

CALCULUS 12 FINAL MOCK B (2024)

MARKS: Part 1 - MC ____ / 22
Part 2 - WS ____ / 53



Name _____
Date _____

PART 1: Multiple Choice Section:

Fill in the **Scan Tron Sheet** with the best answer. Show your work clearly for each question below. Answers may be affected if not done so.

1.

Given $f(x) = \frac{5}{x^2}$ determine $f'(x)$

- A. $-\frac{10}{x}$ B. $-\frac{10}{x^3}$ C. $\frac{3}{x^3}$ D. $\frac{5}{2x}$ E. *none of these*

2.

Evaluate the derivative of the function $f(x) = 3x^2 - 2x - 1$ at the point where $x = 0$

- A. -2 B. -1 C. $\frac{1}{3}$ D. 1 E. *none of these*

3.

Find all values of x such that the function $f(x) = 2x^3 - 3x^2$ is increasing

- A. $x < 1$ B. $x > 0$ C. $0 < x < 1$ D. $x < 0$ or $x > 1$

At which of the following values of x is the function $g(x) = x^3 - 4x^2$ decreasing?

- A. $x = -3$ B. $x = -1$ C. $x = 2$ D. $x = 4$ E. *none of these*

5.

Find the minimum value of the function $f(x) = 2x^2 - 12x + 6$

- A. -24 B. -12 C. 3 D. 6 E. *none of these*

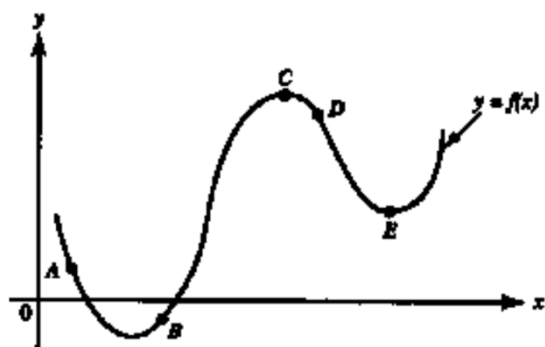
6.

Find the slope of the line tangent to the graph of $y = x^3 - 4x^2 + 2$ at the point where $x = 2$

- A. -10 B. -6 C. -4 D. -2 E. *none of these*

7.

At which point on the graph of $y = f(x)$ is $f'(x) < 0$ and $f''(x) > 0$

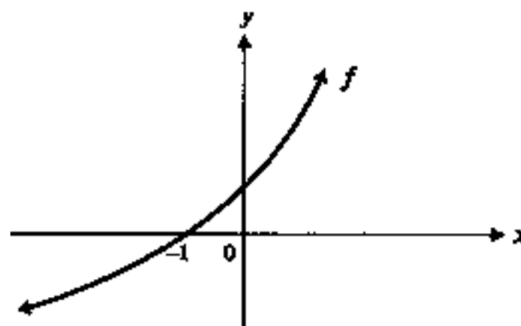


- A. A B. B C. C D. D E. E

8.

The graph of f is shown in the diagram and f is twice differentiable. Which of the following has the smallest value?

- I. $f(-1)$
II. $f'(-1)$
III. $f''(-1)$



- A. I *only* B. II *only* C. III *only* D. I and II E. II and III

9.

If $y = x^2 e^x$ then $\frac{dy}{dx} =$

- A. $2xe^x$ B. $x(x + 2e^x)$ C. $xe^x(x + 2)$ D. $2x + e^x$ E. $2x + e$

10.

If $xy^2 + 2xy = 8$ then at the point $(1, 2)$ $y' =$

- A. $-\frac{5}{2}$ B. $-\frac{4}{3}$ C. -1 D. $-\frac{1}{2}$ E. 0

11.

A function $f(x)$ exists such that $f''(x) = (x-2)^2(x+1)$. How many points of inflection does $f(x)$ have?

- A. none B. one C. two
D. three E. cannot be determined

12.

A particle moves along the x -axis according to the position function $x(t) = 2t^3 - 6t^2 + 9$ where x is in meters and t is in seconds. Find the value(s) of t when the particle is stationary.

- A. $t = 0$ B. $t = 2$ C. $t = 0, t = -2$ D. $t = 0, t = 2$ E. *none of these*

13.

A particle moves along a line according to the distance function $s(t) = 2t^3 - 21t^2 + 60t + 13$. During the time interval from $t = 1$ to $t = 12$, how many times does the particle reverse its direction of movement?

- A. 0 B. 1 C. 2 D. 3 E. 4

14.

Given $f(x) = \begin{cases} x+3 & \text{where } x < 0 \\ x-3 & \text{where } x \geq 0 \end{cases}$ then $\lim_{x \rightarrow 0^+} f(x) =$

- A. -3 B. 0 C. 1 D. 3 E. *does not exist*

15.

Given $f(x) = \begin{cases} e^x & \text{where } x < 1 \\ \ln x & \text{where } x \geq 1 \end{cases}$ then $f(1) =$

- A. 0 B. 1 C. 2 D. e E. *does not exist*

16.

$\lim_{x \rightarrow 3} (x^2 - 2x + 2) =$

- A. -8 B. 4 C. 5 D. 17 E. *none of these*

17. Find the product

$(9 + 8i)(5 - 5i)$

A) $85 - 5i$

C) $85 + 5i$

B) $5 + 85i$

D) $-40i^2 - 5i + 45$

18. The next iterative value of the root of $x^2 + 2x - 4 = 0$ using the Newtons method, if the Initial guess is 1, is

- A. 1.25 B. 1.067 C. 1.167 D. 1.001

19. A carpenter is building a rectangular room with a fixed perimeter of 100 feet. What is its maximum area?

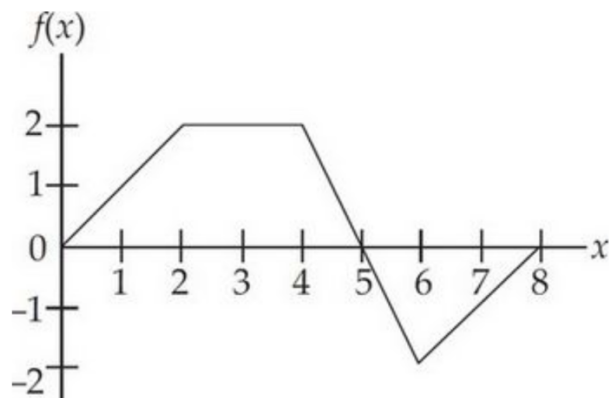
A) 900 ft^2

C) 2500 ft^2

B) 625 ft^2

D) 1875 ft^2

20.



The graph of a piecewise linear function $f(x)$ is above. Evaluate $\int_3^8 f'(x) dx$

- (a) 2 (b) -2 (c) 5 (d) 0

21.

The function f is defined by $f(x) = 4x^2 - 5x + 1$. The application of the Mean Value Theorem to f on the interval $0 < x < 2$ guarantees the existence of a value c , where $0 < c < 2$ such that $f'(c) =$

- (A) 1 (B) 3 (C) 7 (D) 8

22.

Solve the following differential equation. You can assume that $x = 0$ when $y = 0$.

$$3x^2 y^2 \frac{dx}{dy} = 4y^5$$

- a) $x^3 = y^4$ b) $3x^3 = 4y^4$ c) $4x^4 = 4y^5$ d) $y^5 = 4x^2$

PART 2: Written Response Section:

Show all steps clearly to receive full marks. Please **CIRCLE YOUR FINAL ANSWER.**

1. Differentiate (with respect to x) each function: [1 mark each]

a) $f(x) = -3x^3 + 6x^{-2} - 1$

b) $y = 5e^{3x}$

c) $f(x) = 4\sin(x^4 - 5x)$

d. $y = 2^{x^3}$

2. Related Rate Problems. Show all steps and diagram

[3 marks]

You are inflating a spherical balloon at the rate of $7 \text{ cm}^3/\text{sec}$. How fast is its radius increasing when the radius is 4 cm?

3. Find the antiderivative of each of the following functions:

[3 marks]

a) $\int -6x^2 + 8x - 5 \, dx$

b) $\int 2x \ln(3x^2) \, dx$

4. Find the **exact** value of each of the following definite integrals: [2 marks each]

a) $\int_0^{\pi} \cos x \, dx$

b) $\int_{-1}^2 -e^{-x} \, dx$

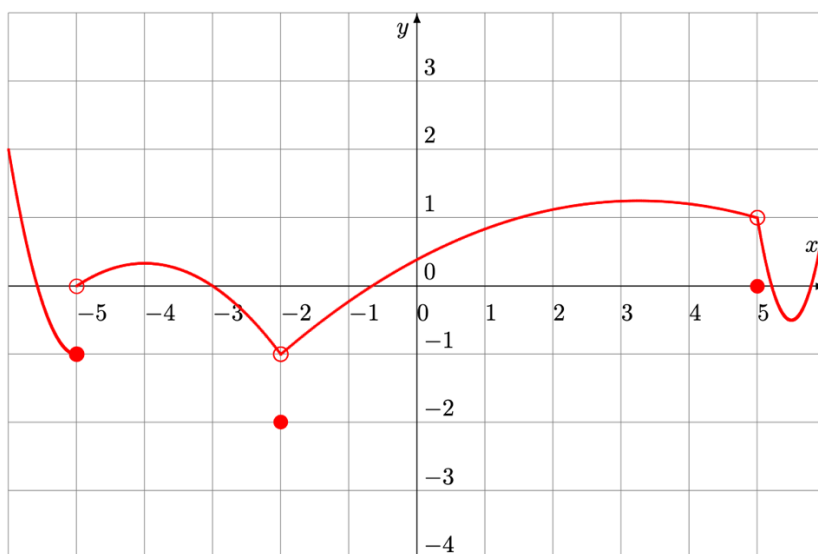
5. Find the **exact** area between the curve $y = x^2 - 3x - 4$ and the x-axis over the interval $-2 \leq x \leq 4$ [3 marks]

6. Find the **exact** area of the region bound between $y = 2x^2 + 10$ and $y = 4x + 16$.

[3 marks]

7. Evaluate each limit:

[3 marks]



Find the following limits:

a) $\lim_{x \rightarrow -2^-} f(x)$ b) $\lim_{x \rightarrow -2^+} f(x)$ c) $\lim_{x \rightarrow -2} f(x)$ d) $\lim_{x \rightarrow -5} f(x)$ e) $\lim_{x \rightarrow 5} f(x)$

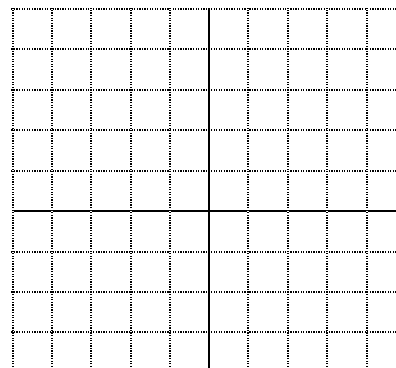
8. Use the **definition** of the derivative to find $\frac{dy}{dx}$ for $y = 2x^2 + x$.

[2 marks]

9. Make a rough sketch of the following function. Indicate x-intercepts, y-intercepts,

vertical and horizontal asymptotes: $y = \frac{2x-1}{x+3}$

[3 marks]



10. Use the **second derivative test** to find and classify all local extrema of :

$$y = \frac{4}{3}x^3 - 36x + 1$$

[2 marks]

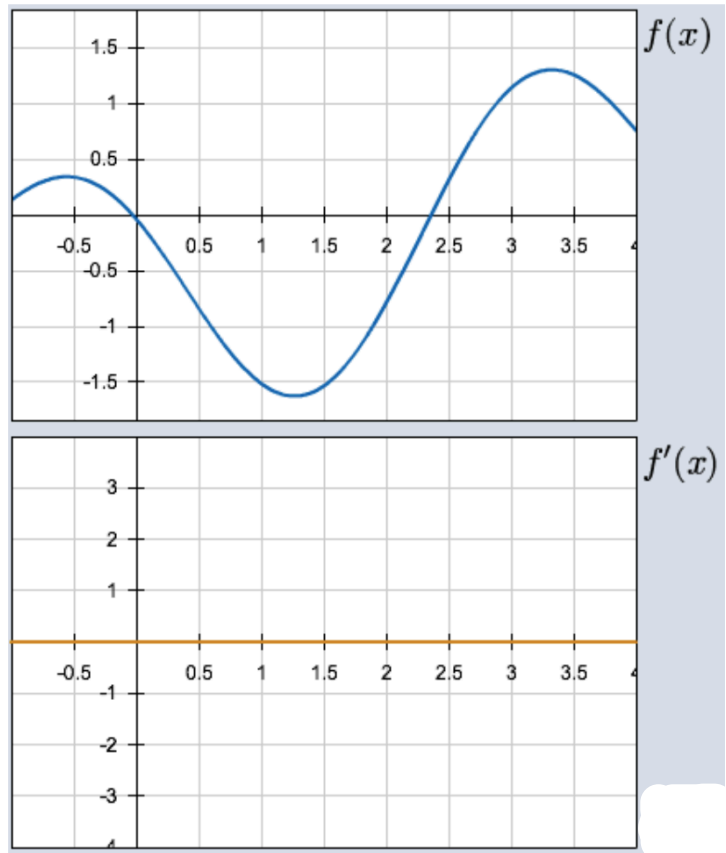
11 a) Given the position-time graph $[0, 4 \text{ seconds}]$ below:

[2 marks]

i) at what time(s) is the velocity 0?

ii) at what time(s) is the acceleration 0?

iii) when is the object slowing down?



12. Solve the following equations **exactly** $2 \ln x = \ln(2x + 24)$.

[2 marks]

13. Given $x^2 - \ln y = 0$, find $\frac{dy}{dx}$ at the point $(1, e)$.

[2 marks]

14. If $y = \sin(xy)$, find $\frac{dy}{dx}$ at the point $\left(1, \frac{\pi}{3}\right)$.

[2 marks]

15. A man has 100 meters of fencing, a large yard, and a small dog. He wants to create a rectangular enclosure for his dog with the fencing that provides the maximum area. What dimensions provide the maximum area?

[3 marks]

16. Use Newton's Method to solve the following equation $-x^3 - 3x + 13 = 0$ [3 marks]
using x -initial value $x = 2$. Just do two iterations $x_3 =$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

17. Evaluate the following **IMPROPER INTEGRALS** and state if convergent or divergent:
[2 marks]

$$\int_0^{\infty} \frac{1}{x+2} dx$$

18. Determine if the Mean Value Theorem applies to the function on the given interval. If it does, find the c-value. If it doesn't, explain why not.
[2 marks]

$$f(x) = x^2 - 2x \quad [1, 3]$$

19. Find the exact value of the following multiple $\int_0^1 \int_2^4 x \, dx \, dy$ integrals: [2 marks]

20. Simplify: $(2 + 5i)(2 - 5i)$

[1 mark]

21. Solve the following equation over the **COMPLEX NUMBERS**:

[2 marks]

$$2x^2 + 2x = 2x - 8$$