MID-TERM REVIEW *Worksheet Package*

Part A:

1. The Calculus 12 Mid-Term test is a cumulative exam based on the concepts in Unit I, II & III. Since there was no assessment for Unit III (Limits), the Mid-Term will consist of a higher percentage compared to Unit I & II. As it is a cumulative exam it may only be written once!

2. Go over your Unit I and Unit II tests paying special attention to your mistakes and questions that you feel that you did not fully understand.

Part B: ***DO NOT SIMPLIFY PRODUCT or QUOTIENT RULE QUESTIONS

1. Find the first derivative of each of the following functions:

a) $y = 8x^5 - 3x^7 + \frac{2}{3}x^6 - 5x$

b) $f(x) = 4\pi x^5 - 2\pi^3 + 6\pi^2 x^5$

c) $h(x) = ax^2 + b^2x^3 + c^3$

d) $y = \frac{5}{x^2} - 6x^{-3} + \frac{2}{x} - 5$

e) $y = 6\sqrt{x} - \frac{5}{\sqrt{x}} + \frac{2}{\sqrt[3]{x}} - \frac{3}{4\sqrt{x}}$

f) $p(x) = \frac{-2x^3}{x - 3x^2}$

g) $y = (2x^2 + 3)(4x - 5)$

h) $(x^2 - 5x + 7)^3$

2. If $y = 8x^6 - 7x^2 + 5x$ find:

$a)\ \dfrac{d^2y}{dx^2}$ $\quad b)\ \left(\dfrac{dy}{dx}\right)^2$ $\quad c)\ y''$ $\quad d)\ \dfrac{d^3y}{dx^3}$

3. If $y = 5K^4x^3 - 2mx^2 + 6K$ find:

$a)\ \dfrac{dy}{dx}$ $\quad b)\ \dfrac{dy}{dK}$ $\quad c)\ \dfrac{dy}{dp}$ $\quad d)\ \dfrac{dy}{dm}$

4. Find the first derivative of each of the following:

$a)\ y = \sqrt{5x^2 - 7x}$

$b)\ y = \left(x^2 - 1\right)^3 \left(2x + 5\right)^2$

$c)\ f(x) = \dfrac{4x^2}{\left(1 - 3x^3\right)^2}$

$d)\ y = \dfrac{1}{\sqrt{4 - u^2}}$

e) $y = 4m^2(3m-2)^5$

f) $h(x) = \left(x + \frac{1}{\sqrt{x}} - \frac{2}{\sqrt[3]{x}}\right)$

g) $p(x) = \sqrt{x}\left(2x - 3x^2\right)^4$

h) $y = \left(\frac{x^2-2}{x^2+2}\right)^2$

5. *If* $s(t) = 6t^4 - 8t + 9$ *is a position function describing motion in a straight line, find the velocity and acceleration as functions of time t.*

6. *If* $x^2 + y^2 = z^2$ *and* $\frac{dx}{dt} = -3,\ \frac{dy}{dt} = 5,\ x = 3$ *and* $y = -4$, *find* $\frac{dz}{dt}$.

7. *How fast is the radius of a circular oil slick changing when the radius is 10 m and the area is increasing at a rate of 100 m²/s ?*

8. *Find the first derivative of each of the following functions:*

$a)\ y = 4x^2 + 2y^2 + 8$

$b)\ y = 6xy^2$

$c)\ f(x) = \cos\left(2x^4\right)$

$d)\ h(x) = \sin^4\left(2x^2 - 5x\right)$

$e)\ m(x) = x\sin x^2$

$f)\ y = \ln\left(2x^2 - 5x\right)$

$g)\ g(x) = \left(x\ln x\right)^3$

$h)\ f(x) = 3^{2x-5}$

9. *Find* $\frac{dy}{dx}$ *for each of the following.*

$a)\ y = x^{\pi} + e^{x} + e^{5} + \pi^{6}$

$b)\ y = e^{\pi x} - e^{\pi} + e^{x} - x^{e}$

$c)\ y = xe^{x} + \pi e^{\pi}$

$d)\ y = e^{6x} - 6^{x} + 6e^{6x}$

10. *Find $\frac{dy}{dx}$ for each of the following.*

$a)\ \ y = \dfrac{\cos 3x}{\sin 3x}$

$b)\ \ y = \tan(x^3) - \tan^3(x)$

$c)\ \ y = \sqrt{x}\ e^{\sqrt{x}}$

$d)\ \ y = Ax^3 - 2Bx^2 + 3c^4x$

$e)\ \ y + e^x = 3y^2$

$f)\ \ y = 5x - \dfrac{2}{x^4} + \dfrac{6}{\sqrt{x}} - \dfrac{2}{5\sqrt{x}}$

$g)\ \ y = \ln\left(x^3 - 2x + 7\right)$

$h)\ \ y = \left(x^2 - 3x^5 + 2\right)^{-3}$

$i)\ \ 6y - 4x^2 = 3xy$

$j)\ \ y = 6^{x^3}$

$k)\ \ y = \sin(xy)$

$l)\ \ y = 5x\sin 4x + \sin^3 x - \sin x^3$

Part C:

3. *If* $y = 6 + 2u^2, \quad u = 4v^2 - 3v, \quad v = -3 + 2x^2, \quad find\ \frac{dy}{dx}\ \ at\ x = -1.$

8. *Find the general antiderivative of each of the following functions:*

$a)\ 6x^3 - 2x^2 + \frac{5}{x^2}$

$b)\ 3e^{4x} - 5e^x$

$c)\ \sin(3x) + 3x$

$d)\ (\cos x)(\sin^3 x)$

$e)\ x^2 \sec^2(x^3)$

$f)\ (3x^2 + 2x)e^{(x^3 + x^2)}$

9. *Find the position function(s) for an object with acceleration function a(t) = t – 2, initial velocity v(0) = 2 and initial position s(0) = 10.*

10. *Simplify:*

a) $\int x^3 \, dx$

b) $\int \pi x^2 \, dx$

c) $\int \frac{1}{1-x} \, dx$

d) $\int e^{4x} - x^2 \, dx$

e) $\int 5y^2 \, dx$

f) $\int \sin(\pi x) \, dx$

* use u-substitution for *g* to *j*

g) $\int (2x+3)^7 \, dx \;,\; u = 2x+3$

h) $\int \frac{1}{(3x-2)^8} \, dx \;,\; u = 3x-2$

i) $\int x^3 \sqrt{2-3x^4} \, dx$

j) $\int \frac{3x}{4x^2-3} \, dx$

11. *Find the <u>exact</u> value of each of the following:*

$a)\ \int_{1}^{\sqrt{2}} 3x \ \ dx$

$b)\ \int_{\pi}^{4\pi} 2e^{x} \ \ dx$

$c)\ \int_{1}^{6} 3m \ \ dx$

$d)\ \int_{0}^{\frac{\pi}{2}} 2\cos x \ \ dx$

$e)\ \int_{-2}^{1} xe^{x^2} \ \ dx$

$f)\ \int_{e}^{4} \frac{\ln x}{x} \ \ dx$

12. *Simplify:*

$a)\ \int_{a}^{b} 4x^2 \ \ dx$

$b)\ \int_{\pi}^{3} \frac{5}{x} \ \ dx$

13. *Find the exact area between y = 4x and the x-axis over the interval* $-2 \le x \le 4$.

14. *Find the exact area between* $y = 4x^2$ *and the y-axis over the interval* $0 \le x \le 3$.

15. *Find the exact area between the two curves*
$f(x) = x - 3$ and $g(x) = -x^2 - 6$ over the interval $-2 \le x \le 5$.

16. *Find the exact area enclosed by the two curves* $f(x) = 6x$ and $g(x) = x^2$.

18. *Simplify:*

a) $\int_{a}^{b} 4x \; dx$

b) $\int_{a}^{2} e^{x} \; dx$

c) $\int_{2m}^{5m} y \; dx$

d) $\int_{-2a}^{5a} k^{2} \; dx$

19. *Solve for x:*

a) $\int_{1}^{x} 2m \; dm = 8$

b) $\int_{x}^{5} 2t + 3 \; dt = 40$

Part D:

1. Evaluate each of the following limits:

$a)\ \lim_{x\to -1} 2x^2-5x+3$

$b)\ \lim_{x\to e}\left(x^3-2x\right)$

$c)\ \lim_{x\to a}\frac{(x+a)^2}{\left(x^2+2a^2\right)}$

$d)\ \lim_{x\to 3}\frac{x^2-9}{x-3}$

$e)\ \lim_{x\to 2}\frac{x-2}{x^3-8}$

$f)\ \lim_{x\to 0}\frac{2\cos x-2}{2x}$

$g)\ \lim_{x\to 0}\frac{\cos x-1}{2e^x-2}$

$h)\ \lim_{x\to 4}\frac{\sqrt{x}-2}{x-4}$

2. If $\lim_{x\to 2} f(x)=1$, use the properties of limits to evaluate each limit:

$a)\ \lim_{x\to 2}\frac{x^2-2}{4f(x)}$

$b)\ \lim_{x\to 2}\sqrt{[f(x)]^2+10-x}$

3. *Use the definition of the derivative to find* $\frac{dy}{dx}$:

$a)\ \ y = \frac{1}{x+3}$

$b)\ \ y = x^2 - 3x$

6. *Evaluate each of the following limits:*

a) $\lim_{x \to \pi} 2x^2 + 5\pi x - 3\pi^2$

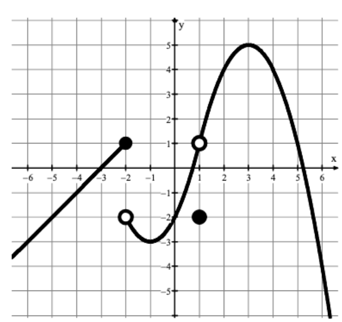
b) $\lim_{x \to 4} \frac{x-4}{\sqrt{x} - 2}$

c) $\lim_{x \to 0} \frac{\sqrt{4+x} - 2}{x}$

d) $\lim_{x \to 0} \frac{e^x - 1}{\sin x}$

7. Give the value of each statement. If the value does not exist, write "does not exist" or "undefined."

a. $\lim_{x \to -2^-} f(x) =$	b. $\lim_{x \to -2^+} f(x) =$	c. $\lim_{x \to -2} f(x) =$
d. $\lim_{x \to 1} f(x) =$	e. $\lim_{x \to 0} f(x) =$	f. $\lim_{x \to 3^-} f(x) =$
g. $\lim_{x \to -1} f(x) =$	h. $f(1) =$	i. $f(-2) =$



8. Evaluate each limit.

a) $\lim_{x \to 3^-} f(x),\ f(x) = \begin{cases} -x+4, & x<3 \\ \dfrac{x}{2}+1, & x \geq 3 \end{cases}$

b) $\lim_{x \to -2^-} f(x),\ f(x) = \begin{cases} -x^2-8x-17, & x \leq -2 \\ 2x-1, & x > -2 \end{cases}$