

MID-TERM REVIEW

Worksheet Package

KEY

Part A:

1. The Calculus 12 Mid-Term test is a cumulative exam based on the concepts in Unit I, II & III. Since there was no assessment for Unit III (Limits), the Mid-Term will consist of a higher percentage compared to Unit I & II. As it is a cumulative exam it may only be written once!

2. Go over your Unit I and Unit II tests paying special attention to your mistakes and questions that you feel that you did not fully understand.

Part B: ***DO NOT SIMPLIFY PRODUCT or QUOTIENT RULE QUESTIONS

1. Find the first derivative of each of the following functions:

a) $y = 8x^5 - 3x^7 + \frac{2}{3}x^6 - 5x$

$$y' = 40x^4 - 21x^6 + 4x^5 - 5$$

b) $f(x) = 4\pi x^5 - 2\pi^3 + 6\pi^2 x^5$

$$f'(x) = 20\pi x^4 + 30\pi^2 x^4$$

c) $h(x) = ax^2 + b^2x^3 + c^3$

$$h'(x) = 2ax + 3b^2x^2$$

d) $y = \frac{5}{x^2} - 6x^{-3} + \frac{2}{x} - 5$ RW $5x^{-2} - 6x^{-3} + 2x^{-1}$

$$y' = -10x^{-3} + 18x^{-4} - 2x^{-2}$$

e) $y = 6\sqrt{x} - \frac{5}{\sqrt{x}} + \frac{2}{\sqrt[3]{x}} - \frac{3}{4\sqrt{x}}$ RW $6x^{1/2} - 5x^{-1/2} + \frac{2}{3}x^{-1/3} - \frac{3}{4}x^{-1/2}$

$$y' = 3x^{-1/2} + \frac{5}{2}x^{-3/2} - \frac{2}{3}x^{-4/3} + \frac{3}{8}x^{-3/2}$$

$$y' = 3x^{-1/2} + \frac{23}{8}x^{-3/2} - \frac{2}{3}x^{-4/3}$$

g) $y = (2x^2 + 3)(4x - 5)$ FS' + F'S

$$y' = (2x^2 + 3)(4) + (4x)(4x - 5)$$

f) $p(x) = \frac{-2x^3}{x - 3x^2}$ BT' - BT
B²

$$p'(x) = \frac{(x - 3x^2)(-6x^2) - (1 - 6x)(-2x^3)}{(x - 3x^2)^2}$$

h) $(x^2 - 5x + 7)^3$

$$3(x^2 - 5x + 7)^2 \cdot (2x - 5)$$

2. If $y = 8x^6 - 7x^2 + 5x$ find: $48x^5 - 14x + 5$

a) $\frac{d^2y}{dx^2}$ b) $\left(\frac{dy}{dx}\right)^2$ c) y'' d) $\frac{d^3y}{dx^3}$

$= 240x^4 - 14$ $(48x^5 - 14x + 5)^2$ Same as a) $960x^3$

3. If $y = 5K^4x^3 - 2mx^2 + 6K$ find:

a) $\frac{dy}{dx}$ b) $\frac{dy}{dK}$ c) $\frac{dy}{dp}$ d) $\frac{dy}{dm}$

$15K^4x^2 - 4mx$ $20K^3x^3 + 6$ 0 $-2x^2$

4. Find the first derivative of each of the following:

a) $y = \sqrt{5x^2 - 7x}$ $(5x^2 - 7x)^{1/2}$

b) $y = (x^2 - 1)^3 (2x + 5)^2$ $F_1 S' + F_2 S'$

$y' = \frac{1}{2}(5x^2 - 7x)^{-1/2} (10x - 7)$ $y' = (x^2 - 1)^3 \cdot (2(2x + 5)(2)) + 3(x^2 - 1)^2 (2x)(2x + 5)^2$

c) $f(x) = \frac{4x^2}{(1 - 3x^3)^2}$ $(1 - 3x^3)^{-2}$

d) $y = \frac{1}{\sqrt{4 - u^2}}$ $(4 - u^2)^{-1/2}$

$f'(x) = (4x^2)(-2(1 - 3x^3)^{-3}(-9x^2)) + (8x)(1 - 3x^3)^2$

$y' = -\frac{1}{2}(4 - u^2)^{-3/2}(-2u)$

$$e) y = 4m^2(3m-2)^5$$

$$y' = 4m^2 \cdot (5(3m-2)^4(3)) + 8m(3m-2)^5$$

$$f) h(x) = \left(x + \frac{1}{\sqrt{x}} - \frac{2}{\sqrt[3]{x}} \right)$$

$$x + x^{-1/2} - 2x^{-1/3}$$

$$h'(x) = 1 - \frac{1}{2}x^{-3/2} + \frac{2}{3}x^{-4/3}$$

$$g) p(x) = \sqrt{x}(2x-3x^2)^4$$

$$p'(x) = x^{1/2} \cdot (4(2x-3x^2)^3(2-6x)) + \frac{1}{2}x^{-1/2}(2x-3x^2)^4$$

$$h) y = \left(\frac{x^2-2}{x^2+2} \right)^2$$

$$2 \left(\frac{x^2-2}{x^2+2} \right)$$

See below

$$y' = 2 \left(\frac{x^2-2}{x^2+2} \right) \left(\frac{(x^2+2)(2x) - (2x)(x^2-2)}{(x^2+2)^2} \right)$$

$$2 \cdot \frac{-1}{3}$$

$$\frac{3-2}{6+2} = \frac{2(-1)}{8}$$

$$\frac{-2}{3} \cdot \frac{8}{9}$$

$$\frac{-16}{27}$$

5. If $s(t) = 6t^4 - 8t + 9$ is a position function describing motion in a straight line, find the velocity and acceleration as functions of time t .

$$v(t) = 24t^3 - 8$$

$$a(t) = 72t^2$$

6. If $x^2 + y^2 = z^2$ and $\frac{dx}{dt} = -3$, $\frac{dy}{dt} = 5$, $x = 3$ and $y = -4$, find $\frac{dz}{dt}$.

$$z = \sqrt{3^2 + (-4)^2}$$

$$z = 5$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}$$

$$2(3)(-3) + 2(-4)(5) = 2(5) \frac{dz}{dt}$$

$$-18 - 40 = 10 \frac{dz}{dt}$$

$$\frac{dz}{dt} = -5.8$$

7. How fast is the radius of a circular oil slick changing when the radius is 10 m and the area is increasing at a rate of 100 m²/s?



$$A = \pi r^2$$

$$r = 10$$

$$\frac{dA}{dt} = 100$$

Find $\frac{dr}{dt}$ @ $r=10$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt} \Rightarrow 100 = 2\pi(10) \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{5}{\pi} \text{ m/s}$$

8. Find the first derivative of each of the following functions:

a) $y = 4x^2 + 2y^2 + 8$

$$y' = 8x + 4yy'$$

$$y' - 4yy' = 8x$$

$$y' \left(\frac{1-4y}{1-4y} \right) = \frac{8x}{1-4y}$$

$$y' = \frac{8x}{1-4y}$$

b) $y = 6xy^2$

$$y' = (6x)(2yy') + (6)(y^2)$$

$$y' = 12xyy' + 6y^2$$

$$y' - 12xyy' = 6y^2$$

$$y' (1 - 12xy) = \frac{6y^2}{1 - 12xy}$$

$$y' = \frac{6y^2}{1 - 12xy}$$

c) $f(x) = \cos(2x^4)$

$$f'(x) = -\sin(2x^4)(8x^3)$$

$$f'(x) = -8x^3 \sin(2x^4)$$

d) $h(x) = (\sin^4(2x^2 - 5x))^5$

$$h'(x) = 4(\sin^4(2x^2 - 5x))^3 \cdot \cos(2x^2 - 5x)(4x - 5)$$

e) $m(x) = x \sin x^2$

$$(x)(\cos(x^2)(2x)) + (1)\sin x^2$$

$$m'(x) = 2x^2 \cos(x^2) + \sin(x^2)$$

f) $y = \ln(2x^2 - 5x)$

$$\frac{1}{2x^2 - 5x} \cdot (4x - 5)$$

$$y' = \frac{4x - 5}{2x^2 - 5x}$$

g) $g(x) = (x \ln x)^3$

$$3(x \ln x)^2 \cdot (x \cdot \frac{1}{x}) + (1)(\ln x)$$

$$g'(x) = 3(x \ln x)^2 \cdot (1 + \ln x)$$

h) $f(x) = 3^{2x-5}$

$$f'(x) = (\ln 3)(3^{2x-5})(2)$$

9. Find $\frac{dy}{dx}$ for each of the following.

a) $y = x^\pi + e^x + e^5 + \pi^6$

$$y' = \pi x^{\pi-1} + e^x$$

b) $y = e^{\pi x} - e^\pi + e^x - x^e$

$$y' = e^{\pi x} \cdot \pi + e^x - ex^{e-1}$$

c) $y = xe^x + \pi e^\pi$

$$y' = xe^x + e^x$$

d) $y = e^{6x} - 6^x + 6e^{6x}$

$$y' = 6e^{6x} - \ln 6 \cdot 6^x + 36e^{6x}$$

10. Find $\frac{dy}{dx}$ for each of the following.

a) $y = \frac{\cos 3x}{\sin 3x}$ $\frac{BT' - BT}{B^2}$

$$y' = \frac{\sin(3x)(-3\sin(3x)) - (3\cos(3x))(\cos(3x))}{\sin^2(3x)}$$

c) $y = \sqrt{x} e^{\sqrt{x}}$ $x^{1/2} \cdot e^{x^{1/2}}$

$$y' = (x^{1/2})(e^{x^{1/2}})(\frac{1}{2}x^{-1/2})$$

b) $y = \tan(x^3) - (\tan^3(x))^3$

$$y' = \sec^2(x^3)(3x^2) - (3(\tan(x))^2 \cdot (\sec^2(x)))$$

d) $y = Ax^3 - 2Bx^2 + 3C^4x$

$$y' = 3Ax^2 - 4Bx + 3C^4$$

e) $y + e^x = 3y^2$

$$y' + e^x = 6yy'$$

$$e^x = 6yy' - y'$$

$$y'(6y - 1)$$

$$y' = \frac{e^x}{6y - 1}$$

g) $y = \ln(x^3 - 2x + 7)$

$$y' = \frac{1}{x^3 - 2x} \cdot 3x^2 - 2$$

$$y' = \frac{3x^2 - 2}{x^3 - 2x}$$

i) $6y - 4x^2 = 3xy$

$$6y' - 8x = 3xy' + 3y$$

$$6y' - 3xy' = \frac{8x + 3y}{6 - 3x}$$

$$y' = \frac{8x + 3y}{6 - 3x}$$

k) $y = \sin(xy)$

$$y' = \cos(xy)(xy' + y)$$

$$y' = xy' \square + y \square$$

$$y' - xy' \square = y \square$$

$$y'(1 - x \square) = \frac{y \square}{1 - x \square}$$

$$y' = \frac{y \cos(xy)}{1 - x \cos(xy)}$$

l) $y = 5x \sin 4x + (\sin^3 x)^3 - \sin(x^3)$

$$y' = (5x) \cdot 4 \cos(4x) + (5)(\sin(4x))(3(\sin(x))^2 \cos(x)) - 3x^2 \cos(x^3)$$

f) $y = 5x - \frac{2x^{-4}}{x^4} + \frac{6x^{-1/2}}{\sqrt{x}} - \frac{2}{5\sqrt{x}} x^{-1/2}$

$$y' = 5 + 8x^{-5} - 3x^{-3/2} + \frac{1}{10}x^{-3/2}$$

$$y' = 5 + 8x^{-5} - \frac{29}{10}x^{-3/2}$$

h) $y = (x^2 - 3x^5 + 2)^{-3}$

$$y' = -3(x^2 - 3x^5 + 2)^{-4}(2x - 15x^4)$$

j) $y = 6^{x^3}$

$$y' = \ln(6) \cdot 6^{x^3} \cdot 3x^2$$

Part C:

3. If $y = 6 + 2u^2$, $u = 4v^2 - 3v$, $v = -3 + 2x^2$, find $\frac{dy}{dx}$ at $x = -1$.
 $u = 7$ $v = -1$

$$\frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dx} = \frac{dy}{dx}$$

$$4u \cdot (8v - 3) \cdot 4x$$

$$4(7) \cdot (8(-1) - 3) \cdot 4(-1)$$

$$28 \cdot (-11) \cdot -4$$

$$= 1056$$

8. Find the general antiderivative of each of the following functions:

DON'T FORGET $+ C$

a) $6x^3 - 2x^2 + \frac{5}{x^2} x^{-2}$

$$\frac{3}{2}x^4 - \frac{2}{3}x^3 - 5x^{-1} + C$$

b) $3e^{4x} - 5e^x$

$$\frac{3}{4}e^{4x} - 5e^x + C$$

c) $\sin(3x) + 3x$

$$-\frac{1}{3}\cos(3x) + \frac{3}{2}x^2 + C$$

d) $\int (\cos x)(\sin^3 x) dx$ $u = \sin x$
 $du = \cos x dx$
 $dx = \frac{du}{\cos x}$

$$\int \cos x \cdot u^3 \cdot \frac{du}{\cos x}$$

$$\frac{1}{4}(\sin x)^4 + C$$

e) $x^2 \sec^2(x^3)$ $3x^2$

$$\frac{1}{3}\tan(x^3) + C$$

f) $(3x^2 + 2x)e^{(x^3 + x^2)}$ $\frac{1}{3x^2 + 2x}$

$$e^{x^3 + x^2} + C$$

9. Find the position function(s) for an object with acceleration function $a(t) = t - 2$, initial velocity $v(0) = 2$ and initial position $s(0) = 10$.

$$v(t) = \frac{1}{2}t^2 - 2t + 2$$

$$s(t) = \frac{1}{6}t^3 - t^2 + 2t + 10$$

10. Simplify: $+ C$

a) $\int x^3 dx$

$$\frac{1}{4}x^3 + C$$

b) $\int \pi x^2 dx$

$$\frac{1}{3}\pi x^3 + C$$

c) $\int \frac{1}{1-x} dx$

$$-\ln(1-x) + C$$

d) $\int e^{4x} - x^2 dx$

$$\frac{1}{4}e^{4x} - \frac{1}{3}x^3 + C$$

e) $\int 5y^2 dx$ don't match

$$5y^2x + C$$

f) $\int \sin(\pi x) dx$

$$-\frac{1}{\pi} \cos(\pi x) + C$$

* use u-substitution for g to j

g) $\int (2x+3)^7 dx$, $u = 2x+3$

$$\int u^7 \quad \begin{matrix} du = 2 dx \\ dx = \frac{du}{2} \end{matrix}$$

$$\frac{1}{2} \cdot \frac{1}{8} (2x+3)$$

$$\frac{1}{16} (2x+3) + C$$

h) $\int \frac{1}{(3x-2)^8} dx$, $u = 3x-2$

$$\frac{1}{3} \int u^{-8} \quad \begin{matrix} du = 3 dx \\ dx = \frac{du}{3} \end{matrix}$$

$$\frac{1}{3} \cdot \frac{-1}{-7} (u)^{-7}$$

$$-\frac{1}{21} (3x-2)^{-7} + C$$

i) $\int x^3 \sqrt{2-3x^4} dx$

$$u = 2-3x^4$$

$$du = -12x^3 dx$$

$$dx = \frac{du}{-12x^3}$$

$$\int \cancel{x^3} u^{1/2} \frac{du}{-12\cancel{x^3}}$$

$$-\frac{1}{12} \int u^{1/2} du$$

$$-\frac{1}{6 \cdot \frac{1}{2} \cdot 3} u^{3/2}$$

$$= -\frac{1}{18} (2-3x^4)^{3/2} + C$$

j) $\int \frac{3x}{4x^2-3} dx$

$$u = 4x^2-3$$

$$du = 8x dx$$

$$dx = \frac{du}{8x}$$

$$\int \frac{3\cancel{x}}{u} \cdot \frac{du}{8\cancel{x}}$$

$$\frac{3}{8} \int u^{-1}$$

$$\frac{3}{8} \ln(4x^2-3) + C$$

11. Find the exact value of each of the following:

$$a) \int_1^{\sqrt{2}} 3x \, dx \quad \frac{3}{2} x^2 \Big|_1^{\sqrt{2}}$$

$$\frac{3}{2} (\sqrt{2})^2 - \frac{3}{2} (1)^2$$

$$3 - \frac{3}{2} = \frac{3}{2}$$

$$b) \int_{\pi}^{4\pi} 2e^x \, dx$$

$$2e^x \Big|_{\pi}^{4\pi}$$

$$2e^{4\pi} - 2e^{\pi}$$

$$c) \int_1^6 3m \, dx$$

$$3m x \Big|_1^6$$

$$[3m(6)] - [3m(1)]$$

$$18m - 3m$$

$$= 15m$$

$$d) \int_0^{\frac{\pi}{2}} 2\cos x \, dx$$

$$= 2$$

$$e) \int_{-2}^1 x e^{x^2} \, dx \quad \text{"u-sub"}$$

$$\frac{e}{2} - \frac{e^4}{2}$$

or

$$-\frac{e^4 - e}{2}$$

$$f) \int_e^4 \frac{\ln x}{x} \, dx \quad \text{"u-sub"}$$

$$\frac{\ln^2(4)}{2} - \frac{1}{2}$$

or

$$\frac{\ln^2(4) - 1}{2}$$

12. Simplify:

$$a) \int_a^b 4x^2 \, dx$$

$$\frac{4}{3} x^3 \Big|_a^b$$

$$\frac{4}{3} b^3 - \frac{4}{3} a^3$$

$$b) \int_{\pi}^3 \frac{5}{x} \, dx$$

$$5 \ln x \Big|_{\pi}^3$$

$$5 \ln(3) - 5 \ln(\pi)$$

13. Find the exact area between $y = 4x$ and the x -axis over the interval $-2 \leq x \leq 4$.

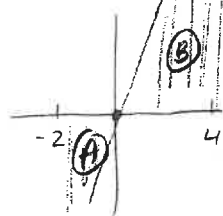
$$\int_{-2}^0 4x \, dx$$

$$2x^2 \Big|_{-2}^0$$

$$[0] - [2(-2)^2] = -8$$

$$A + B$$

$$8 + 32$$

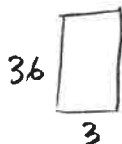
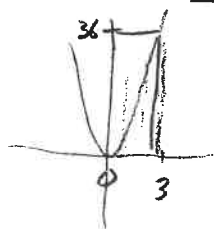


$$\int_0^4 4x \, dx$$

$$2x^2 \Big|_0^4$$

$$= 40 u^2$$

14. Find the exact area between $y = 4x^2$ and the y -axis over the interval $0 \leq x \leq 3$.



$$108 - 36$$

$$- \int_0^3 4x^2 \, dx$$

$$\frac{4}{3} x^3 \Big|_0^3$$

$$= 72 u^2$$

15. Find the exact area between the two curves

$f(x) = x - 3$ and $g(x) = -x^2 - 6$ over the interval $-2 \leq x \leq 5$.

$$\int_{-2}^5 (x - 3 + x^2 + 6) \, dx$$

$$\int_{-2}^5 (x^2 + x + 3) \, dx$$

$$\frac{1}{3} x^3 + \frac{1}{2} x^2 + 3x \Big|_{-2}^5$$

$$\left[\frac{1}{3}(5)^3 + \frac{1}{2}(5)^2 + 3(5) \right] - \left[\frac{1}{3}(-2)^3 + \frac{1}{2}(-2)^2 + 3(-2) \right] = \frac{455}{6} u^2$$

16. Find the exact area enclosed by the two curves $f(x) = 6x$ and $g(x) = x^2$.

$$\int_0^6 (6x - x^2) \, dx$$

$$6x = x^2$$

$$x^2 - 6x$$

$$x(x - 6) = 0$$

$$-\frac{1}{3} x^3 + 3x^2 \Big|_0^6$$

$$\left[-\frac{1}{3}(6)^3 + 3(6)^2 \right] - [0] = 36$$

$$= 36 u^2$$

18. Simplify:

$$a) \int_a^b 4x \, dx \quad 2x^2 \Big|_a^b$$

$$2b^2 - 2a^2$$

$$c) \int_{2m}^{5m} y \, dx$$

$$yx \Big|_{2m}^{5m}$$

$$5my - 2my$$

$$= 3my$$

$$b) \int_a^2 e^x \, dx$$

$$e^x \Big|_a^2$$

$$e^2 - e^a$$

$$d) \int_{-2a}^{5a} k^2 \, dx$$

$$k^2 x \Big|_{-2a}^{5a}$$

$$5ak^2 + 2ak^2$$

$$7ak^2$$

19. Solve for x:

$$a) \int_1^x 2m \, dm = 8$$

$$m^2 \Big|_1^x$$

$$x^2 - 1 = 8$$

$$\sqrt{x^2} = \sqrt{9}$$

$$x = \pm 3$$

$$b) \int_x^5 2t + 3 \, dt = 40$$

$$t^2 + 3t \Big|_x^5$$

$$[5] - [x]$$

$$40 - [x^2 + 3x] = 40$$

$$-x^2 - 3x = 0$$

$$-x(x + 3) = 0$$

$$x = 0, -3$$

Part D:

1. Evaluate each of the following limits:

a) $\lim_{x \rightarrow -1} 2x^2 - 5x + 3$

$$\begin{array}{r} 2(-1)^2 - 5(-1) + 3 \\ 2 + 5 + 3 \\ = 10 \end{array}$$

b) $\lim_{x \rightarrow e} (x^3 - 2x)$

$$e^3 - 2e$$

c) $\lim_{x \rightarrow a} \frac{(x+a)^2}{(x^2+2a^2)}$

$$\frac{(2a)^2}{(a^2+2a^2)} = \frac{4a^2}{3a^2} = \frac{4}{3}$$

e) $\lim_{x \rightarrow 2} \frac{x-2}{x^3-8}$

$$\frac{0}{0}$$

$$\lim_{x \rightarrow 2} \frac{1}{3x^2} = \frac{1}{12}$$

d) $\lim_{x \rightarrow 3} \frac{x^2-9}{x-3}$

$$\frac{0}{0} \lim_{x \rightarrow 3} \frac{2x}{1} = 6$$

f) $\lim_{x \rightarrow 0} \frac{2\cos x - 2}{2x}$

$$\frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{-2\sin x}{2} = \frac{0}{2}$$

$$= 0$$

g) $\lim_{x \rightarrow 0} \frac{\cos x - 1}{2e^x - 2}$

$$\frac{0}{0}$$

$$\frac{-\sin x}{2e^x} = \frac{0}{2}$$

$$= 0$$

h) $\lim_{x \rightarrow 4} \frac{\sqrt{x}-2}{x-4}$

$$\frac{0}{0}$$

$$\frac{1}{2} x^{-1/2}$$

$$\frac{1}{2\sqrt{x}} = \frac{1}{4}$$

2. If $\lim_{x \rightarrow 2} f(x) = 1$, use the properties of limits to evaluate each limit:

a) $\lim_{x \rightarrow 2} \frac{x^2-2}{4f(x)}$

$$\lim_{x \rightarrow 2} \frac{x^2-2}{4} = \frac{1}{2}$$

b) $\lim_{x \rightarrow 2} \sqrt{[f(x)]^2 + 10 - x}$

$$\lim_{x \rightarrow 2} \sqrt{11 - x}$$

$$\sqrt{9}$$

$$x = 3$$

3. Use the definition of the derivative to find $\frac{dy}{dx}$:

a) $y = \frac{1}{x+3}$

$$\lim_{h \rightarrow 0} \frac{\frac{1}{x+h+3} - \frac{1}{x+3}}{h}$$

$$\lim_{h \rightarrow 0} \frac{\cancel{x+3} - \cancel{x} - h - 3}{h (x+h+3)(x+3)}$$

$$\lim_{h \rightarrow 0} \frac{-h}{h (x+h+3)(x+3)}$$

$$\lim_{h \rightarrow 0} \frac{-1}{(x+h+3)(x+3)}$$

$$\frac{dy}{dx} = \frac{-1}{(x+3)^2}$$

b) $y = x^2 - 3x$

$$\lim_{h \rightarrow 0} \frac{(x+h)^2 - 3(x+h) - (x^2 - 3x)}{h}$$

$$\lim_{h \rightarrow 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{3x} - 3h - \cancel{x^2} + \cancel{3x}}{h}$$

$$\lim_{h \rightarrow 0} \frac{2xh + h^2 - 3h}{h}$$

$$\lim_{h \rightarrow 0} \frac{h(2x + h - 3)}{h}$$

$$\lim_{h \rightarrow 0} 2x + \cancel{h} - 3$$

$$\frac{dy}{dx} = 2x - 3$$

6. Evaluate each of the following limits:

a) $\lim_{x \rightarrow \pi} 2x^2 + 5\pi x - 3\pi^2$

$$2\pi^2 + 5\pi \cdot \pi - 3\pi^2$$

$$2\pi^2 + 5\pi^2 - 3\pi^2$$

$$4\pi^2$$

b) $\lim_{x \rightarrow 4} \frac{x-4}{\sqrt{x}-2} (\sqrt{x}+2)$

$$\lim_{x \rightarrow 4} \frac{(x-4)(\sqrt{x}+2)}{(\sqrt{x}-2)(\sqrt{x}+2)}$$

$$\lim_{x \rightarrow 4} \frac{x-4}{\sqrt{x}+2}$$

$$= 4$$

c) $\lim_{x \rightarrow 0} \frac{\sqrt{4+x}-2}{x} (\sqrt{4+x}+2)$

$$\lim_{x \rightarrow 0} \frac{4+x-4}{x(\sqrt{4+x}+2)}$$

$$\lim_{x \rightarrow 0} \frac{1}{\sqrt{4+x}+2}$$

$$= \frac{1}{4}$$

d) $\lim_{x \rightarrow 0} \frac{e^x - 1}{\sin x}$

$$\lim_{x \rightarrow 0} \frac{e^x}{\cos x} = \frac{e^0}{\cos(0)} = \frac{1}{1}$$

$$= 1$$

7. Give the value of each statement. If the value does not exist, write "does not exist" or "undefined."

a. $\lim_{x \rightarrow -2^-} f(x) = 1$

b. $\lim_{x \rightarrow -2^+} f(x) = -2$

c. $\lim_{x \rightarrow -2} f(x) = \text{DNE}$

d. $\lim_{x \rightarrow 1} f(x) = 1$

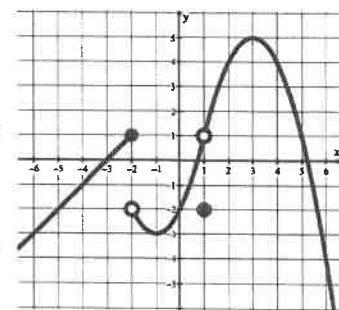
e. $\lim_{x \rightarrow 0} f(x) = -2$

f. $\lim_{x \rightarrow 3^-} f(x) = 5$

g. $\lim_{x \rightarrow -1} f(x) = -3$

h. $f(1) = -2$

i. $f(-2) = 1$



8. Evaluate each limit.

a) $\lim_{x \rightarrow 3^-} f(x), f(x) = \begin{cases} -x+4, & x < 3 \\ \frac{x}{2} + 1, & x \geq 3 \end{cases}$

$$\lim_{x \rightarrow 3^-} -x+4 = 1$$

b) $\lim_{x \rightarrow -2^-} f(x), f(x) = \begin{cases} -x^2 - 8x - 17, & x \leq -2 \\ 2x - 1, & x > -2 \end{cases}$

$$\begin{aligned} \lim_{x \rightarrow -2^-} -x^2 - 8x - 17 \\ = -(-2)^2 - 8(-2) - 17 \\ = -4 + 16 - 17 = -5 \end{aligned}$$