PRE-CALCULUS 11

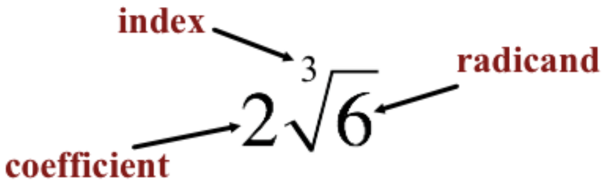
Seminar Notes

Learning Guide 10 & 11

RADICAL FUNCTIONS & EQUATIONS

Frances Kelsey Secondary School – 2019/20

Parts of a Radical



Topic 1

Example 1

Converting Mixed Radicals to Entire Radicals

Example of a Mixed Radicals	Example of a Entire Radical
$2\sqrt{5}$	$\sqrt{20}$

To convert mixed radical to entire radical:

BEWARE of the INDEX!!

Example: $2\sqrt{5}$

1. take the coefficient 2 and square it $2^2 = 4$
2. then multiply the 4 by the radicands $\sqrt{5}$

$$\sqrt{4 \times 5} = \sqrt{20}$$

Try: $a)\ 4\sqrt{3}$

$b)\ v^3\sqrt{v}$

$c)\ 2s^2\sqrt{3s}$

$d)\ 2x^2\sqrt[3]{4x}$

Example 2

Radicals in Simplest Form

(Express Entire Radicals as Mixed Radicals)

Example: $\sqrt{200}$

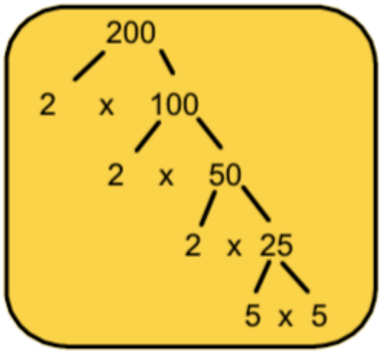
1. Use Prime Factorization

$$\sqrt{\enclose{circle}{2\cdot 2}\cdot 2\cdot \enclose{circle}{5\cdot 5}}$$

2. Circle groups of 2's
   [***because it's a Square Root***]
   and put them out as coefficients

$$2\cdot 5\sqrt{2} = 10\sqrt{2}$$

Do a Factor Tree



Try:

$a)\ \sqrt{52}$

$b)\ \sqrt[4]{c^7}$

$c)\ \sqrt[3]{864m^4n^5}$

Example 3

Compare & Order Radicals

Order the following from least to greatest.

$$8\sqrt{3} \quad 4(13)^{\frac{1}{2}} \quad 14 \quad \sqrt{202} \quad 10\sqrt{2}$$

1st - Express each as an entire radical

$8\sqrt{3}$	$4(13)^{\frac{1}{2}}$	14	$\sqrt{202}$	$10\sqrt{2}$
$=\sqrt{8^2\cdot 3}$	$=\sqrt{4^2\cdot 13}$	$=\sqrt{14^2}$	$=\sqrt{202}$	$=\sqrt{10^2\cdot 2}$
$=\sqrt{192}$	$=\sqrt{208}$	$=\sqrt{196}$	$=\sqrt{202}$	$=\sqrt{200}$

2nd - Now, compare and order the radicands

$$\sqrt{192},\ \sqrt{196},\ \sqrt{200},\ \sqrt{202},\ \sqrt{208}$$

Try: Order from least to greatest

$$5,\ 3\sqrt{3},\ 2\sqrt{6},\ \sqrt{23}$$

Example 4

Add & Subtract Radicals

Simplify and combine like terms.

$a)\ \sqrt{50} + 3\sqrt{2}$ $\qquad$ $b)\ \sqrt{72x} - \sqrt{18x}$

1st - Use your simplifying radical skills [see Example 2]

$a)\ \sqrt{50} + 3\sqrt{2}$

$= \sqrt{5 \cdot 5 \cdot 2} + 3\sqrt{2}$

$= 5\sqrt{2} + 3\sqrt{2}$

$= 8\sqrt{2}$

$b)\ \sqrt{72x} - \sqrt{18x}$

$= \sqrt{6 \cdot 6 \cdot 2 \cdot x} - \sqrt{3 \cdot 3 \cdot 2 \cdot x}$

$= 6\sqrt{2x} - 3\sqrt{2x}$

$= 3\sqrt{2x}$

Try: **Simplify and combine like terms.**

$a)\ -3\sqrt{24} + \sqrt{6}$ $\qquad$ $b)\ \sqrt{20x} - 3\sqrt{45x}$

Topic 2

Example 1

Multiplying Radicals

Multiply, then simplify where possible.

a) $\left(2\sqrt{3}\right)\left(-5\sqrt{6}\right)$

b) $\left(8\sqrt{2} - 5\right)\left(9\sqrt{5} + 6\sqrt{10}\right)$

1st - Use distributive property

$2(-5)\sqrt{3\cdot 6}$

$-10\sqrt{18}$

$\left(8\sqrt{2} - 5\right)\left(9\sqrt{5} + 6\sqrt{10}\right)$

$72\sqrt{10} + 48\sqrt{20} - 45\sqrt{5} - 30\sqrt{10}$

2nd - Simplify the radicals

$= -10\sqrt{3\cdot 3\cdot 2}$

$= -30\sqrt{2}$

$= 72\sqrt{10} + 48\sqrt{2\cdot 2\cdot 5} - 45\sqrt{5} - 30\sqrt{10}$

$= 72\sqrt{10} + 96\sqrt{5} - 45\sqrt{5} - 30\sqrt{10}$

3rd - Collect like radicals

$= -30\sqrt{2}$

$= 42\sqrt{10} + 51\sqrt{5}$

Try: a) $7\sqrt{3}\left(5\sqrt{5} - 6\sqrt{3}\right)$

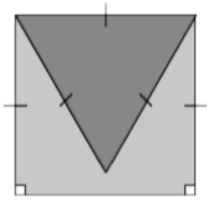
b) $9\sqrt[3]{2m}\left(\sqrt[3]{4m} + 7\sqrt[3]{28}\right)$

c) $\left(4\sqrt{2} + 3\right)\left(\sqrt{7} - 5\sqrt{14}\right)$

Example 2

Apply Radical Multiplication

A equilateral triangle placed inside a square is shown below. The area of the square is 32 cm^2.



a) Find the exact perimeter of the triangle?

$$A = s^2$$

☆ Since the base of the triangle is a side of the square - we can use the Area of a Square formula. $A = s^2$

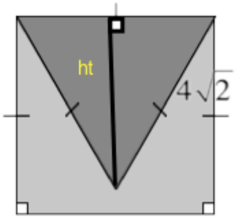
$$32 = s^2$$

$$\sqrt{32} = s$$

$$\sqrt{2\cdot2\cdot2\cdot2\cdot2}$$

Since all sides are the equal - we can now multiply 3 by the side ☆

$$s = 4\sqrt{2} \qquad \Rightarrow \quad 3\left(4\sqrt{2}\right) = 12\sqrt{2}\ cm$$



b) Find the exact height of the triangle?

☺ We need to use pythagorus theorem to find the height.

➡ one side of the triangle is $4\sqrt{2}$

➡ the base is half of the side so:

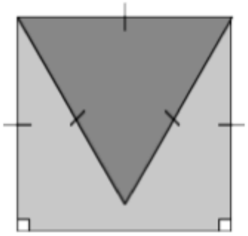
$$\frac{\overset{2}{\cancel{4}}\sqrt{2}}{\cancel{2}} = 2\sqrt{2}$$

$$a = \sqrt{c^2 - b^2}$$

$$a = \sqrt{\left(4\sqrt{2}\right)^2 - \left(2\sqrt{2}\right)^2}$$

$$a = \sqrt{32 - 8}$$

$$a = \sqrt{24} \quad \Rightarrow \quad ht = 2\sqrt{6}$$



c) Find the exact area of the triangle?

☺ We now know the base is $4\sqrt{2}$ and the height is $2\sqrt{6}$. Use the formula:

$$A = \frac{b\cdot h}{2}$$

$$= \frac{\left(4\sqrt{2}\right)\left(2\sqrt{6}\right)}{2} = \frac{8\sqrt{12}}{2}$$

$$= 4\sqrt{12}$$

$$Area = 8\sqrt{3}$$

Try: An isosceles triangle has a base of $\sqrt{20}$ m.
Each of the equal sides is $3\sqrt{7}$ m long.
What is the exact area of the triangle?

Example 3

Divide Radicals

There are three types you'll will run into:

1. Divide

☛ $\dfrac{\sqrt{15m^3}}{\sqrt{5m}}$

$= \sqrt{3m^2}$

2. Rationalize

☛ $\dfrac{2\sqrt{6}}{4\sqrt{8}}$

$= \dfrac{2\sqrt{6}}{4\sqrt{8}}\left(\dfrac{\sqrt{8}}{\sqrt{8}}\right)$

$= \dfrac{2\sqrt{48}}{4\sqrt{64}}$

$= \dfrac{8\sqrt{3}}{32}$

$= \dfrac{\sqrt{3}}{4}$

3. Conjugate

☛ $\dfrac{3}{2\sqrt{2}-5}$

$= \dfrac{3}{2\sqrt{2}-5}\left(\dfrac{2\sqrt{2}+5}{2\sqrt{2}+5}\right)$

$= \dfrac{6\sqrt{2}+15}{4\sqrt{4}+10\sqrt{2}-10\sqrt{2}-25}$

$= \dfrac{6\sqrt{2}+15}{-17}$

$= \dfrac{-6\sqrt{2}-15}{17}$

Try:

$a)\ \dfrac{3\sqrt{24x^3}}{\sqrt{3x}}$

$b)\ \dfrac{-6}{2\sqrt[3]{9}}$

$c)\ \dfrac{6}{\sqrt{4b}+1}$

LEARNING GUIDE 11

Topic 1

Example 1

Solve an Equation with One Radical

Solve algebraically $\sqrt{x+1} + 3 = 5$ - then check by graphing

1st - isolate the radical $\sqrt{x+1} = 5 - 3$

$\sqrt{x+1} = 2$

2nd - square both sides $\left(\sqrt{x+1}\right)^2 = 2^2$

$x + 1 = 4$

3rd - solve $x = 3$

*CHECK FOR EXTRANEOUS

by substituting x = 3 into original equation

$\sqrt{(3)+1} + 3 = 5$

$\sqrt{4} + 3 = 5$

$2 + 3 = 5$ ✓

Try: a) $3 + \sqrt{3x+4} = 7$

b) $2\sqrt{x+6} = -4$

c) $-8 + \sqrt{\frac{3y}{5}} = -2$

d) $r - \sqrt{5-r} = -7$

Topic 2 **Example 2**

Solve an Equation with Two Radicals

Solve $\sqrt{x+2} + \sqrt{3x-2} = 0$ - then check by graphing.

1st isolate the radicals on each side of = sign

$$\sqrt{x+2} = -\sqrt{3x-2}$$

2nd - square both sides

$$\left(\sqrt{x+2}\right)^2 = \left(-\sqrt{3x-2}\right)^2$$

$$x+2 = 3x-2$$

3rd - solve

$$-2x = -4$$

$$x = 2$$

2 does not check out - the solution is **NO SOLUTION**

Try:

a) $\sqrt{2y-3} = \sqrt{y+1}$

b) $\sqrt{19+6x} = \sqrt{2x-5}$

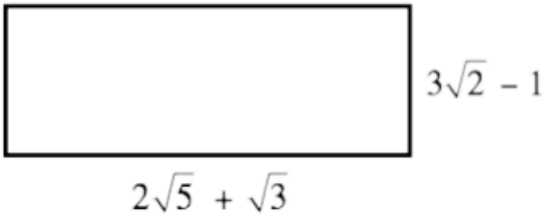
c) $7 + \sqrt{3x} = \sqrt{5x+4} + 5$

d) $3\sqrt{x-2} = \sqrt{2x+3}$

Solve by Graphing
Calculator ONLY!

Some more difficult questions you will come across.

Find the perimeter:



Find the area: