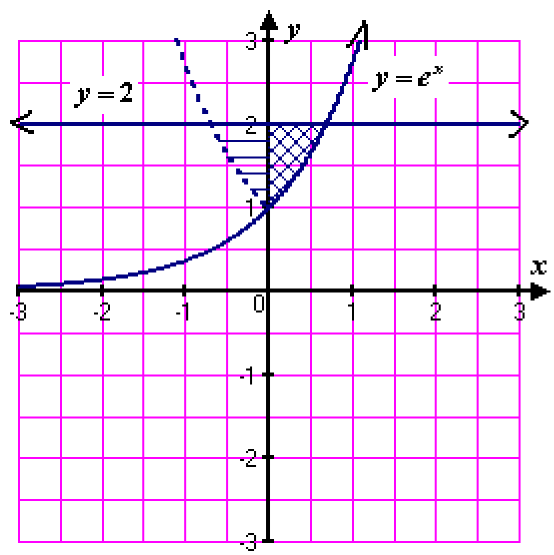






668. *Answer is B.*

Find the volume of the solid generated when the region bounded by the y-axis, $y=e^x$, and $y=2$ is rotated around the y-axis.

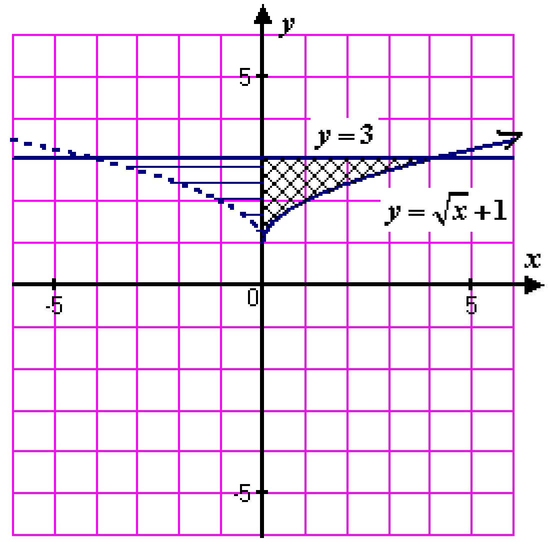


$$y=e^x$$
$$\ln y = x$$
$$\ln y = f(y)$$
$$\text{Volume} = \pi\int_1^2 [f(y)]^2\,dy \quad \leftarrow y\text{-axis}$$
$$=\pi\int_1^2 (\ln y)^2\,dy = \boxed{0.5916}$$

(enter integral using the letter x to evaluate)

669. *Answer is C.*

Find the volume of the solid formed by rotating the region bounded by the graph of $y=\sqrt{x}+1$ the y-axis and the line $y=3$ about the y-axis.



$$y=\sqrt{x}+1$$

$$y-1=\sqrt{x}$$

$$(y-1)^2=x$$

$$(y-1)^2=f(y)$$

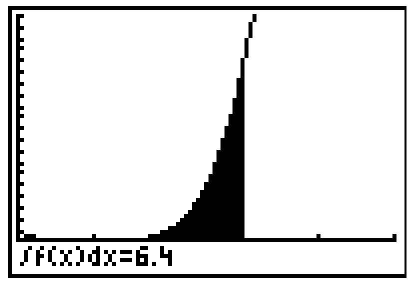
$$\text{Volume}=\pi\int_1^3\left[(y-1)^2\right]^2dy$$

$$=\pi\int_1^3(y-1)^4dy=\boxed{20.106}$$

```
Ans*π
         20.10619298
```

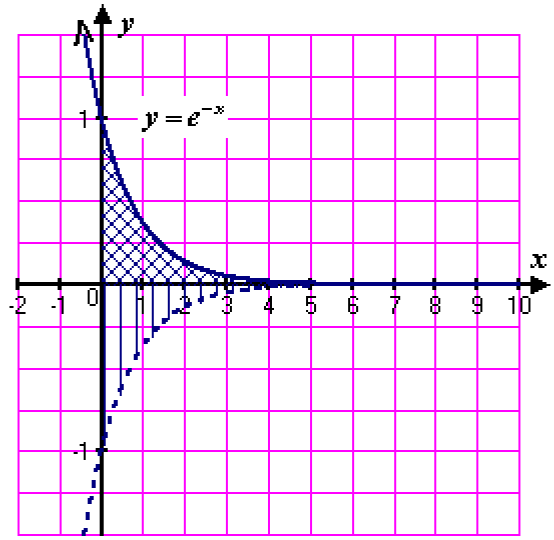
```
  Plot1  Plot2  Plot3
\Y1=(X-1)^4
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

```
WINDOW
 Xmin=0
 Xmax=5
 Xscl=1
 Ymin=-2
 Ymax=20
 Yscl=1
 Xres=1
```



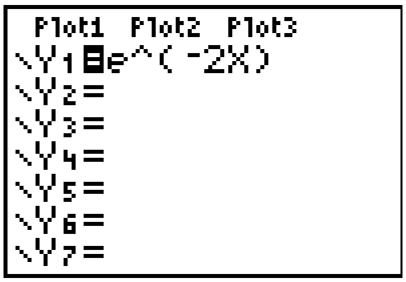
670. *Answer is B.*

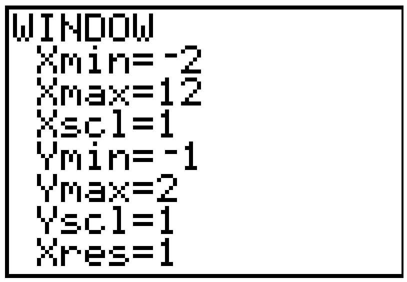
The area bounded by the curve $y = e^{-x}$ and the lines $y = 0$, $x = 0$ and $x = 10$ is rotated about the x-axis. Which of the following is the best approximation for the volume of the solid of revolution so generated ?

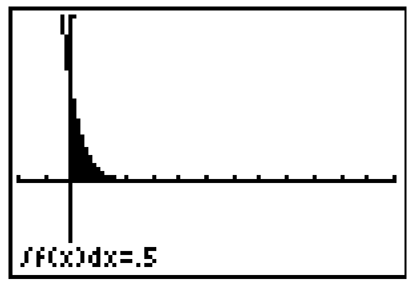


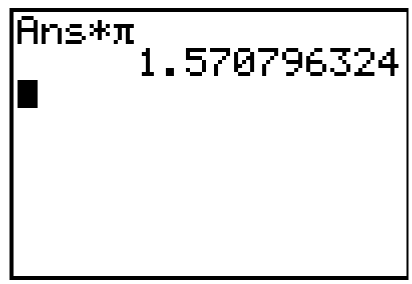
$$\text{Volume} = \pi \int_0^{10} \left[e^{-x} \right]^2 dy$$

$$= \pi \int_1^3 e^{-2x} dx = \boxed{1.5707}$$

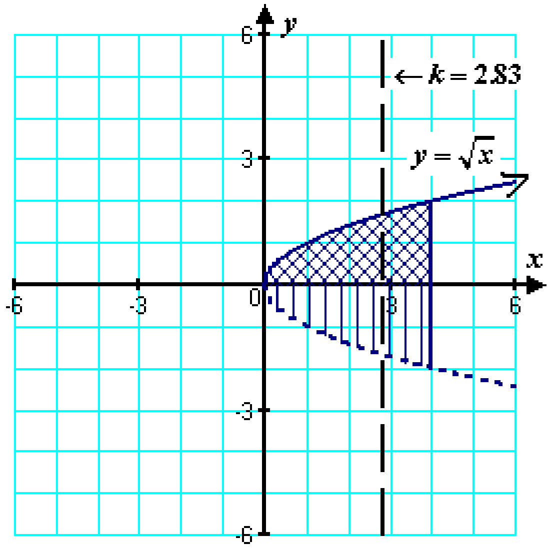








671. *Answer is C.*



The region in the first quadrant bounded above by the graph of $y = \sqrt{x}$ and below by the x-axis on the interval $[\ 0,\ 4]$ is revolved about the x-axis. If a plane perpendicular to the x-axis at the point where $x = k$ divides the solid into parts of equal volume, then $k =$

$$\text{Total volume} = \pi \int_0^4 \left(\sqrt{x}\right)^2 dx = \pi \left[\frac{x^2}{2}\right]_0^4 = 8\pi$$

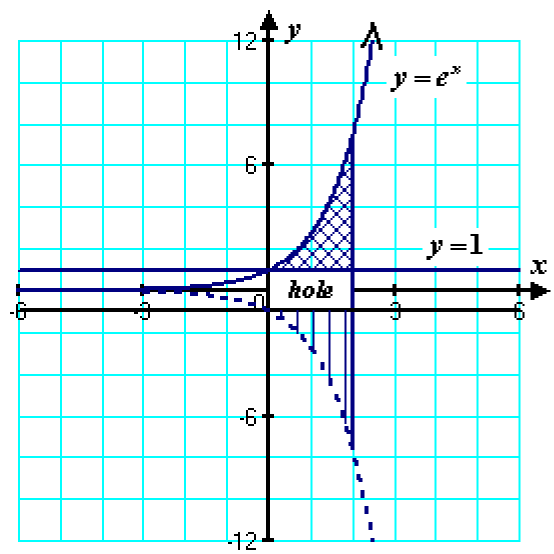
$$\text{Half of volume} = \pi \int_0^k x\,dx = \pi \left[\frac{x^2}{2}\right]_0^k = 4\pi$$

$$\frac{\pi k^2}{2} = 4\pi$$

$$k^2 = 8$$

$$k = \boxed{2.8284}$$

672. *Answer is C.*



The region bounded by $y = e^x$, $y = 1$, and $x = 2$ is rotated about the x-axis. The volume of the solid generated is given by the integral:

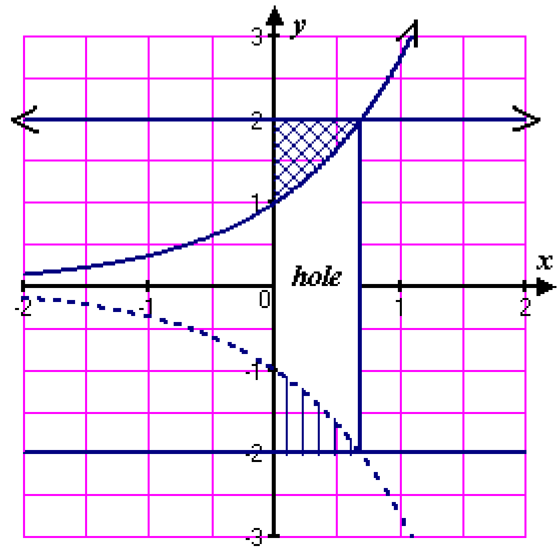
$$\text{Volume} = \pi \int_0^2 (R)^2 - (r)^2\,dx$$

$$= \pi \int_0^2 (e^x)^2 - (1)^2\,dx$$

$$= \pi \int_0^2 (e^{2x} - 1)\,dx$$

673. *Answer is D.*

Let **R** be the region in the first quadrant enclosed by the lines $x = 0$ and $y = 2$ and the graph of $y = e^x$ The volume of the solid generated when **R** is revolved about the x-axis is given by



Limits of integration $e^x = 2$

$$\ln e^x = \ln 2$$

$x = 0$ and $x = \ln 2$

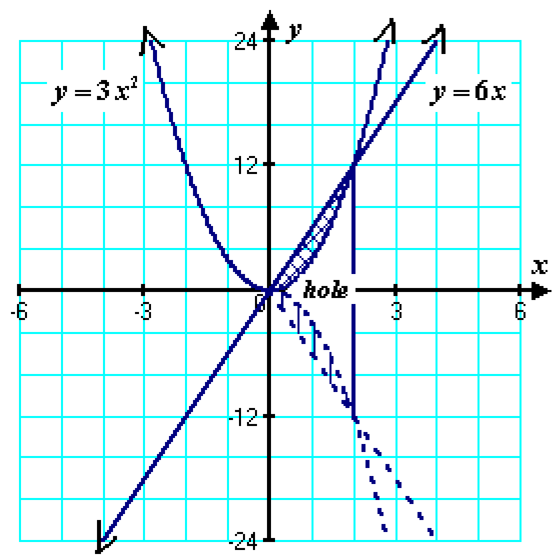
$$\text{Volume} = \pi\int_0^{\ln 2} (R)^2 - (r)^2\,dx$$

$$= \pi\int_0^{\ln 2} \left[(2)^2 - (e^x)^2\right]dx$$

$$= \pi\int_0^{\ln 2} (4 - e^{2x})\,dx$$

674. *Answer is D.*

The volume of the solid generated by rotating about the x-axis the region enclosed between the curve $y = 3x^2$ and the line $y = 6x$ is given by



Limits of integration $3x^2 = 6x$

$$3x^2 - 6x = 0$$

$$3x(x-2) = 0$$

$x = 0$ and $x = 2$

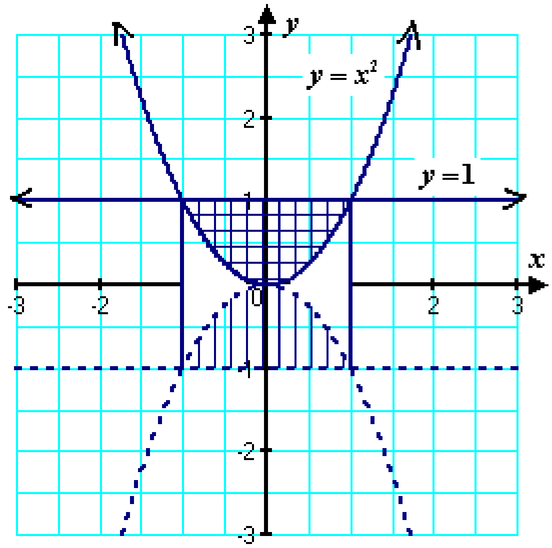
$$\text{Volume} = \pi\int_0^{2} (R)^2 - (r)^2\,dx$$

$$= \pi\int_0^{2} \left[(6x)^2 - (3x^2)^2\right]dx$$

$$= \pi\int_0^{2} (36x^2 - 9x^4)\,dx$$

675. *Answer is E.*

If the region bounded between $y = x^2$ and the horizontal line $y = 1$ is rotated about the x-axis, the volume of the resulting solid of revolution is



Intersection points $y = x^2 \quad 1 = y$

$$x^2 = 1$$

$$x = \pm 1$$

$$\text{Volume} = \pi\int_{-1}^{1}\left[(R)^2 - (r)^2\right]dx = 2\pi\int_{0}^{1}\left[R^2 - r^2\right]a$$

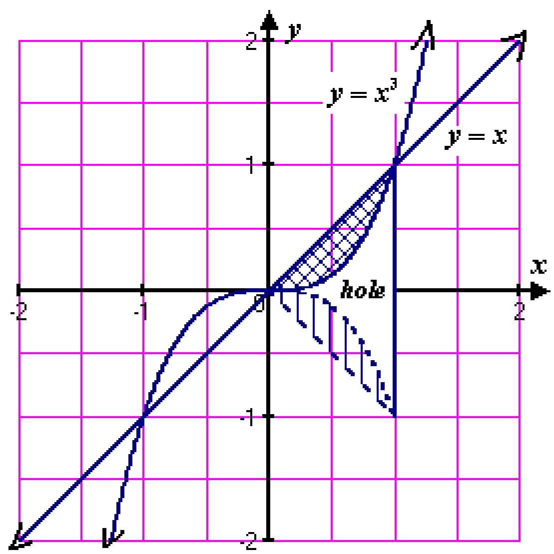
$$= \pi\int_{-1}^{1}\left[(1)^2 - (x^2)^2\right]dx = 2\pi\int_{0}^{1}(1 - x^4)\,dx$$

$$= 2\pi\left[x - \frac{x^5}{5}\right]_0^1 = 2\pi\left(1 - \frac{1}{5}\right)$$

$$= 2\pi\left(\frac{4}{5}\right) = \boxed{\frac{8\pi}{5}}$$

676. *Answer is D.*

The volume of revolution formed by rotating the region bounded by $y = x^3,\ y = x,\ x = 0,\ x = 1$ about the x-axis is represented by



$$\text{Volume} = \pi\int_{0}^{1}(\overbrace{x}^{top})^2 - (\overbrace{x^3}^{bottom})^2\,dx$$

$$= \pi\int_{0}^{1}(x^2 - x^6)\,dx$$

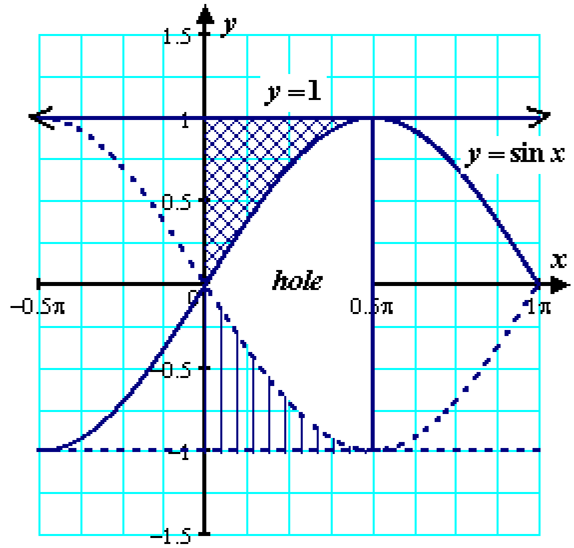
$$= \pi\left[\frac{x^3}{3} - \frac{x^7}{7}\right]_0^1 = \pi\left(\frac{1}{3} - \frac{1}{7}\right) = \boxed{\frac{4}{21}}$$

677. *Answer is E.*

Let **R** be the region between the graphs of

$y = 1$ and $y = \sin x$ from $x = 0$ to $x = \dfrac{\pi}{2}$.

The volume of the solid obtained by revolving **R** about the x-axis is given by

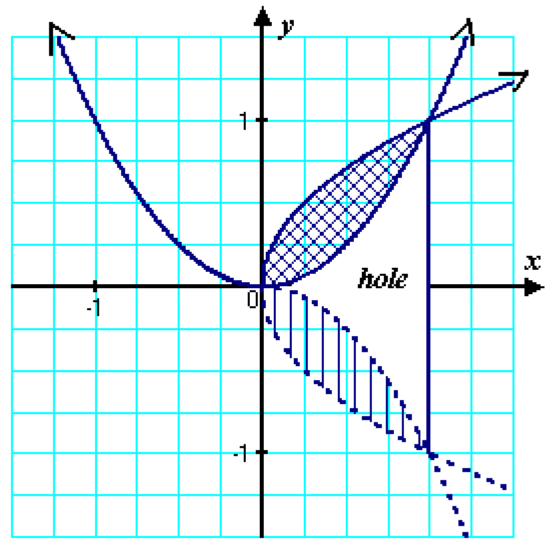


$$\text{Volume} = \pi \int_0^{\frac{\pi}{2}} \left[(R)^2 - (r)^2 \right] dx$$

$$= \pi \int_0^{\frac{\pi}{2}} \left[(1)^2 - (\sin x)^2 \right] dx$$

$$= \pi \int_0^{\frac{\pi}{2}} \left[1 - \sin^2 x \right] dx$$

678. Find the volume of the solid generated by rotating the area bounded by $y = x^2$ and $x = y^2$ about the x-axis.



$$x^2 = \sqrt{x}$$

$$x^4 = x$$

$$x^4 - x = 0$$

$$x(x^3 - 1) = 0$$

$x = 0$ | $x = 1$ ← intersection points

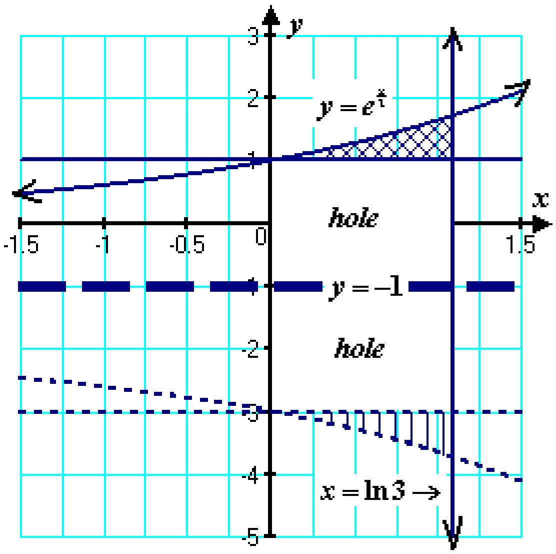
$$\text{Volume} = \pi \int_0^1 \left(\sqrt{x}\right)^2 - (x^2)^2 dx =$$

$$= \pi \int_0^1 x - x^4 dx = \pi \left[\frac{x^2}{2} - \frac{x^5}{5} \right]_0^1$$

$$= \pi \left[\frac{1}{2} - \frac{1}{5} \right] = \boxed{\frac{3\pi}{10}}$$

679. *Answer is A.*

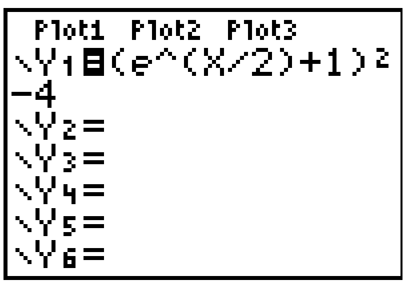
Let **R** be the region in the first quadrant enclosed by the lines $x = \ln 3$ and $y = 1$ and the graph of $y = e^{\frac{x}{2}}$ The volume of the solid generated when **R** is revolved about the line $y = -1$ is

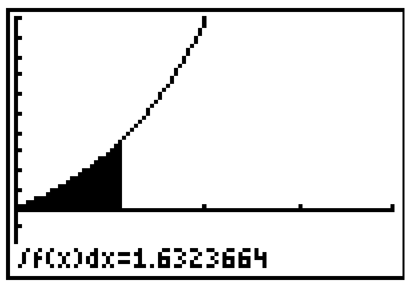


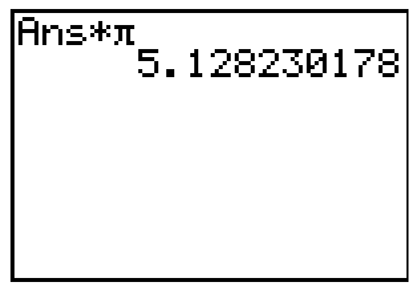
$$\text{Volume} = \pi \int_0^{\ln 3} R^2 - r^2 dx$$

$$= \pi \int_0^{\ln 3} (e^{\frac{x}{2}} - (-1))^2 - (1 - (-1))^2 dx$$

$$= \pi \int_0^{\ln 3} (e^{\frac{x}{2}} + 1)^2 - (2)^2 dx = \boxed{5.1282}$$

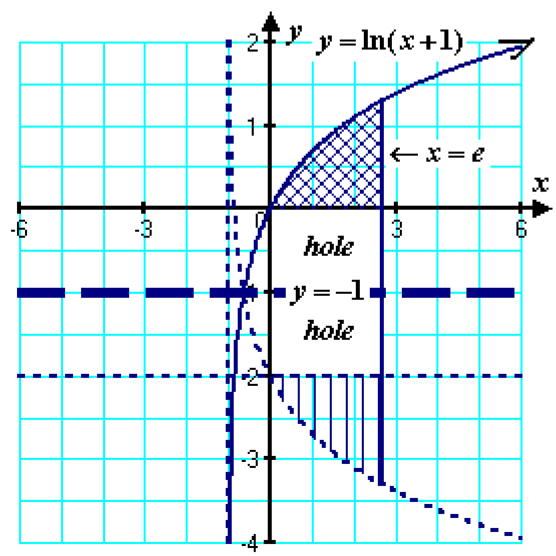






680. *Answer is D.*

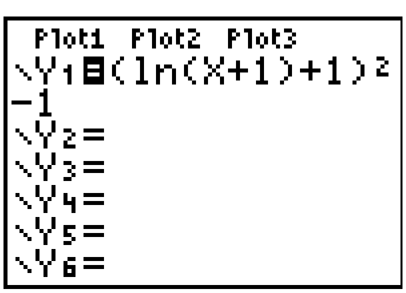
Let **R** be the region in the first quadrant that is enclosed by the graph of $f(x) = \ln(x+1)$, the x-axis and the line $x = e$ What is the volume of the solid generated when **R** is rotated about the line $y = -1$

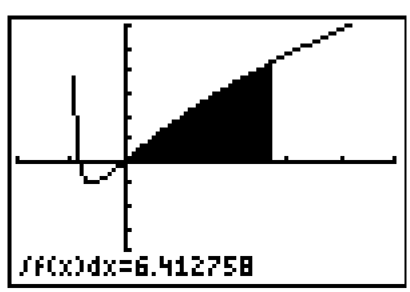


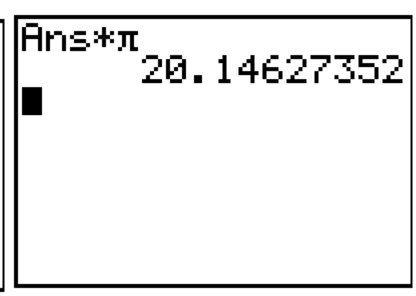
$$\text{Volume} = \pi \int_0^e (R)^2 - (r)^2 dx$$

$$= \pi \int_0^e (\ln(x+1) - (-1))^2 - (0-1)^2 dx$$

$$= \pi \int_0^e (\ln(x+1) + 1)^2 - 1 dx \approx \boxed{20.1462}$$

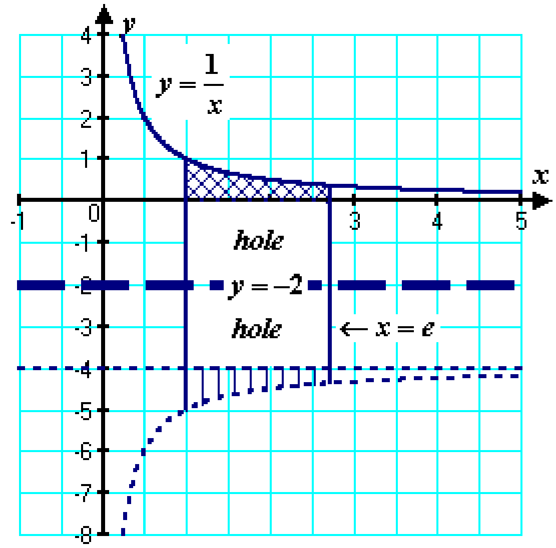






681. *Answer is D.*

If the region bounded between $y=\dfrac{1}{x}$ and the x-axis between the vertical lines $x=1$ and $x=e$ is rotated about the line $y=-2$, the volume of the resulting solid of revolution is represented by



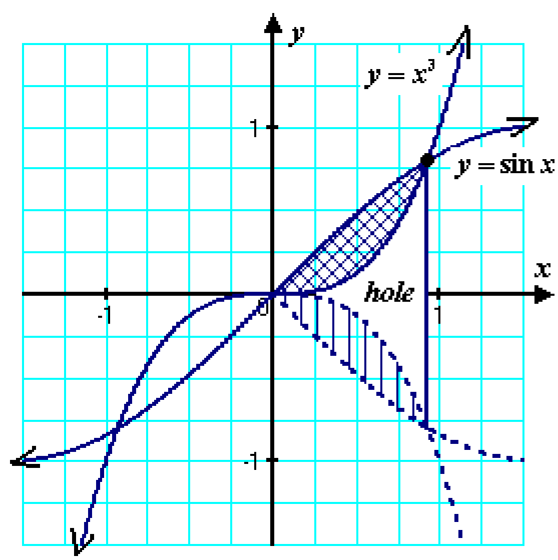
$$\text{Volume}=\pi\int_0^e (R)^2-(r)^2\,dx$$

$$=\pi\int_0^e\left(\frac{1}{x}-(-2)\right)^2-(0-2)^2\,dx$$

$$=\pi\int_0^e\left(\frac{1}{x}+2\right)^2-4\,dx$$

682. *Answer is B.*

What is the approximate volume of the solid obtained by revolving about the x-axis the region in the first quadrant enclosed by the curves $y=x^3$ and $y=\sin x$

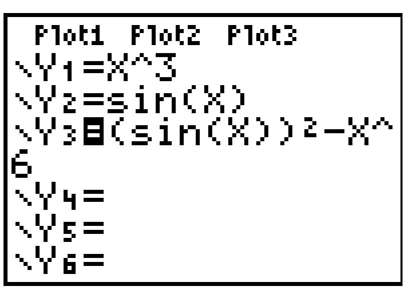


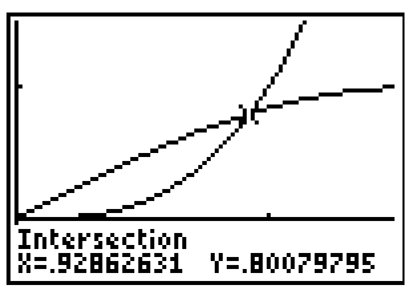
Limits of integration $\quad x^3=\sin x$

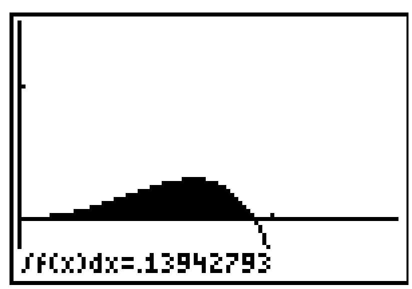
$x=0$ and $x=0.9286 \leftarrow$ calculator

$$\text{Volume}=\pi\int_0^{0.9286}(\sin x)^2-(x^3)^2\,dx$$

$$=\pi\int_0^{0.9286}(\sin^2 x-x^6)\,dx\approx\boxed{0.4380}$$

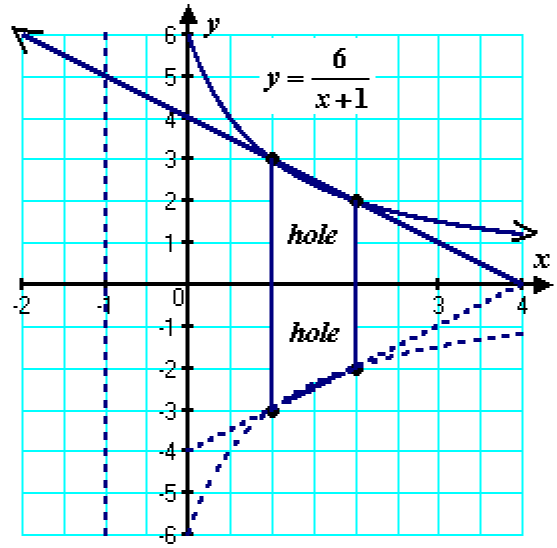






683. *Answer is C.*

What is the volume of the solid obtained by rotating the region between $y = \dfrac{6}{x+1}$ and $y = 4 - x$ around the x-axis ?



Intersections $\dfrac{6}{x+1} = 4 - x$

$$6 = (4-x)(x+1)$$

$$6 = 4 + 3x - x^2$$

$$x^2 - 3x + 2 = 0$$

$$(x-1)(x-2) = 0$$

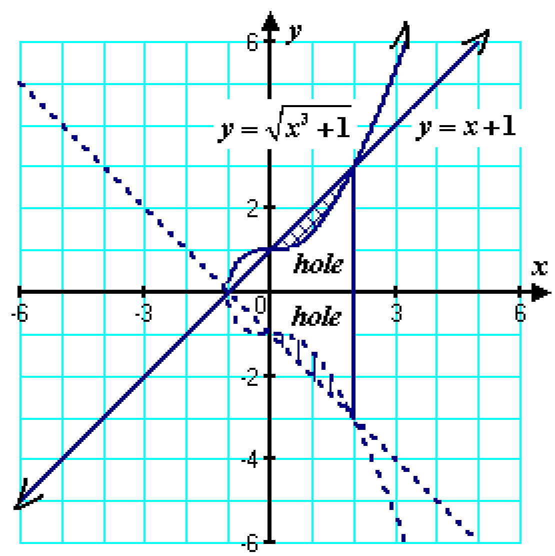
$x = 1$	$x = 2$

$$\text{Volume} = \pi\int_0^e (R)^2 - (r)^2\,dx =$$

$$= \pi\int_1^2 (4-x)^2 - \left(\frac{6}{x+1}\right)^2 dx =$$

684. *Answer is B.*

Let **R** be the region between the curves $y = \sqrt{x^3+1}$ and $y = x + 1$, for which x is positive. What is the volume of the solid obtained by rotating **R** around the x-axis ?



Intersections $\sqrt{x^3+1} = x+1$

$$x^3 + 1 = x^2 + 2x + 1$$

$$x^3 - x^2 - 2x = 0$$

$$x(x^2 - x - 2) = 0$$

$$x(x+1)(x-2) = 0$$

$x = 0$	$x = -1$	$x = 2$

$$\text{Volume} = \pi\int_0^2 (R)^2 - (r)^2\,dx$$

$$= \pi\int_0^2 (x+1)^2 - \left(\sqrt{x^3+1}\right)^2 dx$$

$$= \pi\int_0^2 (-x^3 + x^2 + 2x)\,dx$$

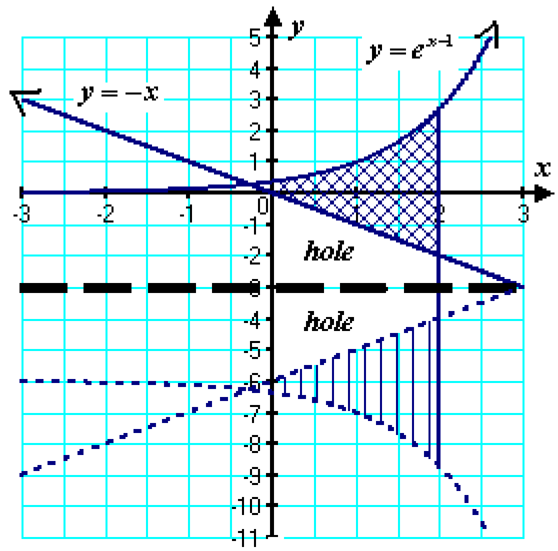
685. *Answer is B.*

The region enclosed by the graphs of $y = e^{x-1}$ and $y = -x$ and the vertical lines $x = 0$ and $x = 2$ is rotated about the line $y = -3$
Which of the following gives the volume of the generated solid ?

$$\text{Volume} = \pi \int_0^2 (R)^2 - (r)^2\,dx =$$

$$= \pi \int_0^2 (e^{x-1} - (-3))^2 - (-x - (-3))^2\,dx =$$

$$= \pi \int_0^2 (e^{x-1} + 3)^2 - (-x + 3)^2\,dx =$$



686. *Answer is C.*

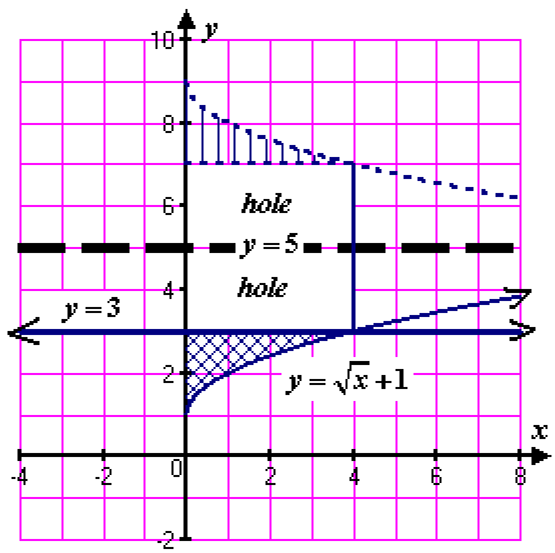
Find the volume of the solid formed by rotating the region bounded by the graph of $y = \sqrt{x} + 1$, the y-axis and the line $y = 3$ about the line $y = 5$

$$\text{Volume} = \pi \int_0^2 (R)^2 - (r)^2\,dx =$$

$$= \pi \int_0^4 \left(5 - (\sqrt{x} + 1)\right)^2 - (5-3)^2\,dx$$

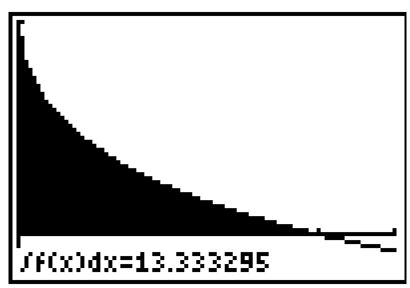
$$= \pi \int_0^4 \left(4 - \sqrt{x}\right)^2 - 4\,dx = \boxed{41.8877}$$

```
Ans*π
          41.88778246
```



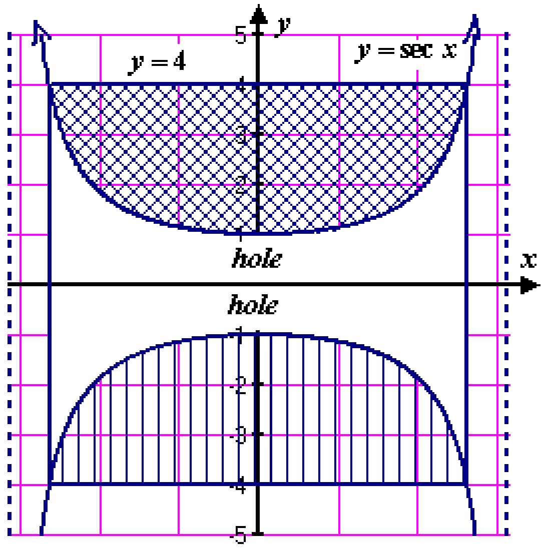

```
 Plot1  Plot2  Plot3
\Y1■(4-√(X))²-4
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

```
WINDOW
 Xmin=0
 Xmax=5
 Xscl=1
 Ymin=-2
 Ymax=12
 Yscl=1
 Xres=1
```



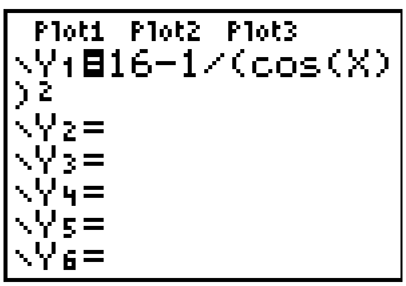
687. *Answer is E.*

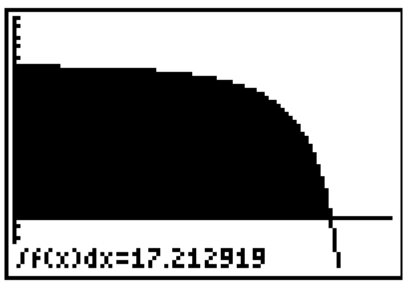
The region **S** in the diagram is bounded by $y = \sec x$ and $y = 4$. What is the volume of the solid formed when **S** is rotated about the x-axis ?

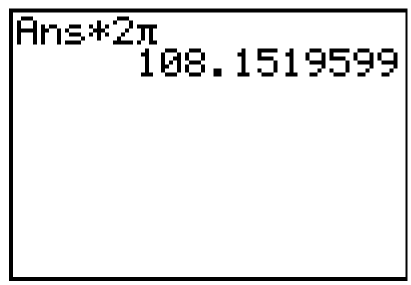


$$\sec x = 4$$
$$\cos x = \tfrac{1}{4}$$
$$x = \pm 1.318116$$

$$\text{Volume} = \pi \int_{-1}^{2} R^2 - r^2 dx =$$
$$= \pi \int_{-1.318116}^{1.318116} 4^2 - (\sec x)^2 dx$$
$$= 2\pi \int_{0}^{1.318116} 16 - \sec^2 x \, dx = \boxed{108.1768}$$

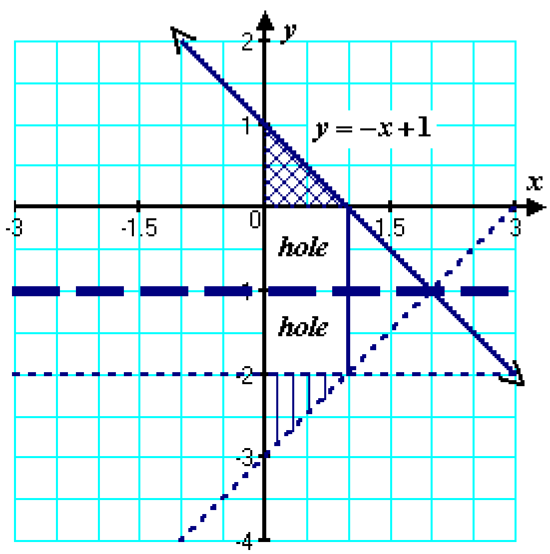






688. *Answer is E.*

The region enclosed by the line $x + y = 1$ and the coordinate axes is rotated about the line $y = -1$. What is the volume of the solid generated ?



$$\text{Volume} = \pi \int_{0}^{1} (R)^2 - (r)^2 dx$$
$$= \pi \int_{0}^{1} [(-x+1) - (-1)]^2 - [0 - (-1)]^2 dx$$
$$= \pi \int_{0}^{1} (-x+2)^2 - (1)^2 dx$$
$$= \pi \int_{0}^{1} (x^2 - 4x + 3) \, dx$$
$$= \pi \left[\frac{x^3}{3} - 2x^2 + 3x \right]_0^1 = \pi \left[\frac{1}{3} - 2 + 3 \right] = \boxed{\frac{4\pi}{3}}$$

689. *Answer is D.*

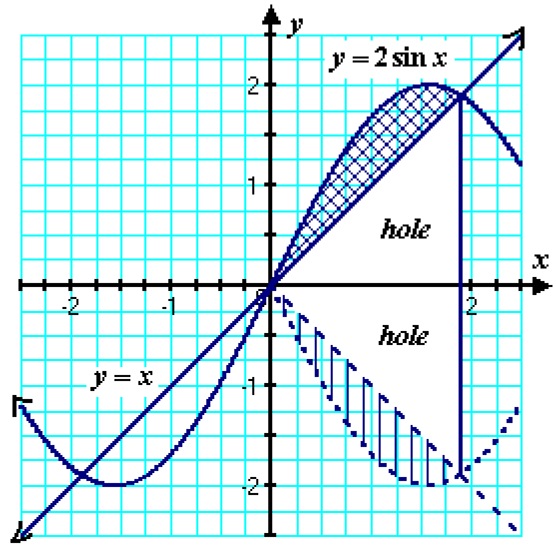
The region in the first quadrant enclosed by the graphs of $y = x$ and $y = 2\sin x$ is revolved about the x-axis. The volume of the solid generated is

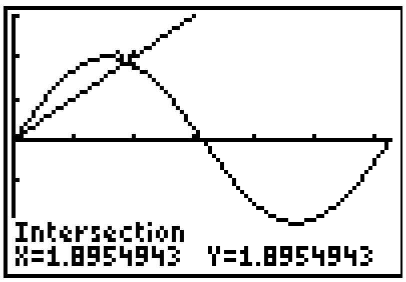
Intersections $x = 0,\ 1.8954943$ ← calculator

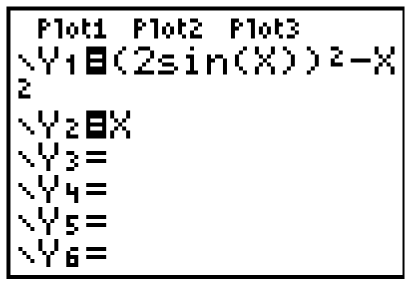
$$\text{Volume} = \pi\int_0^{1.8954943} (R)^2 - (r)^2\,dx =$$

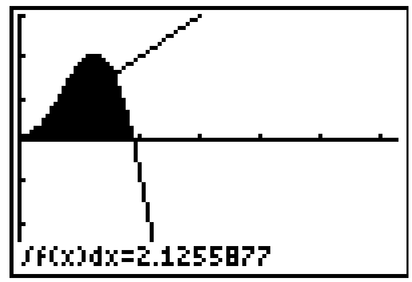
$$= \pi\int_0^{1.8954943} (2\sin x)^2 - (x)^2\,dx$$

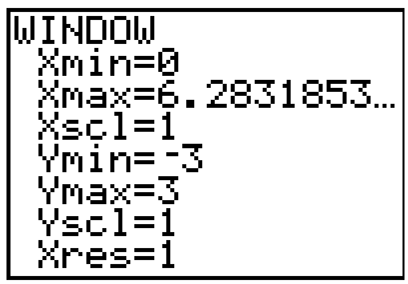
$$\approx \boxed{6.67773}$$

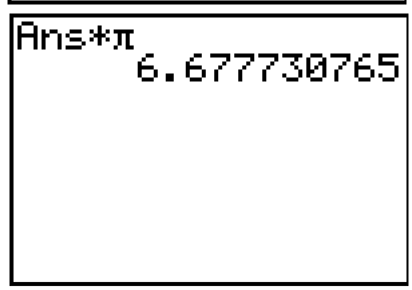












690. *Answer is A.*

The volume of the solid generated by revolving about the y-axis the region bounded by the graphs of $y=\sqrt{x}$ and $y=x$ is

$$y = x \quad \sqrt{x} = y$$

right $\rightarrow$ $y = x \qquad x = y^2$

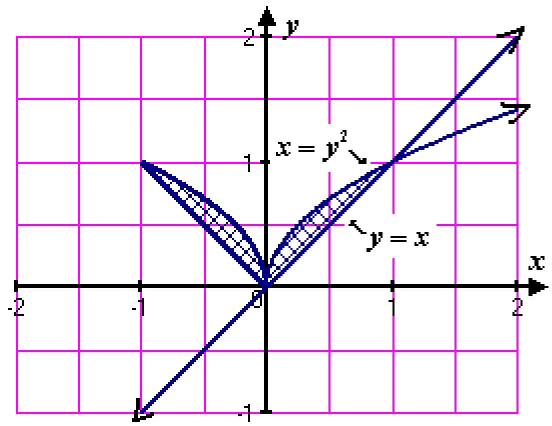
$$y = y^2$$
$$0 = y^2 - y$$
$$0 = y(y-1)$$

(washer) $\quad y = 0 \mid y = 1 \leftarrow$ intersection points

$$\text{Volume} = \pi\int_0^1 R^2 - r^2 dy = \pi\int_0^1 (y)^2 - (y^2)^2 dy$$

$$\text{Volume} = \pi\int_0^1 (y^2 - y^4)\,dy$$

$$= \pi\left[\frac{y^3}{3} - \frac{y^5}{5}\right]_0^1 = \boxed{\frac{2\pi}{15}}$$



691. *Answer is E.*

What is the volume of the solid obtained by revolving about the y-axis the region enclosed by the graphs of $x = y^2$ and $x = 9$

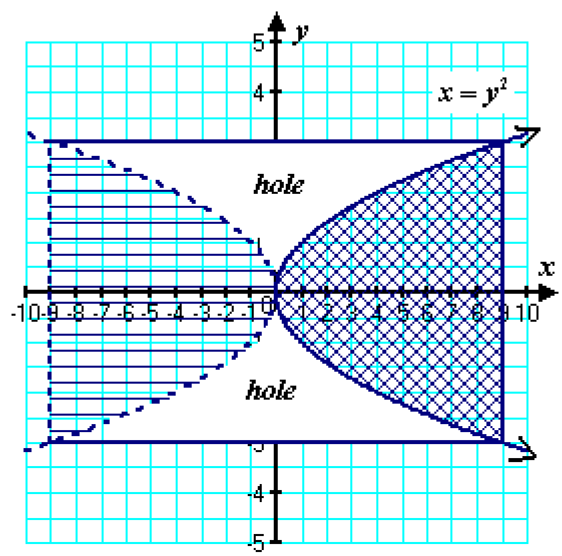
$$y^2 = 9$$
$$y = \pm 3 \leftarrow \text{intersection points}$$

$$f(y) = y^2 \quad \text{(washer)} \quad \rightarrow \int_{-3}^{3} R^2 - r^2 dy$$

$$\text{Volume} = \pi\int_{-3}^{3} \left[(9)^2 - (y^2)^2\right] dy$$

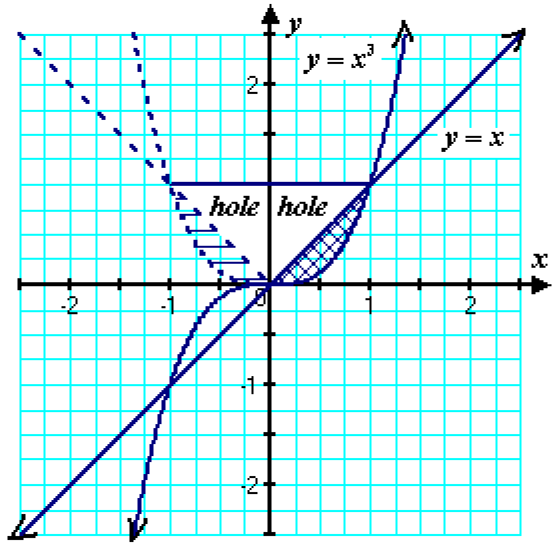
$$= 2\pi\int_0^3 (81 - y^4)\,dy$$

$$= 2\pi\left[81y - \frac{y^5}{5}\right]_0^3 = \boxed{\frac{1944\pi}{5}}$$



692. *Answer is D.*

Identify the definite integral that computes the volume of the solid generated by revolving the region bounded by the graph of $y = x^3$ and the line $y = x$ between $x = 0$ and $x = 1$, about the y-axis.



Intersections $x = 0,\ 1$

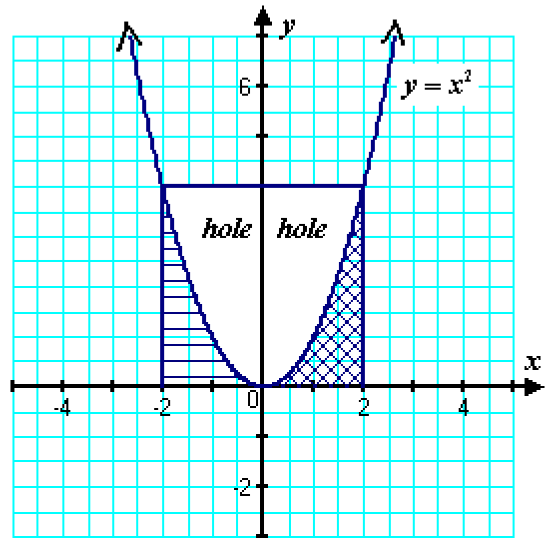
$$\text{Volume} = \pi\int_0^1 (R)^2 - (r)^2\,dy =$$

$$= \pi\int_0^1 (\sqrt[3]{y})^2 - (y)^2\,dy$$

$$= \pi\int_0^1 (y^{\frac{2}{3}} - y^2)\,dy$$

693. *Answer is A.*

The region enclosed by the graph of $y = x^2$, the line $x = 2$, and the x-axis is revolved about the y-axis. The volume of the solid generated is



$$\text{Volume} = \pi\int_0^4 2^2 - \left(\sqrt{y}\right)^2 dy = \pi\int_0^4 4 - y\,dy$$

$$= \pi\left[4y - \frac{y^2}{2}\right]_0^4 = \pi(16-8) = \boxed{8\pi}$$

694. *Answer is E.*

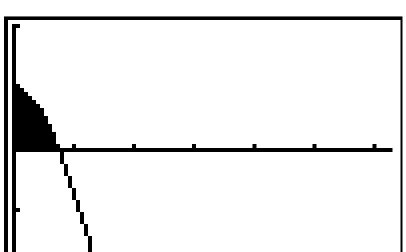
Difficulty $= \mathbf{0.64}$

The region enclosed by the graph of $y = x^2$, the line $x = 2$, and the x-axis is revolved about the y-axis. The volume of the solid generated is

$$\text{Volume} = \pi\int_0^4 (R)^2 - (r)^2\,dy =$$

$$\text{Volume} = \pi\int_0^4 (2)^2 - \left(\sqrt{y}\right)^2 dy = \pi\int_0^4 4 - y\,dy$$

$$= \pi\left[4y - \frac{y^2}{2}\right]_0^4 = \pi(16-8) = \boxed{8\pi}$$



695. *Answer is C.*

The region in the first quadrant enclosed by the y-axis and the graph of $y = \cos x$ and $y = x$ is rotated about the x-axis. The volume of the solid generated is

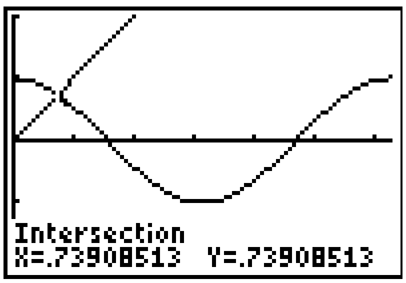
Intersection $\cos x = x$

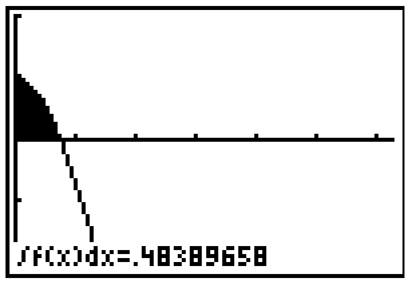
calculator $x = 0.73908$

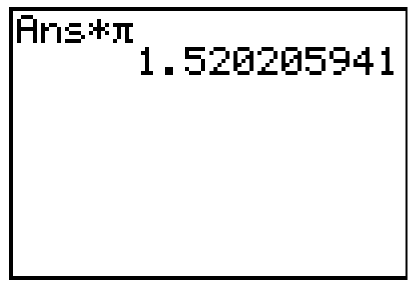
$$\text{Volume} = \pi \int_0^{.73908} (R)^2 - (r)^2\,dx$$

$$= \pi \int_0^{0.73908} (\cos x)^2 - (x)^2\,dx$$

$$= \pi \int_0^{0.73908} \cos^2 x - x^2\,dx \approx \boxed{1.5202}$$







696. Let **R** denote the region enclosed between the graph of $y = x^2$ and the graph of $y = 2x$

a) Find the area of region **R**

b) Find the volume of the solid obtained by revolving the region **R** about the y-axis.

Intersection: $x^2 = 2x$ when $x = 0$ and $x = 2$

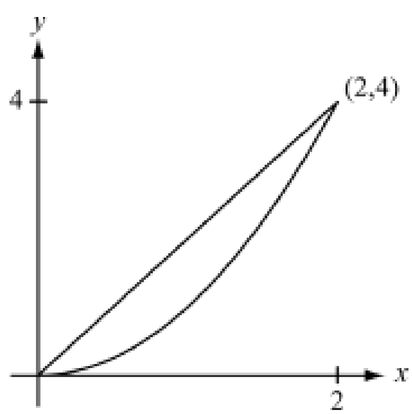
(a) $\text{Area} = \int_0^2 (2x - x^2)\,dx$

$$= x^2 - \frac{x^3}{3}\Big|_0^2 = 4 - \frac{8}{3} = \frac{4}{3}$$

or

$\text{Area} = \int_0^4 \left(y^{1/2} - \frac{y}{2}\right)dy$

$$= \frac{2}{3}y^{3/2} - \frac{y^2}{4}\Big|_0^4 = \frac{16}{3} - 4 = \frac{4}{3}$$



Disks:

$$\text{Volume} = \pi\int_0^4 \left(\left(y^{1/2}\right)^2 - \left(\frac{y}{2}\right)^2\right)dy = \pi\int_0^4 \left(y - \frac{y^2}{4}\right)dy$$

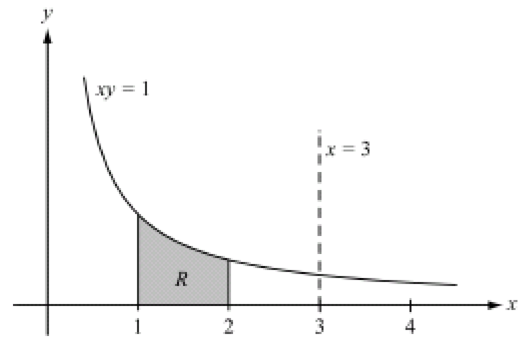
$$= \pi\left(\frac{y^2}{2} - \frac{y^3}{12}\right)\Big|_0^4 = \pi\left(8 - \frac{64}{12}\right) = \frac{8}{3}\pi$$

697.

In the figure, the shaded region **R** is bounded by the graphs of $xy = 1$, $x = 1$, $x = 2$ and $y = 0$

***a*)** Find the volume of the solid generated by revolving the region **R** about the x-axis.

***b*)** Find the volume of the solid generated by revolving the region **R** about the line $x = 3$



(a) $$\text{Volume} = \pi\int_1^2 y^2\,dx = \pi\int_1^2 \left(\frac{1}{x}\right)^2 dx = -\pi\frac{1}{x}\Big|_1^2 = -\frac{\pi}{2}+\pi = \frac{\pi}{2}$$

(b) $$\text{Volume} = 2\pi\int_1^2 (3-x)y\,dx = 2\pi\int_1^2 (3-x)\frac{1}{x}\,dx = 2\pi\int_1^2\left(\frac{3}{x}-1\right)dx$$

$$= 2\pi(3\ln x - x)\Big|_1^2 = 2\pi((3\ln 2-2)-(-1)) = 2\pi(3\ln 2 - 1)$$

698.

Let **R** be the region bounded by the curves $f(x) = \dfrac{4}{x}$ and $g(x) = (x-3)^2$

***a*)** Find the area of **R**

***b*)** Find the volume of the solid generated by revolving **R** about the x-axis.

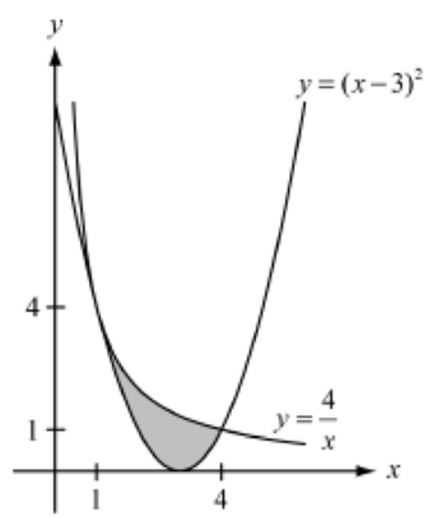
1976 AB3/BC2
Solution

Intersection points occur when

$$\frac{4}{x} = (x-3)^2$$

$$0 = x^3 - 6x^2 + 9x - 4 = (x-4)(x-1)^2$$

Thus the intersection points are at $(1,4)$ and $(4,1)$.



(a) $$\text{Area} = \int_1^4 \left(\frac{4}{x}-(x-3)^2\right)dx$$

$$= \left(4\ln x - \frac{(x-3)^3}{3}\right)\Bigg|_1^4 = 4\ln 4 - 3$$

(b) $$\text{Volume} = \int_1^4 \pi\left(\left(\frac{4}{x}\right)^2 - (x-3)^4\right)dx$$

$$= \pi\left(-\frac{16}{x} - \frac{(x-3)^5}{5}\right)\Bigg|_1^4 = \pi\left(\left(-4-\frac{1}{5}\right)-\left(-16-\frac{-32}{5}\right)\right)$$

$$= \frac{27\pi}{5}$$

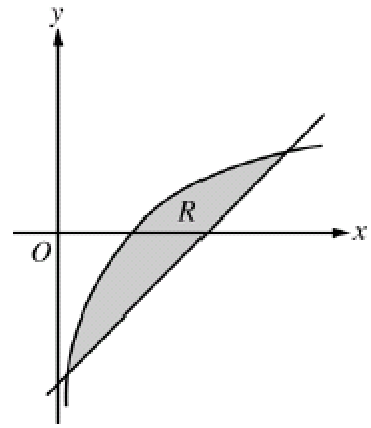
699.

Let **R** be the shaded region bounded by the graph $y = \ln x$ and the line $y = x - 2$, as shown on the right

***a*)** Find the area of **R**

***b*)** Find the volume of the solid generated when **R** is rotated about the horizontal line $y = -3$

***c*)** Write, but do not evaluate, an integral expression that can be used to find the volume of the solid generated when **R** is rotated about the y-axis



$\ln(x) = x - 2$ when $x = 0.15859$ and 3.14619.
Let $S = 0.15859$ and $T = 3.14619$

(a) Area of $R = \int_S^T (\ln(x) - (x-2))\,dx = 1.949$

$$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$$

(b) Volume $= \pi \int_S^T \left((\ln(x) + 3)^2 - (x - 2 + 3)^2 \right) dx$
$= 34.198$ or 34.199

$$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits, constant, and answer} \end{cases}$$

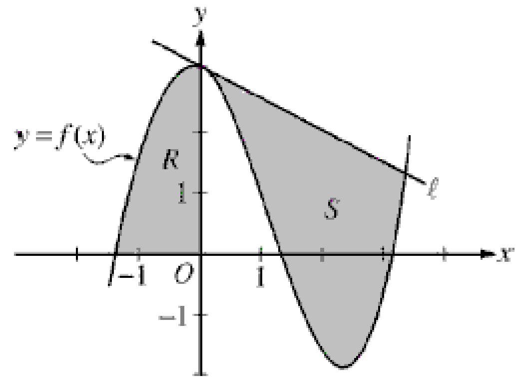
(c) Volume $= \pi \int_{S-2}^{T-2} \left((y+2)^2 - \left(e^y\right)^2 \right) dy$

$$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits and constant} \end{cases}$$

700. Let f be the function given by

$$f(x)=\frac{x^3}{4}-\frac{x^2}{3}-\frac{x}{2}+3\cos x$$

Let **R** be the shaded region in the second quadrant bounded by the graph of f, and let **S** be the shaded region bounded by the graph of f and the line ℓ, the line tangent to the graph of f at $x=0$, as shown.



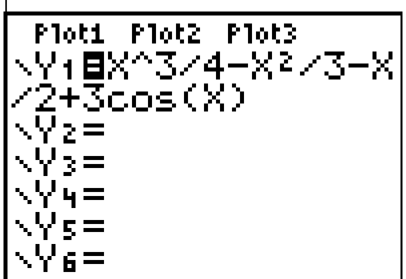
a) Find the area of **R**

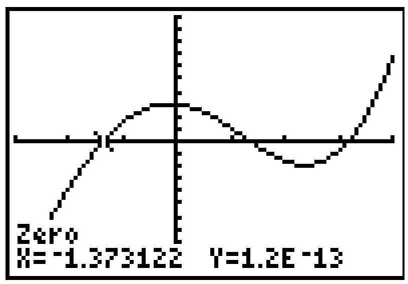
b) Find the volume of the solid generated when **R** is rotated about the horizontal line $y=-2$

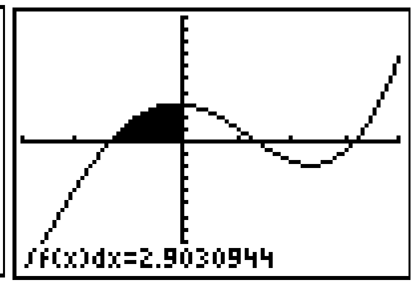
c) Write, but do not evaluate, an integral expression that can be used to find the area of **S**

a) x-intercept left of **0** at $\mathbf{P}=-1.373122$

$$\text{Area } \mathbf{R}=\int_P^0 \frac{x^3}{4}-\frac{x^2}{3}-\frac{x}{2}+3\cos x\,dx=\boxed{2.903}$$



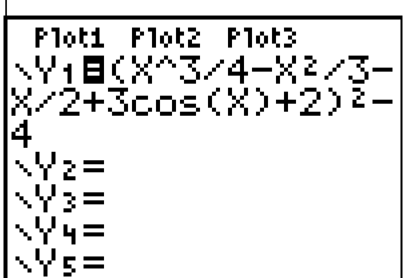


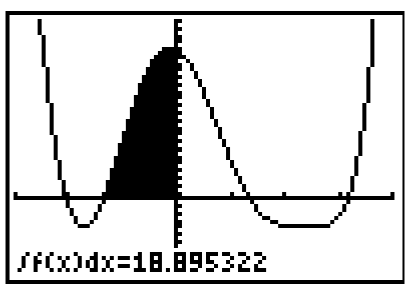


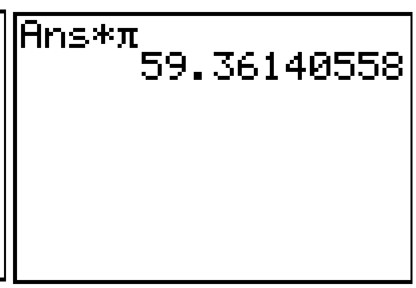
b) $y=-2$ (washer) $\rightarrow \pi\int_P^0 R^2-r^2 \rightarrow dx=$

$$\text{Volume}=\pi\int_P^0\left[(f(x)+2)^2-(2)^2\right]dx=$$

$$=\pi\int_P^0\left[\left(\frac{x^3}{4}-\frac{x^2}{3}-\frac{x}{2}+3\cos x+2\right)^2-4\right]dx=\boxed{59.361}$$





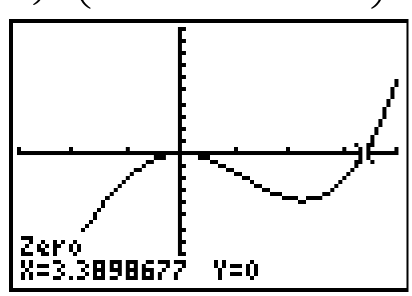


c) $f(x)=\frac{x^3}{4}-\frac{x^2}{3}-\frac{x}{2}+3\cos x$ Area $\mathbf{S}=\int_0^{3.389677}\left(3-\frac{1}{2}x\right)-\left(\frac{x^3}{4}-\frac{x^2}{3}-\frac{x}{2}+3\cos x\right)dx$

$$f'(x)=\frac{3x^2}{4}-\frac{2x}{3}-\frac{1}{2}-3\sin x$$

$$f'(0)=-\frac{1}{2} \rightarrow \ell=-\frac{1}{2}x+3$$

$$\frac{x^3}{4}-\frac{x^2}{3}-\frac{x}{2}+3\cos x+\frac{1}{2}x-3=0$$



Intersection of ℓ and $f(x)$ $\rightarrow x=3.389677$

701. Consider the closed curve in the xy-plane given by $x^2+2x+y^4+4y=5$

a) Show that $\dfrac{dy}{dx}=\dfrac{-(x+1)}{2(y^3+1)}$

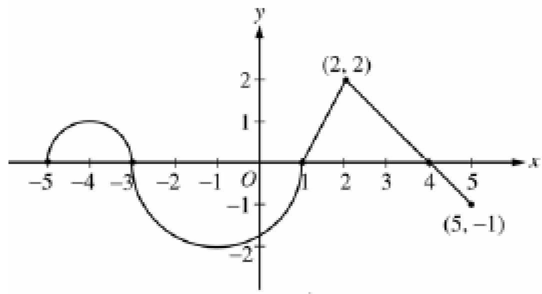
b) Write an equation for the line tangent to the curve at the point $(-2,\ 1)$

c) Find the coordinates of the two points on the curve where the line tangent to the curve is vertical.

d) Is it possible for this curve to have a horizontal tangent at points where it intersects the x-axis ? Explain your reasoning.

(a) $2x+2+4y^3\dfrac{dy}{dx}+4\dfrac{dy}{dx}=0$ $\left(4y^3+4\right)\dfrac{dy}{dx}=-2x-2$ $\dfrac{dy}{dx}=\dfrac{-2(x+1)}{4\left(y^3+1\right)}=\dfrac{-(x+1)}{2\left(y^3+1\right)}$	$2:\begin{cases}1: \text{implicit differentiation}\\1: \text{verification}\end{cases}$
(b) $\left.\dfrac{dy}{dx}\right\rvert_{(-2,\,1)}=\dfrac{-(-2+1)}{2(1+1)}=\dfrac{1}{4}$ Tangent line: $y=1+\frac{1}{4}(x+2)$	$2:\begin{cases}1: \text{slope}\\1: \text{tangent line equation}\end{cases}$
(c) Vertical tangent lines occur at points on the curve where $y^3+1=0$ (or $y=-1$) and $x\neq -1$. On the curve, $y=-1$ implies that $x^2+2x+1-4=5$, so $x=-4$ or $x=2$. Vertical tangent lines occur at the points $(-4,-1)$ and $(2,-1)$.	$3:\begin{cases}1: y=-1\\1: \text{substitutes } y=-1 \text{ into the equation of the curve}\\1: \text{answer}\end{cases}$
(d) Horizontal tangents occur at points on the curve where $x=-1$ and $y\neq -1$. The curve crosses the x-axis where $y=0$. $(-1)^2+2(-1)+0^4+4\cdot 0\neq 5$ No, the curve cannot have a horizontal tangent where it crosses the x-axis.	$2:\begin{cases}1: \text{works with } x=-1 \text{ or } y=0\\1: \text{answer with reason}\end{cases}$

702. Let f be a function defined on the closed interval $-5 \le x \le 5$ with $f(1) = 3$. The graph of f', the derivative of f, consists of two semicircles and two line segments, as shown above.



Graph of f'

(a) For $-5 < x < 5$, find all values x at which f has a relative maximum. Justify your answer.

(b) For $-5 < x < 5$, find all values x at which the graph of f has a point of inflection. Justify your answer.

(c) Find all intervals on which the graph of f is concave up and also has positive slope. Explain your reasoning.

(d) Find the absolute minimum value of $f(x)$ over the closed interval $-5 \le x \le 5$. Explain your reasoning.

(a) $f'(x) = 0$ at $x = -3,\ 1,\ 4$

f' changes from positive to negative at -3 and 4.

Thus, f has a relative maximum at $x = -3$ and at $x = 4$.

$$2 : \begin{cases} 1 : x\text{-values} \\ 1 : \text{justification} \end{cases}$$

(b) f' changes from increasing to decreasing, or vice versa, at $x = -4,\ -1,$ and 2. Thus, the graph of f has points of inflection when $x = -4,\ -1,$ and 2.

$$2 : \begin{cases} 1 : x\text{-values} \\ 1 : \text{justification} \end{cases}$$

(c) The graph of f is concave up with positive slope where f' is increasing and positive: $-5 < x < -4$ and $1 < x < 2$.

$$2 : \begin{cases} 1 : \text{intervals} \\ 1 : \text{explanation} \end{cases}$$

(d) Candidates for the absolute minimum are where f' changes from negative to positive (at $x = 1$) and at the endpoints ($x = -5, 5$).

$$f(-5) = 3 + \int_1^{-5} f'(x)\,dx = 3 - \frac{\pi}{2} + 2\pi > 3$$

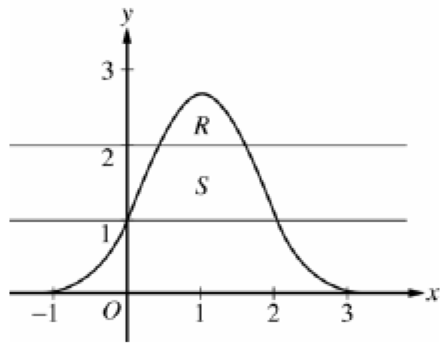
$$f(1) = 3$$

$$f(5) = 3 + \int_1^{5} f'(x)\,dx = 3 + \frac{3 \cdot 2}{2} - \frac{1}{2} > 3$$

The absolute minimum value of f on $[-5, 5]$ is $f(1) = 3$.

$$3 : \begin{cases} 1 : \text{identifies } x = 1 \text{ as a candidate} \\ 1 : \text{considers endpoints} \\ 1 : \text{value and explanation} \end{cases}$$

703. Let R be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal line $y = 2$, and let S be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal lines $y = 1$ and $y = 2$, as shown above.



(a) Find the area of R.

(b) Find the area of S.

(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 1$.

$e^{2x-x^2} = 2$ when $x = 0.446057, 1.553943$

Let $P = 0.446057$ and $Q = 1.553943$

(a) Area of $R = \int_P^Q \left(e^{2x-x^2} - 2\right) dx = 0.514$

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$

(b) $e^{2x-x^2} = 1$ when $x = 0,\ 2$

Area of $S = \int_0^2 \left(e^{2x-x^2} - 1\right) dx - \text{Area of } R$

$= 2.06016 - \text{Area of } R = 1.546$

OR

$\int_0^P \left(e^{2x-x^2} - 1\right) dx + (Q - P) \cdot 1 + \int_Q^2 \left(e^{2x-x^2} - 1\right) dx$

$= 0.219064 + 1.107886 + 0.219064 = 1.546$

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$

(c) Volume $= \pi \int_P^Q \left(\left(e^{2x-x^2} - 1\right)^2 - (2 - 1)^2\right) dx$

$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{constant and limits} \end{cases}$

704. A particle moves along the x-axis so that its velocity v at time t for $0 \le t \le 5$ is given by $v(t) = \ln(t^2 - 3t + 3)$ The particle is at position $x = 8$ at time $t = 0$

a) Find the acceleration of the particle at time $t = 4$

b) Find all times t in the open interval $0 < t < 5$ at which the particle changes direction. During which time intervals, for $0 \le t \le 5$, does the particle travel to the left ?

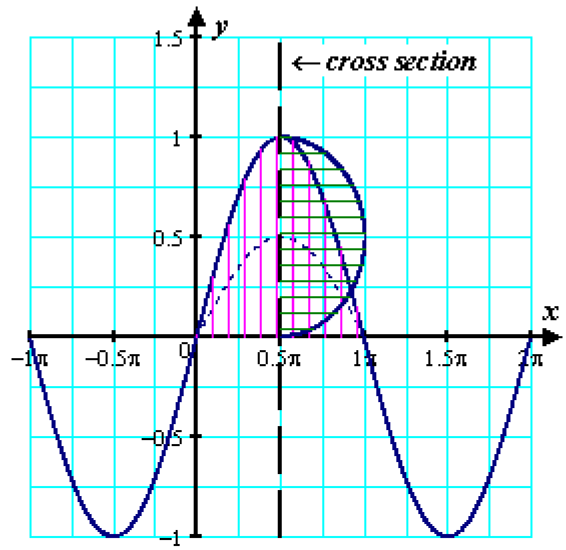
c) Find the position of the particle at time $t = 2$

d) Find the average speed of the particle over the interval $0 \le t \le 2$

(a) $a(4) = v'(4) = \frac{5}{7}$	1 : answer
(b) $v(t) = 0$ $t^2 - 3t + 3 = 1$ $t^2 - 3t + 2 = 0$ $(t-2)(t-1) = 0$ $t = 1, 2$ $v(t) > 0$ for $0 < t < 1$ $v(t) < 0$ for $1 < t < 2$ $v(t) > 0$ for $2 < t < 5$ The particle changes direction when $t = 1$ and $t = 2$. The particle travels to the left when $1 < t < 2$.	$3 : \begin{cases} 1 : \text{sets } v(t) = 0 \\ 1 : \text{direction change at } t = 1, 2 \\ 1 : \text{interval with reason} \end{cases}$
(c) $s(t) = s(0) + \int_0^t \ln\left(u^2 - 3u + 3\right) du$ $s(2) = 8 + \int_0^2 \ln\left(u^2 - 3u + 3\right) du$ $= 8.368$ or 8.369	$3 : \begin{cases} 1 : \int_0^2 \ln\left(u^2 - 3u + 3\right) du \\ 1 : \text{handles initial condition} \\ 1 : \text{answer} \end{cases}$
(d) $\frac{1}{2}\int_0^2 \lvert v(t) \rvert\, dt = 0.370$ or 0.371	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

705. *Answer is A.*

The base of a solid is the region enclosed by $y = \sin x$ and the x-axis on the interval $[\,0,\ \pi\,]$ Cross sections perpendicular to the x-axis are ***semicircles*** with diameter in the plane of the base. Write an integral that represents the volume of the solid.



Circle $\rightarrow$ $\mathbf{A} = \pi r^2$ Semicircle $\rightarrow$ $\mathbf{A} = \frac{\pi}{2} r^2$

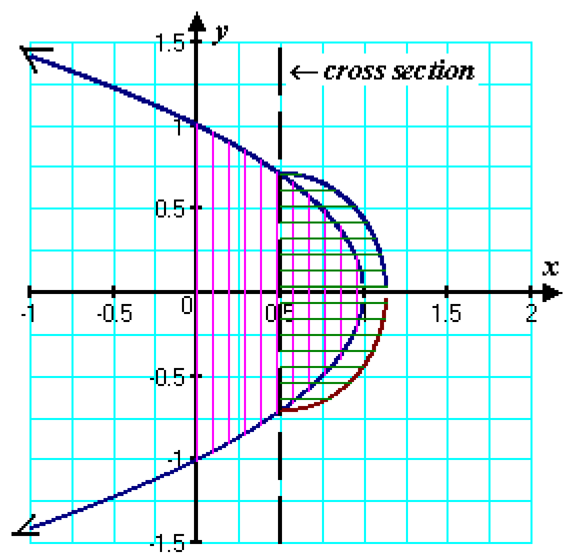
Radius of semicircle $r = \frac{\sin x}{2}$

$$\text{Volume} = \frac{\pi}{2}\int_0^{\pi}\left(\frac{\sin x}{2}\right)^2 dx$$

$$= \frac{\pi}{8}\int_0^{\pi}(\sin x)^2\, dx$$

706. *Answer is B.*

The base of a solid is the region enclosed by the graph of $x = 1 - y^2$ and the y-axis. If all plane cross-sections perpendicular to the x-axis are ***semicircles*** with diameters parallel to the y-axis, then the volume is:



Circle $\rightarrow$ $\mathbf{A} = \pi r^2$ Semicircle $\rightarrow$ $\mathbf{A} = \frac{\pi}{2} r^2$

Radius of semicircle $r = \sqrt{1 - x}$

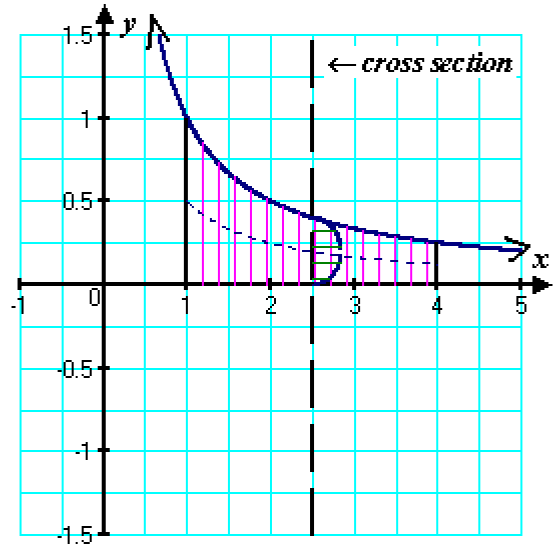
$$\text{Volume} = \frac{\pi}{2}\int_0^{1}\left(\sqrt{1-x}\right)^2 dx$$

$$= \frac{\pi}{2}\int_0^{1}(1 - x)\, dx$$

$$= \frac{\pi}{2}\left[x - \frac{x^2}{2}\right]_0^1 = \frac{\pi}{2}\left[1 - \frac{1}{2}\right] = \boxed{\frac{\pi}{4}}$$

707. *Answer is B.*

Let the first quadrant region enclosed by the graph of $y = \frac{1}{x}$ and the lines $x = 1$ and $x = 4$ be the base of a solid. If cross sections perpendicular to the x-axis are ***semicircles***, the volume of the solid is



Circle $\rightarrow$ $\mathbf{A} = \pi r^2$ Semicircle $\rightarrow$ $\mathbf{A} = \frac{\pi}{2} r^2$

Radius of semicircle $r = \frac{1}{2}\left(\frac{1}{x}\right) = \frac{1}{2x}$

$$\text{Volume} = \int_1^4 \frac{\pi}{2}(r)^2\,dx = \frac{\pi}{2}\int_1^4 (r)^2\,dx$$

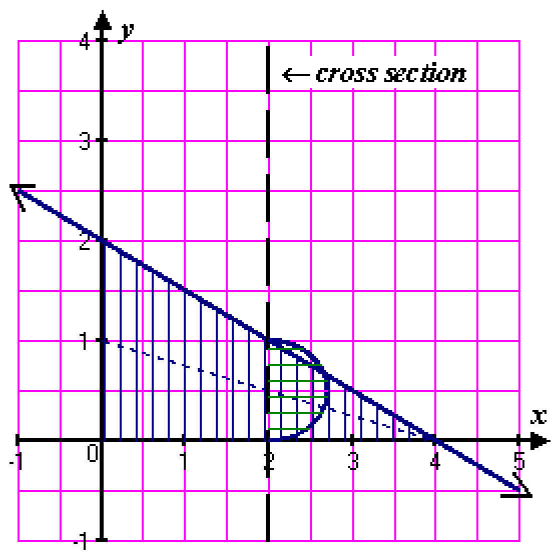
$$= \frac{\pi}{2}\int_1^4 \left(\frac{1}{2x}\right)^2 dx = \frac{\pi}{2}\int_1^4 \frac{1}{4x^2}\,dx$$

$$= \frac{\pi}{8}\int_1^4 x^{-2}\,dx = -\frac{\pi}{8}\left[\frac{1}{x}\right]_1^4 = \frac{\pi}{8}\left[\frac{1}{4} - \frac{4}{4}\right]$$

$$= -\frac{\pi}{8}\left[-\frac{3}{4}\right] = \boxed{\frac{3\pi}{32}}$$

708. *Answer is A.*

The base of a solid is the region in the first quadrant bounded by the line $x + 2y = 4$ and the coordinate axes. What is the volume of the solid if every cross section perpendicular to the x-axis is a semicircle ?



$$x + 2y = 4$$

$$2y = -x + 4$$

$$y = -\tfrac{1}{2}x + 2 \qquad \rightarrow \text{Radius} = \tfrac{1}{2}(-\tfrac{1}{2}x + 2)$$

$$\mathbf{A} = \pi r^2 = \pi(-\tfrac{x}{4} + 1)^2 \quad \rightarrow \text{Semicircle} = \tfrac{\pi}{2}(1 - \tfrac{x}{4})^2$$

$$\text{Volume} = \frac{\pi}{2}\int_0^4 (1 - \tfrac{x}{4})^2\,dx$$

$$= \frac{\pi}{2}\int_0^4 (1 - \tfrac{x}{2} + \tfrac{x^2}{16})\,dx$$

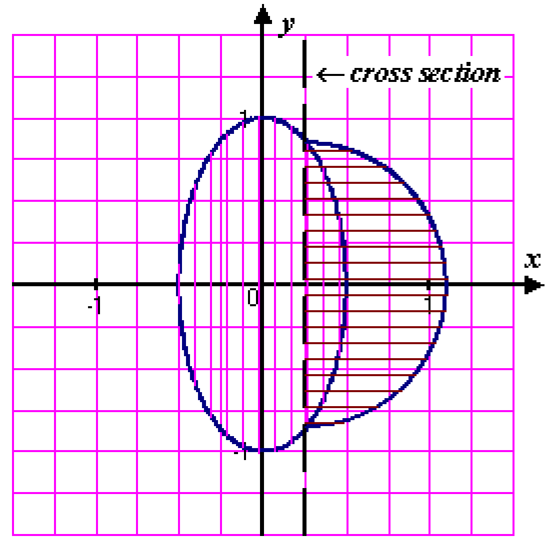
$$= \frac{\pi}{2(16)}\int_0^4 (16 - 8x + x^2)\,dx$$

$$= \frac{\pi}{32}\left[16x - 4x^2 + \frac{x^3}{3}\right]_0^4$$

$$= \frac{\pi}{32}\left[\cancel{64} - \cancel{64} + \frac{64}{3}\right] = \boxed{\frac{2\pi}{3}}$$

709. *Answer is C.*

The base of a solid is the region enclosed by the ellipse $4x^2 + y^2 = 1$ If all plane cross sections perpendicular to the x-axis are semicircles, then its volume is



$$4x^2 + y^2 = 1$$

$$y = \pm\sqrt{1-4x^2} \quad \rightarrow \quad \text{Radius} = \sqrt{1-4x^2}$$

$$A = \pi r^2 = \pi\left(\sqrt{1-4x^2}\right)^2$$

$$\text{Semicircle} = \frac{\pi}{2}\left(\sqrt{1-4x^2}\right)^2$$

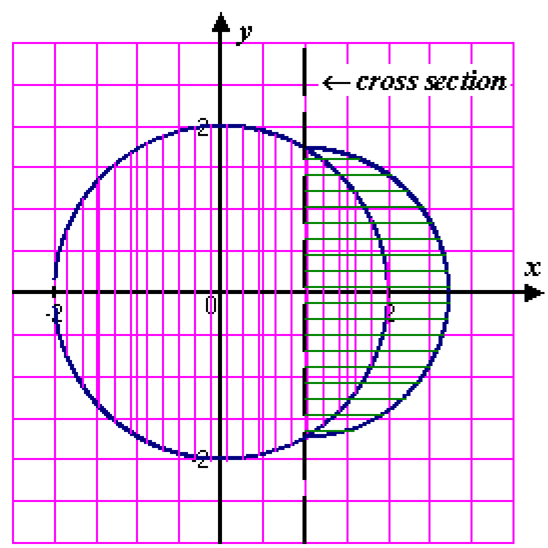
$$\text{Volume} = \frac{\pi}{2}\int_{-\frac{1}{2}}^{\frac{1}{2}}\left(\sqrt{1-4x^2}\right)^2 dx$$

$$= \pi\int_0^{\frac{1}{2}}(1-4x^2)\,dx = \pi\left[x - \frac{4x^3}{3}\right]_0^{\frac{1}{2}}$$

$$= \pi\left[\frac{1}{2} - \frac{4(\frac{1}{2})^3}{3}\right] = \pi\left[\frac{1}{2} - \frac{1}{6}\right] = \boxed{\frac{\pi}{3}}$$

710. *Answer is B.*

The base of a solid is a region enclosed by the circle $x^2 + y^2 = 4$ What is the approximate volume of the solid if the cross sections of the solid perpendicular to the x-axis are semicircles ?



$$x^2 + y^2 = 4$$

$$\text{Radius} \rightarrow y = \sqrt{4-x^2}$$

$$\text{Area circle} = \pi r^2 = \pi\left(\sqrt{4-x^2}\right)^2 = \pi(4-x^2)$$

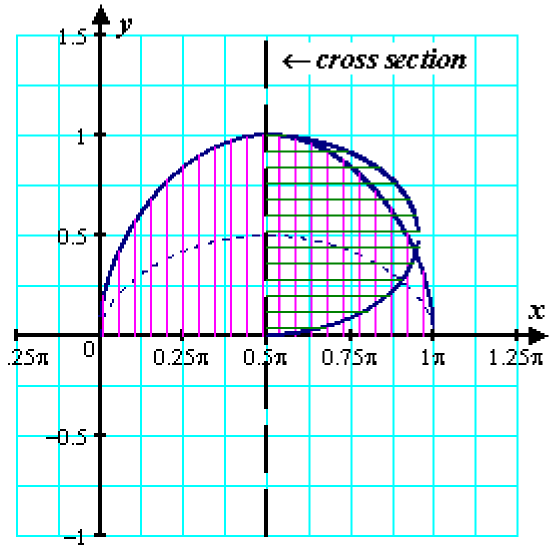
$$\text{Area semicircle} = \frac{\pi}{2}(4-x^2)$$

$$\text{Volume} = \frac{\pi}{2}\int_{-2}^{2}(4-x^2)\,dx = \pi\int_0^2(4-x^2)\,dx$$

$$= \pi\left[4x - \frac{x^3}{3}\right]_0^2 = \pi\left[8 - \frac{8}{3}\right] = \boxed{\frac{16\pi}{3}}$$

711. *Answer is C.*

The base of a solid is the region in the first quadrant bounded by the curve $y=\sqrt{\sin x}$ for $0 \le x \le \pi$ If each cross section of the solid perpendicular to the x-axis is a semicircle, the volume of the solid is



Circle $\rightarrow$ $\mathbf{A}=\pi r^2$ Semicircle $\rightarrow$ $\mathbf{A}=\frac{\pi}{2}r^2$

Radius of semicircle $r=\frac{\sqrt{\sin x}}{2}$

$$\text{Volume}=\frac{\pi}{2}\int_0^{\pi}\left(\frac{\sqrt{\sin x}}{2}\right)^2 dx$$

$$=\frac{-\pi}{8}\int_0^{\pi}(-\sin x)\,dx$$

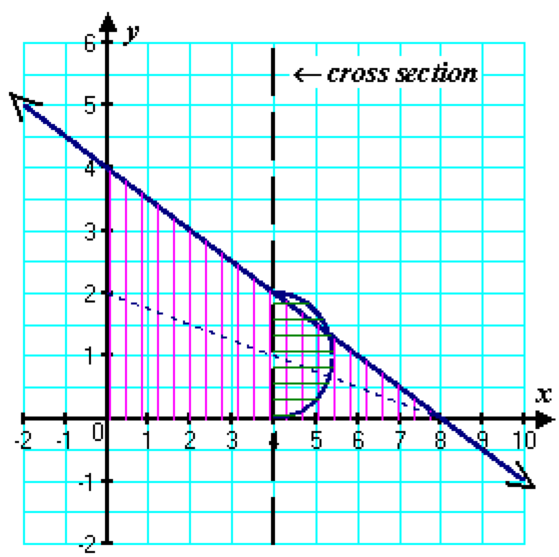
$$=\frac{-\pi}{8}\left[\cos x\right]_0^{\pi}=\frac{-\pi}{8}\left[\cos\pi-\cos 0\right]$$

$$=\frac{-\pi}{8}\left[-1-1\right]=\boxed{\frac{\pi}{4}}$$

712. *Answer is C.*

Difficulty $= \mathbf{0.19}$

The base of a solid is a region in the first quadrant bounded by the x-axis, the y-axis, and the line $x+2y=8$, as shown in the figure on the right. If cross sections of the solid perpendicular to the x-axis are semicircles, what is the volume of the solid ?

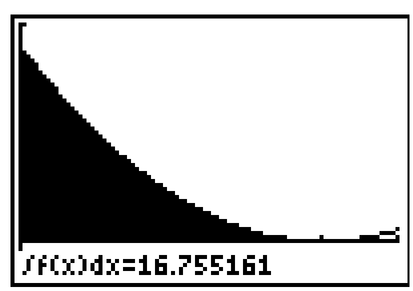


$x+2y=8$ $\rightarrow$ x-intercept $=8$

$y=-\frac{1}{2}x+4$ $\rightarrow$ Radius $=-\frac{1}{4}x+2$

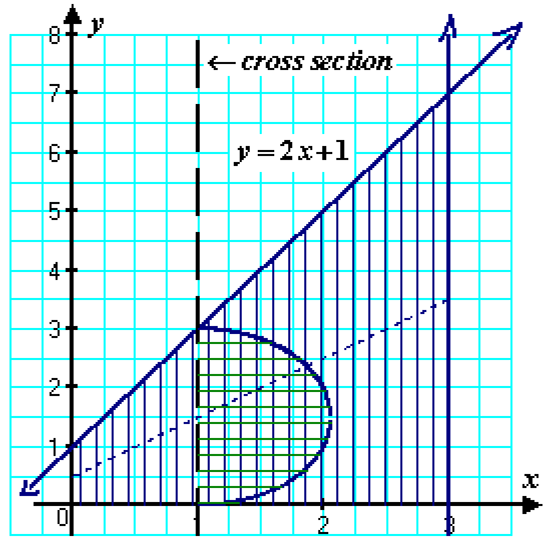
$$\text{Volume}=\frac{\pi}{2}\int_0^{8}\left(-\tfrac{1}{4}x+2\right)^2 dx=\boxed{16.755161}$$

↑ divide by $\mathbf{2}$ because of semicircle



713. *Answer is B.*

The base of a solid is the region in the first quadrant bounded by the x-axis, y-axis, and the lines $y = 2x + 1$ and $x = 3$, as shown in the diagram. If cross-sections of the solid perpendicular to the x-axis are semicircles, what is the volume of the solid ?

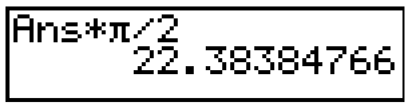


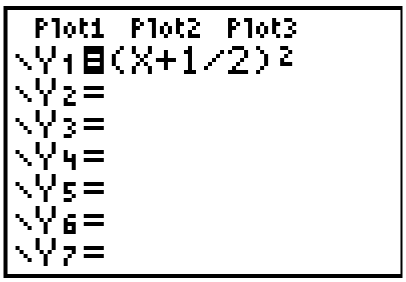
Radius $y = \frac{2x+1}{2} = x + \frac{1}{2}$

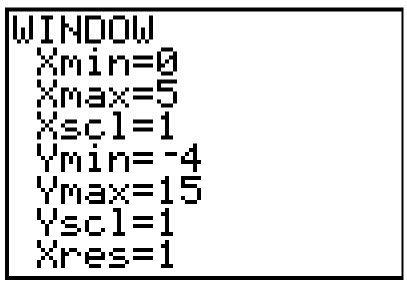
Circle $= \pi r^2$ Semicircle $= \dfrac{\pi r^2}{2}$

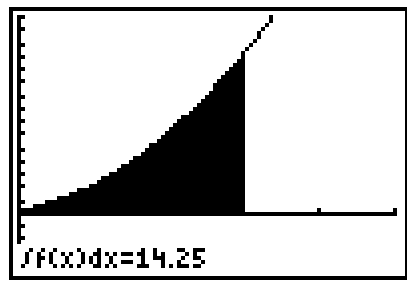
$$\text{Volume} = \frac{\pi}{2}\int_0^3 (x + \tfrac{1}{2})^2\,dx$$

$$= 14.25(\tfrac{\pi}{2}) \approx \boxed{22.383}$$



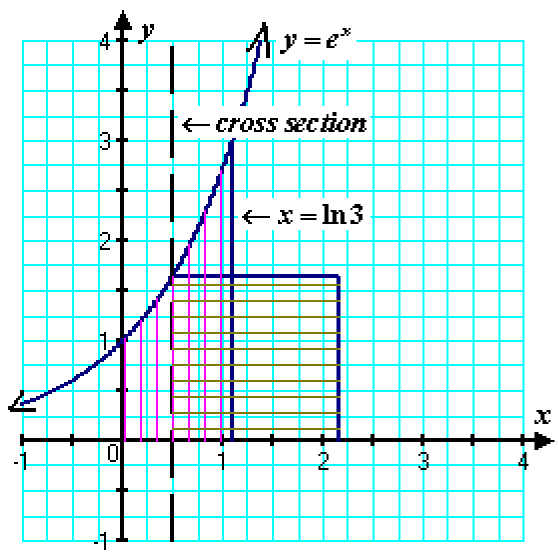






714. *Answer is C.*

The base of a solid is the region enclosed by $y = e^x$, the x-axis, the y-axis and the line $x = \ln 3$ Cross sections perpendicular to the x-axis are ***squares***. Write an integral that represents the volume of the solid.



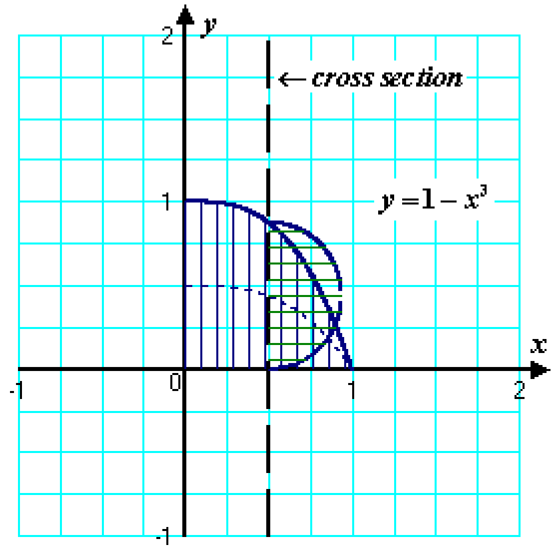
$$\text{Volume} = \int_0^{\ln 3} (e^x)^2\,dx$$

$$= \frac{1}{2}\int_0^{\ln 3} e^{2x}(2)dx = \frac{1}{2}\left[e^{2x}\right]_0^{\ln 3}$$

$$= \frac{1}{2}\left[e^{2\ln 3} - e^0\right] = \frac{1}{2}\left[9 - 1\right] = \boxed{4}$$

715. *Answer is A.*

The base of a solid is a region in the first quadrant bounded by the x-axis, the y-axis, and the graph of $y = 1 - x^3$, as shown in the diagram. If cross-sections of the solid perpendicular to the x-axis are semicircles, what is the volume of the solid ?

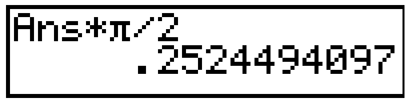


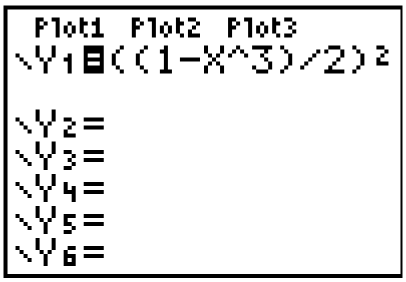
Radius $y = \frac{1-x^3}{2}$

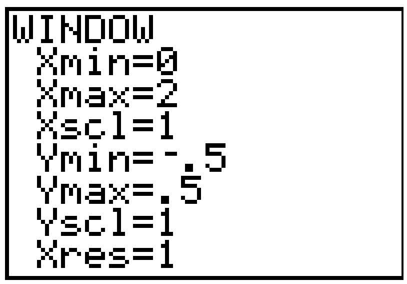
Circle $= \pi r^2$ Semicircle $= \dfrac{\pi r^2}{2}$

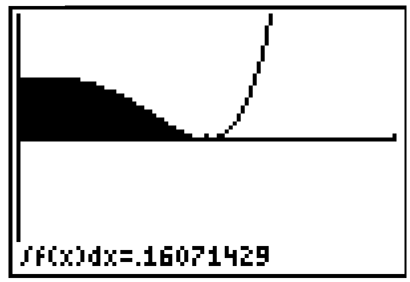
Volume $= \dfrac{\pi}{2}\displaystyle\int_0^3 \left(\tfrac{1-x^3}{2}\right)^2 dx$

$= 0.1607\left(\tfrac{\pi}{2}\right) \approx \boxed{0.2524}$







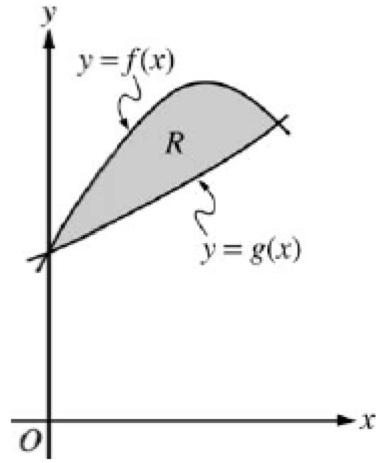


716. Let f and g be the functions given by $f(x)=1+\sin(2x)$ and $g(x)=e^{\frac{x}{2}}$ Let **R** be the shaded region in the first quadrant enclosed by the graphs of f and g as shown in the diagram.

a) Find the area of **R**

b) Find the volume of the solid generated when **R** is revolved about the x-axis.

c) The region **R** is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles with diameters extending from $y=f(x)$ to $y=g(x)$. Find the volume of this solid.



The graphs of f and g intersect in the first quadrant at $(S,T)=(1.13569, 1.76446)$.

1 : correct limits in an integral in (a), (b), or (c)

(a) Area $= \int_0^S (f(x)-g(x))\,dx$

$= \int_0^S \left(1+\sin(2x)-e^{x/2}\right)dx$

$= 0.429$

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(b) Volume $= \pi\int_0^S \left((f(x))^2-(g(x))^2\right)dx$

$= \pi\int_0^S \left((1+\sin(2x))^2-\left(e^{x/2}\right)^2\right)dx$

$= 4.266$ or 4.267

$3 : \begin{cases} 2 : \text{integrand} \\ \quad \langle -1 \rangle \text{ each error} \\ \quad \text{Note: } 0/2 \text{ if integral not of form} \\ \qquad c\int_a^b \left(R^2(x)-r^2(x)\right)dx \\ 1 : \text{answer} \end{cases}$

(c) Volume $= \int_0^S \frac{\pi}{2}\left(\frac{f(x)-g(x)}{2}\right)^2 dx$

$= \int_0^S \frac{\pi}{2}\left(\frac{1+\sin(2x)-e^{x/2}}{2}\right)^2 dx$

$= 0.077$ or 0.078

$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

717. Let R be the region in the first and second quadrants bounded above by the graph of $y = \frac{20}{1+x^2}$ and below by the horizontal line $y = 2$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the x-axis.

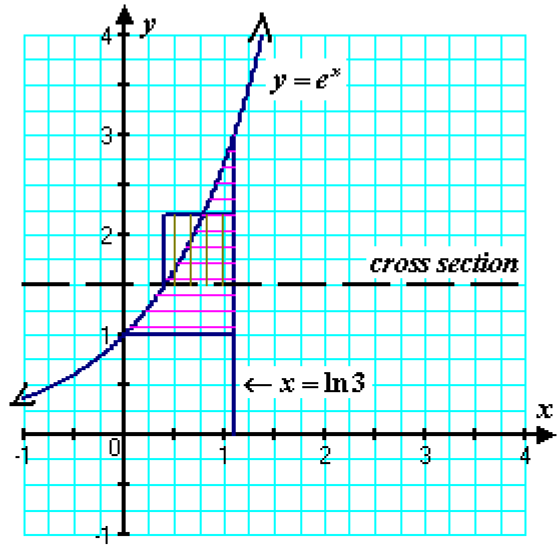
(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles. Find the volume of this solid.

$\frac{20}{1+x^2} = 2$ when $x = \pm 3$	1 : correct limits in an integral in (a), (b), or (c)
(a) Area $= \int_{-3}^{3} \left(\frac{20}{1+x^2} - 2 \right) dx = 37.961$ or 37.962	$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$
(b) Volume $= \pi \int_{-3}^{3} \left(\left(\frac{20}{1+x^2} \right)^2 - 2^2 \right) dx = 1871.190$	$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$
(c) Volume $= \frac{\pi}{2} \int_{-3}^{3} \left(\frac{1}{2} \left(\frac{20}{1+x^2} - 2 \right) \right)^2 dx$ $= \frac{\pi}{8} \int_{-3}^{3} \left(\frac{20}{1+x^2} - 2 \right)^2 dx = 174.268$	$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

718. *Answer is B.*

The base of a solid is the region enclosed by $y = e^x$ and the lines $y = 1$ and $x = \ln 3$ Cross sections perpendicular to the y-axis are squares. Write an integral that represents the volume of the solid.

$$\text{Volume} = \int_1^3 (\ln 3 - \ln y)^2 \, dy$$



719. *Answer is E.*

A solid has as its base the region bounded by $y = \sqrt{x}$, the x-axis, and the vertical line $x = 4$. Each cross-section of the solid perpendicular to the y-axis is a square. Which one of the following expressions represents the volume of the solid ?

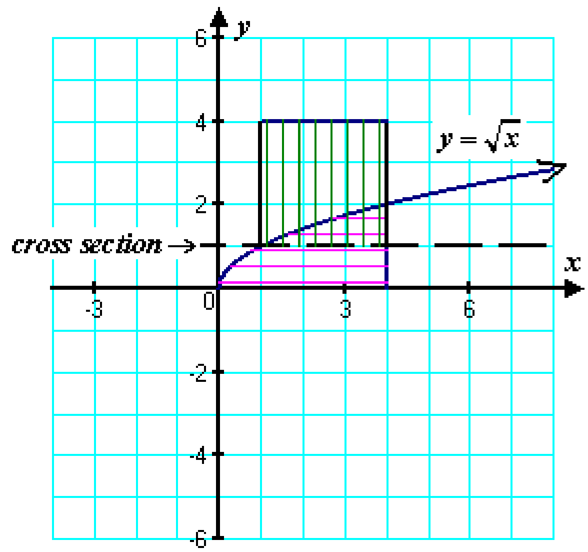
$$y = \sqrt{x}$$
$$y^2 = x$$

Cross section perpendicular to y-axis $\rightarrow \int dy$

Side of square $= 4 - y^2$

Area of square $= (4 - y^2)^2$

$$\text{Volume} = \int_0^2 (4 - y^2)^2 \, dy$$

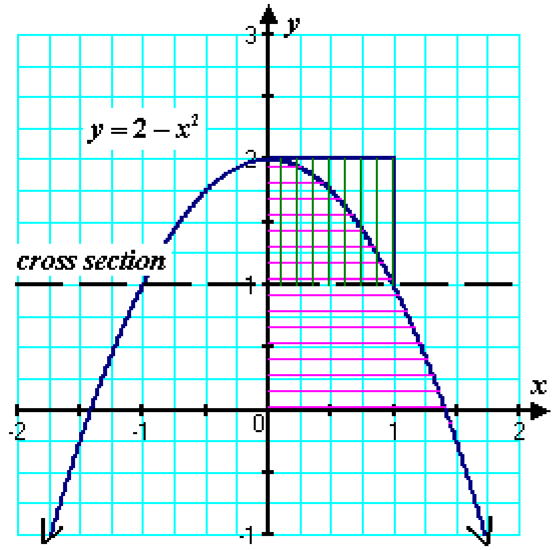


720. *Answer is B.*

Difficulty $= \mathbf{0.34}$

The base of a solid is the region in the first quadrant enclosed by the graph of $y = 2 - x^2$ and the coordinate axes. If every cross section of the solid perpendicular to the y-axis is a square, the volume of the solid is given by

$$y = 2 - x^2$$
$$x^2 = 2 - y$$
$$x = \sqrt{2-y}$$
$$\text{Volume} = \int_0^2 \left(\sqrt{2-y}\right)^2 dy$$
$$= \int_0^2 (2-y)\,dy$$

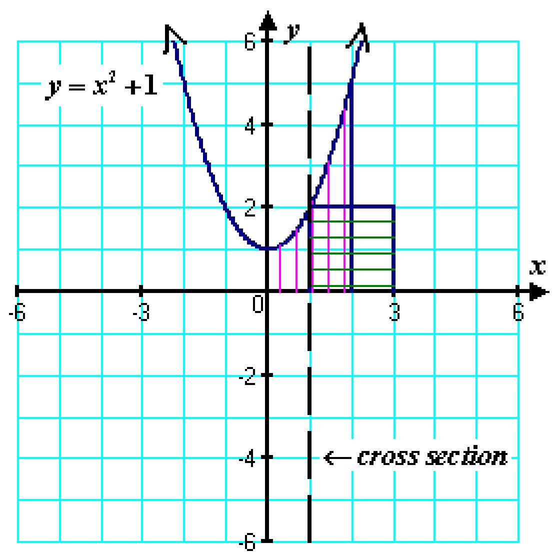


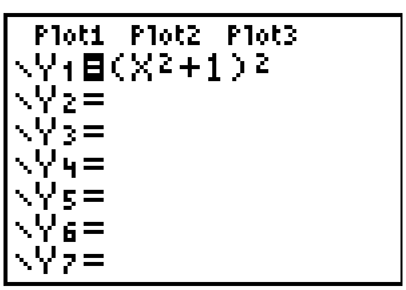
721. *Answer is D.*

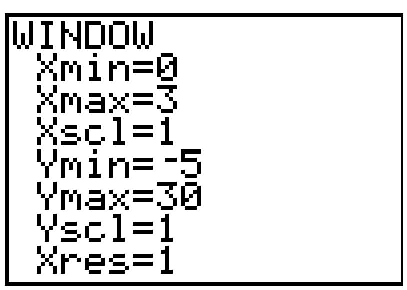
The base of a solid is a region in the first quadrant bounded by the x-axis, the y-axis, the graph of $y = x^2 + 1$, and the vertical line $x = 2$ If cross sections perpendicular to the x-axis are squares, what is the volume of the solid ?

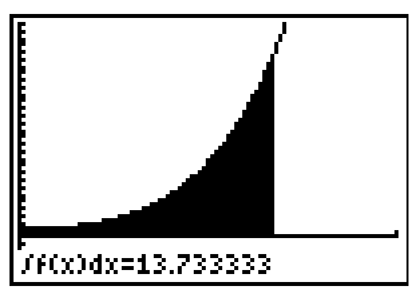
Side of square $= x^2 + 1$ Area square $= (x^2 + 1)^2$

$$\text{Volume} = \int_0^2 (x^2+1)^2\,dx \approx \boxed{13.733}$$









722. *Answer is E.*

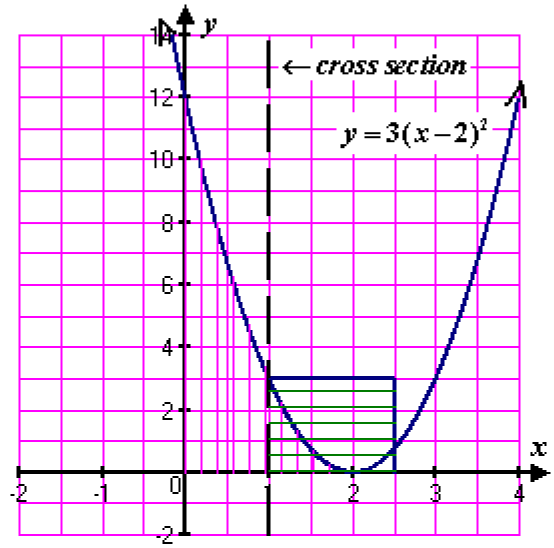
The base of a solid is the region enclosed by the graph of $y = 3(x-2)^2$ and the coordinate axes. If every cross section perpendicular to the x-axis is a square, then the volume of the solid is

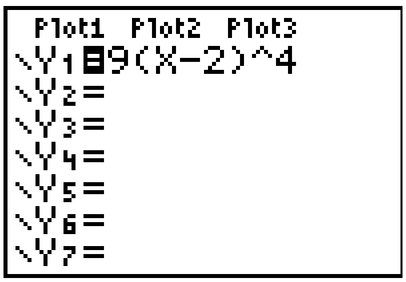
Side of square $= 3(x-2)^2$

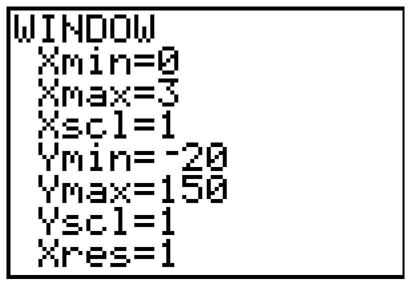
Area of square $= \left[3(x-2)^2\right]^2 = 9(x-2)^4$

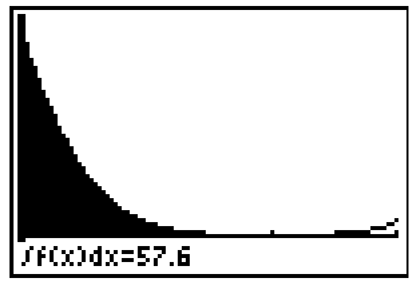
Volume $= \int_0^2 9(x-2)^4\,dx = \boxed{57.6}$

Used calculator to evaluate









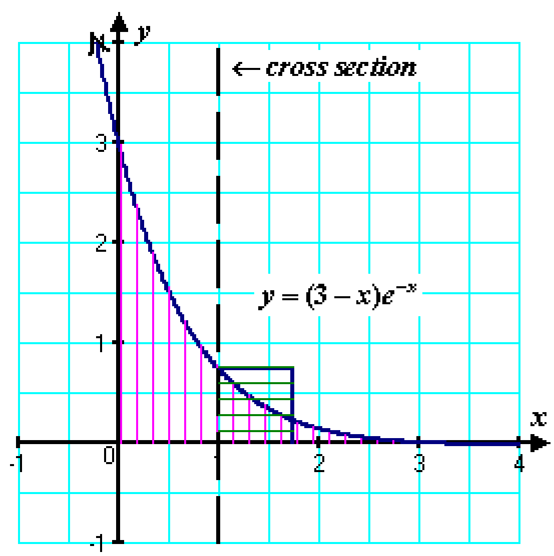
723. *Answer is C.*

The base of a solid is the region in the first quadrant bounded by the x-axis, the y-axis, and the graph of $y = (3-x)e^{-x}$ as shown in the diagram. If cross sections of the solid perpendicular to the x-axis are squares, what is the volume of the solid ?

Side of square $= (3-x)e^{-x}$

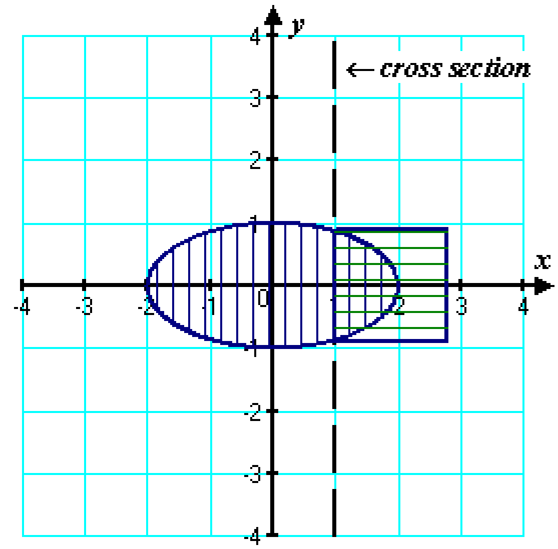
Area of square $= \left[(3-x)e^{-x}\right]^2$

Volume $= \int_0^3 \left[(3-x)e^{-x}\right]^2 dx \approx \boxed{3.2493}$



724. *Answer is D.*

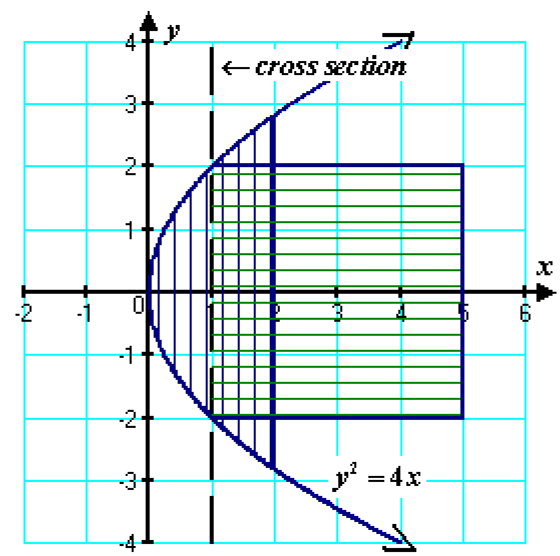
The base of a solid is the region enclosed by the graph of $x^2+4y^2=4$ Cross sections of the solid perpendicular to the x-axis are squares. Find the volume of the solid.



$$x^2+4y^2=4$$
$$4y^2=4-x^2$$
$$y=\sqrt{1-\tfrac{x^2}{4}}$$
$$\text{Volume}=2\int_0^2\left(2\sqrt{1-\tfrac{x^2}{4}}\right)^2dx$$
$$=8\int_0^2 1-\tfrac{x^2}{4}\,dx=8\left[x-\frac{x^3}{12}\right]_0^2$$
$$=8\left[2-\frac{8}{12}\right]=\boxed{\frac{32}{3}}$$

725. *Answer is E.*

The base of a solid is the region bounded by the parabola $y^2=4x$ and the line $x=2$. Each plane section perpendicular to the x-axis is a square. The volume of the solid is



$$y^2=4x$$
$$y=\sqrt{4x}$$
$$\text{Side of square}=2\sqrt{4x}$$
$$\text{Volume}=\int_0^2\left(2\sqrt{4x}\right)^2dx$$
$$=4\int_0^2 4x\,dx=16\left[\frac{x^2}{2}\right]_0^2=\boxed{32}$$

726.

Let **R** be the region in the first quadrant under the graph of $y = \dfrac{1}{\sqrt{x}}$ for $4 \le x \le 9$

a) Find the area of **R**

b) If the line $x = k$ divides the region **R** into two regions of equal area, what is the value of k

c) Find the volume of the solid whose base is the region **R** and whose cross sections cut by planes perpendicular to the x-axis are squares.

a) Find the area of **R**

$$\int_4^9 \frac{1}{\sqrt{x}}\,dx = \int_4^9 x^{-\frac{1}{2}}\,dx = \left[2\sqrt{x}\right]_4^9$$

$$= \left[2\sqrt{9} - 2\sqrt{4}\right] = \left[6-4\right] = \boxed{2}$$

b) Value of $k =$

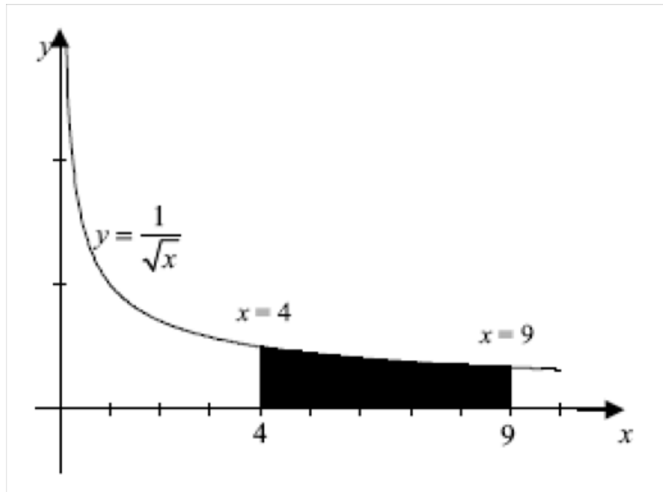
$$\int_4^k \frac{1}{\sqrt{x}}\,dx = \int_4^k x^{-\frac{1}{2}}\,dx = \left[2\sqrt{x}\right]_4^k$$

$$= \left[2\sqrt{k} - 2\sqrt{4}\right] = \boxed{1}$$

$$2\sqrt{k} - 4 = 1$$

$$\sqrt{k} = \tfrac{5}{2}$$

$$\boxed{k = \tfrac{25}{4}}$$

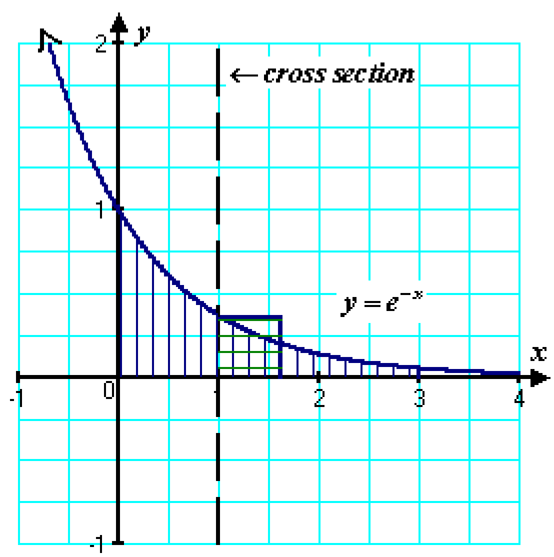


c) Find the volume

$$\text{Volume} \int_4^9 \left(\frac{1}{\sqrt{x}}\right)^2 dx = \int_4^9 \frac{1}{x}\,dx = \left[\ln x\right]_4^9 = \left[\ln 9 - \ln 4\right] = \boxed{\ln\tfrac{9}{4} \approx 0.811}$$

727. *Answer is A.*

The base of a solid is the region enclosed by the graph of $y = e^{-x}$, the coordinate axes, and the line $x = 3$. If all plane cross sections perpendicular to the x-axis are squares, then its volume is



$$\text{Volume} = \int_0^3 (e^{-x})^2\,dx$$

$$= -\frac{1}{2}\int_0^3 (e^{-2x})(-2)\,dx$$

$$= -\frac{1}{2}\left[e^{-2x}\right]_0^3 = -\frac{1}{2}\left[e^{-6x} - e^{0}\right]_0^3$$

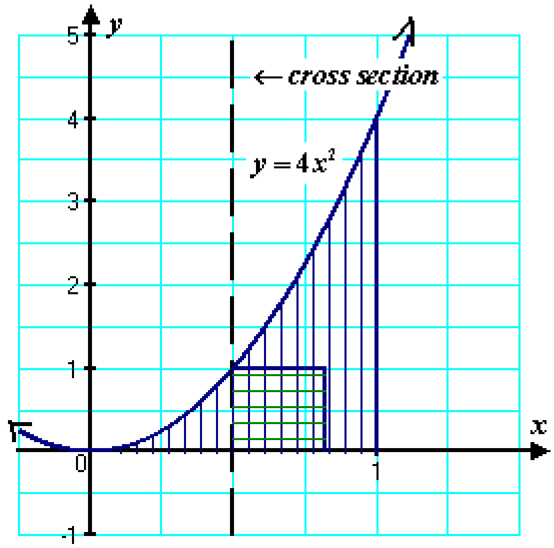
$$= -\frac{1}{2}\left[e^{-6x} - 1\right] = \boxed{\frac{1-e^{-6x}}{2}}$$

728. *Answer is D.*

The base of a solid is the region in the first quadrant enclosed by the parabola $y = 4x^2$, the line $x = 1$, and the x-axis. Each plane section of the solid perpendicular to the x-axis is a square. The volume of the solid is

$$\text{Volume} = \int_0^1 (4x^2)^2\,dx$$

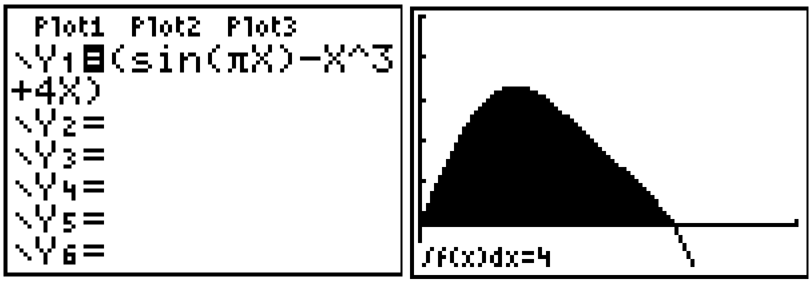
$$= 16\int_0^1 x^4\,dx = 16\left[\frac{x^5}{5}\right]_0^1 = \boxed{\frac{16}{5}}$$



729. Let **R** be the region bounded by the graphs of $y = \sin(\pi x)$ and $y = x^3 - 4x$ as shown in the diagram.

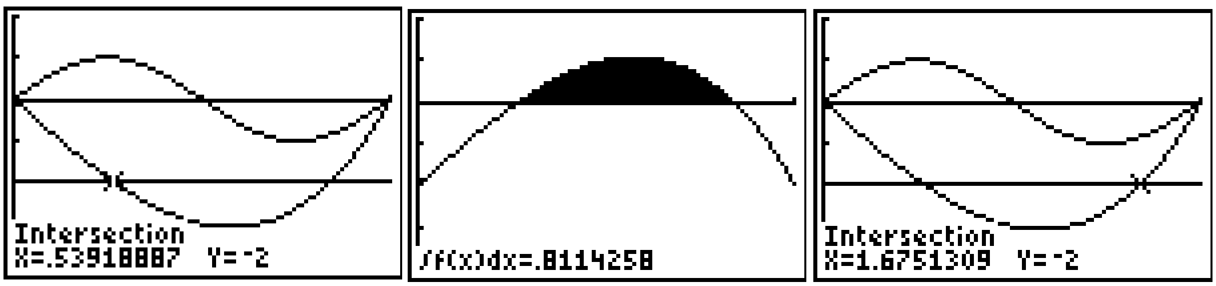
a) Find the area of **R**

$$\text{Area } \mathbf{R} = \int_0^2 \left[\sin(\pi x) - (x^3 - 4x)\right] dx = \boxed{4}$$



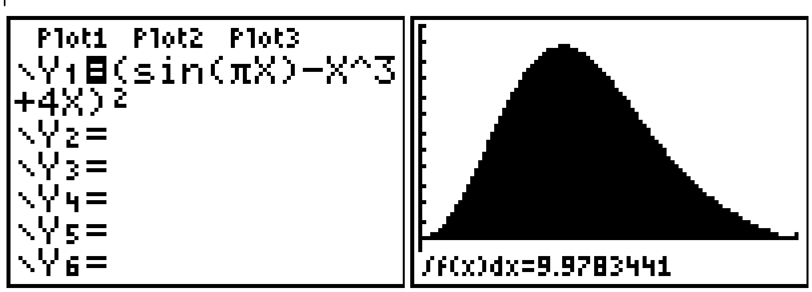
b) The horizontal line $y = -2$ splits the region **R** into two parts. Write, but do not evaluate, an integral expression for the area of the part of **R** that is below this horizontal line.

$$\int_{0.53918887}^{1.6751309} \left[(-2) - (x^3 - 4x)\right] dx = \boxed{0.8114258}$$



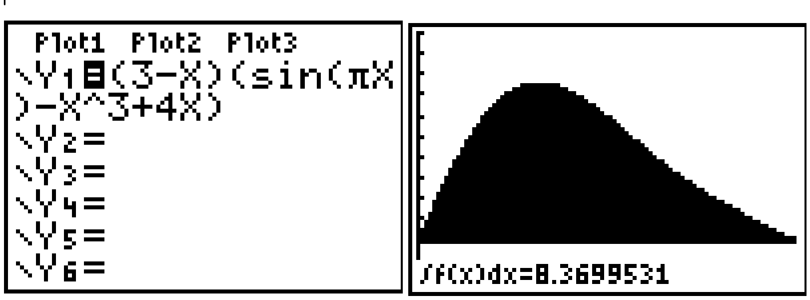
c) The region **R** is the base of a solid. For this solid, each cross section perpendicular to the *x*-axis is a square. Find the volume of this solid.

$$\int_0^2 \left[\sin(\pi x) - (x^3 - 4x)\right]^2 dx = \boxed{9.978}$$



d) The region **R** models the surface of a small pond. At all points in **R** at a distance x from the *y*-axis, the depth of the water is given by $h(x) = 3 - x$. Find the volume of water in the pond.

$$\int_0^2 (3 - x)\left[\sin(\pi x) - (x^3 - 4x)\right] dx = \boxed{8.370}$$



730. Let **R** be the region in the first quadrant bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{3}$

***a*)** Find the area of **R**

***b*)** Find the volume of the solid generated when **R** is rotated about the vertical line $x = -1$

c*)** The region **R** is the base of a solid. For this solid, the cross sections perpendicular to the ***y-axis are squares. Find the volume of this solid.

The graphs of $y = \sqrt{x}$ and $y = \frac{x}{3}$ intersect at the points $(0, 0)$ and $(9, 3)$.

(a) $\int_0^9 \left(\sqrt{x} - \frac{x}{3}\right) dx = 4.5$

OR

$\int_0^3 \left(3y - y^2\right) dy = 4.5$

$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(b) $\pi \int_0^3 \left((3y+1)^2 - \left(y^2+1\right)^2\right) dy$

$= \frac{207\pi}{5} = 130.061 \text{ or } 130.062$

$4 : \begin{cases} 1 : \text{constant and limits} \\ 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

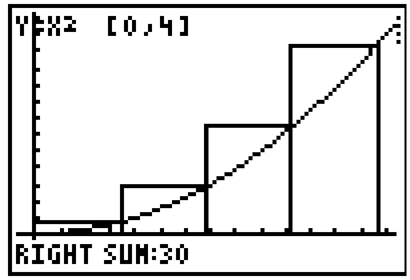
(c) $\int_0^3 \left(3y - y^2\right)^2 dy = 8.1$

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and answer} \end{cases}$

731.

Definite Integrals (exact)

$$\int_0^4 x^2 dx = \left[\frac{x^3}{3}\right]_0^4 = \frac{64}{3} = \boxed{21\tfrac{1}{3}} \leftarrow \textit{exact}!!!$$

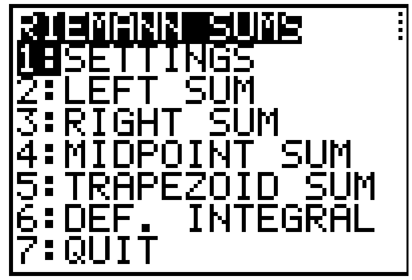


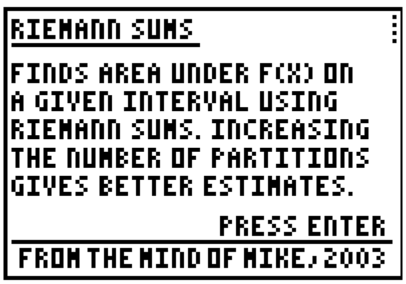
Riemann Sums (approximations)

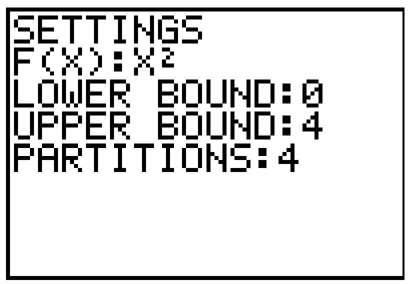
→ increasing/decreasing functions

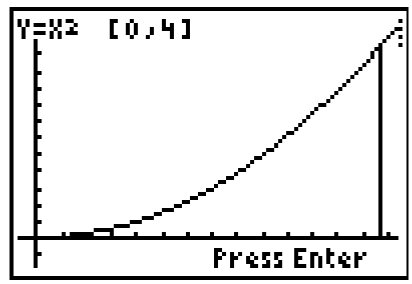
→ concavity/number of partitions

Partitions	4	10	100
Left(under)	14	18.24	21.0144
Right(over)	30	24.64	21.6544
Midpoint	21	21.28	21.3328
Trapezoid	22	21.44	21.3344



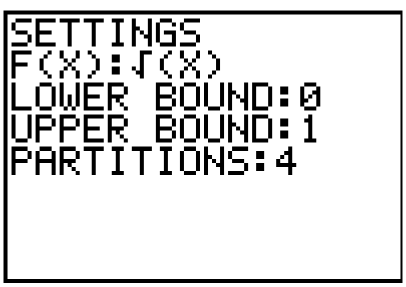


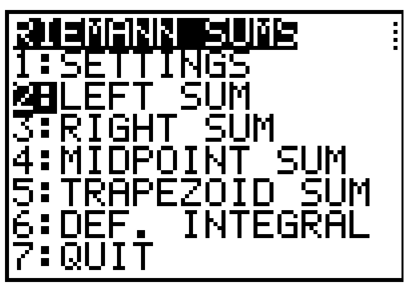


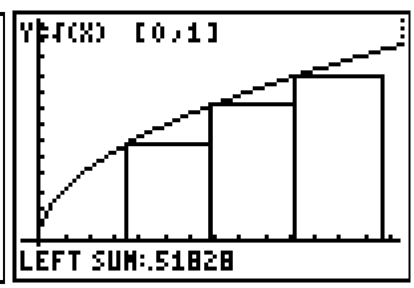


732. *Answer is B.*

The lower sum of $f(x) = \sqrt{x}$ on the interval $[\ 0,\ 1\ ]$ with four equal subintervals is

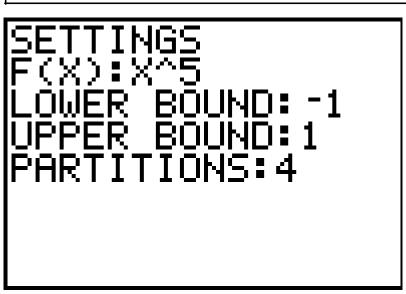


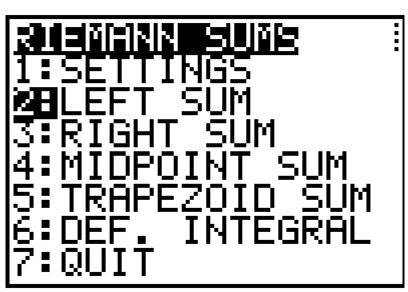


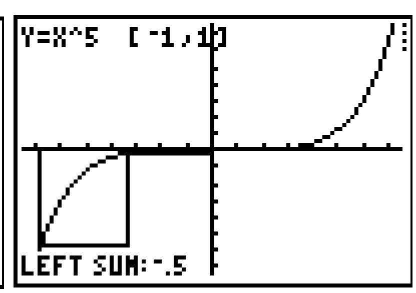


733. *Answer is B.*

The left-hand sum for $f(x) = x^5$ on the interval $[-1,\ 1\ ]$ using four equal subintervals is

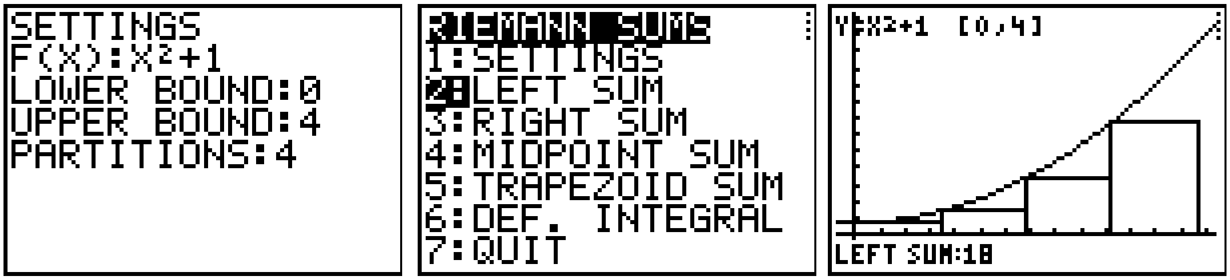






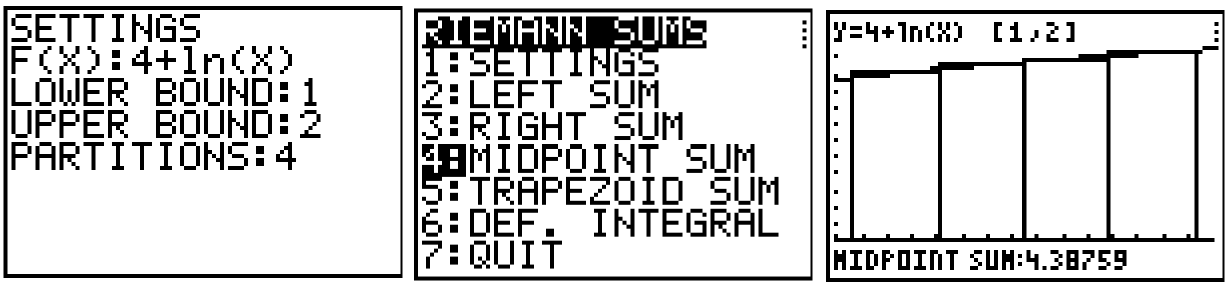
734. *Answer is A.*

Use a Riemann sum and four inscribed rectangles to approximate $\int_0^4 x^2 + 1\,dx =$



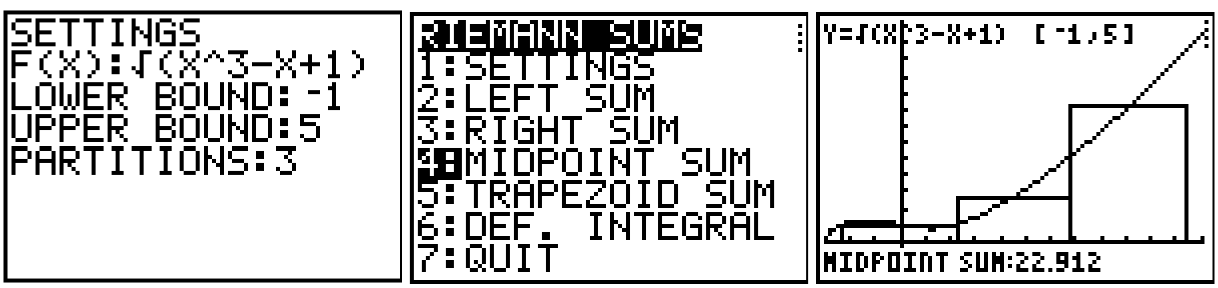
735. *Answer is B.*

If $\int_1^2 (4 + \ln x)\,dx$ is approximated by a midpoint Riemann sum with four subintervals of equal length, then the value is



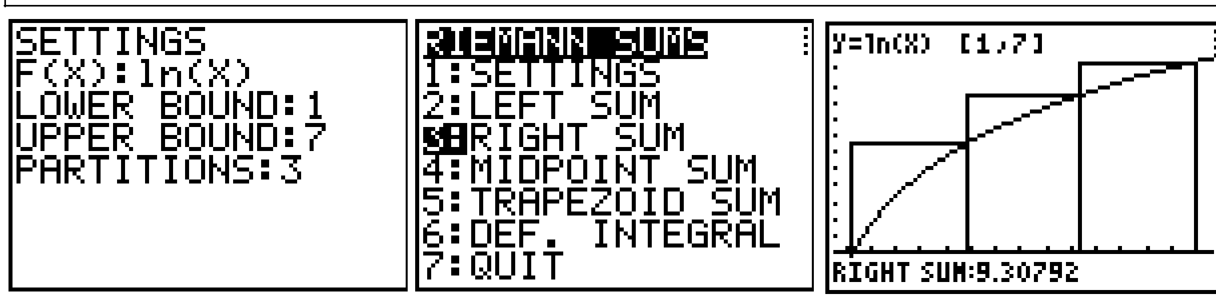
736. *Answer is B.*

When $\int_{-1}^5 \sqrt{x^3 - x + 1}\,dx$ is approximated by using the mid-points of **3** rectangles of equal width, then the approximation is nearest to



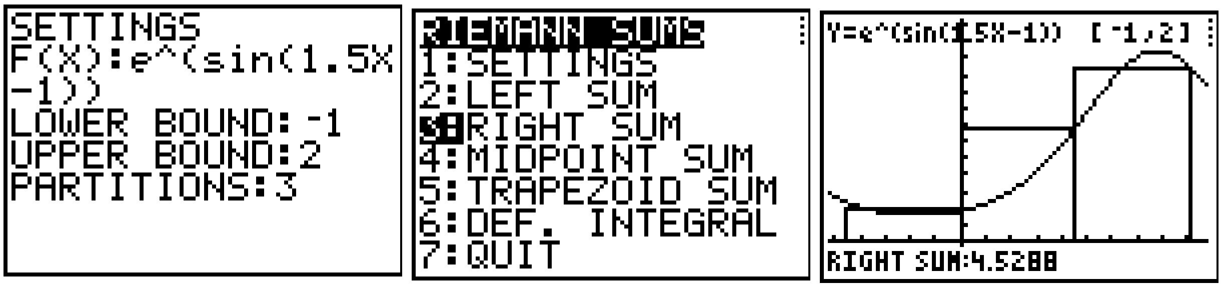
737. *Answer is C.*

If $\int_1^7 \ln x\,dx$ is approximated by **3** circumscribed rectangles of equal width on the x-axis, then the approximation is



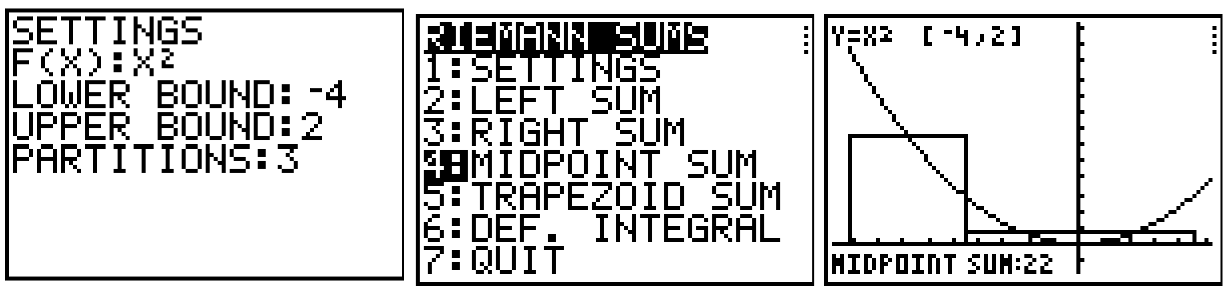
738. *Answer is C.*

An approximation for $\int_{-1}^{2} e^{\sin(1.5x-1)}dx$ using a right-hand Riemann sum with three equal subdivisions is nearest to

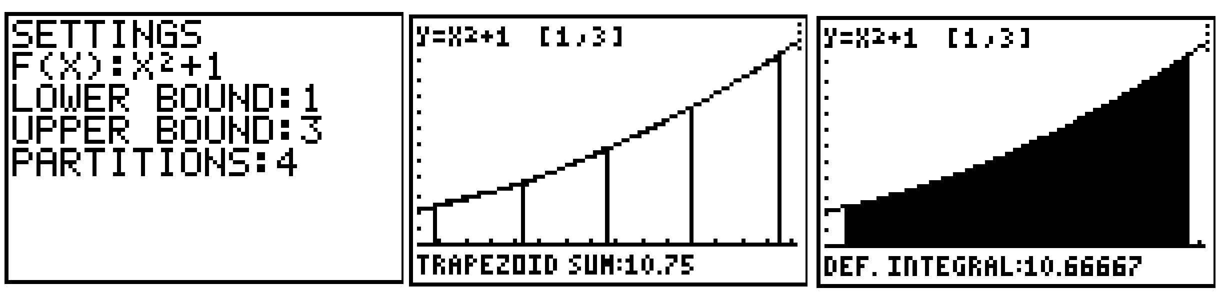


739. *Answer is C.*

The midpoint-sum approximation for $\int_{-4}^{2} x^2\,dx$ using three subintervals of equal length is



740. *Answer is C.*

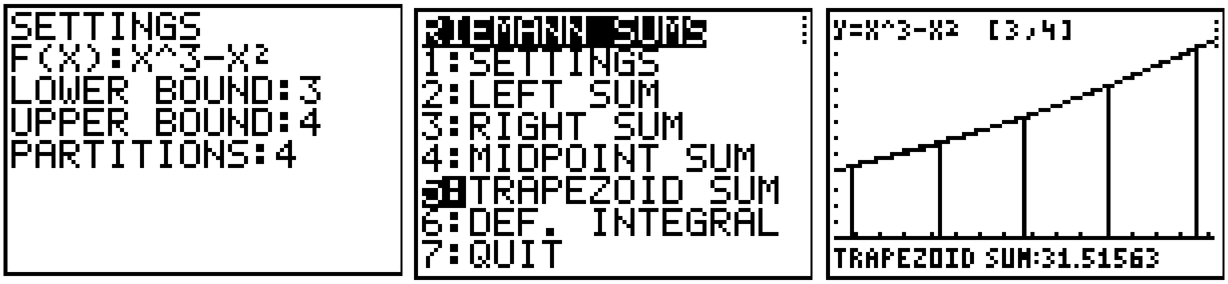


If the definite integral $\int_{1}^{3}(x^2+1)\,dx$ is approximated by using the Trapezoid Rule with $n=4$, the error is

$$\text{Error} = 10\tfrac{3}{4} - 10\tfrac{2}{3} = \tfrac{9}{12} - \tfrac{8}{12} = \boxed{\tfrac{1}{12}}$$

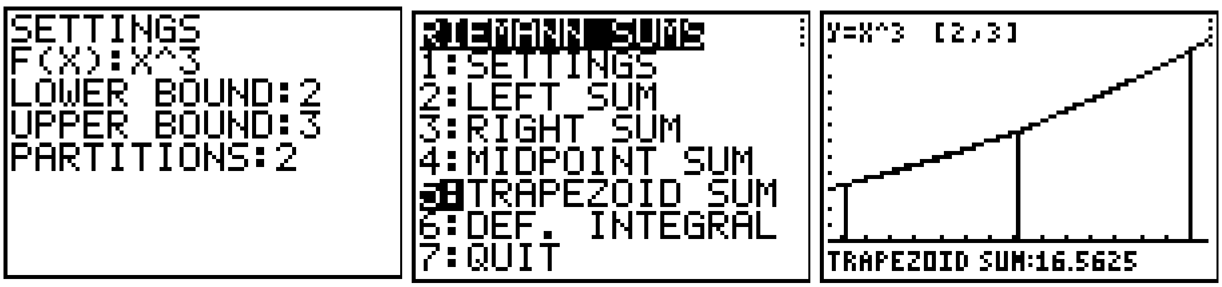
741. *Answer is D.*

Use the trapezoid rule with $n = 4$ to approximate the area between the curve $y = x^3 - x^2$ and the x-axis from, $x = 3$ to $x = 4$



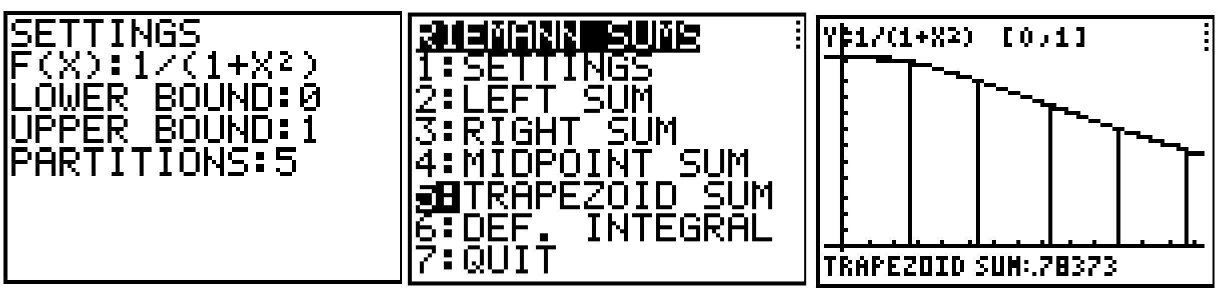
742. *Answer is E.*

If the trapezoidal rule is applied to $\int_2^3 x^3\,dx$ with $\Delta x = \frac{1}{2}$, the approximate value for the integral is



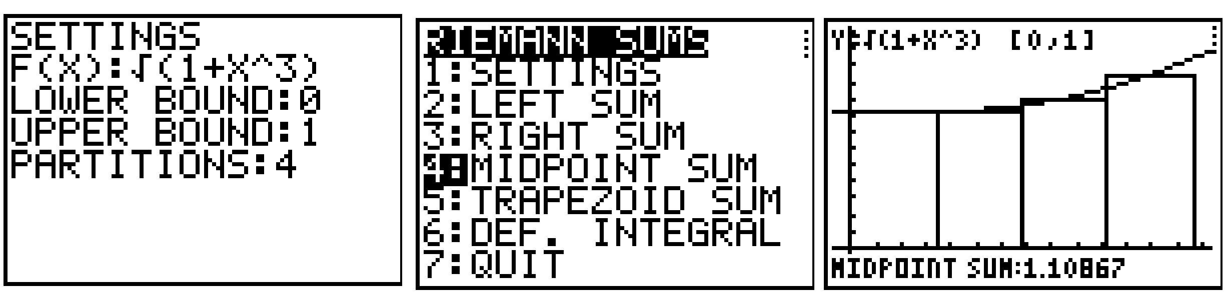
743. *Answer is A.*

If the Trapezoidal Rule is used with $n = 5$, then $\int_0^1 \frac{dx}{1+x^2}$ is equal, to three decimal places, to



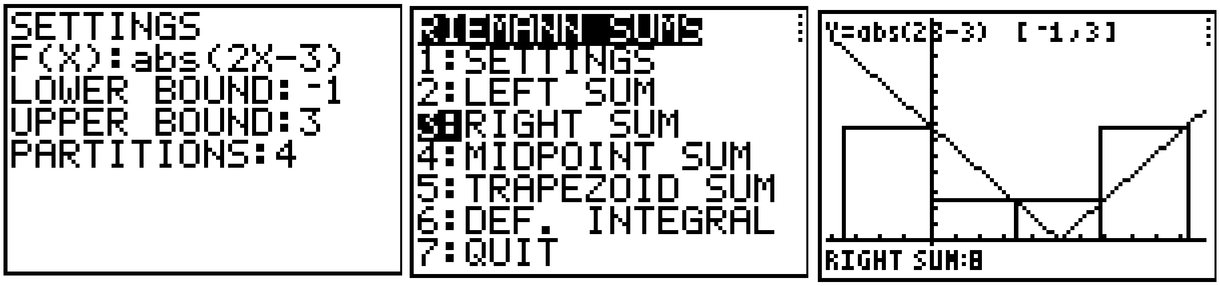
744. *Answer is B.*

If $\mathbf{M(4)}$ is used to approximate $\int_0^1 \sqrt{1+x^3}\,dx$, then the definite integral is equal, to two decimal places, to



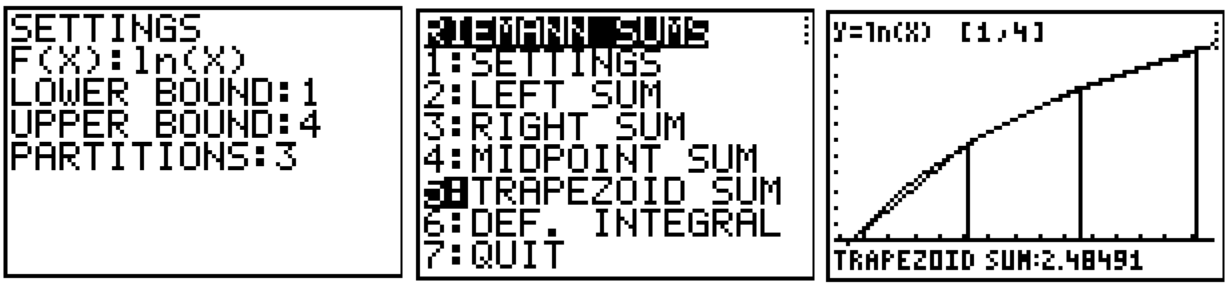
745. *Answer is D.*

Use a right-hand Riemann sum with **4** equal subdivisions to approximate the integral $\int_{-1}^{3} \lvert 2x-3 \rvert\, dx$



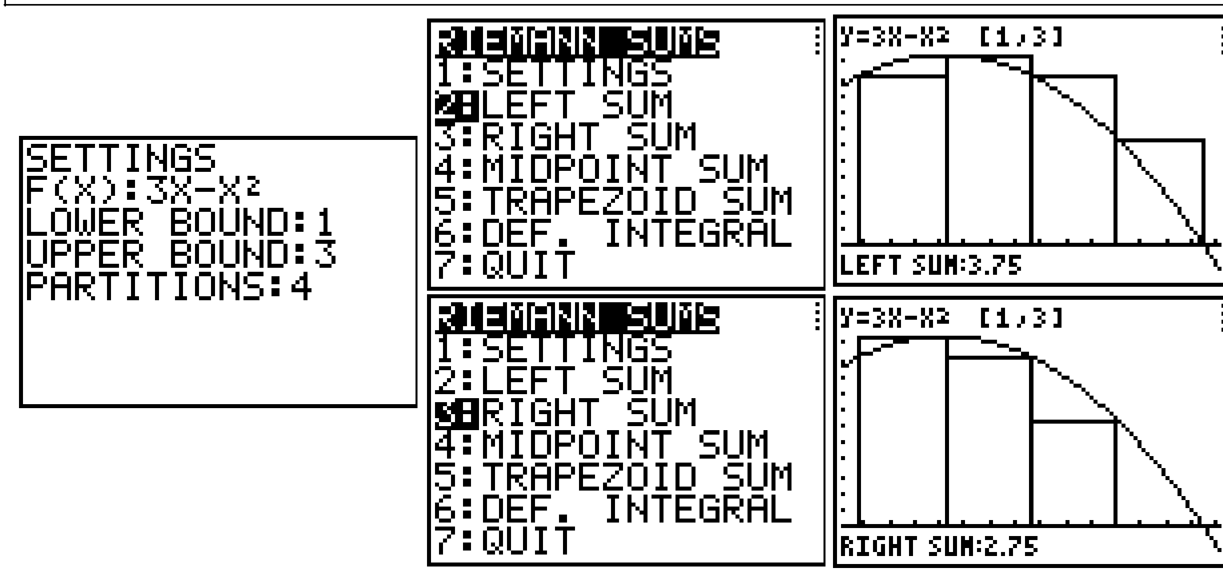
746. *Answer is D.*

Let ***R*** be the region in the first quadrant enclosed by the x-axis and the graph of $y = \ln x$ from $x = 1$ to $x = 4$ If the Trapezoid Rule with **3** subdivisions is used to approximate the area of ***R***, the approximation is



747. *Answer is C.*

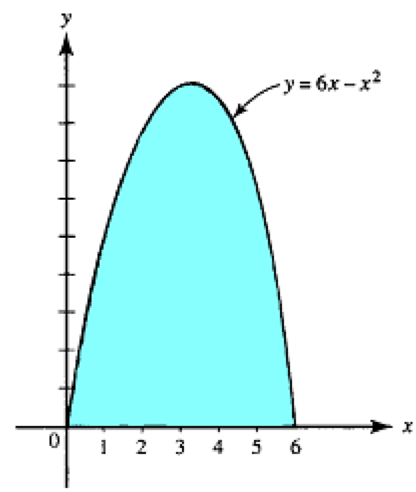
L and **R** are the left-hand and right-hand Riemann sums, respectively, of $f(x) = 3x - x^2$ on $[\,1,\ 3\,]$, divided into **4** subintervals of equal length. Which of the following statements is true ?

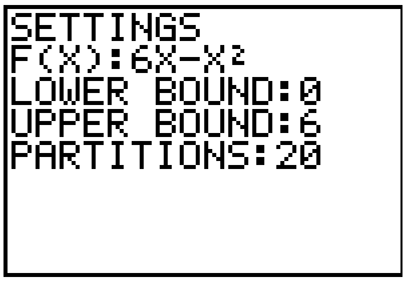


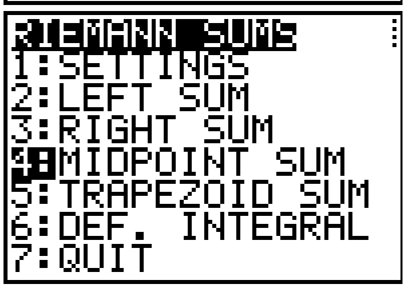
748. *Answer is D.*

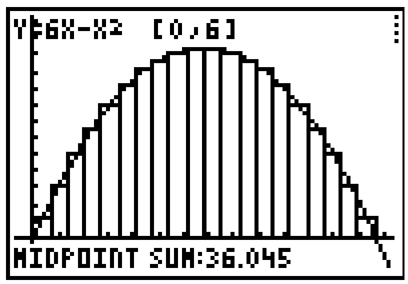
If we approximate the area of the shaded region by $\boldsymbol{M(20)}$ (that is, the midpoint sum with **20** subintervals), then the difference $M(20)-\int_0^6 f(x)\,dx$ is equal to

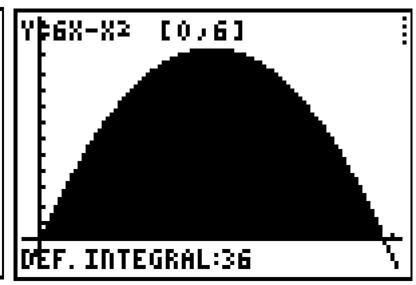
$$M(20)-\int_0^6 f(x)\,dx = 36.045-36=\boxed{0.045}$$







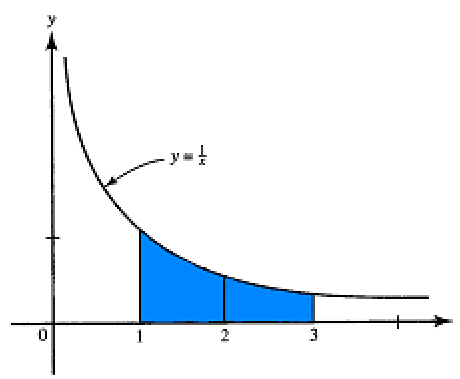


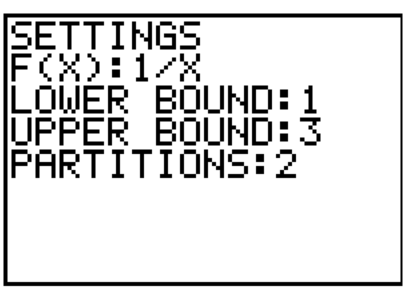


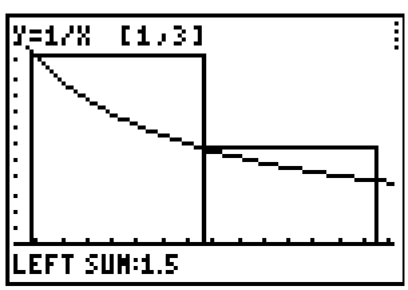
749. *Answer is E.*

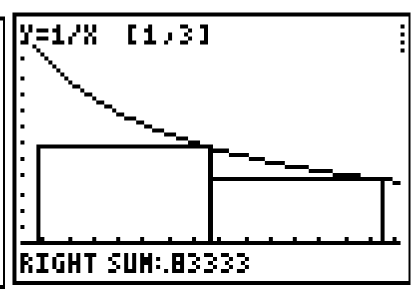
The area of the following shaded region is equal exactly to $\ln 3$. If we approximate $\ln 3$ using $\boldsymbol{L(2)}$ and $\boldsymbol{R(2)}$, which of the inequalities follows ?

$$R(2)=0.8333\overline{3}=\frac{5}{6}<\int_1^3 \ln x\,dx<\frac{3}{2}=1.5=L(2)$$









750. *Answer is B.*

If $\int_0^6 (x^2 - 2x + 2)\,dx$ is approximated by three ***inscribed*** rectangles of equal width on the x-axis, then the approximation is

$f(x) = x^2 - 2x + 2$

$f(0) = 2$

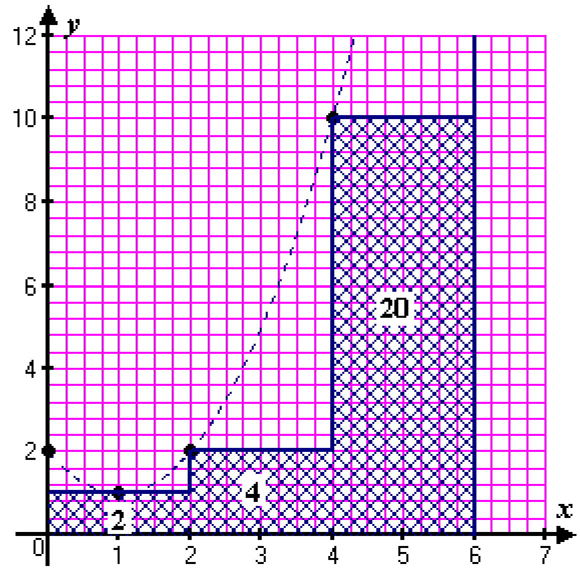
$f(1) = (1)^2 - 2(1) + 2 = \boxed{1}$ ← minimum point

$f(2) = (2)^2 - 2(2) + 2 = \boxed{2}$

$f(4) = (4)^2 - 2(4) + 2 = \boxed{10}$

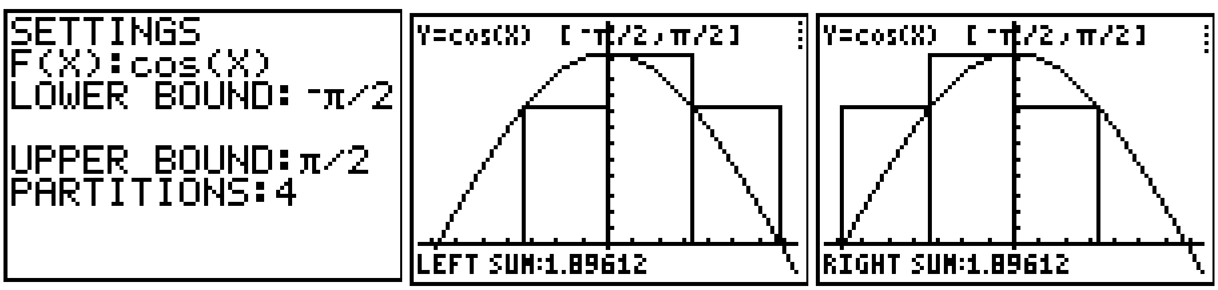
$f(6) = (6)^2 - 2(6) + 2 = 26$

Approximate area $= 2(1) + 2(2) + 2(10) = \boxed{26}$



751. *Answer is E.*

Which of the following is true for $f(x) = \cos x$ on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ using four equal subintervals ?



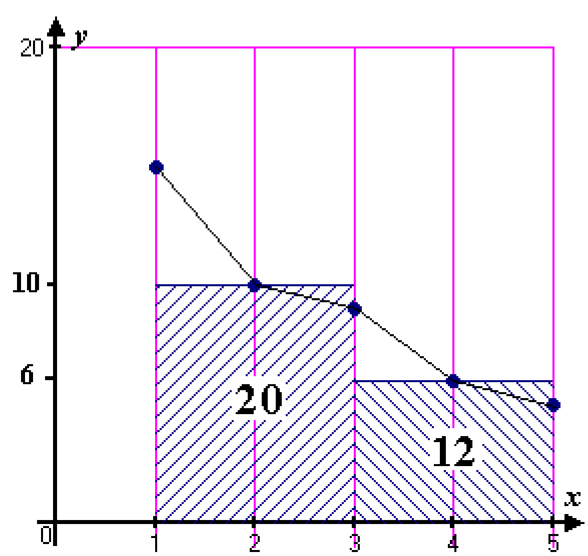
752. *Answer is D.*

The function f is continuous in the closed interval $[\ 1,\ 5\ ]$ and has values that are given in the table. If two subintervals of equal length are used, what is the midpoint Riemann sum approximation of $\int_1^5 f(x)\,dx =$

x	1	2	3	4	5
$f(x)$	15	10	9	6	5

Area by midpoint method graphically

$\int_1^5 f(x)dx \approx (20) + (12) = \boxed{32}$



753. *Answer is D.*

Difficulty = **0.62**

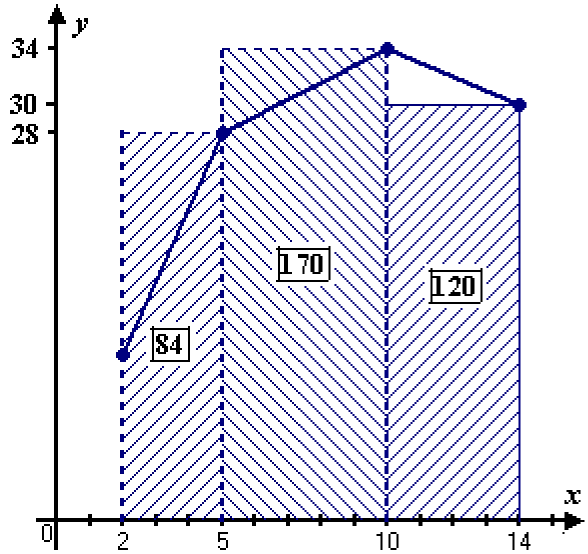
The function f is continuous on the closed interval $[\ 2,\ 14\ ]$ and has values as shown in the table. Using the subintervals $[\ 2,\ 5\ ]$, $[\ 5,\ 10\ ]$ and $[\ 10,\ 14\ ]$ what is the approximation of $\int_2^{14} f(x)\,dx$ found by using a right Riemann sum ?

x	2	5	10	14
$f(x)$	12	28	34	30

Area by right Riemann method graphically

$$\int_2^{14} f(x)dx \approx (84) + (170) + (120) = \boxed{374}$$

*** Note different interval widths



754. *Answer is B.*

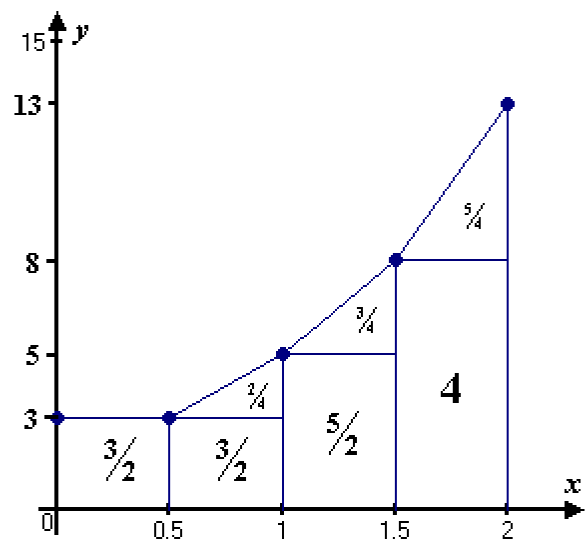
Difficulty = **0.47**

A table of values for a continuous function f is shown. If four equal subintervals of $[\ 0,\ 2\ ]$ are used, which of the following is the trapezoidal approximation of $\int_0^2 f(x)\ dx$

x	0.0	0.5	1.0	1.5	2.0
$f(x)$	3	3	5	8	13

Area by trapezoidal method graphically

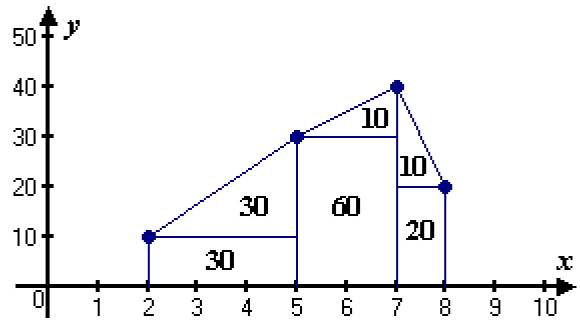
$$\int_0^2 f(x)dx \approx (3 + \tfrac{5}{2} + 4) + (\tfrac{2}{4} + \tfrac{3}{4} + \tfrac{5}{4}) = \boxed{12}$$



755. *Answer is C.*

Difficulty = **0.46**

The function f is continuous on the closed interval $[\,2,\ 8\,]$ and has values that are given in the table. Using the subintervals $[\,2,\ 5\,]$, $[\,5,\ 7\,]$ and $[\,7,\ 8\,]$ what is the trapezoidal approximation of $\int_2^8 f(x)\,dx$



x	2	5	7	8
$f(x)$	10	30	40	20

Area by trapezoidal method graphically

$$\int_2^8 f(x)\,dx \approx 60+70+30 = \boxed{160}$$

756. *Answer is D.*

For the function whose values are given in the table, $\int_0^6 f(x)\,dx$ is approximated by a Riemann Sum using the value at the midpoint of each of three intervals of width **2**

The approximation is

x	0	1	2	3	4	5	6
$f(x)$	0	0.25	0.48	0.68	0.84	0.95	1

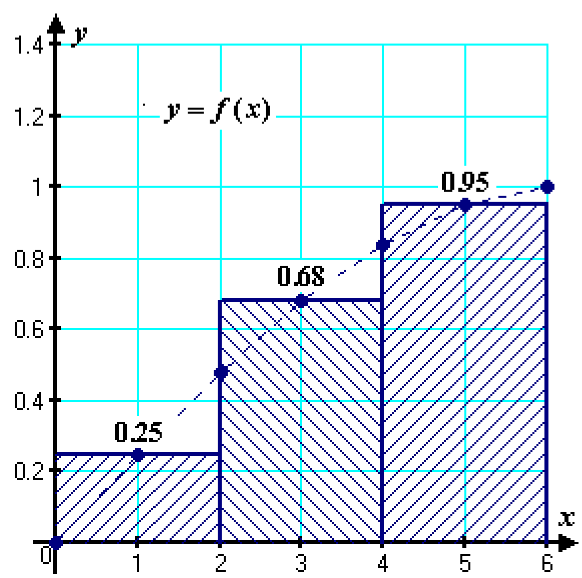
x	0	1	2	3	4	5	6
$f(x)$	0	0.25	0.48	0.68	0.84	0.95	1

$0-2 \rightarrow$ area $= 2(0.25) = 0.5$

$2-4 \rightarrow$ area $= 2(0.68) = 1.36$

$4-6 \rightarrow$ area $= 2(0.95) = 1.9$

$$\int_0^6 f(x)\,dx \approx \boxed{3.76}$$



757. *Answer is E.*

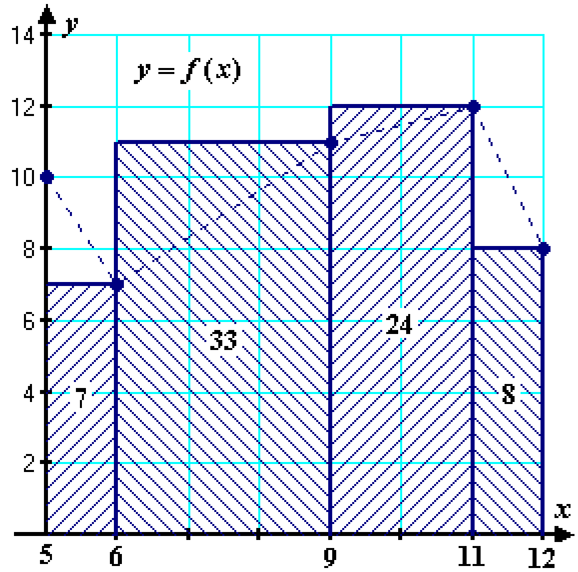
The function f is continuous on the closed interval $[\,5, 12\,]$ and differentiable on the open interval $(\,5, 12)$ and f has the values given the the table. Using the subintervals $[\,5, 6\,]$, $[\,6, 9\,]$, $[\,9, 11\,]$, and $[\,11, 12\,]$, what is the right-hand Riemann sum approximation to $\int_5^{12} f(x)\,dx$

x	5	6	9	11	12
$f(x)$	10	7	11	12	8

$\text{Area} = 1(7) + 3(11) + 2(12) + 1(8)$

$\text{Area} = 7 + 33 + 24 + 8 = 72$

$$\int_5^{12} f(x)\,dx \approx \boxed{72}$$



758. *Answer is E.*

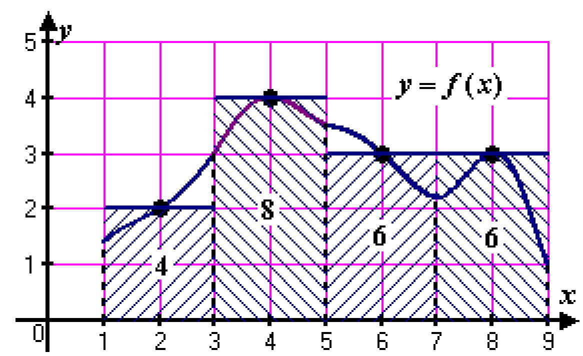
The graph of f over the interval $[\,1, 9\,]$ is shown in the figure. Using the data in the figure, find a midpoint approximation with 4 equal subdivisions for $\int_1^9 f(x)\,dx$

$\text{Area} = 2(2) + 2(4) + 2(3) + 2(3) = 24$

$$\int_1^9 f(x)\,dx \approx \boxed{24}$$

Midpoint approximation

$\rightarrow$ value of $f(x)$ at interval midpoints



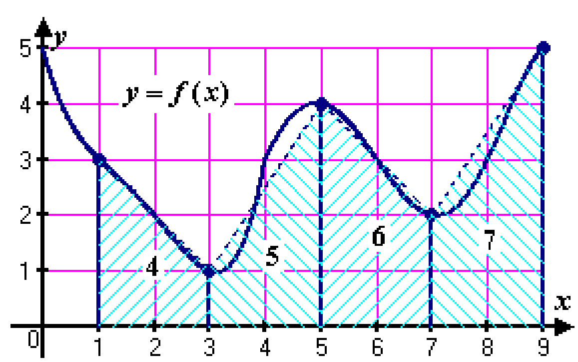
759. *Answer is B.*

Consider the function f whose graph is shown at the right. Use the Trapezoid Rule with $n = 4$ to estimate the value of $\int_1^9 f(x)\,dx$

$\text{Trapezoid} = w\left(\frac{h_1 + h_2}{2}\right)$

$\text{Area} = 2\left[\frac{(1+3)}{2} + \frac{(1+4)}{2} + \frac{(4+2)}{2} + \frac{(2+5)}{2}\right] = 22$

$$\int_1^9 f(x)\,dx \approx \boxed{22}$$



760. *Answer is C.*

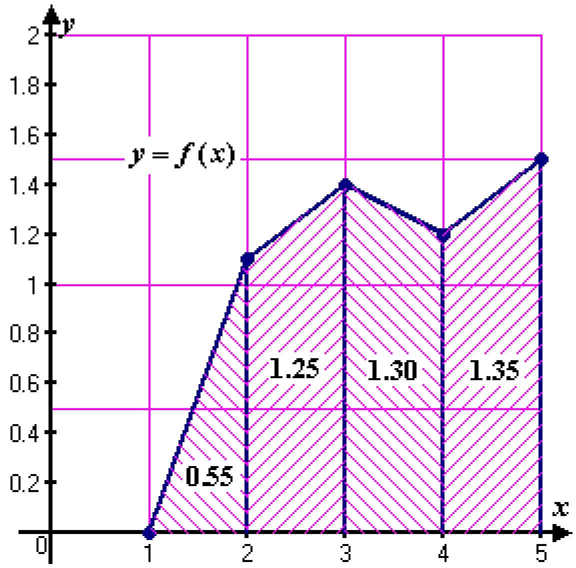
The following table lists the known values of a function f If the Trapezoid Rule is used to approximate $\int_1^5 f(x)\,dx$ the result is

x	1	2	3	4	5
$f(x)$	0	1.1	1.4	1.2	1.5

Trapezoid $= w\left(\frac{h_1+h_2}{2}\right)$

Area $= 1\left[\frac{(0+1.1)}{2}+\frac{(1.1+1.4)}{2}+\frac{(1.4+1.2)}{2}+\frac{(1.2+1.5)}{2}\right] = 4.45$

$$\int_1^5 f(x)\,dx \approx \boxed{4.45}$$



761. *Answer is B.*

The table contains values of a continuous function f at several values of x. Estimate $\int_2^5 f(x)\,dx$ using a trapezoidal approximation with three equal subintervals.

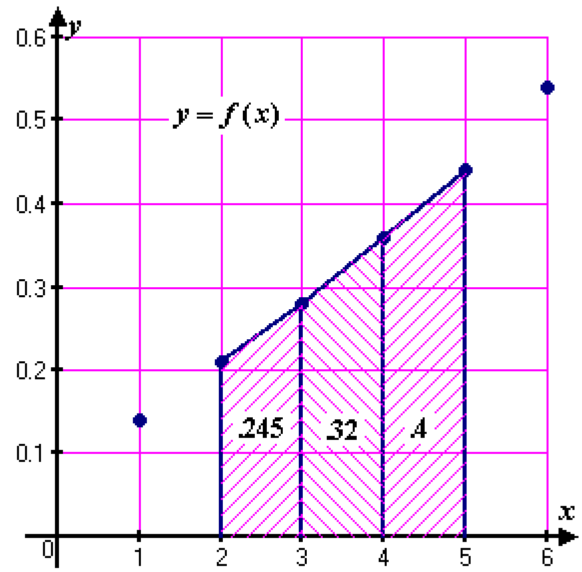
x	1	2	3	4	5	6
$f(x)$	0.14	0.21	0.28	0.36	0.44	0.54

Trapezoid $= w\frac{(h_1+h_2)}{2}$

Area $= \frac{0.21+0.28}{2}+\frac{0.28+0.36}{2}+\frac{0.36+0.44}{2}$

Area$= .245+.32+.4 = 0.965$

$$\int_2^5 f(x)\,dx \approx \boxed{0.965}$$



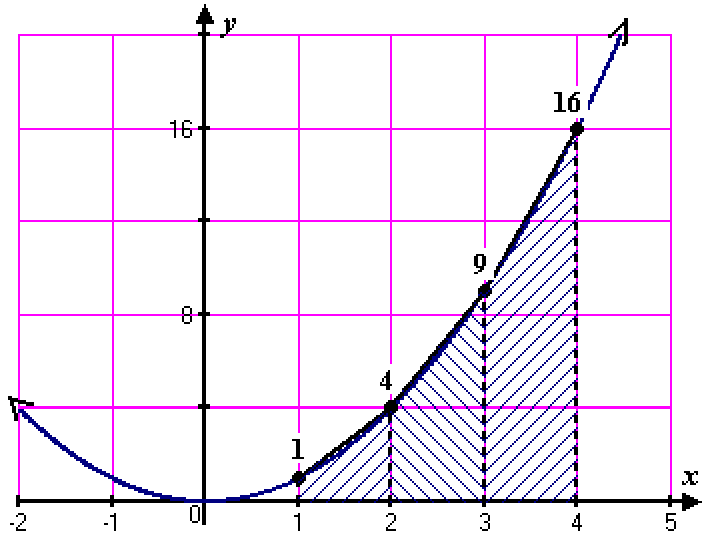
762. *Answer is C.* *NO calculator*

Use the Trapezoid Rule with $n = 3$ to approximate the area under $y = x^2$ from $x = 1$ to $x = 4$

$$\text{Trapezoid} = w\frac{(h_1 + h_2)}{2}$$

$$A = 1\left[\tfrac{(1+4)}{2} + \tfrac{(4+9)}{2} + \tfrac{(9+16)}{2}\right]$$

$$A = \frac{1}{2}[5 + 13 + 25] = \boxed{\frac{43}{2}}$$



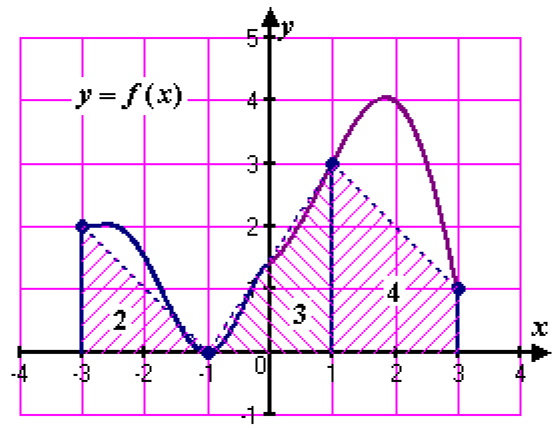
763. *Answer is C.*

The graph of f is shown at the right. Approximate $\int_{-3}^{3} f(x)\,dx$ using the Trapezoid Rule with **3** equal subdivisions.

$$\text{Trapezoid} = w\frac{(h_1 + h_2)}{2}$$

$$\text{Area} = 2\left[\frac{(2+0)}{2} + \frac{(0+3)}{2} + \frac{(3+1)}{2}\right] = 9$$

$$\int_{-3}^{3} f(x)\,dx \approx \boxed{9}$$



764. *Answer is E.*

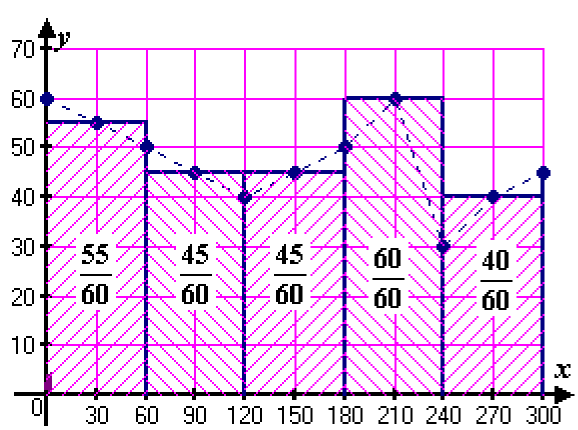
The table shows the velocity readings of a car taken every **30** seconds of a five-minute interval.

Time (sec)	**0**	**30**	**60**	**90**	**120**	**150**	**180**	**210**	**240**	**270**	**300**
Velocity (mph)	**60**	**55**	**50**	**45**	**40**	**45**	**50**	**60**	**30**	**40**	**45**

What is the approximate distance (in miles) traveled by the car during this five-minute interval, using a midpoint Riemann sum with **60**-second subintervals ?

$$D = rt$$

$$\text{Area} = \left[\frac{55 + 45 + 45 + 60 + 40}{60}\right] = 4.083 \text{ miles}$$



765. *Answer is D.*

Water drains continuously from a tank. The rate (in gallons per second) at which the water drains out is measured at the times (in seconds) given in the table. What is the trapezoidal approximation, based on all of the data in the table, for the total amount of water that has drained from the tank in the first ten seconds ?

Time (sec)	0	3	8	10
Rate (gal/sec)	16	10	6	5

$V = rt \qquad \text{Trapezoid} = w\dfrac{(h_1+h_2)}{2}$

$\text{Area} = 3\dfrac{(16+10)}{2} + 5\dfrac{(10+6)}{2} + 2\dfrac{(6+5)}{2}$

$\text{Area} = 39 + 40 + 11 = \boxed{90 \text{ gallons}}$

766. *Answer is B.*

Let f be a continuous function on $[\ 0,\ 6\ ]$ and have the selected values as shown in the table. If you use the subintervals $[\ 0,\ 2\ ]$, $[\ 2,\ 4\ ]$ and $[\ 4,\ 6\ ]$, what is the ***trapezoidal*** approximation of $\int_0^6 f(x)\,dx$

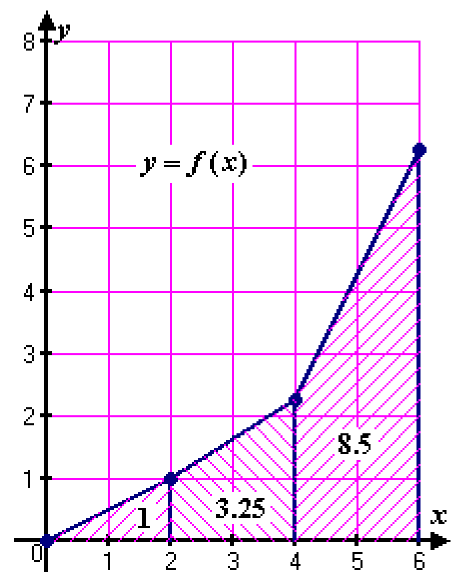
x	0	2	4	6
$f(x)$	0	1	2.25	6.25

$\text{Trapezoid} = w\dfrac{(h_1+h_2)}{2}$

$\text{Area} = 2\left[\dfrac{(0+1)}{2} + \dfrac{(1+2.25)}{2} + \dfrac{(2.25+6.25)}{2}\right]$

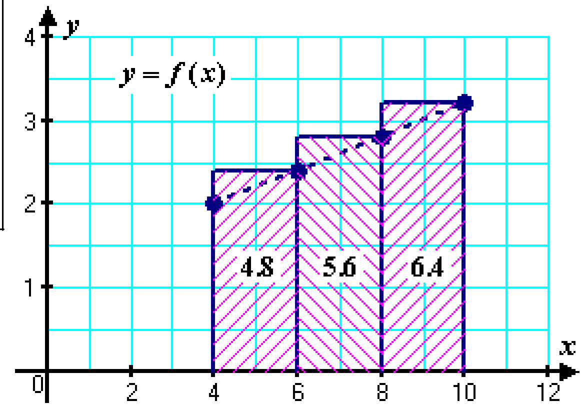
$\text{Area} = 1 + 3.25 + 8.5 = 12.75$

$\int_0^6 f(x)\,dx \approx \boxed{12.75}$



767. *Answer is D.*

Let f be a continuous function on $[\ 4,\ 10\ ]$ and has selected values as shown in the table. Using three ***right endpoint rectangles*** of equal length, what is the approximate value of $\int_4^{10} f(x)\,dx =$



x	4	6	8	10
$f(x)$	2	2.4	2.8	3.2

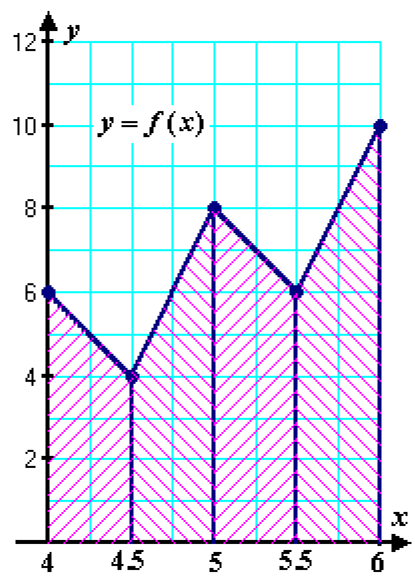
Area $= 2(2.4) + 2(2.8) + 2(3.2) = 16.8$

$$\int_4^{10} f(x)\,dx \approx \boxed{16.8}$$

Approximation by right endpoint rectangles

768. *Answer is B.*

The function f is continuous on the closed interval $[\ 4,\ 6\ ]$ and has values that are given in the table. Using four equal subintervals, what is the trapezoidal approximation to $\int_4^{6} f(x)\,dx$



x	4	4.5	5	5.5	6
$f(x)$	6	4	8	6	10

$$\text{Trapezoid} = w\frac{(h_1 + h_2)}{2}$$

$$\text{A} = \frac{1}{2}\left[\tfrac{(6+4)}{2} + \tfrac{(4+8)}{2} + \tfrac{(8+6)}{2} + \tfrac{(6+10)}{2}\right] = 13$$

$$\int_4^{6} f(x)\,dx \approx \boxed{13}$$

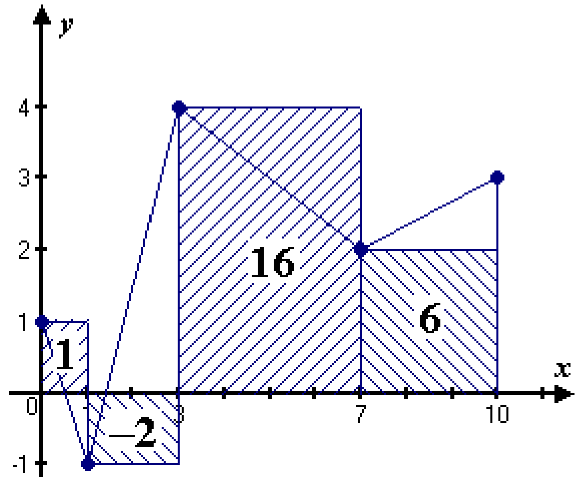
769. *Answer is D.*

The function f is continuous on the closed interval $[\ 0,\ 10\ ]$ and has the values given in the table. Using the subintervals $[\ 0,\ 1\ ]$, $[\ 1,\ 3\ ]$, $[\ 3,\ 7\ ]$, $[\ 7,\ 10\ ]$ what is the left Riemann sum estimate for $\int_0^{10} f(x)dx =$

x	0	1	3	7	10
$f(x)$	1	–1	4	2	3

Area by left Riemann method graphically

$$\int_0^{10} f(x)\,dx \approx (1)+(-2)+(16)+(6) = \boxed{21}$$



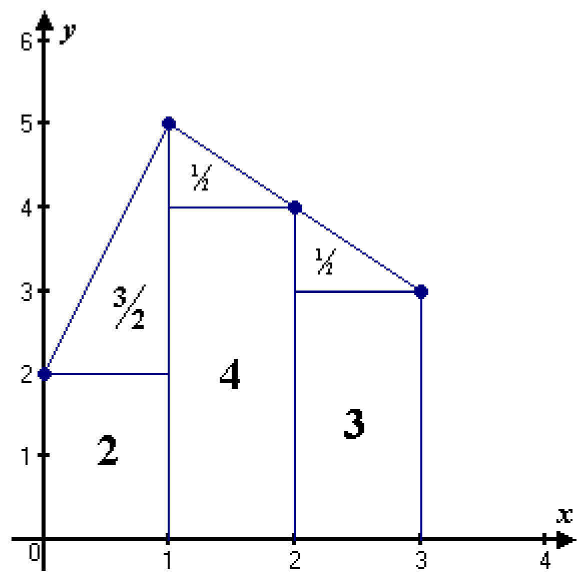
770. *Answer is B.*

The function f is continuous on the closed interval $[\ 0,\ 3\ ]$ and has values that are given in the table. Using the subintervals $[\ 0,\ 1\ ]$, $[\ 1,\ 2\ ]$ and $[\ 2,\ 3\ ]$, what is the trapezoidal approximation to $\int_0^{3} f(x)dx$

x	0	1	2	3
$f(x)$	2	5	4	3

Area by trapezoidal method graphically

$$\int_0^{3} f(x)\,dx \approx (2+\tfrac{3}{2})+(4+\tfrac{1}{2})+(3+\tfrac{1}{2}) = \boxed{11\tfrac{1}{2}}$$



771. *Answer is B.*

The table gives values for the velocity of a particle at certain times t between $t = 0$ and $t = 60$ The approximation of the total distance traveled by the particle during the time period $0 \le t \le 60$, computed using a right-hand Riemann sum with four equal subintervals, is

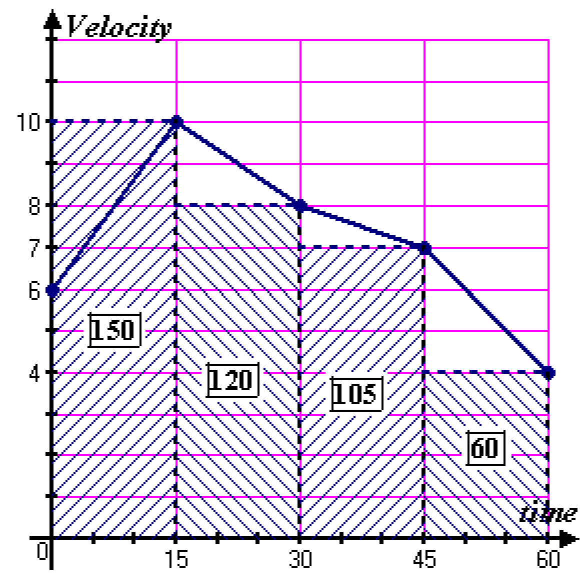
time (seconds)	**0**	**15**	**30**	**45**	**60**
velocity (feet per second)	**6**	**10**	**8**	**7**	**4**

$$\int f(t)\,dt = amount$$

Estimate by ***right*** Riemann sum

$$\int_0^{60} v(t)\,dx \approx 15(10) + 15(8)$$

$$+ 15(7) + 15(4) = \boxed{435}$$



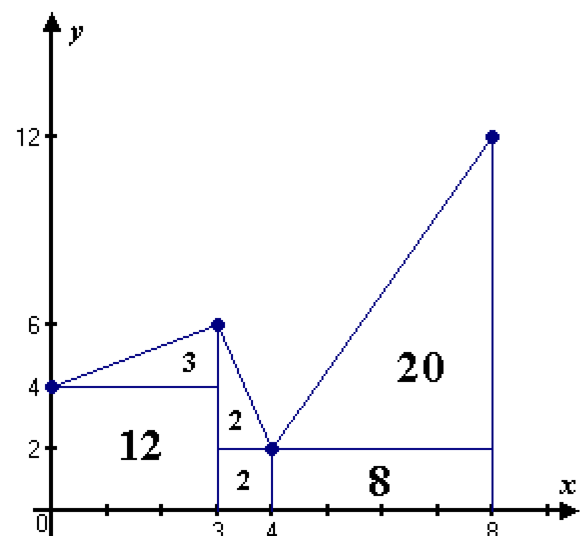
772. *Answer is C.*

The function f is continuous on the interval $[\ 0,\ 8\ ]$ and has values that are given in the table. Using the subintervals $[\ 0,\ 3\ ]$, $[\ 3,\ 4\ ]$, $[\ 4,\ 8\ ]$, what is the trapezoidal approximation of $\int_0^8 f(x)\,dx$

x	0	3	4	8
$f(x)$	4	6	2	12

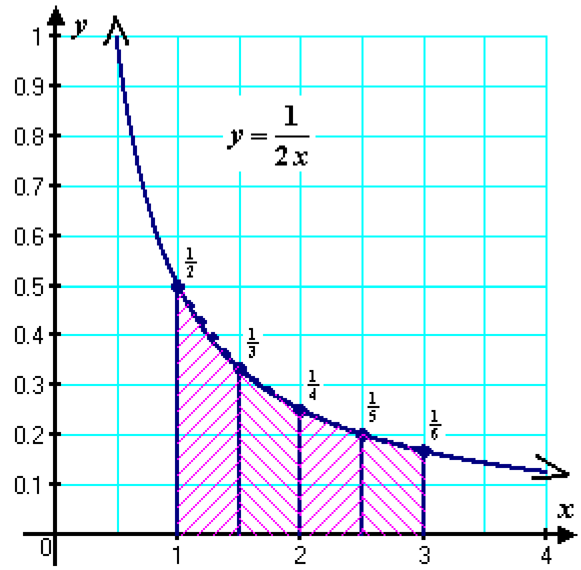
Area by trapezoidal method graphically

$$\int_0^8 f(x)\,dx \approx (12 + 3) + (2 + 2) + (8 + 20) = \boxed{47}$$



773. *Answer is B. This is a NO calculator question*

Use the trapezoidal method with **4** divisions to approximate the area of the region bounded by the graph of $y = \dfrac{1}{2x}$, the lines $x = 1$ and $x = 3$, and the x-axis



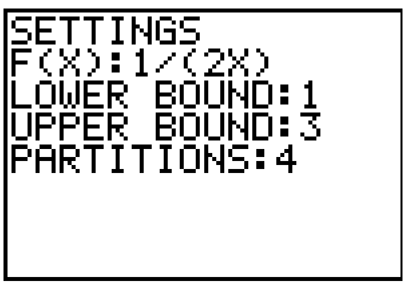
$$\text{Trapezoid} = w\frac{(h_1 + h_2)}{2}$$

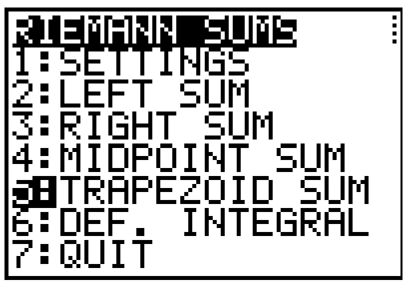
$$\mathbf{A} = \frac{1}{2}\left[\frac{\frac{1}{2}+\frac{1}{3}}{2} + \frac{\frac{1}{3}+\frac{1}{4}}{2} + \frac{\frac{1}{4}+\frac{1}{5}}{2} + \frac{\frac{1}{5}+\frac{1}{6}}{2}\right]$$

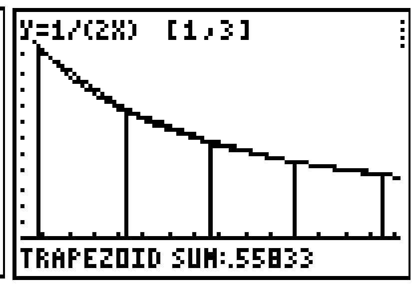
$$= \frac{1}{2}\left[\frac{5}{12} + \frac{7}{24} + \frac{9}{40} + \frac{11}{60}\right]$$

$$= \frac{1}{2}\left[\frac{50+35+27+22}{120}\right] = \frac{1}{2}\left[\frac{134}{120}\right]$$

$$= \boxed{\frac{67}{120}}$$







774. *Answer is B.*

What is the approximation of the area under the graph of $f(x) = \sqrt{1+x^3}$ using the trapezoidal sum with all the points in the partition $\{\ 1,\ \frac{5}{4},\ 2,\ 3\ \}$

$$\text{Trapezoid} = w\frac{(h_1 + h_2)}{2}$$

$$\mathbf{A} = \frac{1}{4}\left[\frac{\sqrt{2} + \sqrt{1+\frac{5}{4}^3}}{2}\right] + \frac{3}{4}\left[\frac{\sqrt{1+\frac{5}{4}^3} + 3}{2}\right] + 1\left[\frac{3+\sqrt{28}}{2}\right]$$

$$\mathbf{A} = \frac{1}{8}\left[\sqrt{2} + \sqrt{1+\frac{5}{4}^3}\right] + \frac{3}{8}\left[\sqrt{1+\frac{5}{4}^3} + 3\right] + \frac{1}{2}\left[3+\sqrt{28}\right]$$

$$\mathbf{A} = 0.391584931 + 1.769424707 + 4.145751311 \approx \boxed{6.306760949}$$

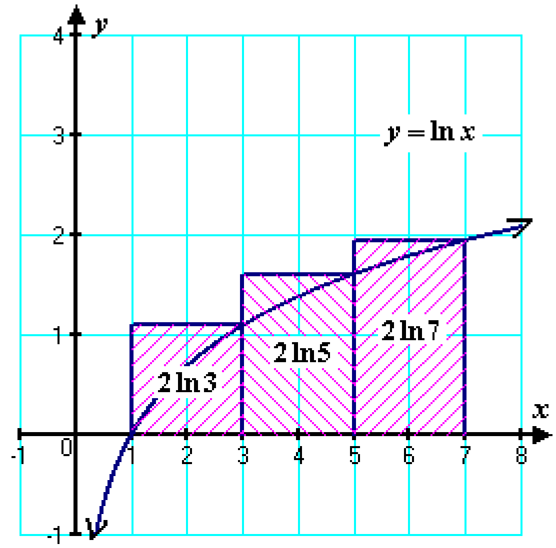
775. *Answer is C.*

If the definite integral $\int_1^7 \ln x\,dx$ is approximated by **3** ***circumscribed*** rectangles of equal width on the x-axis, then the approximation is

Sketch graph of situation $f(x) \to$ increasing concave down

$\underbrace{\textbf{\textit{circumscribed}}\text{ rectangles}}_{\text{right-hand Reimann sum}}$

$\mathbf{A = 2[\ln 3 + \ln 5 + \ln 7]}$



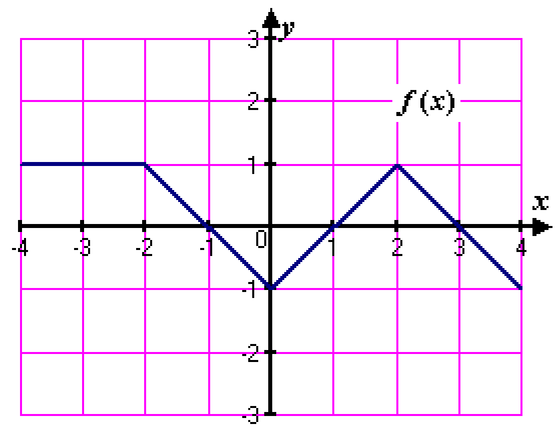
776. *Answer is D.*

The graph of f is shown at the right. Which of the following statements must be true ?

I. $f'(3) > f'(1)$

II. $\int_0^2 f(x)\,dx > f'(3.5)$

III. $\int_1^0 f(x)\,dx = \int_2^3 f(x)\,dx$



I. $f'(3) > f'(1)$ ☒ False $f'(3) = -1$ $f'(1) = 1$

II. $\int_0^2 f(x)\,dx > f'(3.5)$ ☑ True $\int_0^2 f(x)\,dx = 0$ $f'(3.5) = -1$

III. $\int_1^0 f(x)\,dx = \int_2^3 f(x)\,dx$ ☑ True $\int_1^0 f(x)\,dx = \frac{1}{2}$ $\int_2^3 f(x)\,dx = \frac{1}{2}$

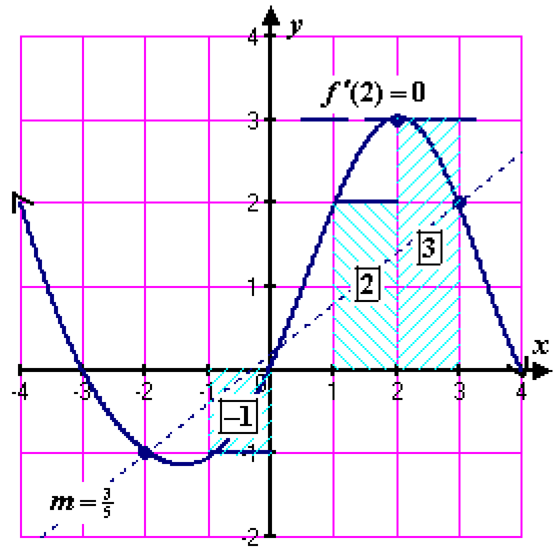
777. *Answer is C.*

The graph of a function f whose domain is the interval $[-4,\ 4]$ is shown in the figure. Which of the following statements are true ?

I. The average rate of change of f over the interval from $x=-2$ to $x=3$ is $\frac{1}{5}$

II. The slope of the tangent line at the point where $x=2$ is 0

III. The left-sum approximation of $\int_{-1}^{3} f(t)dt$ with 4 equal subdivisions is 4



I. The average rate of change of f over the interval from $x=-2$ to $x=3$ is $\frac{1}{5}$ ☒ False

Points $(-2,-1)$ and $(\ 3,\ 2)$ Average slope $=\dfrac{2-(-1)}{3-(-2)}=\dfrac{3}{5}$

II. The slope of the tangent line at the point where $x=2$ is 0 $f'(2)=0$ ☑ True

III. The left-sum approximation of $\int_{-1}^{3} f(t)dt$ with 4 equal subdivisions is 4

$\int_{-1}^{3} f(t)dt=-1+0+2+3=4$ ☑ True

778. *Answer is B.*

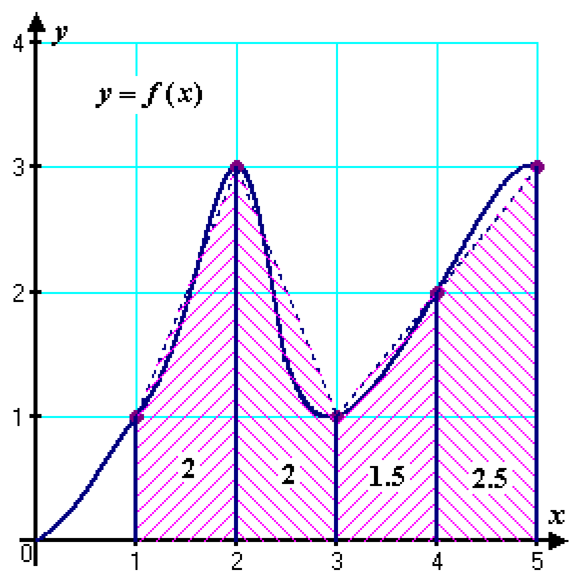
Use the Trapezoid Rule with $n=4$ to approximate the integral $\int_{1}^{5} f(x)\,dx$ for the function f whose graph is shown on the right.

Trapezoid $=w\dfrac{(h_1+h_2)}{2}$

$A=1\left[\frac{1+3}{2}+\frac{3+1}{2}+\frac{1+2}{2}+\frac{2+3}{2}\right]$

$A=\frac{1}{2}\left[1+3+3+1+1+2+2+3\right]=8$

$\int_{1}^{5} f(x)\,dx\approx\boxed{8}$



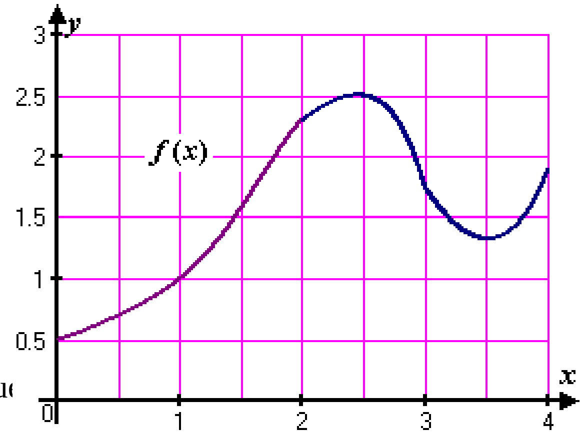
779. *Answer is E.*

A graph of the function is shown on the right.
Which of the following statements are true ?

I. $f(1) > f'(3)$

II. $\int_1^2 f(x)\,dx > f'(3.5)$

III. $\lim_{h\to 0}\dfrac{f(2+h)-f(2)}{h} > \dfrac{f(2.5)-f(2)}{2.5-2}$



I. $f(1) > f'(3)$ ☑ True

$f(1) = 1 \qquad f'(3) = negative$

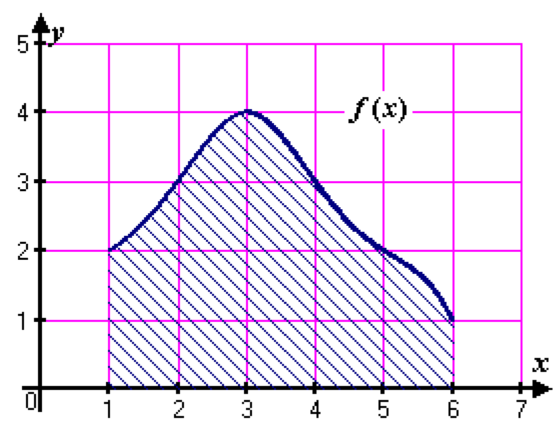
II. $\int_1^2 f(x)\,dx > f'(3.5)$ ☑ True $\int_1^2 f(x)\,dx \approx 6 \qquad f'(3.5) = 0$

III. $\lim_{h\to 0}\dfrac{f(2+h)-f(2)}{h} > \dfrac{f(2.5)-f(2)}{2.5-2}$ ☑ True slope is decreasing over interval $(2,\ 2.5)$

$$\underbrace{\lim_{h\to 0}\frac{f(2+h)-f(2)}{h} = f'(2)\approx 1}_{\text{instantaneous slope at } x=2} \qquad \underbrace{\frac{f(2.5)-f(2)}{2.5-2} = \frac{2.5-2.25}{2.5-2} = \frac{0.25}{0.5} \approx \frac{1}{2}}_{\text{average slope over interval } (2,\ 2.5)}$$

780. *Answer is C.*

The region shaded in the figure on the right is rotated about the x-axis. Using the Trapezoid Rule with **5** equal subdivisions, the approximate volume of the resulting solid is



$$\text{Trapezoid} = w\frac{(h_1+h_2)}{2}$$

$$\text{Volume} = \pi\int_1^6 \left(\frac{h_1+h_2}{2}\right)^2 dx$$

$$\text{Volume} \approx \pi\left[\left(\frac{2+3}{2}\right)^2 + \left(\frac{3+4}{2}\right)^2 + \left(\frac{4+3}{2}\right)^2 + \left(\frac{3+2}{2}\right)^2 + \left(\frac{2+1}{2}\right)^2\right]$$

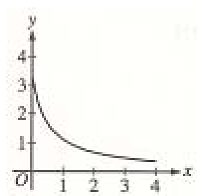
$$\approx \frac{\pi}{4}[25+49+49+25+9] \approx \frac{157\pi}{4} \approx 122 \approx \boxed{127}$$

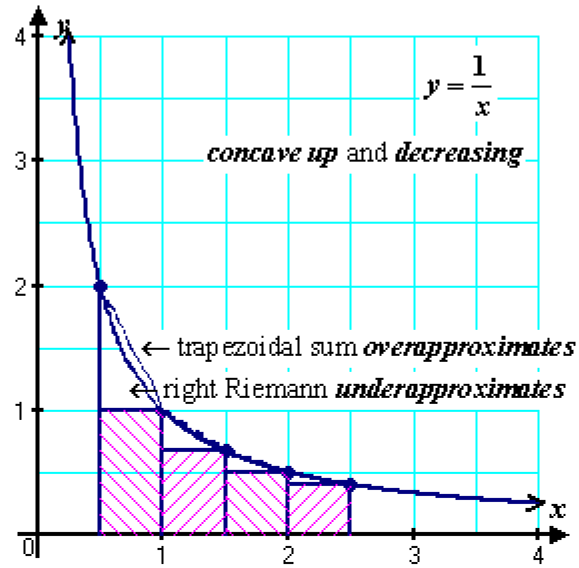
781. *Answer is A.*

Difficulty $= \mathbf{0.46}$

If a trapezoidal sum ***overapproximates*** $\int_0^4 f(x)\,dx$ and a right Riemann sum ***underapproximates*** $\int_0^4 f(x)\,dx$, which of the following could be the graph of $y = f(x)$

Any graph that is ***concave up*** and ***decreasing*** has this behavior





782. *Answer is D.*

Difficulty $= \mathbf{0.10}$

If the definite integral $\int_0^2 e^{x^2}\,dx$ is first approximated by using two <u>inscribed</u> rectangles of equal width and then approximated by using the trapezoidal rule with $n = 2$, the difference between the two approximations is

$y = e^{x^2}$ ← solid black curve

Hatched left ← **2** inscribed rectangles

(***under*** approximates value of integral)

Rectangles area $= \mathbf{1(1) + 1(}e\mathbf{) = 1 +} e$

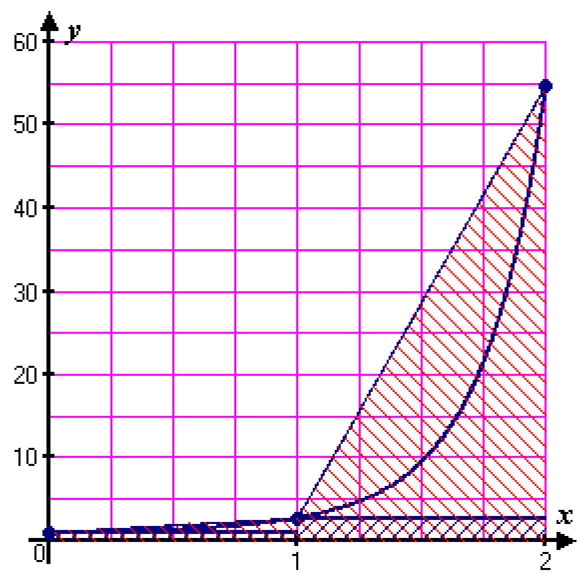
Hatched right ← extra added by trapezoid

(***over*** approximates value of integral)

Trapezoid area $= 1\left(\frac{1+e}{2}\right) + 1\left(\frac{e+e^4}{2}\right) = \frac{1+2e+e^4}{2}$

Difference of approximations is

$$\frac{1+2e+e^4}{2} - (1+e) \approx \boxed{26.799}$$



783.

Distance from the river's edge (feet)	0	8	14	22	24
Depth of the water (feet)	0	7	8	2	0

A scientist measures the depth of the Doe River at Picnic Point. The river is **24** feet wide at this location. The measurements are taken in a straight line perpendicular to the edge of the river. The data are shown in the table above. The velocity of the water at Picnic Point, in feet per minute, is modeled by $v(t) = 16 + 2\sin\left(\sqrt{t+10}\right)$ for $0 \le t \le 120$ minutes.

a) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the area of the cross section of the river at Picnic Point, in square feet. Show the computations that lead to your answer.

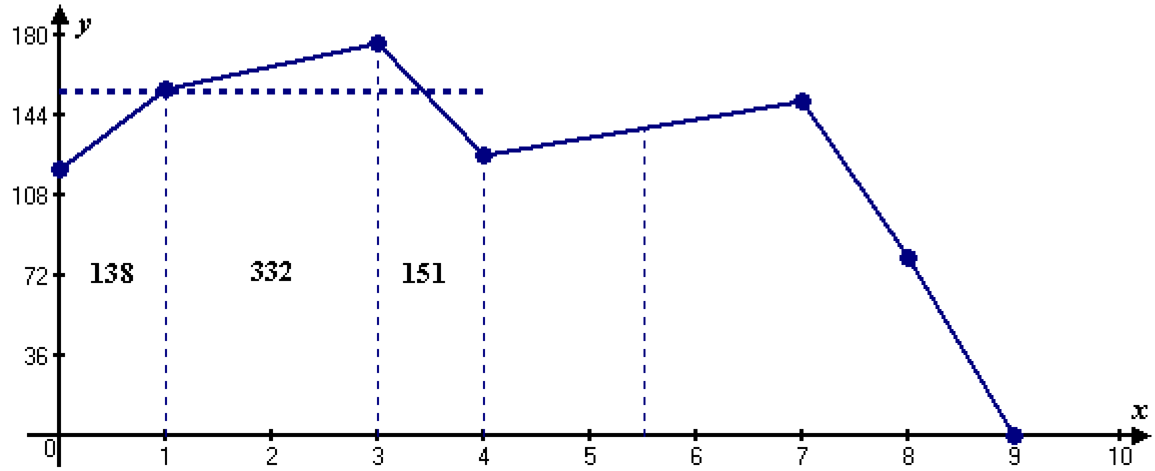
b) The volumetric flow at a location along the river is the product of the cross-sectional area and the velocity of the water at that location. Use you approximation from part (***a***) to estimate the average value of the volumetric flow at Picnic Point, in cubic feet per minute, from $t = 0$ to $t = 120$ minutes.

c) The scientist proposes the function f, given by $f(x) = 8\sin\left(\dfrac{\pi x}{24}\right)$, as a model for the depth of the water, in feet, at Picnic Point x feet from the river's edge. Find the area of the cross section of the river at Picnic Point based on this model.

d) Recall that the volumetric flow is the product of the cross-sectional area and the velocity of the water at a location. To prevent flooding, water must be diverted if the average value of the volumetric flow at Picnic Point exceeds **2100** cubic feet per minute for a **20**-minute period. Using your answer from part (***c***), find the average value of the volumetric flow during the time interval $40 \le t \le 60$ minutes. Does this value indicate that the water mnust be diverted ?

(a) $\dfrac{(0+7)}{2}\cdot 8 + \dfrac{(7+8)}{2}\cdot 6 + \dfrac{(8+2)}{2}\cdot 8 + \dfrac{(2+0)}{2}\cdot 2$ $= 115\text{ ft}^2$	1 : trapezoidal approximation
(b) $\dfrac{1}{120}\displaystyle\int_0^{120} 115v(t)\,dt$ $= 1807.169$ or $1807.170\ \text{ft}^3/\text{min}$	3 : { 1 : limits and average value constant; 1 : integrand; 1 : answer }
(c) $\displaystyle\int_0^{24} 8\sin\left(\frac{\pi x}{24}\right)dx = 122.230$ or $122.231\ \text{ft}^2$	2 : { 1 : integra1; 1 : answer }
(d) Let C be the cross-sectional area approximation from part (c). The average volumetric flow is $\dfrac{1}{20}\displaystyle\int_{40}^{60} C\cdot v(t)\,dt = 2181.912$ or $2181.913\ \text{ft}^3/\text{min}$. Yes, water must be diverted since the average volumetric flow for this 20-minute period exceeds $2100\ \text{ft}^3/\text{min}$.	3 : { 1 : volumetric flow integral; 1 : average volumetric flow; 1 : answer with reason }

784.



t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

Concert tickets went on sale at noon $(t = 0)$ and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0 \le t \le 9$. Values of $L(t)$ at various times t are shown in the table above.

(a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. $(t = 5.5)$. Show the computations that lead to your answer. Indicate units of measure.

(b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.

(c) For $0 \le t \le 9$, what is the fewest number of times at which $L'(t)$ must equal 0 ? Give a reason for your answer.

(d) The rate at which tickets were sold for $0 \le t \le 9$ is modeled by $r(t) = 550te^{-t/2}$ tickets per hour. Based on the model, how many tickets were sold by 3 P.M. $(t = 3)$, to the nearest whole number?

(a) $L'(5.5) \approx \dfrac{L(7) - L(4)}{7 - 4} = \dfrac{150 - 126}{3} = 8$ people per hour

$2: \begin{cases} 1: \text{estimate} \\ 1: \text{units} \end{cases}$

(b) The average number of people waiting in line during the first 4 hours is approximately

$$\frac{1}{4}\left(\frac{L(0)+L(1)}{2}(1-0) + \frac{L(1)+L(3)}{2}(3-1) + \frac{L(3)+L(4)}{2}(4-3)\right)$$

$= 155.25$ people

$2: \begin{cases} 1: \text{trapezoidal sum} \\ 1: \text{answer} \end{cases}$

(c) L is differentiable on $[0, 9]$ so the Mean Value Theorem implies $L'(t) > 0$ for some t in $(1, 3)$ and some t in $(4, 7)$. Similarly, $L'(t) < 0$ for some t in $(3, 4)$ and some t in $(7, 8)$. Then, since L' is continuous on $[0, 9]$, the Intermediate Value Theorem implies that $L'(t) = 0$ for at least three values of t in $[0, 9]$.

$3: \begin{cases} 1: \text{considers change in sign of } L' \\ 1: \text{analysis} \\ 1: \text{conclusion} \end{cases}$

OR

The continuity of L on $[1, 4]$ implies that L attains a maximum value there. Since $L(3) > L(1)$ and $L(3) > L(4)$, this maximum occurs on $(1, 4)$. Similarly, L attains a minimum on $(3, 7)$ and a maximum on $(4, 8)$. L is differentiable, so $L'(t) = 0$ at each relative extreme point on $(0, 9)$. Therefore $L'(t) = 0$ for at least three values of t in $[0, 9]$.

OR

$3: \begin{cases} 1: \text{considers relative extrema of } L \text{ on } (0, 9) \\ 1: \text{analysis} \\ 1: \text{conclusion} \end{cases}$

[*Note: There is a function L that satisfies the given conditions with $L'(t) = 0$ for exactly three values of t.*]

(d) $\int_0^3 r(t)\,dt = 972.784$

There were approximately 973 tickets sold by 3 P.M.

$2: \begin{cases} 1: \text{integrand} \\ 1: \text{limits and answer} \end{cases}$

785.

t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

Rocket A has positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t = 0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \le t \le 80$ seconds, as shown in the table above.

(a) Find the average acceleration of rocket A over the time interval $0 \le t \le 80$ seconds. Indicate units of measure.

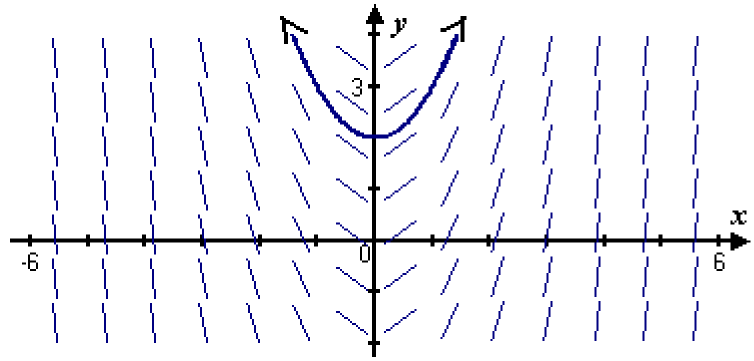
(b) Using correct units, explain the meaning of $\int_{10}^{70} v(t)\,dt$ in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t)\,dt$.

(c) Rocket B is launched upward with an acceleration of $a(t) = \frac{3}{\sqrt{t+1}}$ feet per second per second. At time $t = 0$ seconds, the initial height of the rocket is 0 feet, and the initial velocity is 2 feet per second. Which of the two rockets is traveling faster at time $t = 80$ seconds? Explain your answer.

(a) Average acceleration of rocket A is $\frac{v(80)-v(0)}{80-0} = \frac{49-5}{80} = \frac{11}{20}\ \text{ft}/\text{sec}^2$	1 : answer
(b) Since the velocity is positive, $\int_{10}^{70} v(t)\,dt$ represents the distance, in feet, traveled by rocket A from $t = 10$ seconds to $t = 70$ seconds. A midpoint Riemann sum is $20[v(20)+v(40)+v(60)]$ $= 20[22+35+44] = 2020$ ft	$3 : \begin{cases} 1 : \text{explanation} \\ 1 : \text{uses } v(20),\ v(40),\ v(60) \\ 1 : \text{value} \end{cases}$
(c) Let $v_B(t)$ be the velocity of rocket B at time t. $v_B(t) = \int \frac{3}{\sqrt{t+1}}\,dt = 6\sqrt{t+1} + C$ $2 = v_B(0) = 6 + C$ $v_B(t) = 6\sqrt{t+1} - 4$ $v_B(80) = 50 > 49 = v(80)$ Rocket B is traveling faster at time $t = 80$ seconds.	$4 : \begin{cases} 1 : 6\sqrt{t+1} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{finds } v_B(80), \text{ compares to } v(80), \\ \quad \text{and draws a conclusion} \end{cases}$
Units of ft/sec^2 in (a) and ft in (b)	1 : units in (a) and (b)

786. *Answer is B.*

Which equation has the slope field shown on the right ?



In ***each*** Vertical column slopes = same !!!

(y does not affect the derivative)

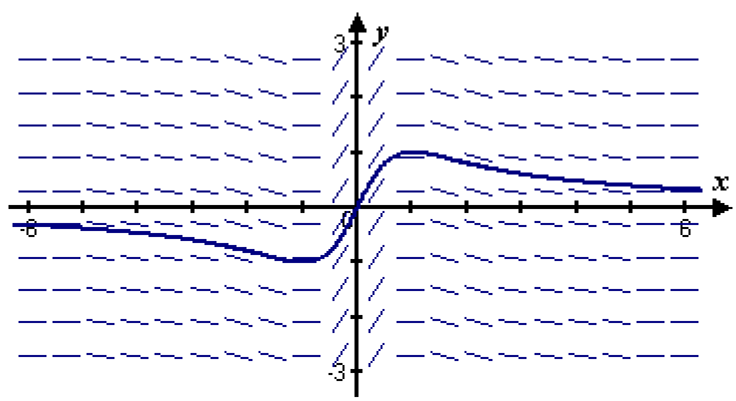
$$\frac{dy}{dx} = f(x)$$

$x = 0$ slope is horizontal

$y = x + 1$	$y' = 1$	☒ constant positive slope
$y = x^2 + 2$	$y' = 2x$	☑ slope ***negative*** if $x < 0$
$y = x^3 - 2$	$y' = 3x^2$	☒ positive slope only
$y = \ln(x+1)$	$y' = \frac{1}{x+1}$	☒ positive slope only
$y = 2e^x$	$y' = 2e^x$	☒ positive slope only

787. *Answer is B*

Which function could be a particular solution of the differential equation whose slope field is shown on the right ?



In ***each*** vertical column slopes = same !!!

(y does not affect the derivative)

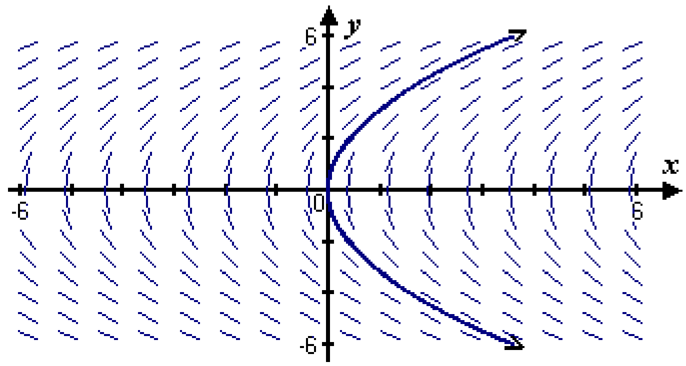
$$\frac{dy}{dx} = f(x)$$

At $x = \pm 1 \quad \rightarrow \quad \frac{dy}{dx} = 0$

$y = \cancel{x^3}$	$y' = 3x^2$	☒	$\leftarrow$ always positive
$y = \dfrac{2x}{x^2+1}$	$y' = \dfrac{2 - 2x^2}{(x^2+1)^2}$	☑	$\leftarrow \begin{cases} \text{odd function} \\ \text{horizontal asymptote} \end{cases}$
$y = \cancel{\dfrac{x^2}{x^2+1}}$	$y' = \dfrac{2x}{(x^2+1)^2}$	☒	$\leftarrow y' \neq 0$ at $x = \pm 1$
$y = \cancel{\sin x}$	$y' = \cos x$	☒	$\leftarrow y' \neq 0$ at $x = \pm 1$
$y = \cancel{e^{x^2}}$	$y' = -\dfrac{2x}{e^{x^2}}$	☒	$\leftarrow$ always negative for $x > 0$

788. *Answer is A.*

Which equation has the slope field shown on the right ?



In ***each*** horizontal row slopes = same !!!

(x does not affect the derivative)

$$\frac{dy}{dx} = f(y)$$

$y = 0$ slope is vertical (undefined)

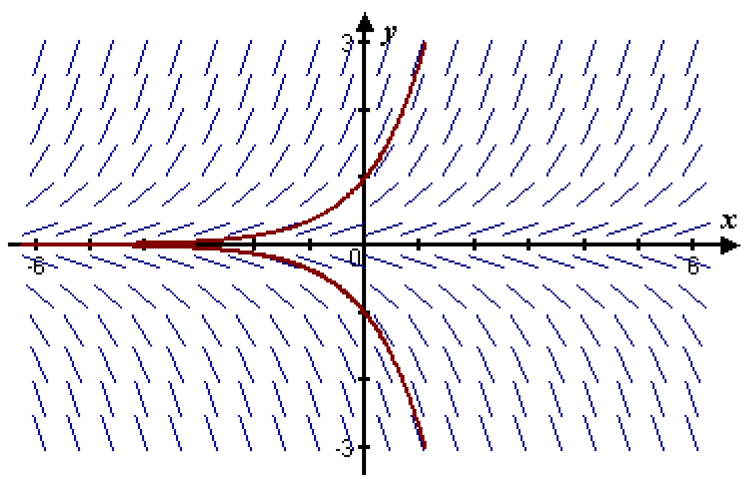
$\frac{dy}{dx} = \cancel{\frac{5}{x}}$ $\frac{dy}{dx} = \cancel{\frac{x}{y}}$ $\frac{dy}{dx} = \cancel{x * y}$ $\leftarrow \frac{dy}{dx} \neq f(y)$

$\frac{dy}{dx} = 5y$ ☒ $\leftarrow y = 0$ slope $= 0$

$\frac{dy}{dx} = \frac{5}{y}$ ☑ $\leftarrow \begin{cases} \text{no } x\text{-term} \\ y = 0 \leftarrow \text{vertical slope } \textbf{\textit{undefined}} \end{cases}$

789. *Answer is C.*

The slope field for $\frac{dy}{dx} = y$ shows that the solutions to the differential equation



$$\frac{dy}{y} = dx$$

$$\int \frac{dy}{y} = \int dx$$

$$\ln|y| = x + C$$

$$y = e^{x+C} = Ce^{x}$$

have y-intercept $(0,\ 1)$ ☒ true only if $C = 1$

have a positive y-intercept ☒ true only if $C > 0$

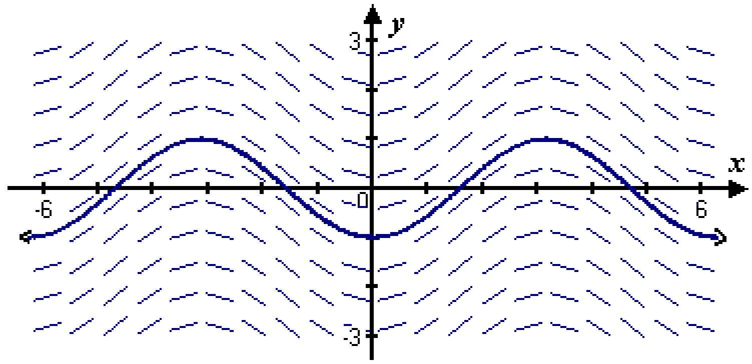
have a horizontal asymptote ☑ ***always*** true $y = 0$

are even functions ☒ false, no symmetry

are odd functions ☒ false, no symmetry

790. *Answer is B.*

The slope field for a differential equation $\dfrac{dy}{dx} = f(x, y)$ is given in the diagram. The slope field corresponds to which of the following differential equations ?



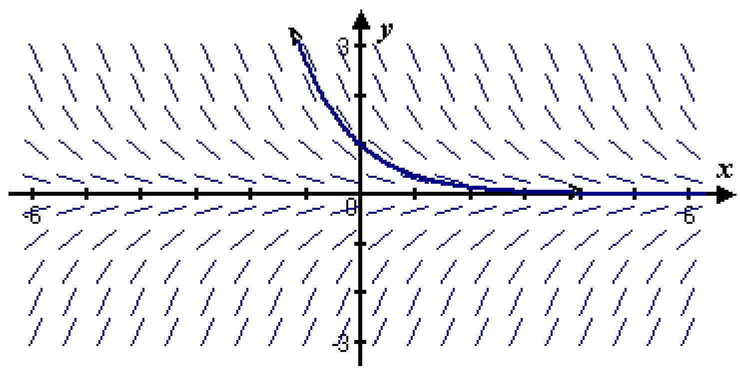
At $x = 0$ slope $= 0$ and concave ***up*** and no vertical asymptotes $\dfrac{dy}{dx} \neq \tan x \sec x$

$y \neq \sec x$

$$y = -\cos x + C$$
$$y' = -(-\sin x) = \sin x$$
$$\frac{dy}{dx} = \sin x \quad ☑$$

791. *Answer is C.*

The slope field for a differential equation $\dfrac{dy}{dx} = f(x, y)$ is given in the diagram. The slope field corresponds to which of the following differential equations ?



In ***each*** horizontal row slopes = same !!!

(x does not affect the derivative)

$$\frac{dy}{dx} = f(y)$$

Asymptote at $y = 0$

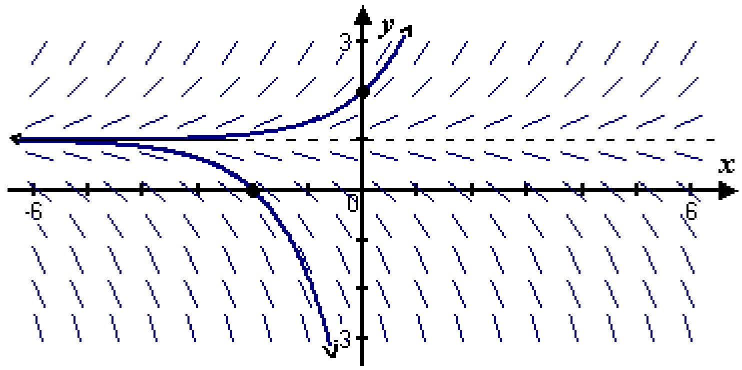
$\dfrac{dy}{dx} = \cancel{x+y}$ $\quad \dfrac{dy}{dx} = \cancel{e^x}$ $\quad \dfrac{dy}{dx} = \cancel{1-\ln x}$ $\quad \leftarrow \dfrac{dy}{dx} \neq f(y)$

$\dfrac{dy}{dx} = y^2$ ☒ $\leftarrow$ slope always positive

$\dfrac{dy}{dx} = -y$ ☑ $\leftarrow \begin{cases} \text{no } x\text{-term} \\ \text{negative slope above } y = 0 \\ \text{positive slope below } y = 0 \end{cases}$

792. *Answer is D.*

The slope field for a differential equation $\dfrac{dy}{dx} = f(x, y)$ is given in the diagram. Which of the following statements are true ?



I. A solution curve that contains the point $(\ 0,\ 2)$ also contains the point $(-2,\ 0)$

II. As y approaches 1, the rate of change of y approaces zero.

III. All solution curves for the differential equation have the same slope for a given value of y

Each horizontal row has constant slope

(x does not affect the derivative)

$$\frac{dy}{dx} = f(y)$$

Asymptote $y = 1$

Above $y = 1$ $(\ 0,\ 2)$ $\rightarrow$ $y = e^x + 1$

Below $y = 1$ $(-2,\ 0)$ $\rightarrow$ $y = -e^{x+2} + 1$

$$\frac{dy}{dx} = y - 1$$

$$\int \frac{dy}{y-1} = \int dx$$

$$\ln|y-1| = x + C$$

$$|y-1| = e^{x+C}$$

$$y = Ke^x + 1 \quad (\text{asymptote } y = 1\ )$$

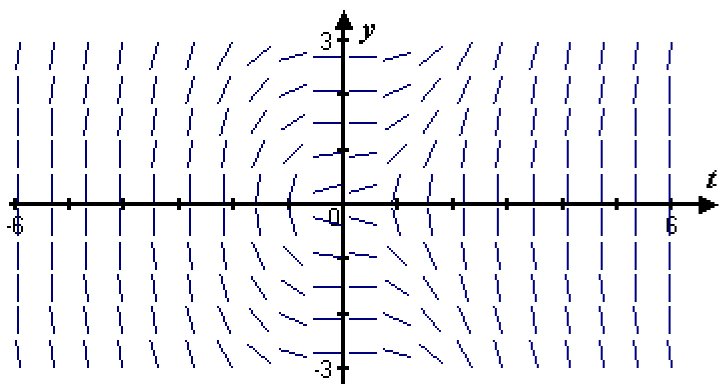
I. A solution curve that contains the point $(\ 0,\ 2)$ also contains the point $(-2,\ 0)$ ☒ impossible

II. As y approaches 1, the rate of change of y approaces zero. ☑ true

III. All solution curves for the differential equation have the same slope for a given value of y ☑

793. *Answer is A.*

On the $\underbrace{\text{positive } y\text{-axis}}_{t=0}$, the slope field for the differential equation $\dfrac{dy}{dt} = \dfrac{t^2}{y}$ has

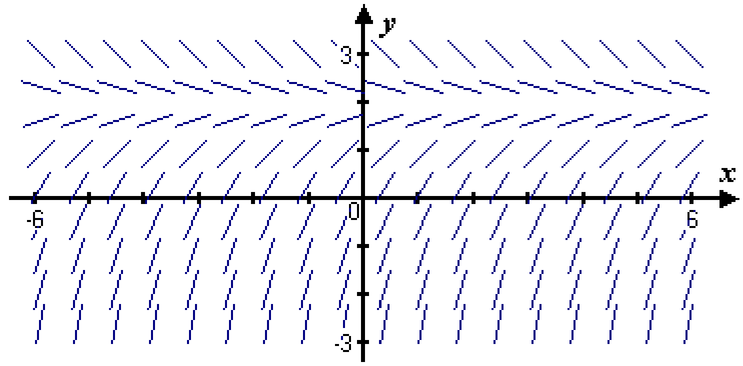


$$\frac{dy}{dt} = \frac{0^2}{y} = 0$$

(horizontal tangents/segments)

794. *Answer is D.*

The slope field for a differential equation $\frac{dy}{dx} = f(x, y)$ is given in the diagram. The slope field corresponds to which of the following differential equations ?



Each horizontal row has constant slope (x does not affect the derivative)

$$\frac{dy}{dx} = f(y)$$

Asymptote $y = 2$

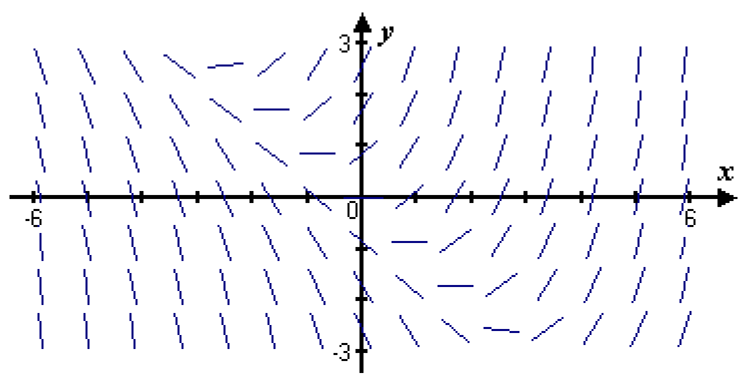
$\frac{dy}{dx} = \cancel{2 - \ln x}$ $\frac{dy}{dx} = \cancel{2 - e^{-x}}$ $\frac{dy}{dx} = \cancel{x^2}$ $\leftarrow \frac{dy}{dx} \neq f(y)$

$\frac{dy}{dx} = y - 2y^2$ ☒ $\leftarrow$ if $y < 0$ slope negative

$\frac{dy}{dx} = 2 - y$ ☑ $\left\{ \begin{array}{l} \text{asymptote at } y = 2 \\ \textbf{\textit{negative}} \text{ slope above } y = 2 \\ \textbf{\textit{positive}} \text{ slope below } y = 2 \end{array} \right.$

795. *Answer is A.*

The slope field for a differential equation $\frac{dy}{dx} = f(x, y)$ is given in the diagram. The slope field corresponds to which of the following differential equations ?



$$\frac{dy}{dx} = f(x, y)$$

Line $y = -x$ makes $\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \cancel{-y}$ $\frac{dy}{dx} = \cancel{y - \frac{1}{2}y^2}$ $\frac{dy}{dx} = \cancel{y^2}$ $\leftarrow \frac{dy}{dx} \neq f(x, y)$

$\frac{dy}{dx} = x^2 + y^2$ ☒ $\leftarrow$ graphs slope always ≥ 0

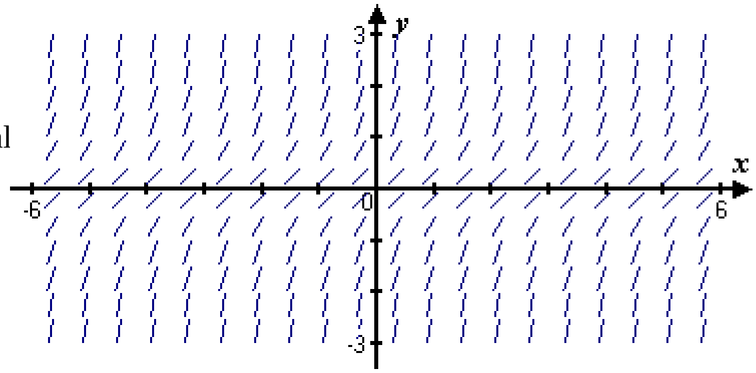
$\frac{dy}{dx} = x + y$ ☑ $\leftarrow \frac{dy}{dx} = 0$ along line $y = -x$

796. *Answer is E.*

797. *Answer is B.*

798. *Answer is D.*

Which of the following differential equations could be represented by this slope field ?



In ***each*** horizontal row slopes = same !!!

(x does not affect the derivative)

$$\frac{dy}{dx} = f(y)$$

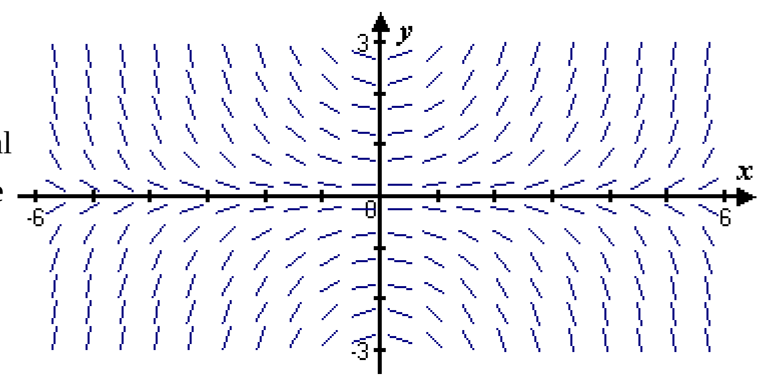
$\frac{dy}{dx} = \cancel{x^3+1}$ $\quad \frac{dy}{dx} = \cancel{\tan x}$ $\quad \frac{dy}{dx} = \cancel{\frac{1}{1+x^2}}$ $\leftarrow \frac{dy}{dx} \neq f(y)$

$\frac{dy}{dx} = \cancel{\tan^{-1} y}$ ☒ $\leftarrow$ ***not*** always positive

$\frac{dy}{dx} = 1 + y^2$ ☑ $\leftarrow \left\{ \begin{array}{l} \text{no } x\text{-term} \\ \text{slope always positive} \end{array} \right.$

799. *Answer is A.*

Which of the following differential equations corresponds to the slope field shown in the diagram ?



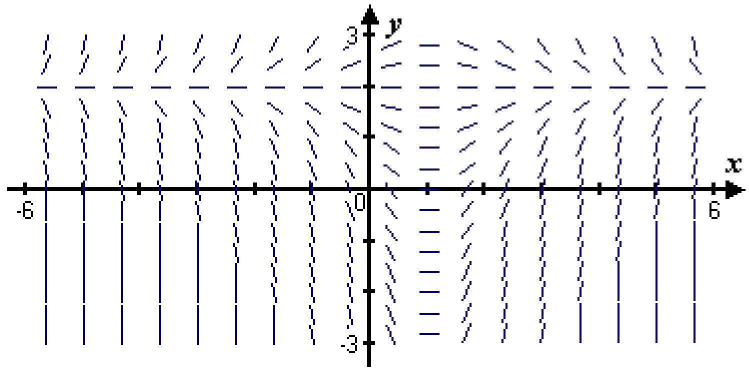
Asymptote $y = 0$ makes $\frac{dy}{dx} = 0$

When $x = 0$ makes $\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{xy}{2}$ Obvious only one that fits !!!

800. *Answer is C.*

The graph is a slope field for which of the following differential equations ?



Asymptote $y = 2$ makes $\frac{dy}{dx} = 0$

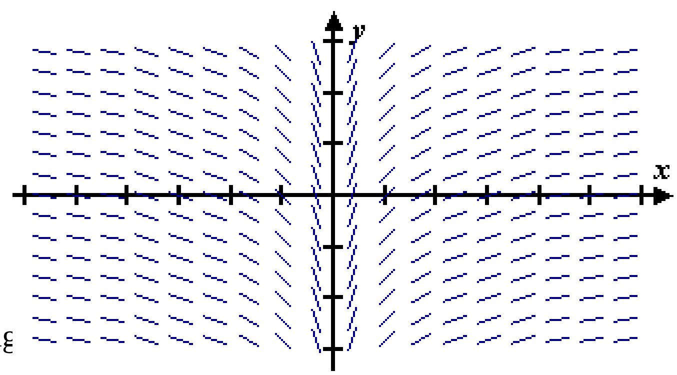
When $x = 1$ makes $\frac{dy}{dx} = 0$

$\frac{dy}{dx} = (1 - x)(y - 2)$ Obvious only one that fits !!!

801. *Answer is E.*

802. *Answer is A.*

The slope field matches which differential equation ?



$\frac{dy}{dx} = \textbf{\textit{undefined}}$ when $x = 0$

Value of y does not affect derivative

$\frac{dy}{dx} = \frac{1}{x}$ ☑ no y, undefined at x=0, chang

$\frac{dy}{dx} = \frac{1}{x^2}$ ☒ always positive

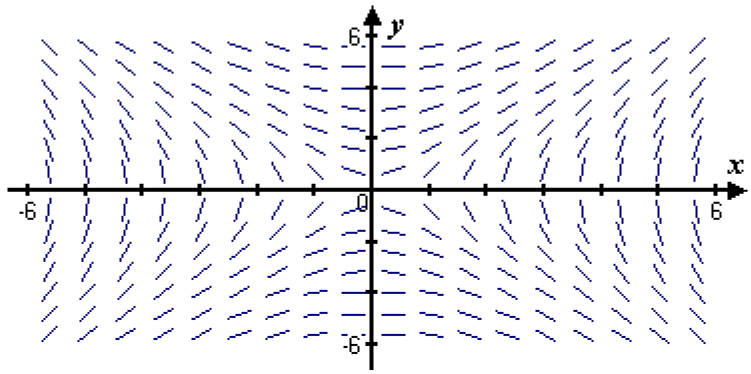
$\frac{dy}{dx} = \frac{y}{x}$ ☒ y does not affect derivative

$\frac{dy}{dx} = \frac{\ln x}{x}$ ☒ x cannot be negative

$\frac{dy}{dx} = \frac{\sin x}{x}$ ☒ slope is never undefined

803. *Answer is C.*

Which of the following equations has the slope field shown ?



Slope changes along rows/columns

(slope depends on both x and y)

$\dfrac{dy}{dx} = f(x, y)$ ☑

$y = 0$ $\leftarrow$ vertical slope (undefined)

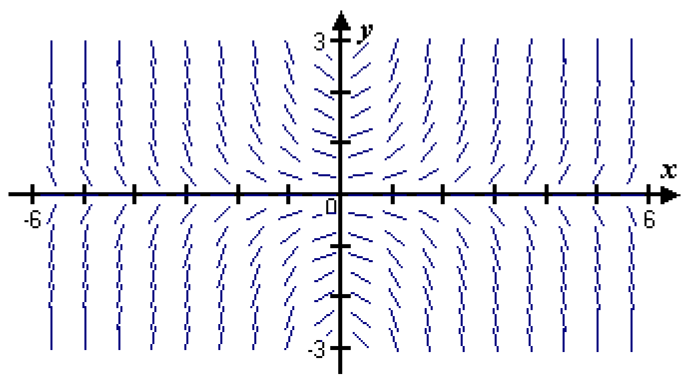
$\dfrac{dy}{dx} = \cancel{2x}$ $\dfrac{dy}{dx} = \cancel{2y}$ $\leftarrow$ $\dfrac{dy}{dx} \neq f(x, y)$

$\dfrac{dy}{dx} = \cancel{xy}$ $\dfrac{dy}{dx} = \cancel{\dfrac{2y}{x}}$ ☒ $\leftarrow$ $y = 0$ ***not*** vertical slope (undefined)

$\dfrac{dy}{dx} = \boxed{\dfrac{2x}{y}}$ ☑ $\leftarrow$ $y = 0$ vertical slope (undefined)

804. *Answer is A.*

Which of the following differential equations generates the slope field shown in the diagram ?



$\dfrac{dy}{dx} = f(x, y)$ $\dfrac{dy}{dx} = 0$ when $x = 0$

Horizontal asymptote $y = 0$

$\dfrac{dy}{dx} = xy$ ☑ $\dfrac{dy}{dx} = 0$ when $x = 0$, $\dfrac{dy}{dx} = 0$ when $y = 0$

$\dfrac{dy}{dx} = x$ ☒ no y variable

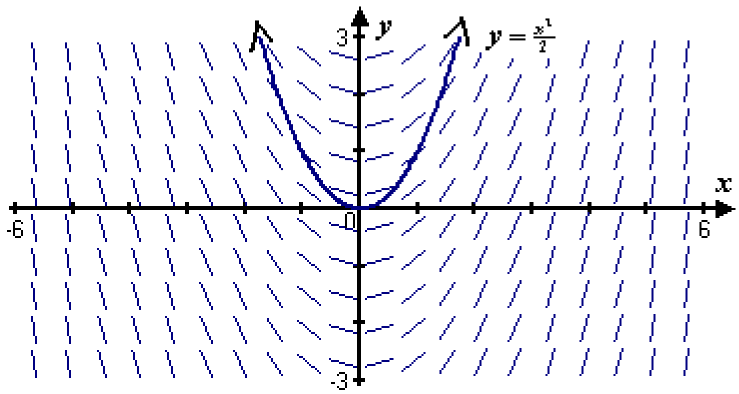
$\dfrac{dy}{dx} = y$ ☒ no x variable

$\dfrac{dy}{dx} = x + y$ ☒ if $y = 0$ then $\dfrac{dy}{dx} = 0$ for all x

$\dfrac{dy}{dx} = x^2$ ☒ no y variable

805. *Answer is A.*

The slope field for the differential equation $\dfrac{dy}{dx} = x$



$$dy = x\,dx$$

$$\int dy = \int x\,dx$$

$$y = \tfrac{x^2}{2} + C$$

has line segments symmetric to the y-axis ☑ true

shows that the solutions to the differential equation are odd functions ☒ false, even functions

shows that the solutions to the differential are straight lines ☒ false, parabolas

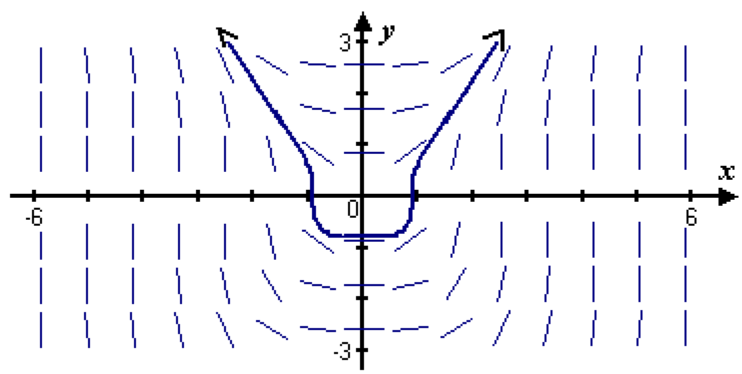
shows that the solutions to the differential equation are decreasing for increasing x ☒ only if $x < 0$

shows that there is a horizontal asymptote ☑ false, parabolas do not have asymptotes

806. *Answer is E.*

Difficulty = **0.45**

Shown in the diagram is a slope field for which of the following differential equations ?



Pick critical points and check each of possible solutions

$(\ 1,\ 1)\ \ m = 1\ \ \rightarrow$ fits ***all*** equations

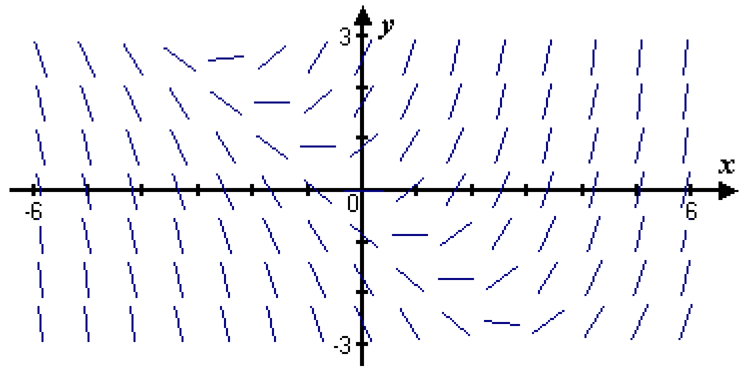
$(-1,\ 1)\ m = -1\ \ \rightarrow$ fits $\dfrac{x}{y}, \dfrac{x^3}{y}, \dfrac{x^3}{y^2}$ but ***not*** $\cancel{\dfrac{x^2}{y^2}}, \cancel{\dfrac{x^2}{y}}$

$(-1,-1)\ m = -1\ \ \rightarrow$ fits $\boxed{\dfrac{dy}{dx} = \dfrac{x^3}{y^2}}$ but ***not*** $\cancel{\dfrac{x}{y}}, \cancel{\dfrac{x^3}{y}}$

807. *Answer is C.*

Difficulty = **0.38**

Shown on the right is a slope field for which of the following differential equations ?



Slope changes along rows/columns
(slope depends on both x and y)

$\frac{dy}{dx}=f(x,y)$ ☑

Line $y=-x$ $\leftarrow$ slope $=0$

Check slopes at $(1,-1)$, $(2,-2)$, ...

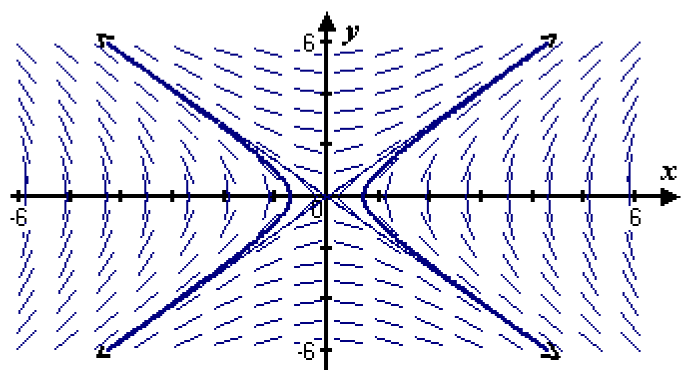
$\frac{dy}{dx}=\cancel{1+x}$ $\frac{dy}{dx}=\cancel{x^2}$ $\frac{dy}{dx}=\cancel{\ln y}$ $\leftarrow$ $\frac{dy}{dx}\neq f(x,y)$

$\frac{dy}{dx}=\frac{x}{y}$ ☒ $\leftarrow$ slope $\neq 0$ along $y=-x$

$\frac{dy}{dx}=\boxed{x+y}$ ☑ $\leftarrow$ slope $=0$ along $y=-x$

808. *Answer is D.*

Which of the following equations can be a solution of the differential equation whose slope field is shown on the right.



$y=\pm 1.41x$ lines asymptotes

$y=0$ undefined slopes

$x=0$ zero slopes

Fits a hyperbola with center $(0,\ 0)$

$2xy=1$ ☒ $\leftarrow$ not a hyperbola of form $y=\frac{1}{2x}$

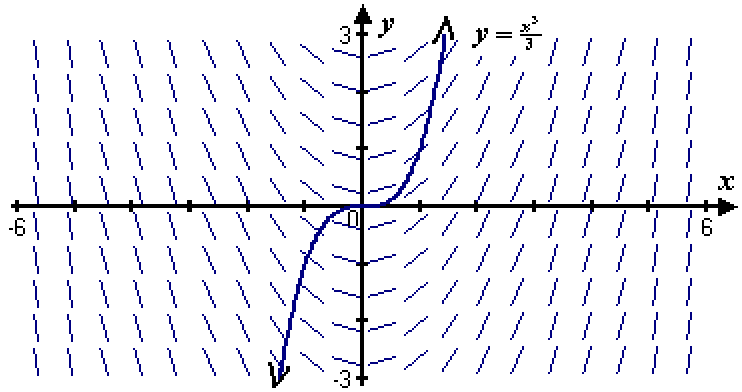
$2x+y=1$ ☒ $\leftarrow$ not a straight line

$2x^2+y^2=1$ ☒ $\leftarrow$ not an ellipse

$2x^2-y^2=1$ ☑ $\leftarrow$ a hyperbola opening sideways

$y=2x^2+1$ ☒ $\leftarrow$ not a parabola

809. *Answer is C.*



The slope field for the differential equation $\frac{dy}{dx}=x^2$

$$dy = x^2 dx$$
$$\int dy = \int x^2 dx$$
$$y = \tfrac{x^3}{3} + C$$

has line segments symmetric to the y-axis ☒ careful they are same slope but ***not*** symmetric wrt y-axis

shows that the solutions to the differential equation are even functions ☒ no, not symmetric wrt y-axis

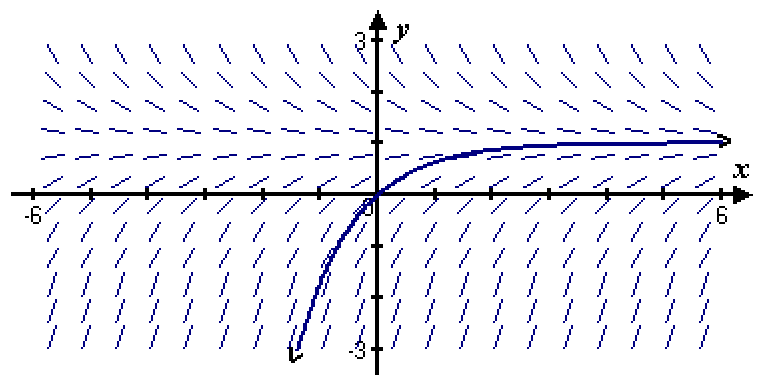
shows that the graphs of the solutions are increasing for increasing x ☑ true $y=\tfrac{x^3}{3}+C$ are increasing

shows that the graphs of the solutions are decreasing for increasing x ☑ false $y=\tfrac{x^3}{3}+C$ are increasing

shows that there are solutions that have a horizontal asymptote ☒ false $y=\tfrac{x^3}{3}+C$ has no asymptotes

810. *Answer is E.*

Shown on the right is the slope field for which differential equation ?



In ***each*** horizontal row slopes = same !!!
(x does not affect the derivative)

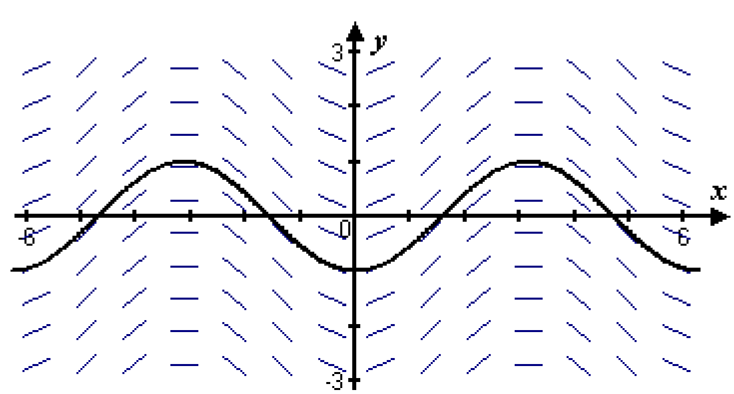
$$\frac{dy}{dx} = f(y) \leftarrow \textbf{\textit{no}}\ x\text{-terms}$$

Asymptote at $y=1$

$\frac{dy}{dx}=\cancel{1-x}$ $\frac{dy}{dx}=\cancel{x-y}$ $\frac{dy}{dx}=\cancel{-\frac{x}{y}}$ $\frac{dy}{dx}=\cancel{\frac{y}{x}}$

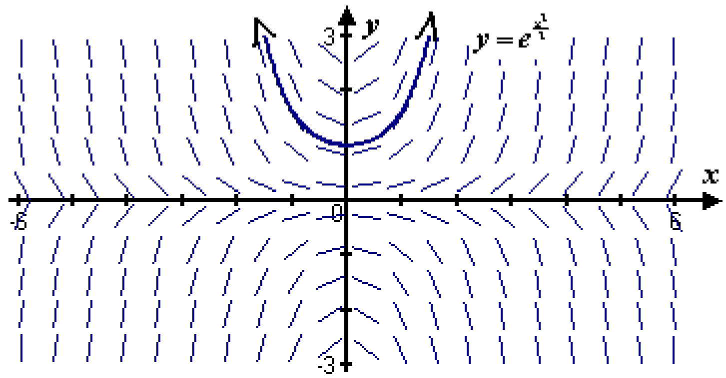
$$\frac{dy}{dx}=1-y \ ☑ \leftarrow \begin{cases} \text{no } x\text{-term} \\ \text{negative slope above } y=1 \\ \text{positive slope below } y=1 \end{cases}$$

811. *Answer is B.*



812. *Answer is C.*

This slope field is for which of the following differential equations ?



$$\frac{dy}{dx} = f(x,y)$$

$$\frac{dy}{dx} = 0 \text{ when } x = 0 \text{ or } y = 0$$

Quadrants slope $\begin{array}{c|c} - & + \\ \hline + & - \end{array}$

$\frac{dy}{dx} = x^2 + y^2$ ☒ $\frac{dy}{dx} =$ always positive

$\frac{dy}{dx} = x(x-1)$ ☒ $\frac{dy}{dx} =$ no y term

$\frac{dy}{dx} = xy$ ☑ $\frac{dy}{dx} =$ has all characteristics

$\frac{dy}{dx} = \frac{x}{2y}$ ☒ $\frac{dy}{dx} = \textbf{\textit{undefined}}$ when $y = 0$

$\frac{dy}{dx} = y$ ☒ $\frac{dy}{dx} =$ no x term

$$\frac{dy}{dx} = xy$$

$$\frac{dy}{y} = xdx$$

$$\ln|y| = \tfrac{x^2}{2} + C$$

$$y = Ce^{\frac{x^2}{2}}$$

813. *Answer is E.*

The slope field for the differential equation $\frac{dy}{dx} = \frac{3y}{xy+5x}$ will have ***vertical*** segments when

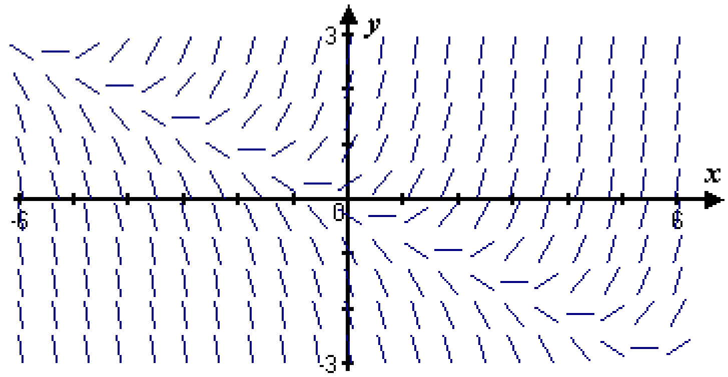
vertical segments $\rightarrow$ undefined slope $\rightarrow$ $\frac{dy}{dx} = \frac{3y}{xy+5x} =$ undefined

$$xy + 5x = 0$$

$$x(y+5) = 0$$

$$\boxed{x = 0} \quad \boxed{y = -5}$$

814. *Answer is C.*



The slope field is for which of the following differential equations ?

$\dfrac{dy}{dx} = f(x,y)$

$\dfrac{dy}{dx} = 0$ when $y = -\tfrac{1}{2}x$

$\dfrac{dy}{dx} = y - 2$ ☒ no x variable

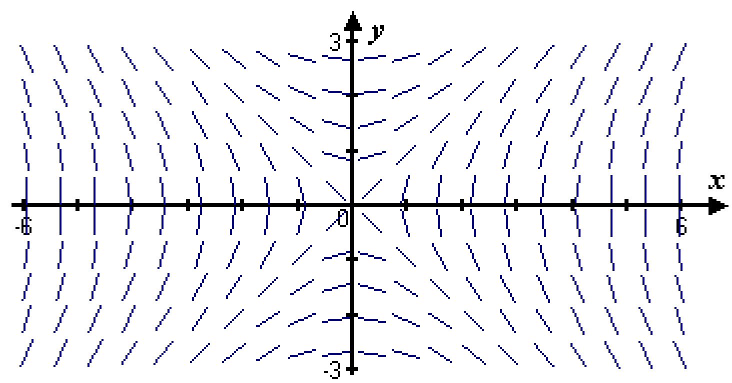
$\dfrac{dy}{dx} = x - 2$ ☒ no y variable

$\dfrac{dy}{dx} = x + 2y = x + 2(-\tfrac{1}{2}x) = 0$ ☑ $\dfrac{dy}{dx} = 0$ when $y = -\tfrac{1}{2}x$ for all values of x

$\dfrac{dy}{dx} = xy = x(-\tfrac{1}{2}x) = -\tfrac{1}{2}x^2$ ☒ slope is always negative

$\dfrac{dy}{dx} = x + y = x + (-\tfrac{1}{2}x) = \tfrac{1}{2}x$ ☒ $\dfrac{dy}{dx} \neq 0$ when $y = -\tfrac{1}{2}x$

815. *Answer is E.*



This is a slope field for which of the following differential equations ?

$\dfrac{dy}{dx} = f(x,y)$

$\dfrac{dy}{dx} = 1$ when $y = x$

$\dfrac{dy}{dx} = -1$ when $y = -x$

$\dfrac{dy}{dx} = \textit{undefined}$ when $y = 0$

$\dfrac{dy}{dx} = xy = x(x) = x^2 \neq 1$ ☒ $\dfrac{dy}{dx} \neq 1$ when $y = x$

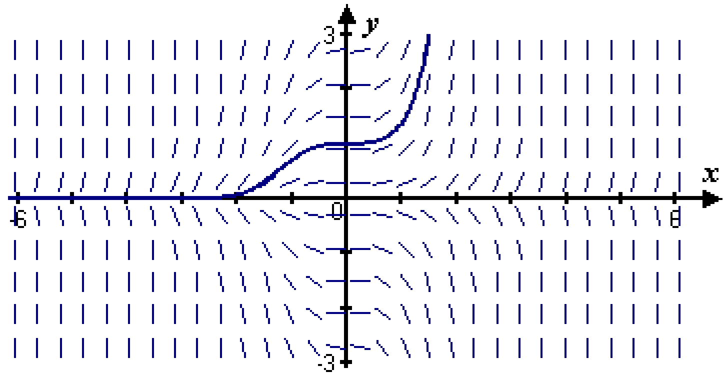
$\dfrac{dy}{dx} = \dfrac{x^2}{y} = \dfrac{x^2}{x} = x \neq 1$ ☒ $\dfrac{dy}{dx} \neq 1$ when $y = x$

$\dfrac{dy}{dx} = x^2 y = x^2(x) = x^3 \neq 1$ ☒ $\dfrac{dy}{dx} \neq 1$ when $y = x$

$\dfrac{dy}{dx} = \dfrac{y}{x} = \dfrac{x}{x} = 1$ ☒ $\dfrac{dy}{dx} \neq \textit{undefined}$ when $y = 0$

$\dfrac{dy}{dx} = \dfrac{x}{y} = \dfrac{x}{x} = 1$ ☑ $\dfrac{dy}{dx} \neq 0$ when $y = -\tfrac{1}{2}x$ and $\dfrac{dy}{dx} = \textit{undefined}$ when $y = 0$

816. *Answer is E.*



This is a slope field for which of the following differential equations ?

$$\frac{dy}{dx} = f(x,y)$$

$$\frac{dy}{dx} = 0 \text{ when } x = 0 \text{ or } y = 0$$

$$\textit{Quadrants} \text{ slope } \begin{array}{c|c} + & + \\ \hline - & - \end{array}$$

$\frac{dy}{dx} = xy^2$ ☒ $\frac{dy}{dx} =$ wrong sign quadrant **II**

$\frac{dy}{dx} = xy$ ☒ $\frac{dy}{dx} =$ wrong sign quadrant **II**

$\frac{dy}{dx} = \frac{x}{y}$ ☒ $\frac{dy}{dx} = \textit{undefined}$ when $y = 0$

$\frac{dy}{dx} = \frac{x^2}{y}$ ☒ $\frac{dy}{dx} = \textit{undefined}$ when $y = 0$

$\frac{dy}{dx} = x^2 y$ ☑ $\frac{dy}{dx} = \textit{sign}$ correct all $\textit{quadrants}$

$$\frac{dy}{dx} = x^2 y$$

$$\frac{dy}{y} = x^2 dx$$

$$\ln|y| = \tfrac{x^3}{3} + C$$

$$y = Ce^{\frac{x^3}{3}}$$

817. *Answer is E.*

The slope field for the differential equation $\frac{dy}{dx} = \frac{x^2 y + y^2 x}{3x + y}$ will have ***horizontal*** segments when

$\textit{horizontal}$ segments $\rightarrow$ slope $= 0 \rightarrow \frac{dy}{dx} = \frac{x^2 y + y^2 x}{3x + y} = 0$

$$x^2 y + y^2 x = 0$$

$$xy(x + y) = 0$$

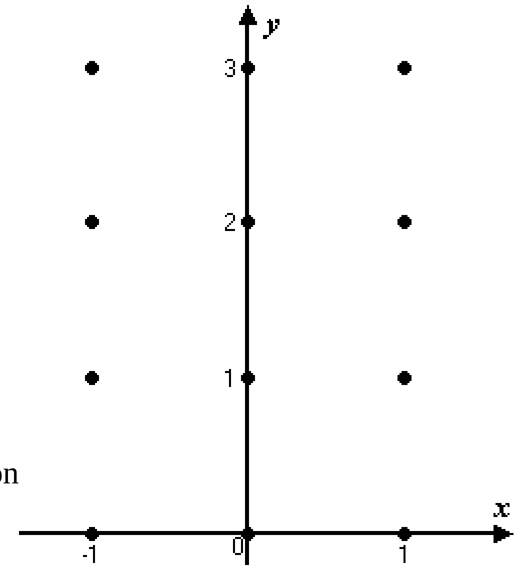
$$\boxed{x = 0} \quad \boxed{y = 0} \quad \boxed{y = -x}$$

818. Consider the differential equation $\frac{dy}{dx} = x^2(y-1)$

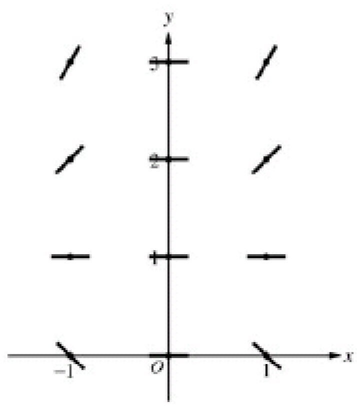
a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.

b) While the slope field in part (***a***) is drawn at only twelve points, it is defined at every point in the ***xy***-plane. Describe all points in the ***xy***-plane for which the slopes are positive.

c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$



(a)



2 : $\begin{cases} 1 : \text{zero slope at each point } (x, y) \text{ where } x = 0 \text{ or } y = 1 \\ 1 : \begin{cases} \text{positive slope at each point } (x, y) \text{ where } x \neq 0 \text{ and } y > 1 \\ \text{negative slope at each point } (x, y) \text{ where } x \neq 0 \text{ and } y < 1 \end{cases} \end{cases}$

(b) Slopes are positive at points (x, y) where $x \neq 0$ and $y > 1$.

1 : description

(c) $\frac{1}{y-1}dy = x^2 dx$

$\ln|y-1| = \frac{1}{3}x^3 + C$

$|y-1| = e^C e^{\frac{1}{3}x^3}$

$y - 1 = Ke^{\frac{1}{3}x^3}$, $K = \pm e^C$

$2 = Ke^0 = K$

$y = 1 + 2e^{\frac{1}{3}x^3}$

6 : $\begin{cases} 1 : \text{separates variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \\ \quad 0/1 \text{ if } y \text{ is not exponential} \end{cases}$

Note: max $3/6$ [1-2-0-0-0] if no constant of integration

Note: $0/6$ if no separation of variables

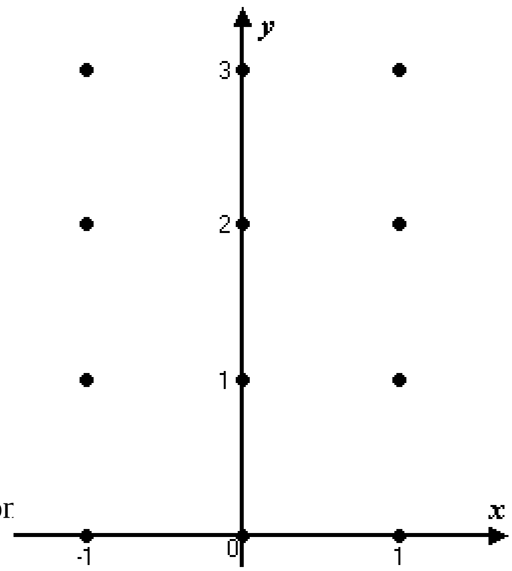
819.

Consider the differential equation $\dfrac{dy}{dx} = x^4(y-2)$

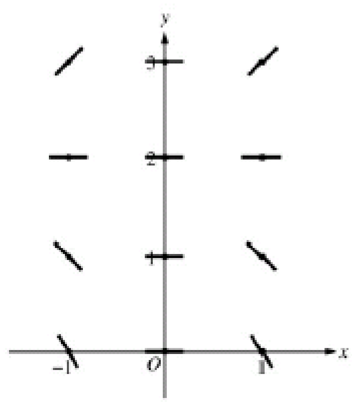
***a*)** On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.

b*)** While the slope field in part (a***) is drawn at only twelve points, it is defined at every point in the ***xy***-plane. Describe all points in the ***xy***-plane for which the slopes are negative.

***c*)** Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 0$



(a)



2 : { 1 : zero slope at each point (x, y) where $x = 0$ or $y = 2$
1 : { positive slope at each point (x, y) where $x \neq 0$ and $y > 2$
negative slope at each point (x, y) where $x \neq 0$ and $y < 2$ }

(b) Slopes are negative at points (x, y) where $x \neq 0$ and $y < 2$.

1 : description

(c) $\dfrac{1}{y-2}\,dy = x^4 dx$

$\ln|y-2| = \frac{1}{5}x^5 + C$

$|y-2| = e^C e^{\frac{1}{5}x^5}$

$y - 2 = Ke^{\frac{1}{5}x^5}, \; K = \pm e^C$

$-2 = Ke^0 = K$

$y = 2 - 2e^{\frac{1}{5}x^5}$

6 : { 1 : separates variables
2 : antiderivatives
1 : constant of integration
1 : uses initial condition
1 : solves for y
0/1 if y is not exponential }

Note: max 3/6 [1-2-0-0-0] if no constant of integration

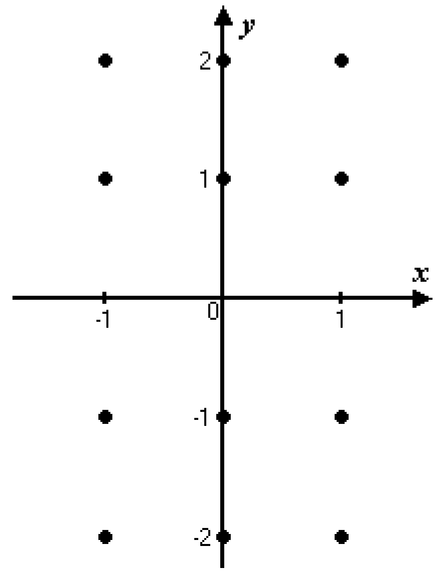
Note: 0/6 if no separation of variables

820. Consider the differential equation $\frac{dy}{dx} = -\frac{2x}{y}$

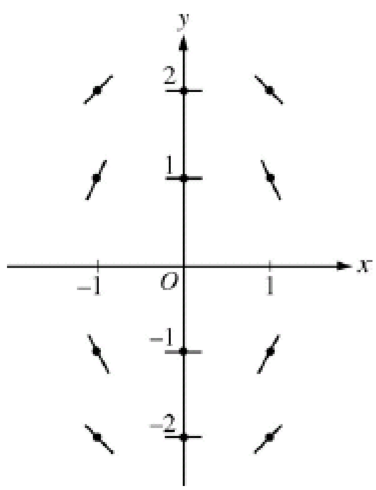
a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.

b) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = -1$ Write an equation for the line tangent to the graph of f at $(1, -1)$ and use it to approximate $f(1.1)$

c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(1) = -1$



(a)



$2 : \begin{cases} 1 : \text{zero slopes} \\ 1 : \text{nonzero slopes} \end{cases}$

(b) The line tangent to f at $(1, -1)$ is $y + 1 = 2(x - 1)$.
Thus, $f(1.1)$ is approximately -0.8.

$2 : \begin{cases} 1 : \text{equation of the tangent line} \\ 1 : \text{approximation for } f(1.1) \end{cases}$

(c) $\frac{dy}{dx} = -\frac{2x}{y}$

$y\,dy = -2x\,dx$

$\frac{y^2}{2} = -x^2 + C$

$\frac{1}{2} = -1 + C;\ C = \frac{3}{2}$

$y^2 = -2x^2 + 3$

Since the particular solution goes through $(1, -1)$, y must be negative.

Thus the particular solution is $y = -\sqrt{3 - 2x^2}$.

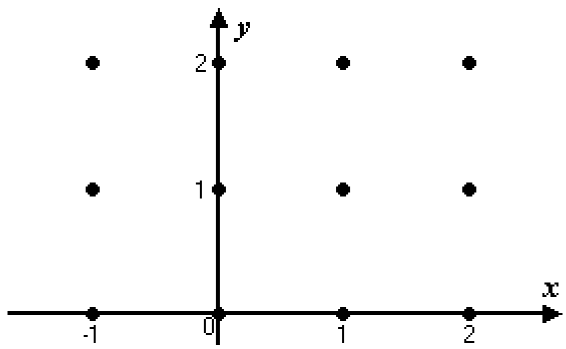
$5 : \begin{cases} 1 : \text{separates variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max $2/5$ [1-1-0-0-0] if no constant of integration

Note: $0/5$ if no separation of variables

821. Consider the differential equation $\frac{dy}{dx} = -\frac{xy^2}{2}$

Let $y = f(x)$ be the particular solution to this differential equation with the initial condition $f(-1) = 2$

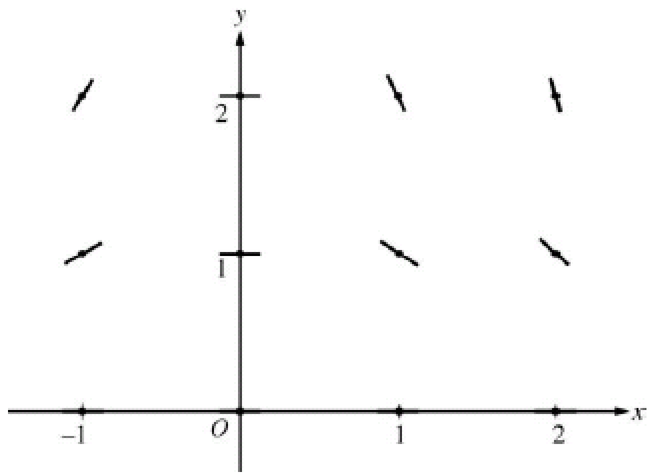


a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.

b) Write an equation for the line tangent to the graph of f at $x = -1$

c) Find the solution $y = f(x)$ to the given differential equation with the initial condition $f(-1) = 2$

(a)



$2 : \begin{cases} 1 : \text{zero slopes} \\ 1 : \text{nonzero slopes} \end{cases}$

(b) Slope $= \frac{-(-1)4}{2} = 2$

$y - 2 = 2(x+1)$

1 : equation

(c) $\frac{1}{y^2}dy = -\frac{x}{2}dx$

$-\frac{1}{y} = -\frac{x^2}{4} + C$

$-\frac{1}{2} = -\frac{1}{4} + C;\ C = -\frac{1}{4}$

$y = \frac{1}{\frac{x^2}{4} + \frac{1}{4}} = \frac{4}{x^2+1}$

$6 : \begin{cases} 1 : \text{separates variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

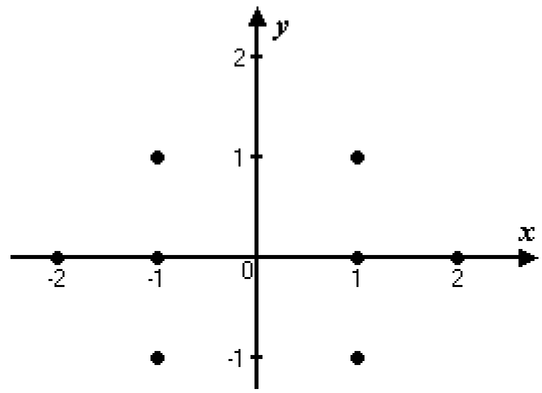
Note: max $3/6$ [1-2-0-0-0] if no constant of integration

Note: $0/6$ if no separation of variables

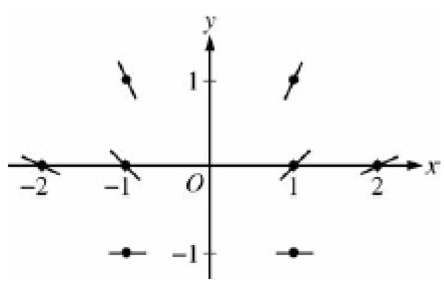
822. Consider the differential equation $\frac{dy}{dx} = \frac{1+y}{x}$ where $x \neq 0$

***a*)** On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.

***b*)** Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(-1) = 1$ and state its domain.



(a)



2 : sign of slope at each point and relative steepness of slope lines in rows and columns

(b) $\frac{1}{1+y}\,dy = \frac{1}{x}\,dx$

$\ln|1+y| = \ln|x| + K$

$|1+y| = e^{\ln|x| + K}$

$1 + y = C|x|$

$2 = C$

$1 + y = 2|x|$

$y = 2|x| - 1$ and $x < 0$

or

$y = -2x - 1$ and $x < 0$

7 :

6 :
- 1 : separates variables
- 2 : antiderivatives
- 1 : constant of integration
- 1 : uses initial condition
- 1 : solves for y

Note: max $3/6$ [1-2-0-0-0] if no constant of integration

Note: $0/6$ if no separation of variables

1 : domain

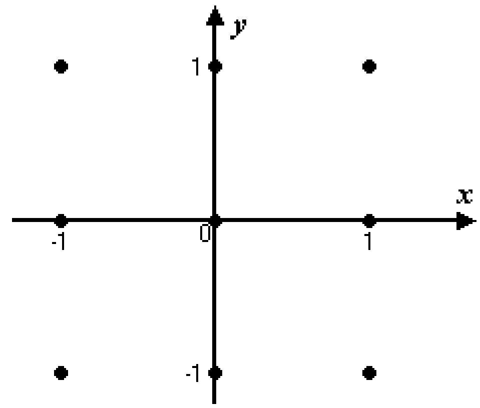
823.

Consider the differential equation $\frac{dy}{dx} = (y-1)^2 \cos(\pi x)$

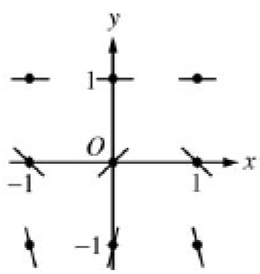
a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

b) There is a horizontal line with equation $y = c$ that satisfies this differential equation. Find the value of c

c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 0$



(a)



$2: \begin{cases} 1: \text{zero slopes} \\ 1: \text{all other slopes} \end{cases}$

(b) The line $y = 1$ satisfies the differential equation, so $c = 1$.

$1: c = 1$

(c) $\frac{1}{(y-1)^2}\,dy = \cos(\pi x)\,dx$

$-(y-1)^{-1} = \frac{1}{\pi}\sin(\pi x) + C$

$\frac{1}{1-y} = \frac{1}{\pi}\sin(\pi x) + C$

$1 = \frac{1}{\pi}\sin(\pi) + C = C$

$\frac{1}{1-y} = \frac{1}{\pi}\sin(\pi x) + 1$

$\frac{\pi}{1-y} = \sin(\pi x) + \pi$

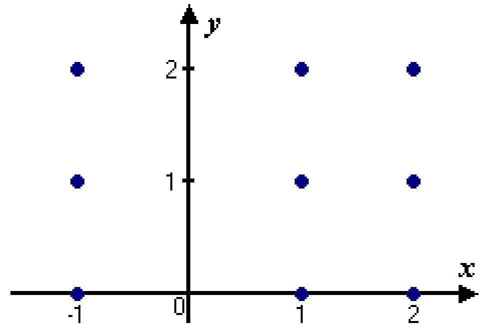
$y = 1 - \frac{\pi}{\sin(\pi x) + \pi}$ for $-\infty < x < \infty$

$6: \begin{cases} 1: \text{separates variables} \\ 2: \text{antiderivatives} \\ 1: \text{constant of integration} \\ 1: \text{uses initial condition} \\ 1: \text{answer} \end{cases}$

Note: max $3/6$ [1-2-0-0-0] if no constant of integration

Note: $0/6$ if no separation of variables

824. Consider the differential equation $\dfrac{dy}{dx} = \dfrac{y-1}{x^2}$ where $x \neq 0$

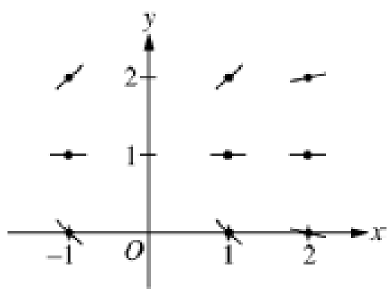


a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(2) = 0$

c) For the particular solution $y = f(x)$ described in part (***b***), find $\lim\limits_{x \to \infty} f(x)$

(a)



$2 : \begin{cases} 1 : \text{zero slopes} \\ 1 : \text{all other slopes} \end{cases}$

(b) $\dfrac{1}{y-1}\,dy = \dfrac{1}{x^2}\,dx$

$\ln|y-1| = -\dfrac{1}{x} + C$

$|y-1| = e^{-\frac{1}{x}+C}$

$|y-1| = e^{C}e^{-\frac{1}{x}}$

$y - 1 = ke^{-\frac{1}{x}}$, where $k = \pm e^{C}$

$-1 = ke^{-\frac{1}{2}}$

$k = -e^{\frac{1}{2}}$

$f(x) = 1 - e^{\left(\frac{1}{2}-\frac{1}{x}\right)}$, $x > 0$

$6 : \begin{cases} 1 : \text{separates variables} \\ 2 : \text{antidifferentiates} \\ 1 : \text{includes constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

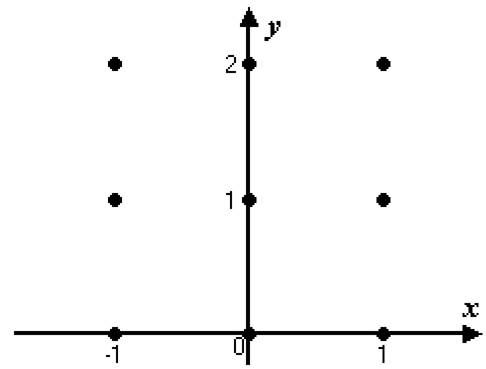
Note: max 3/6 [1-2-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables

(c) $\lim\limits_{x \to \infty} 1 - e^{\left(\frac{1}{2}-\frac{1}{x}\right)} = 1 - \sqrt{e}$

1 : limit

825. Consider the differential equation $\frac{dy}{dx} = \frac{1}{2}x + y - 1$



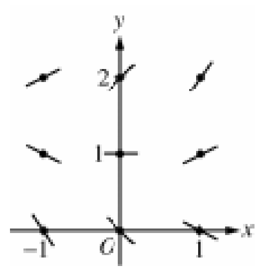
a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

b) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Describe the region in the xy-plane in which all solution curves to the differential equation are concave up.

c) Let $y = f(x)$ be a particular solution to the differential equation with the initial condition $f(0) = 1$. Does f have a relative minimum, a relative maximum, or neither at $x = 0$ Justify your answer.

d) Find the values of the constants m and b, for which $y = mx + b$ is a solution to the differential equation.

(a)



2 : Sign of slope at each point and relative steepness of slope lines in rows and columns.

(b) $\frac{d^2y}{dx^2} = \frac{1}{2} + \frac{dy}{dx} = \frac{1}{2}x + y - \frac{1}{2}$

Solution curves will be concave up on the half-plane above the line $y = -\frac{1}{2}x + \frac{1}{2}$.

$3 : \begin{cases} 2 : \frac{d^2y}{dx^2} \\ 1 : \text{description} \end{cases}$

(c) $\left.\frac{dy}{dx}\right|_{(0,1)} = 0 + 1 - 1 = 0$ and $\left.\frac{d^2y}{dx^2}\right|_{(0,1)} = 0 + 1 - \frac{1}{2} > 0$

Thus, f has a relative minimum at $(0, 1)$.

$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

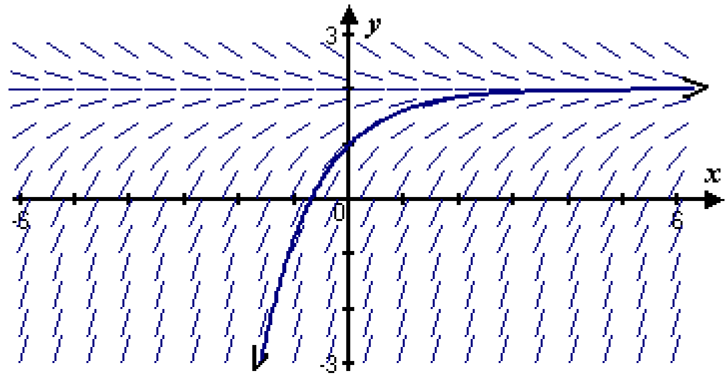
(d) Substituting $y = mx + b$ into the differential equation:

$m = \frac{1}{2}x + (mx + b) - 1 = \left(m + \frac{1}{2}\right)x + (b - 1)$

Then $0 = m + \frac{1}{2}$ and $m = b - 1$: $m = -\frac{1}{2}$ and $b = \frac{1}{2}$.

$2 : \begin{cases} 1 : \text{value for } m \\ 1 : \text{value for } b \end{cases}$

826. *Answer is D.*



A slope field for a differential equation $\frac{dy}{dx} = f(x, y)$ is given at the right. Which of the following could be a solution ?

Horizontal asymptote at $y = 2$

Horizontal segments same $\frac{dy}{dx} = f(y)$

Negative slope if $y > 0$ (decay graph)

$$\frac{dy}{dx} = 2 - y = 2 - (2 - e^{-x}) = e^{-x} \ ☑$$

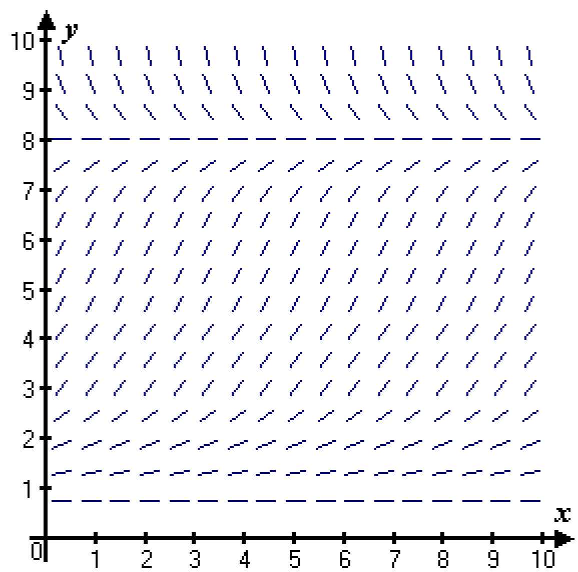
827. *Answer is C.*

A slope field for a differential equation $\frac{dy}{dx} = f(x, y)$ is given in the figure on the right. Which of the following statements are true ?

I. The value of $\frac{dy}{dx}$ at the point $(3,\ 3)$ is approximately **1**

II. As y approaches **8** the rate of change of y approaches zero.

III. All solution curves for the differential equation have the same slope for a given value of x



I. The value of $\frac{dy}{dx}$ at the point $(3,\ 3)$ is approximately **1** ☑ True, observe graph

II. As y approaches **8** the rate of change of y approaches zero. ☑ True, horizontal asymptote

III. All solution curves for the differential equation have the same slope for a given value of x ☒ False, observe graph

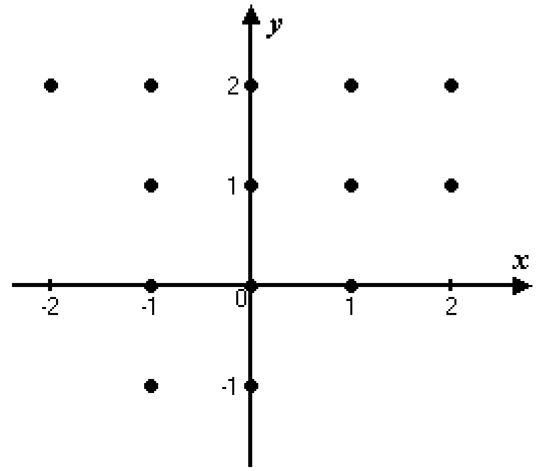
828. Consider the differential equation $\frac{dy}{dx} = x - y$

a) On the axes provided, sketch a slope field for the given differential equation at the fourteen points indicated.

b) Sketch the solution curve that contains the point $(-1,\ 1)$

c) Find an equation for the straight line solution through the point $(\ 1,\ 0)$

d) Show that if **C** is a constant, then $y = x - 1 + Ce^{-x}$ is a solution of the differential equation



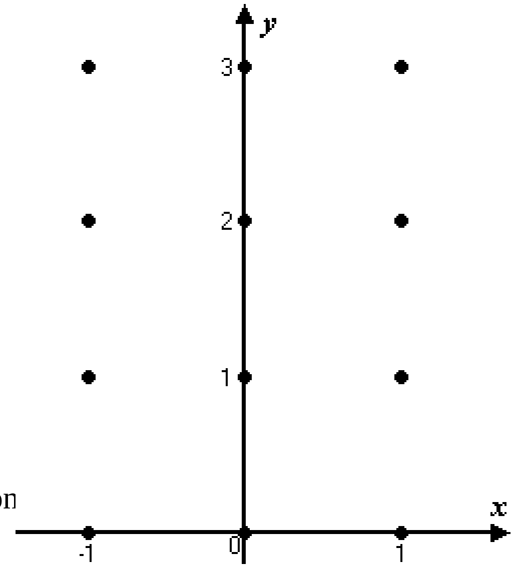
829.

Consider the differential equation $\dfrac{dy}{dx} = x^2(2y+1)$

a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.

b) Although the slope field in part (***a***) is drawn at only **12** points, it is defined at every point in the ***xy***-plane. Describe all points in the ***xy***-plane for which the slopes are positive.

c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 2$



$$\frac{dy}{dx} = x^2(2y+1)$$

$$\frac{1}{2}\int \frac{2dy}{(2y+1)} = \int x^2 dx$$

$$\frac{1}{2}\ln|2y+1| = \frac{x^3}{3} + C_2$$

$$\ln|2y+1| = \frac{2x^3}{3} + 2C_2 = \frac{2x^3}{3} + C_1 \qquad \leftarrow \text{ where } 2C_2 = C_1$$

$$|2y+1| = e^{\frac{2x^3}{3}+2C_2} \qquad = \left(e^{C_1}\right)\left(e^{\frac{2x^3}{3}}\right) = Ke^{\frac{2x^3}{3}} \qquad \leftarrow \text{where } K = \pm e^{C_1} \text{ can be } \textbf{\textit{positive}} \text{ or } \textbf{\textit{negative}}$$

$$2y+1 = Ke^{\frac{2x^3}{3}} \qquad \leftarrow \text{ point } (0,\ 2) \text{ makes } K = 5 \quad \left(\text{now drop } |\ |\right)$$

$$2y+1 = 5e^{\frac{2x^3}{3}}$$

$$y = \frac{1}{2}\left(5e^{\frac{2x^3}{3}} - 1\right)$$