CALCULUS 12 – Mid Term Exra Practice

Practice problems:

Find the area of the region bounded by the given curves. Decide whether to integrate with respect to x or y.

1. $y = 3x^2$, $y = 0$, and $x = 3$

2. $y = x^2$, $y = 2 - x$, $x = 0$ and $x \geq 0$

3. $y = e^x$, $y = xe^x$, and $x = 0$

4. $x = 2y^2$ and $x = 4 + y^2$

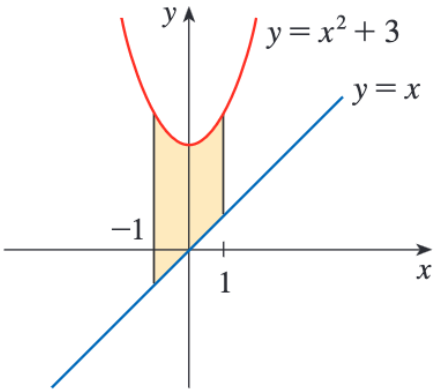
5. $y = \cos x$, $y = \sin 2x$, $x = 0$ and $x = \dfrac{\pi}{2}$

Answers:

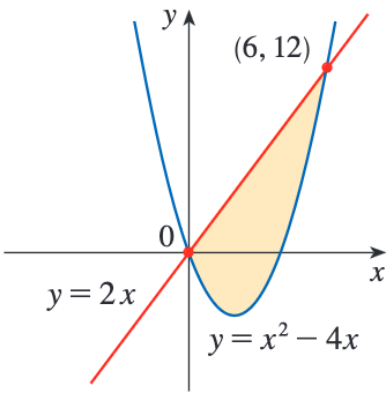
1. $A = 27$
2. $A = \dfrac{7}{6}$
3. $A = e - 2$
4. $A = \dfrac{32}{3}$
5. $A = \dfrac{1}{2}$

1–4 ▪ Find the area of the shaded region.

1.



2.



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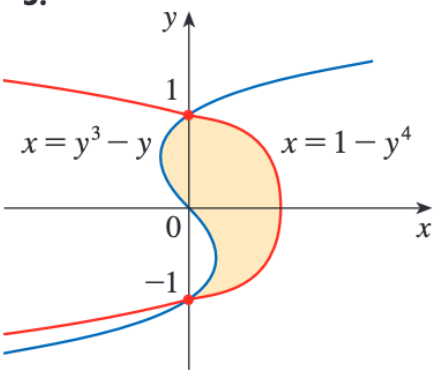
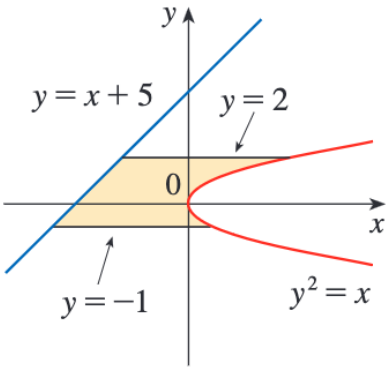


4.



1.	2.
3.	4.

Find the region bounded.

5. $y = 4x^2, \quad y = x^2 + 3$

6. $y = x + 1, \quad y = (x - 1)^2, \quad x = -1, \quad x = 2$

7. $y = x^2 + 1, \quad y = 3 - x^2, \quad x = -2, \quad x = 2$

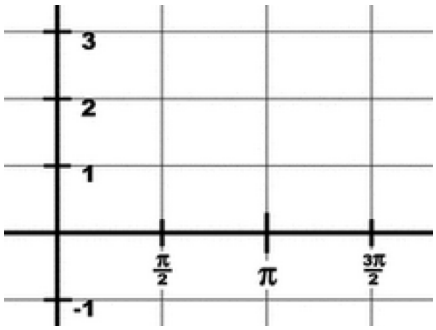
8. $y^2 = x, \quad x - 2y = 3$

5.	6.
7.	8.

37. Evaluate

$$\int_0^{\pi} \left| \sin x - \frac{2}{\pi} x \right| dx$$

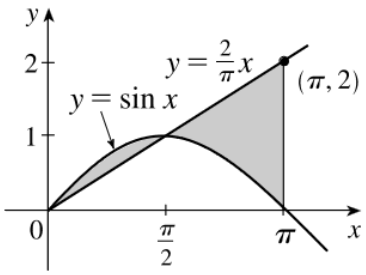
and interpret it as the area of a region. Sketch the region.



ANSWERS

1. $\frac{20}{3}$
2. 36
3. $\frac{8}{5}$
4. $\frac{33}{2}$
5. 4
6. $\frac{31}{6}$
7. 8
8. $\frac{32}{3}$

37. $\frac{\pi}{2}$



Limit

1. $\lim_{x \to 2} \frac{x^2 - 4}{x^2 + 4}$ is

(A) 1 **(B)** 0 **(C)** $-\frac{1}{2}$ **(D)** -1 **(E)** ∞

2. $\lim_{x \to \infty} \frac{4 - x^2}{x^2 - 1}$ is

(A) 1 **(B)** 0 **(C)** -4 **(D)** -1 **(E)** ∞

3. $\lim_{x \to 3} \frac{x - 3}{x^2 - 2x - 3}$ is

(A) 0 **(B)** 1 **(C)** $\frac{1}{4}$ **(D)** ∞ **(E)** none of these

4. $\lim_{x \to 0} \frac{x}{x}$ is

(A) 1 **(B)** 0 **(C)** ∞ **(D)** -1 **(E)** nonexistent

5. $\lim_{x \to 2} \frac{x^3 - 8}{x^2 - 4}$ is

(A) 4 **(B)** 0 **(C)** 1 **(D)** 3 **(E)** ∞

6. $\lim_{x \to \infty} \frac{4 - x^2}{4x^2 - x - 2}$ is

(A) -2 **(B)** $-\frac{1}{4}$ **(C)** 1 **(D)** 2

(E) nonexistent

1. B
2. D
3. C
4. A
5. D
6. **B**

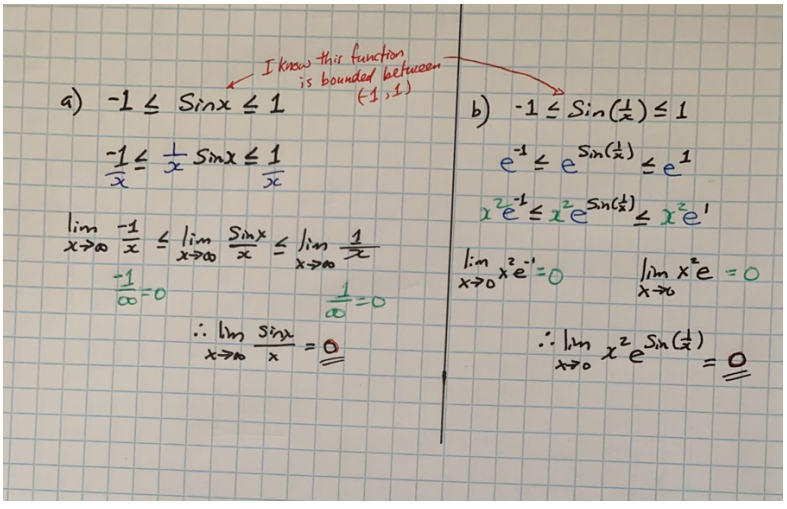
Squeeze Theorem

Use the squeeze theorem to find the limit for each.

a) $\lim_{x\to\infty} \frac{sinx}{x}$

b) $\lim_{x\to 0} x^2 e^{\sin\left(\frac{1}{x}\right)}$

ANSWER



Set 5: Multiple-Choice Questions on Integration

1. $\int (3x^2 - 2x + 3)\, dx =$

(A) $x^3 - x^2 + C$ **(B)** $3x^3 - x^2 + 3x + C$ **(C)** $x^3 - x^2 + 3x + C$

(D) $\frac{1}{2}(3x^2 - 2x + 3)^2 + C$ **(E)** none of these

2. $\int \left(x - \frac{1}{2x}\right)^2 dx =$

(A) $\frac{1}{3}\left(x - \frac{1}{2x}\right)^3 + C$ **(B)** $x^2 - 1 + \frac{1}{4x^2} + C$ **(C)** $\frac{x^3}{3} - 2x - \frac{1}{4x} + C$

(D) $\frac{x^3}{3} - x - \frac{4}{x} + C$ **(E)** none of these

3. $\int \sqrt{4 - 2t}\ dt =$

(A) $-\frac{1}{3}(4 - 2t)^{3/2} + C$ **(B)** $\frac{2}{3}(4 - 2t)^{3/2} + C$ **(C)** $-\frac{1}{6}(4 - 2t)^3 + C$

(D) $+\frac{1}{2}(4 - 2t)^2 + C$ **(E)** $\frac{4}{3}(4 - 2t)^{3/2} + C$

4. $\int (2 - 3x)^5\, dx =$

(A) $\frac{1}{6}(2 - 3x)^6 + C$ **(B)** $-\frac{1}{2}(2 - 3x)^6 + C$ **(C)** $\frac{1}{2}(2 - 3x)^6 + C$

(D) $-\frac{1}{18}(2 - 3x)^6 + C$ **(E)** none of these

5. $\int \frac{1 - 3y}{\sqrt{2y - 3y^2}}\, dy =$

(A) $4\sqrt{2y - 3y^2} + C$ **(B)** $\frac{1}{4}(2y - 3y^2)^2 + C$ **(C)** $\frac{1}{2}\ln\sqrt{2y - 3y^2} + C$

(D) $\frac{1}{4}(2y - 3y^2)^{1/2} + C$ **(E)** $\sqrt{2y - 3y^2} + C$

6. $\int \frac{dx}{3(2x-1)^2} =$

(A) $\frac{-3}{2x-1} + C$ **(B)** $\frac{1}{6-12x} + C$ **(C)** $+\frac{6}{2x-1} + C$

(D) $\frac{2}{3\sqrt{2x-1}} + C$ **(E)** $\frac{1}{3}\ln|2x-1| + C$

7. $\int \frac{2\,du}{1+3u} =$

(A) $\frac{2}{3}\ln|1+3u| + C$ **(B)** $-\frac{1}{3(1+3u)^2} + C$ **(C)** $2\ln|1+3u| + C$

(D) $\frac{3}{(1+3u)^2} + C$ **(E)** none of these

8. $\int \frac{t}{\sqrt{2t^2-1}}\,dt =$

(A) $\frac{1}{2}\ln\sqrt{2t^2-1} + C$ **(B)** $4\ln\sqrt{2t^2-1} + C$ **(C)** $8\sqrt{2t^2-1} + C$

(D) $-\frac{1}{4(2t^2-1)} + C$ **(E)** $\frac{1}{2}\sqrt{2t^2-1} + C$

9. $\int \cos 3x\, dx =$

(A) $3 \sin 3x + C$ **(B)** $-\sin 3x + C$ **(C)** $-\frac{1}{3}\sin 3x + C$

(D) $\frac{1}{3}\sin 3x + C$ **(E)** $\frac{1}{2}\cos^2 3x + C$

10. $\int \frac{x\, dx}{1+4x^2} =$

(A) $\frac{1}{8}\ln(1+4x^2) + C$ **(B)** $\frac{1}{8(1+4x^2)^2} + C$ **(C)** $\frac{1}{4}\sqrt{1+4x^2} + C$

(D) $\frac{1}{2}\ln\left|1+4x^2\right| + C$ **(E)** $\frac{1}{2}\tan^{-1} 2x + C$

11. $\int \frac{dx}{1+4x^2} =$

(A) $\tan^{-1}(2x) + C$ **(B)** $\frac{1}{8}\ln(1+4x^2) + C$ **(C)** $\frac{1}{8(1+4x^2)^2} + C$

(D) $\frac{1}{2}\tan^{-1}(2x) + C$ **(E)** $\frac{1}{8x}\ln\left|1+4x^2\right| + C$

1. C
2. E
3. A
4. D
5. E
6. B
7. A
8. E
9. D
10. A
11. D

Derivatives:

1 Differentiate. $f(x) = 3x^5 - 2x^2 - 7x^{-3} + 9x^{-4}$

a. $f(x) = 15x^4 - 4x - 21x^{-2} + 36x^{-3}$

b. $f(x) = 15x^4 - 4x + 21x^{-4} - 36x^{-5}$

c. $f(x) = 15x^4 - 4x + 21x^2 - 36x^5$

d. $f(x) = 15x^4 - 4x - 21x^{-4} + 36x^{-5}$

2 What is the derivative of $f(x) = 3x^2 - x + 8$?

a. $f'(x) = 6x - 1$

b. $f'(x) = 6x + 8$

c. $f'(x) = 3x$

d. $f'(x) = 9x + 7$

3 What is the derivative of $f(x) = 23\pi$?

a. $f'(x) = 23\pi$

b. $f'(x) = \pi$

c. $f'(x) = 0$

d. $f'(x) = 23$

4 What is the derivative of $f(x) = 2\sqrt{x^7} - 5\sqrt[4]{x^3} + 9\sqrt[3]{x^2} - \dfrac{6}{\sqrt[6]{x^5}}$?

a. $f'(x) = x^3 - \dfrac{5}{2}\sqrt{x} + \dfrac{9}{2}\sqrt[6]{x} + \dfrac{3}{\sqrt[3]{x^4}}$

b. $f'(x) = 7\sqrt{x^5} - \dfrac{20}{3\sqrt[4]{x}} + \dfrac{3}{2\sqrt[3]{x}} - \dfrac{5}{\sqrt[6]{x^{11}}}$

c. $f'(x) = 7\sqrt{x^5} - \dfrac{15}{4\sqrt[4]{x}} + \dfrac{6}{\sqrt[3]{x}} + \dfrac{5}{\sqrt[6]{x^{11}}}$

d. $f'(x) = 14x^3 - 15\sqrt{x} + 18\sqrt[3]{x} + \dfrac{30}{\sqrt[3]{x^2}}$

5 What is the derivative of $f(x) = 3x^4$?

a. $f'(x) = 3x^3$

b. $f'(x) = 12x$

c. $f'(x) = 12x^3$

d. $f'(x) = 4x$

6 Find the derivative. $f(x) = 4\sqrt{x} + 3$

a. $f'(x) = \dfrac{4}{\sqrt{x}}$

b. $f'(x) = \dfrac{4}{\sqrt{x}} + \dfrac{1}{3}$

c. $f'(x) = 2$

d. $f'(x) = \dfrac{2}{\sqrt{x}}$

7 Find the derivative. $f(x) = 99x$

a. $f'(x) = 99$

b. $f'(x) = x$

c. $f'(x) = 99x$

d. $f'(x) = 100$

8 Differentiate. $f(x) = 2x^6$

a. $f'(x) = 6$

b. $f'(x) = 6x^5$

c. $f'(x) = 12x^5$

d. $f'(x) = 12$

9 Calculate the derivative. $f(x) = 8x^2 - 5x + 3$

a. $f'(x) = 8x$

b. $f'(x) = 16x - 5x$

c. $f'(x) = 2x - 5$

d. $f'(x) = 16x - 5$

10 Find the derivative. $f(x) = 4x^3 - 3\sqrt{x^3} + \sqrt{x}$

a. $f'(x) = 12x^2 - \dfrac{9}{2}\sqrt{x} + \dfrac{1}{2\sqrt{x}}$

b. $f'(x) = 12x^2 + \dfrac{2}{\sqrt{x}}$

c. $f'(x) = 4x^2 - 9\sqrt{x} + 1$

d. $f'(x) = 64x^2 + 9\sqrt{x} - \dfrac{1}{\sqrt{x}}$

11 What is the slope of the tangent line of the function $f(x) = 2x^3 - 4x^2 - 3x$ at $x = 2$?

a. 2
b. 4
c. 5
d. 7

12 Find $f'(1)$ when $f(x) = \dfrac{x-7}{x+2}$.

a. 1
b. 2
c. 7
d. 10

13 What is the derivative of $f(x) = \sin(3x^2)$?

a. $f'(x) = 3x^2 \cos(3x^2)$
b. $f'(x) = 6x \cos(3x^2)$
c. $f'(x) = 3x^2 \sin(3x^2)$
d. $f'(x) = 6x \sin(3x^2)$

14 What is the second derivative of $f(x) = 3x^3 + 2x^2$?

a. $f''(x) = 18x + 4$
b. $f''(x) = 9x^2 + 4x$
c. $f''(x) = 18$
d. $f''(x) = 0$

15 Find the derivative. $f(x) = \sqrt[5]{x^7}$

a. $f'(x) = \sqrt[5]{7x^6}$
b. $f'(x) = \dfrac{7}{5}\sqrt[5]{x^2}$
c. $f'(x) = \sqrt[4]{x^6}$
d. $f'(x) = \sqrt[5]{x^2}$

16

Find $f'(0)$ if $f(x) = 7xe^{x^2}$.

a. e

b. 6

c. 7

d. 8

17 Differentiate. $f(x) = 3x^{-2} + 5x^{-3} - 2x^{-5}$

a. $f'(x) = -6x^{-1} - 15x^{-2} + 10x^{-4}$

b. $f'(x) = 3x^{-3} + 5x^{-4} - 2x^{-6}$

c. $f'(x) = 3x^{-1} + 5x^{-2} - 2x^{-4}$

d. $f'(x) = -6x^{-3} - 15x^{-4} + 10x^{-6}$

18 What is the derivative of $f(x) = 2\sqrt{x^7} - 5\sqrt[4]{x^3} + 9\sqrt[3]{x^2} - \frac{6}{\sqrt[6]{x^5}}$?

a. $f'(x) = x^3 - \frac{5}{2}\sqrt{x} + \frac{9}{2}\sqrt[6]{x} + \frac{3}{\sqrt[3]{x^4}}$

b. $f'(x) = 7\sqrt{x^5} - \frac{20}{3\sqrt[4]{x}} + \frac{3}{2\sqrt[3]{x}} - \frac{5}{\sqrt[6]{x^{11}}}$

c. $f'(x) = 7\sqrt{x^5} - \frac{15}{4\sqrt[4]{x}} + \frac{6}{\sqrt[3]{x}} + \frac{5}{\sqrt[6]{x^{11}}}$

d. $f'(x) = 14x^3 - 15\sqrt{x} + 18\sqrt[3]{x} + \frac{30}{\sqrt[3]{x^2}}$

19 Find $\frac{d}{dx}\left(\ln e^{2x}\right)$.

a. $\frac{1}{e^{2x}}$

b. $\frac{2}{e^{2x}}$

c. $2x$

d. 1

e. 2

20 The slope of the line tangent to the graph of $\ln(xy) = x$ at the point where $x = 1$ is which of the following?

a. 0
b. 1
c. e
d. e^2
e. $1 - e$

21 If $y = x^2 e^x$, then $\frac{dy}{dx} =$

a. $2xe^x$
b. $x(x + 2e^x)$
c. $xe^x(x + 2)$
d. $2x + e^x$
e. $2x + e$

22 If $y = x^5 \cos x$, find $\frac{d^2y}{dx^2}$.

a. $-20x^3 \cos x$
b. $20x^3 \cos x - x^5 \cos x$
c. $20x^3 \cos x - 10x^4 \sin x - x^5 \cos x$
d. $20x^3 \cos x + x^5 \cos x$
e. $20x^3$

23 Differentiate. $f(x) = 2x^6$

a. $f'(x) = 6$

b. $f'(x) = 6x^5$

c. $f'(x) = 12x^5$

d. $f'(x) = 12$

24 If $y = 10^{x^2-1}$, then $\frac{dy}{dx} =$

a. $(\ln 10)10^{x^2-1}$

b. $(2x)10^{x^2-1}$

c. $\left(x^2 - 1\right)10^{x^2-2}$

d. $2x(\ln 10)10^{x^2-1}$

e. $x^2(\ln 10)10^{x^2-1}$

25 If $x^2 + xy + y^3 = 0$, then, in terms of x and y, $\frac{dy}{dx} =$

a. $-\frac{2x + y}{x + 3y^2}$

b. $-\frac{x + 3y^2}{2x + y}$

c. $\frac{-2x}{1 + 3y^2}$

d. $\frac{-2x}{x + 3y^2}$

e. $-\frac{2x + y}{x + 3y^2 - 1}$

ANSWERS

1. **b**
2. **a**
3. **c**
4. **c**
5. **c**
6. **d**
7. **a**
8. **c**
9. **d**
10. **a**
11. **c**
12. **a**
13. **b**
14. **a**
15. **b**
16. **c**
17. **d**
18. **c**
19. **e**
20. **a**
21. **c**
22. **c**
23. **c**
24. **d**
25. **a**