PRE-CALCULUS 11

Seminar Notes
Learning Guide 12 & 13

RATIONAL FUNCTIONS & EQUATIONS

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Example 1

Determine Non-Permissible Values

For each rational expression, determine all non-permissible values.

a) $\dfrac{-2x^3}{5y^2z}$ b) $\dfrac{4m}{m(3m-1)}$ c) $\dfrac{5x+2}{x^2+5x+6}$

1ˢᵗ take the denominator and make it equal zero.

$5y^2 = 0 \;;\; z = 0$ $m = 0 \;;\; 3m - 1 = 0$ $x^2 + 5x + 6 = 0$

⭐ **factor**

2ⁿᵈ then solve equation to find non-permissible values.

$y = 0 \;;\; z = 0$ $m = 0 \;;\; m = \dfrac{1}{3}$ $(x+2)(x+3) = 0$

$x = -2 \;;\; x = -3$

Example 2
Simplify a Rational Expression

☆ When simplifying a rational expression, always state the non-permissible values.

Simplify and state the non-permissible values.

$$\dfrac{3x - 6}{x^2 + x - 6}$$

1st - determine any non-permissible values

$x^2 + x - 6 = 0 \Rightarrow (x+3)(x-2) = 0 \Rightarrow x = -3, -2 \Rightarrow x \neq -3, 2$

2nd - factor to cancel out equivalent expressions

$$\dfrac{3x - 6}{x^2 + x - 6} = \dfrac{3\cancel{(x-2)}}{(x+3)\cancel{(x-2)}} = \dfrac{3}{x+3} \;;\; x \neq -3, 2$$

Example 3
Rational Expressions With Pairs of Non-Permissible Values

Simplify and determine non-permissible values.

$$\frac{16x^2 - 9y^2}{8x - 6y}$$

To simplify:

1st - factor

$$\frac{(4x+3y)(4x-3y)}{2(4x-3y)}$$

$$4x - 3y = 0$$

$$\frac{4x}{4} = \frac{3y}{4}$$

$$x \neq \frac{3y}{4}$$

2nd - then look to cancel out equivalent expression

$$\frac{(4x+3y)(4x-3y)}{2(4x-3y)} = \frac{(4x+3y)}{2}$$

Watch out for the "Opposite Rule"

Simplify and state the non-permissibles.

$$\frac{x^2 + 3x - 10}{2 - x}$$

Try: Simplify and determine non-permissible values.

$$\frac{2m^2 + 6mn - 36n^2}{6m + 36n}$$

Example 1

Multiplying Rational Expressions

Multiply and write your answer in simplest form. Identify all non-permissible values.

$$\frac{x^2 - x - 12}{x^2 - 9} \times \frac{x^2 - 4x + 3}{x^2 - 4x}$$

1st - factor
$$= \frac{(x-4)(x+3)}{(x-3)(x+3)} \times \frac{(x-1)(x-3)}{x(x-4)}$$

$$= \frac{(x-4)(x+3)(x-1)(x-3)}{(x-3)(x+3)x(x-4)} \star \quad \text{non-permissible values}$$

2nd - look to cancel out equivalent expression, then state all non-permissible values

$$= \frac{\cancel{(x-4)}\cancel{(x+3)}(x-1)\cancel{(x-3)}}{\cancel{(x-3)}\cancel{(x+3)}x\cancel{(x-4)}} \quad = \quad \frac{x-1}{x} \; ; \quad x \neq -3, 0, 3, 4$$

Try: Multiply and write your answer in simplest form. Identify all non-permissible values.

a) $\dfrac{2a-10}{a^2-4a-5} \times \dfrac{a^2-1}{4a-4}$

b) $\dfrac{2-x}{m^2} \times \dfrac{2m}{3n-6}$

Example 2
Divide Rational Expressions

Dividing Rational Expressions is pretty much the same as multiplying, except you must first reciprocate (flip) the rational expression that comes <u>after</u> the $\div$ sign.

Determine the quotient in simplest form.
Identify all non-permissible values.

$$\frac{b^2 - 4}{6} \div \frac{b-2}{3}$$

$$= \frac{(b-2)(b+2)}{6_2} \times \frac{{}^1 3}{b-2} = \frac{b+2}{2} \; ; \; b \neq 2$$

Try: Determine the quotient in simplest form.
Identify all non-permissible values.

$$\frac{c^2 - 6c - 7}{c^2 - 49} \div \frac{c^2 + 8c + 7}{c^2 + 7c}$$

Example 3
Multiply and Divide
Rational Expressions

Now it is time to put both your
multiplying and dividing skills together
to simplify a Rational Expression.

Try:

Simplify. What are the non- permissible
values?

$$\frac{3x+12}{3x^2-5x-12} \div \frac{12}{3x+4} \times \frac{2x-6}{x+4}$$

LEARNING GUIDE 13

Topic 1 **Example 1**

Add or Subtract Rational Expressions With Common Denominators

Determine each sum or difference. Express each answer in simplest form. Identify all non-permissible values.

a) $\dfrac{2a}{b} - \dfrac{a-1}{b}$

$= \dfrac{2a-(a-1)}{b}$ ◄··► don't forget bracket

$= \dfrac{2a-a+1}{b}$ collect like terms

$= \dfrac{a+1}{b}$; $b \neq 0$

b) $\dfrac{2x}{x+4} + \dfrac{8}{x+4}$

$= \dfrac{2x+8}{x+4}$ ◄·► factor

$= \dfrac{2(x+4)}{x+4}$

$= 2$; $x \neq -4$

Try:

a) $\dfrac{2}{m-2} + \dfrac{-7}{m-2}$

b) $\dfrac{4x}{x+5} + \dfrac{-7}{x+5}$

Try:

Determine each sum or difference. Express each answer in simplest form. Identify all non-permissible values.

$$\dfrac{2w^2 - w}{(w-3)(w+1)} + \dfrac{3-2w}{(w-3)(w+1)} - \dfrac{8}{(w-3)(w+1)}$$

Example 2
Add or Subtract Rational Expressions With Unlike Denominators

Simplify. Express each answer in simplest form.

a) $\dfrac{2x}{xy} + \dfrac{4}{x^2} - 3$

1st find LCD = $x^2 y$, then multiply each numerator by the missing term.

$= \dfrac{2x(x)}{xy(x)} + \dfrac{4(y)}{x^2(y)} - \dfrac{3(x^2 y)}{1(x^2 y)}$

$= \dfrac{2x^2 + 4y - 3x^2 y}{x^2 y}$

b) $\dfrac{1 - \dfrac{1}{x}}{x - \dfrac{1}{x}}$ ⭐ this is a complex fraction

[a fraction in the numerator/denominator]

Find LCD in the Complex Fraction = x, then multiply LCD by every term.

$= \dfrac{(x)1 - \dfrac{1}{\cancel{x}}(\cancel{x})}{(x)x - \dfrac{1}{\cancel{x}}(\cancel{x})}$ This simplifies a complex fraction.

$= \dfrac{x - 1}{x^2 - 1} = \dfrac{\cancel{x-1}^{1}}{\cancel{(x-1)}(x+1)}$ factor

$= \dfrac{1}{x + 1}$

Try: Simplify. What are the non-permissible values.

a) $\dfrac{4}{p^2 - 1} + \dfrac{3}{p + 1}$

b) $\dfrac{2 - \dfrac{4}{n}}{n - \dfrac{4}{n}}$

c) $\dfrac{5}{2x - 8} - \dfrac{3}{x - 4}$

d) $\dfrac{-1}{c + 1} + \dfrac{3c + 1}{c^2 - 2c - 15}$

Example 1
Solve a Rational Equation

Solve the following equation. What values are non-permissible.

$$\frac{2}{a^2-4} + \frac{10}{6a+12} = \frac{1}{a-2}$$

1st Factor denominator to find LCD

$$= \frac{2}{(a-2)(a+2)} + \frac{10}{6(a+2)} = \frac{1}{a-2}$$

LCD = $6(a-2)(a+2)$

⭐ From the factors, the non-permissible values are:

$$x \neq -2, 2$$

2nd To clear fractions multiply each term by the LCD, then cancel out like terms

$$= \ ^{6(a-2)(a+2)}\left[\frac{2}{(a-2)(a+2)}\right] + \ ^{6(a-2)(a+2)}\left[\frac{10}{6(a+2)}\right] = \ ^{6(a-2)(a+2)}\left[\frac{1}{a-2}\right]$$

3rd Solve equation

$$12 + 10a - 20 = 6a + 12$$

$$4a = 20$$

$$a = 5 \qquad \text{Check your answer:}$$

Substitute a = 5 into the original equation.

Left Side	Right Side
$\dfrac{2}{a^2-4} + \dfrac{10}{6a+12}$	$\dfrac{1}{a-2}$
$= \dfrac{2}{(5)^2-4} + \dfrac{10}{6(5)+12}$	$= \dfrac{1}{(5)-2}$
$= \dfrac{2}{21} + \dfrac{10}{42}$	$= \dfrac{1}{3}$
$= \dfrac{1}{3}$ ✓	

Try: Solve the following equation. What values are non-permissible.

$$\frac{9}{x-3} - \frac{4}{x-6} = \frac{18}{x^2 - 9x + 18}$$

Example 2
Solve a Rational Equation With an Extraneous Root

Solve the following equation. What values are non-permissible.

$$\frac{2x}{x+3} + \frac{x}{x-3} = \frac{18}{x^2-9}$$

1 ˢᵗ Find RESTRICTION , $x \neq 3, -3$ ⭐ Then find LCD $= (x+3)(x-3)$

2 ⁿᵈ multiply each rational expression by LCD (this eliminates denominator)

$$\overset{(x+3)(x-3)}{\left(\frac{2x}{x+3}\right)} + \overset{(x+3)(x-3)}{\left(\frac{x}{x-3}\right)} = \overset{(x+3)(x-3)}{\left(\frac{18}{x^2-9}\right)} \text{⭐ factor}$$

$$(x+3)(x-3)$$

$$2x(x-3) + x(x+3) = 18 \quad (expand)$$

$$2x^2 - 6x + x^2 + 3x = 18 \qquad (collect\ like\ terms)$$

$$3x^2 - 3x - 18 = 0 \qquad (factor)$$

$$3(x^2 - x - 6) = 0$$

$$3(x-3)(x+2) = 0$$

$$x = 3 , \quad x = -2$$

**CHECK FOR EXTRANEOUS

$$\boxed{x = -2,} \quad \overset{⭐}{x = 3} \ is \ extraneous$$

Solution

EXTRANEOUS is when a Solution equals a Restriction

Try: Solve the following equation. What values are non-permissible.

$$\frac{3x}{x+2} - \frac{5}{x-3} = \frac{-25}{x^2 - x - 6}$$

Example 3
Use a Rational Equation to Solve a Problem

Two brothers share in cutting their lawn. Tim can cut the lawn in 40 min. Jim can cut the same lawn in 50 min. How long will it take to cut the lawn if they work together?

1ˢᵗ Make a Table

	Time to cut lawn (min)	Fraction of work in 1 min	Fraction of work done in t minutes
Tim	40	$\frac{1}{40}$	$\frac{t}{40}$
Jim	50	$\frac{1}{50}$	$\frac{t}{50}$
Together	t	$\frac{1}{t}$	1

From the table, the equation for Tim and Jim to cut the lawn together is:

$$\frac{t}{40} \; + \; \frac{t}{50} \; = \; 1$$

Tim's Fraction of Work Time   Jim's Fraction of Work Time   The 1 lawn they cut together

Find the LCD = **200**, now multiply each term by LCD

$$\overset{5}{\cancel{200}}\left(\frac{t}{\cancel{40}}\right) \; + \; \overset{4}{\cancel{200}}\left(\frac{t}{\cancel{50}}\right) \; = \; \overset{200}{}(1)$$

$$5t \; + \; 4t \; = \; 200$$

$$9t \; = \; 200$$

$$t \; = \; \frac{200}{9} \;\; or \;\; 22.2 \; min.$$

Don't forget to do a Check !

Try: Mary takes 4 h to paint a room. It takes Sue 3 h to paint the same area. How long will the paint job take if they together?

Example 4
Use a Rational Equation to Solve a Problem

A bike race goes from Victoria to Chemainus and back. The total distance was 140 km. Conditions were excellent on the way from Victoria to Chemainus. However, bad weather caused the winner's average speed to decrease by 6 km/h on the return trip. The total time for the trip was 8.5 h. What was the winning rider's average speed on the way to Chemainus?

- Use the formula *distance = rate x time*, or $time = \dfrac{dist.}{rate}$

- Let x represent the average speed, in km/h, on the trip from Victoria to Chemainus.

Make a Table

	Distance (km)	Rate (km/h)	Time (h)
Trip to Chemainus	70	x	$\dfrac{70}{x}$
Return from Chemainus	70	$x - 6$	$\dfrac{70}{x-6}$
		Total	$8\frac{1}{2}$ or $\frac{17}{2}$

$$\underset{\substack{\text{Victoria to}\\\text{Chemainus}}}{\dfrac{70}{x}} + \underset{\substack{\text{Chemainus to}\\\text{Victoria}}}{\dfrac{70}{x-6}} = \underset{\substack{\text{Total time}\\\text{for Race}}}{\dfrac{17}{2}}$$

$$\frac{70}{x} + \frac{70}{x-6} = \frac{17}{2}$$

Find LCD = $2(x)(x-6)$, then multiply each term by LCD

$$2(x)(x-6)\left(\frac{70}{x}\right) + 2(x)(x-6)\left(\frac{70}{x-6}\right) = 2(x)(x-6)\left(\frac{17}{2}\right)$$

$$2(x-7)(70) + 2(x)(70) = (x)(x-6)(17)$$

$$140x - 840 + 140x = 17x^2 - 102x$$

$$0 = 17x^2 - 382x + 840$$

Use Quadratic Program on Calculator.

$$x = 20 \text{ km/h} \quad or \quad x = 2.471 \text{ km/h}$$

This answer will not work because you will get a negative answer.

TRY:

Simon Whitfield won a Gold medal in the 2000 Summer Olympics in Triathlon. He swam 1.5 km, biked 40 km and ran 10 km. He ran at an average speed of x, biked at an average speed of $2x$, and swam at an average speed of $\frac{x}{4}$, where x is in km per hours.

a) Determine an expression for the total time taken to complete the race.

b) Simon can swim at 5km/hr. How long will it take him to complete the race?