CALCULUS 12 "PRACTICE EXAM"

Name ___________________________ Date __________________

1. Differentiate each function:

a) $g(x) = \frac{1}{3}x^6 - \frac{1}{2}x^8 + \frac{3}{5}x^5$

b) $y = 8p^2 - 2kp^3 + 5m^2$ find $\frac{dy}{dp}$

c) $y = \sin^3 4x + 4\tan 5x$

d. $g(x) = \sqrt{6x^2 - 2x + 8}$

e) $h(x) = 6\cos 4x$

f) $y = \sin\left(2x^4 + 3x\right)$

2a) Find $\frac{dy}{dx}$ $x^2 + 4xy - y^2 = 10$

2b) Find $f'(x)$, **simplified** for the following.

$$f(x) = \frac{(x+3)^4}{(x^2+5)^{\frac{1}{2}}}$$

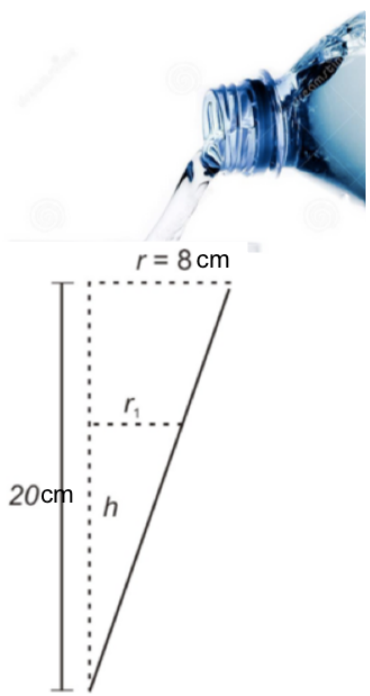
3a) If $f(x) = 3x^3 - 2x^2 + 7$ find $f'(x)$ at the point (-1, 2)

3b) If $f(x) = k\sqrt[3]{x}$ determine the value of the constant $\boldsymbol{k}$ so that $f'(8) = 2$

4. Related Rate Problems. Show all steps and diagram

a) The hypotenuse of a right triangle is of fixed length but the lengths of the other two sides x and y depend on time. How fast is x changing when $\frac{dy}{dt} = 5$ and $y = 12$ if the length of the hypotenuse is 13?

b) A conical cup is being filled a rate of 3 cm^3/s. Find the rate the water level is rising at 12 cm. See diagram below.



5. Use the **<u>second</u> derivative test** to find and classify all local extrema for:

$$y = x^3 - 12x - 2$$

6. Find the general antiderivative of each of the following functions:

a) $5x^3 - 4x^2 + 3$

b) $x^4 + \frac{1}{x^3} + 2\pi$

c) $\frac{12}{x} + 3\cos x - 2$

d) $\cos(4x) + 4x^2$

e) $6e^x - 4e^{3x}$

f) $x^3 \cos(x^4)$

7. Find the **exact** value of each of the following definite integrals:

a) $\int_1^{\pi} x^2 \, dx$

b) $\int_0^3 e^x \, dx$

8. Find the **exact** area between the curve $y = 4x$ and the *x*-axis over the interval $-2 \leq x \leq 5$

9. Find the area of the region bounded by $y = -2 + x$ *and* $x = y^2$ integrating with **respect to *y*.**

10. Find the antiderivative:

a) $\int \frac{12x}{\left(4-3x^2\right)^3}$

b) $\int x^2 \cos x \; dx$

11. Find the **exact** value of each of the following:

a) $\int_0^1 xe^{x^2}\ dx$

12. Find the area of the region bound between two functions $f(x) = 2x \ \ and \ \ g(x) = 5x - x^2$.

13. Evaluate each limit:

a) $\lim_{x \to -2} 3x^2 + 1$

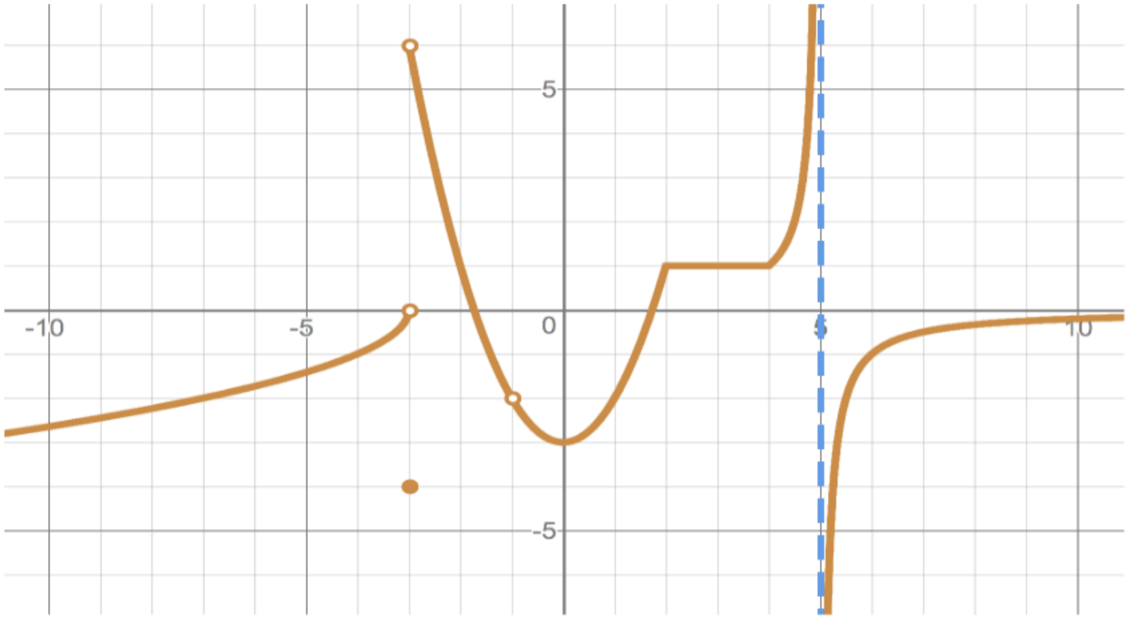
b) $\lim_{x \to a} \frac{(x+2a)^2}{x^2 + a^2}$

c) $\lim_{x \to 3} \frac{9 - x^2}{x - 3}$

d) $\lim_{x \to 0} \frac{4\sin 2x}{4x}$

13b)

Use the graph below to evaluate the following limits:



1. $\lim_{x\to -3^-} f(x) =$

2. $\lim_{x\to -3^+} f(x) =$

3. $\lim_{x\to -3} f(x) =$

4. $f(-3) =$

5. $\lim_{x\to 5^-} f(x) =$

6. $\lim_{x\to 5^+} f(x) =$

7. $\lim_{x\to 5} f(x) =$

8. $f(5) =$

9. $\lim_{x\to -1^-} f(x) =$

10. $\lim_{x\to -1^+} f(x) =$

11. $\lim_{x\to -1} f(x) =$

12. $f(-1) =$

13. $\lim_{x\to 2^-} f(x) =$

14. $\lim_{x\to 2^+} f(x) =$

15. $\lim_{x\to 2} f(x) =$

16. $f(2) =$

17. $\lim_{x\to +\infty} f(x) =$

18. $\lim_{x\to -\infty} f(x) =$

14. Use the **definition** of the derivative to find $\frac{dy}{dx}$ for

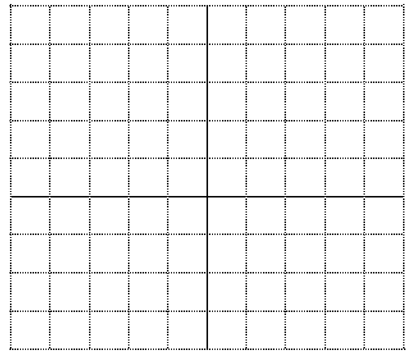
a) $y = x^2 + 3x$

b) $y = \frac{2x}{x^2 + 4}$

15. Evaluate each of the following: ****Must show steps!!**

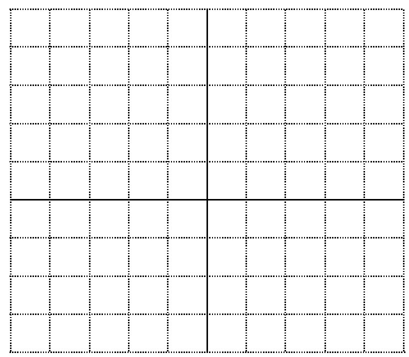
$$\lim_{h \to 0} (1 + 4h)^{\frac{1}{h}}$$

16. Make a rough sketch of the following function. Indicate x-intercepts, y-intercepts, vertical and horizontal asymptotes: $y = \dfrac{x+3}{x-1}$



17a) Find the: x and y intercepts, coordinates of the critical points, coordinates of the inflection points, open intervals where the function is concave up and concave down, and relative minima and maxima. Using this information, sketch the graph of the function.

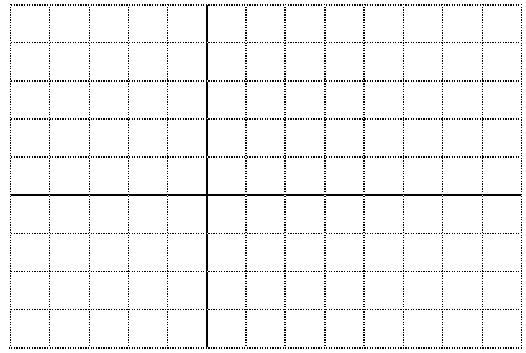
$f(x) = 12x - x^3$



x-int.	
y-int	
Local Max.	
Local Min.	
Inflection Pt(s)	
Interval Concave up	
Interval Concave down	

17b) Find the: x and y intercepts, coordinates of the critical points, coordinates of the inflection points, open intervals where the function is concave up and concave down, and relative minima and maxima. Using this information, sketch the graph of the function.

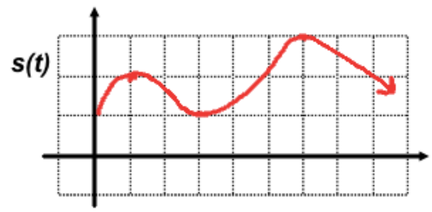
$$f(x) = \frac{1}{5}(x-4)^{\frac{2}{3}}$$



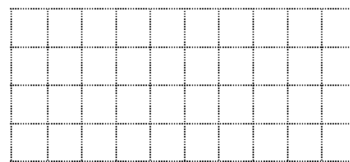
x-int.	
y-int	
Local Max.	
Local Min.	
Inflection Pt(s)	
Interval Concave up	
Interval Concave down	

18. Given the position-time graph below:

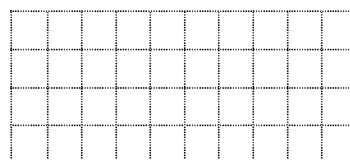
i) at what time(s) is the velocity 0?



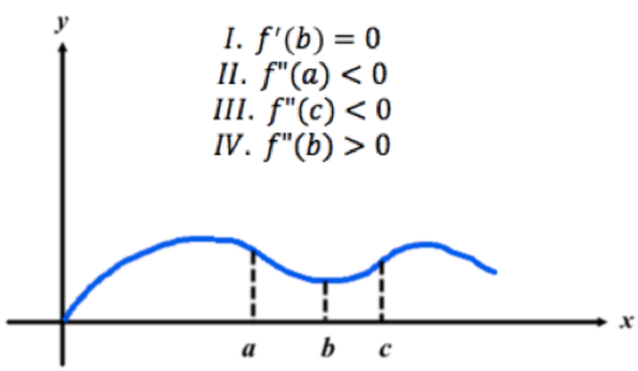
ii) at what time(s) is the acceleration 0?



iii) when is the object slowing down?

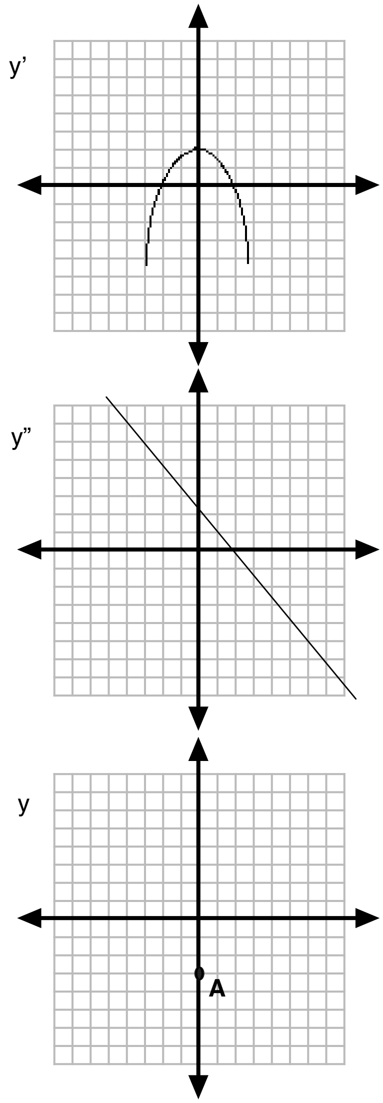


19. Given the function shown below. Identify which of following statements below are true?

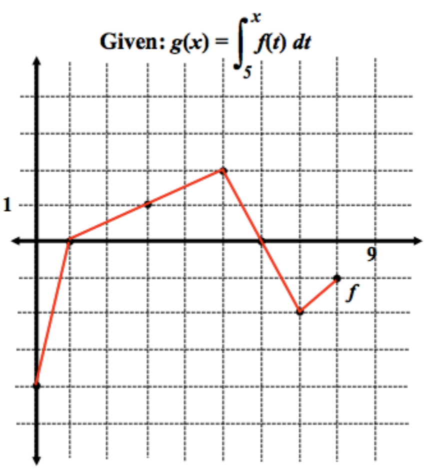


$I.\ f'(b) = 0$
$II.\ f''(a) < 0$
$III.\ f''(c) < 0$
$IV.\ f''(b) > 0$

20. Given the graphs of the first and second derivatives of a function f(x) sketch the graph of f(x) passing through the indicated point



21 a. Use the given graph to find the following items below:



$g(3) =$

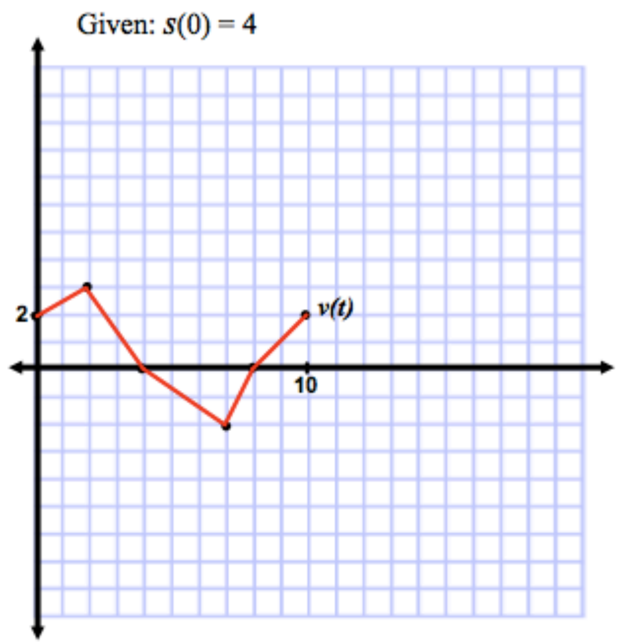
$g'(x) =$ $g'(3) =$

$g''(x) =$ $g''(3) =$

Where is the g increasing?

Where is g concave up?

21 b.



Find: $s(10) =$ $v(9) =$ $a(9) =$

Total distanced traveled?

When is speed increasing?

When does it travel to the right?

When does it change direction?

When is it farthest to the right?

When is it the fastest?

22. Solve the following equations **exactly** $2 \ln x = \ln(x + 6)$

23. Given $x - \ln y = 0$, find $\frac{dy}{dx}$ at the point $(1, e)$.

24. Find the equation of the normal line to the curve $y = 2\ln(x^2)$ at the point $(e, 1)$.

25 a) Pete and Repete create an open box by cutting squares of equal size from the corner of a 48 *cm* by 30 *cm* piece of tin and folding up the sides. Determine the size of the cutout that maximizes the volume of the box.

25 b) A manufacturer needs to make a cylindrical can that will hold 1.5 liters of liquid. Determine the dimensions of the can that will minimize the amount of material used in the construction.