PRE-CALCULUS 11

Seminar Notes

Learning Guide 14 & 15

SYSTEM OF EQUATIONS & INEQUALITIES

Frances Kelsey Secondary School – 2019/20

Topic 1

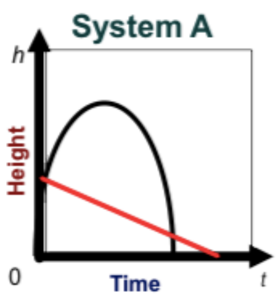
Example 1

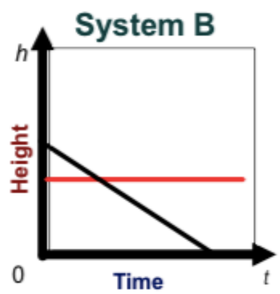
Relate a System of Equations to a Context

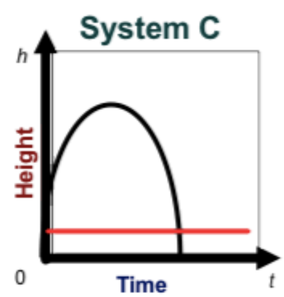
A springboard diver practices her dives from a 3-m springboard. Her coach uses video analysis to plot her height above the water.

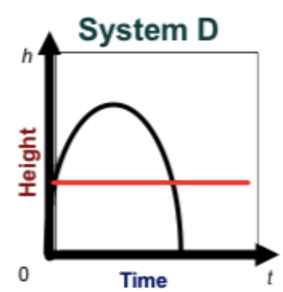
a) Which system below could represent the scenario? Explain.

b) Interpret the point(s) of intersection in the system you chose.



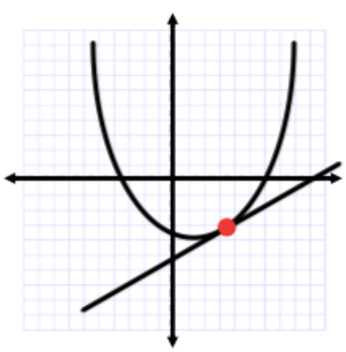




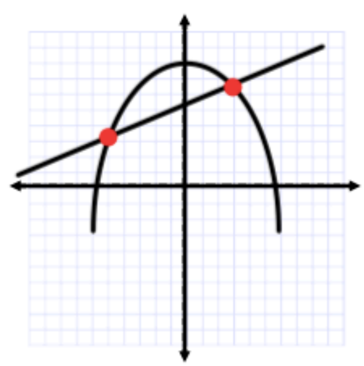


Number of Solutions

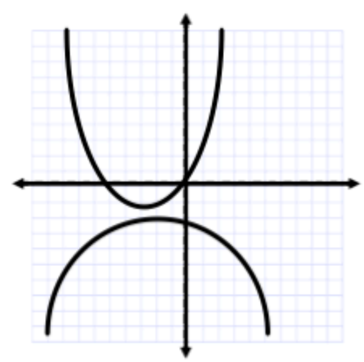
one solution



two solution



no solution



Example 2

Solve a System of Equations Graphically

Solve the following equations graphically.

$4x - y + 3 = 0$

$2x^2 + 8x - y + 3 = 0$

1st - put each system in y =

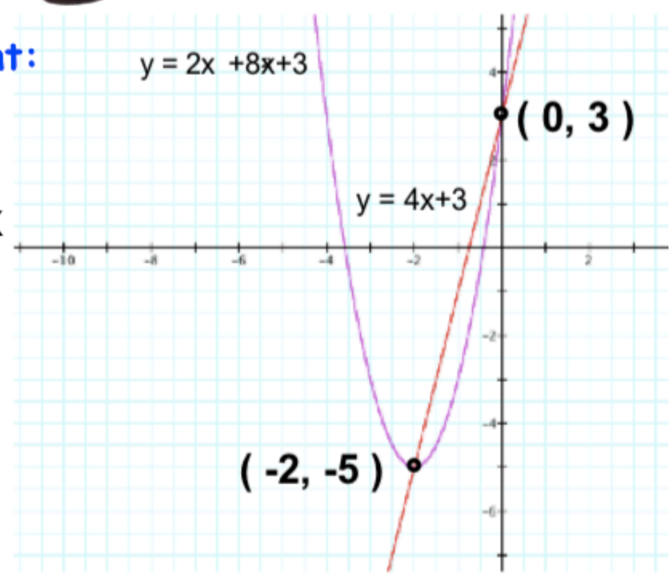
2nd - type into Graph. Calc.

3rd - find the intersection points

Steps to find Intersection Point:

2nd TRACE 5

- move cursor to a point of intersection, then hit ENTER 3X
- repeat for other points of intersection



Solutions are (-2,-5) and (0,3)

Try: Solve each system of equations graphically.

a) $x - y + 1 = 0$
$x^2 - 6x + y + 3 = 0$

b) $2x^2 + 16x + y = -26$
$x^2 + 8x - y = -19$

Topic 2

Example 1

Solve a System of Linear-Quadratic Equations Algebraically

Solve the following system of equations.

$$5x - y = 10$$
$$x^2 + x - 2y = 0$$

Method 1: Substitution

1. Solve the linear equation for y.

   $5x - y = 10$

   $y = 5x - 10$

2. Substitute 5x - 10 for y in the quadratic equation and simplify.

   $x^2 + x - 2(5x - 10) = 0$

   $x^2 - 9x + 20 = 0$

3. Solve quadratic by factoring or Quad. Prgm.

   $(x - 4)(x - 5) = 0$

   $x = 4$ or $x = 5$

Method 2: Elimination

1. Align the terms with the same degree.

   $5x - y = 10$ ①

   $x^2 + x - 2y = 0$ ②

2. Multiply ① by -2 so you get opposite term to -2y in ①.

   $-2(5x - y) = -2(10)$

   $-10x + 2y = -20$ ③

3. Add ③ and ② to eliminate y-terms.

   $0x^2 - 10x + 2y = -20$

   $x^2 + x - 2y = 0$

   $x^2 - 9x = -20$

   $x^2 - 9x + 20 = 0$

Try: Solve the following system of equations algebraically.

$$3x + y = -9$$
$$4x^2 - x + y = -9$$

Example 2

Model a Situation With a System of Equations

Determine two integers such that the sum of the smaller number and twice the larger number is 46. Also, when the square of the smaller is decreased by three times the larger, the result is 93.

a) Write a system of equations.

Solution:

a) Let S represent the smaller number.
Let L represent the larger number.

First statement: "the smaller number and twice the larger number is 46"

$S + 2L = 46$

Second statement: "the square of the smaller is decreased by three times the larger, the result is 93"

$S^2 - 3L = 93$

b) Solve algebraically.

Hint: use Elimination Method

$S + 2L = 46$ (1)

$S^2 - 3L = 93$ (2)

- Multiply system (1) by 3 and (2) by 2

$3(S + 2L) = 3(46)$ — new system: $3S + 6L = 138$ (3)

$2(S^2 - 3L) = 2(93)$ — new system: $2S^2 - 6L = 186$ (4)

- Add (3) and (4) to eliminate L.

$0 + 3S + 6L = 138$

$2S^2 - 0S - 6L = 186$

~~$2S^2 + 3S = 324$~~

$2S^2 + 3S - 324 = 0$ [use quad. prgm on Calc.]

$S =$ ~~-13.5~~ or 12 — numbers are supposed to be integers.

$S = 12$

- Substitute $S = 12$ into linear equation to find L.

$S + 2L = 46$

$12 + 2L = 46$

$2L = 34$

$L = 17$

Solution

(12, 17)

Try: Determine two integers that have the following relationship: Fourteen more than twice the first integer gives the second integer. The second integer increased by one is the square of the first integer.

a) Write a system of equations.
b) Solve algebraically.

Example 3

Solve a System of Quadratic-Quadratic Equations Algebraically

Solve the following systems of equations.

$3x^2 - x - y - 2 = 0$ ①

$6x^2 + 4x - y = 4$ ②

• Must eliminate y, so multiply ① by -1 then add ① and ② .

$-3x^2 + x + y = -2$

$6x^2 + 4x - y = 4$

$3x^2 + 5x = 2$

• Solve the quadratic equation by factoring or Quad. Prgm.

$x = -2$ or $x = \frac{1}{3}$

• Now substitute these values into $3x^2 - x - y = 2$

$3(-2)^2 - (-2) - y = 2$

$12 + 2 - y = 2$

$y = 12$

$3\left(\frac{1}{3}\right)^2 - \left(\frac{1}{3}\right) - y = 2$

$\frac{1}{3} - \frac{1}{3} - y = 2$

$y = -2$

$$Solutions: \ (-2, 12) \ and \ \left(\frac{1}{3}, -2\right)$$

Try: **a)** **Solve the system algebraically.**

$$6x^2 - x - y = -1$$
$$4x^2 - 4x - y = -6$$

b) **Two paths for an "alley-oop" in basketball can be modelled by these two system of equations, where d is horizontal distance and h is height.**

The pass: $d^2 - 2d + 3h = 9$

The jump for the dunk: $5d^2 - 10d + h = 0$

i) Solve the system of equations algebraically.

ii) Interpret your results.

LEARNING GUIDE 15

Inequality Reference Table

Types	Describtion	Graph Line		Shading	Graph. Calc. Feature
>	Greater than	Broken	- - - - -→	Above line	◥
≥	Greater than and =	solid	⟶	Above line	◥
<	Less than	Broken	- - - - -→	Below line	◣
≤	Less than and =	solid	⟶	Below line	◣

Topic 1

Example 1

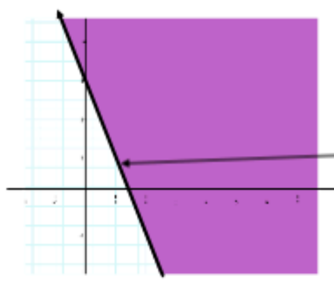
Graph a Linear Inequality

Graph $2x + y \geq 3$

The triangles above give you the correct shading. They go in front of the Y1 =. You hit the ENTER key as many times until you see the appropriate one.

Method 1: Graphing Calculator

- put into y= form ⟶ $y \geq -2x + 3$
- type ◥ Y1 = -2x + 3,
- then hit GRAPH
- hit 2nd GRAPH to get two points to plot on graph paper
- draw the <u>solid</u> line between them



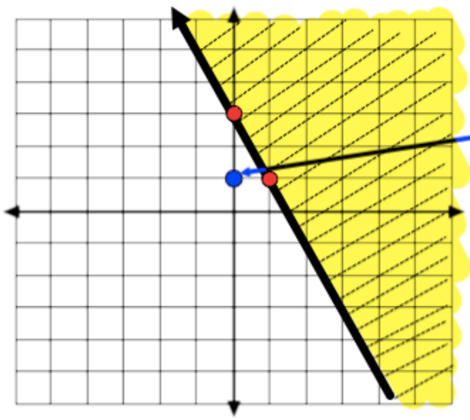
The calculator will not indicate if the line is solid or broken.

Method 2: Draw using Slope-Intercept Form

$$y \geq -2x + 3$$

Slope: -2   y-intercept: 3

- plot y-intercept
- now from that point, get another point by using the slope: drop 2, run right 1
- connect the two points with a solid line and **shade above the line.**



☆ b) Determine if the point (0, 1) is a part of the solution.

No. The point does not lie in the shaded region.

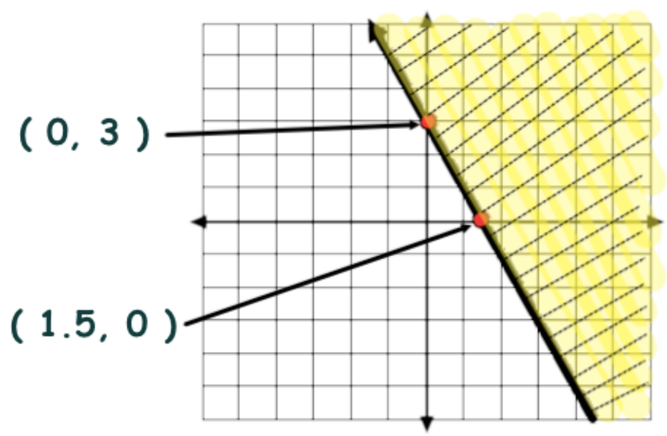
Method 3: Draw using the Intercepts

$$2x + y \geq 3$$

For x = 0
2(0) + y = 3
y = 3

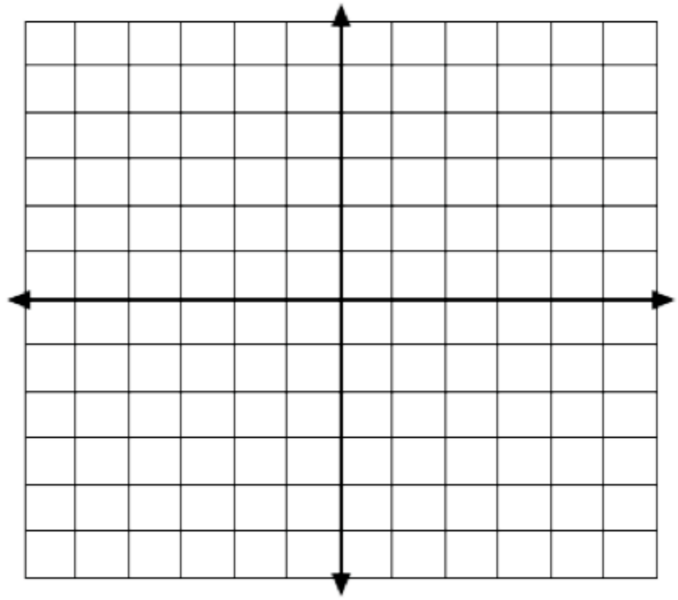
For y = 0
2x + (0) = 3
x = 1.5

- plot the points (0, 3) and (1.5, 0)
- connect the two points with a solid line and **shade above the line.**



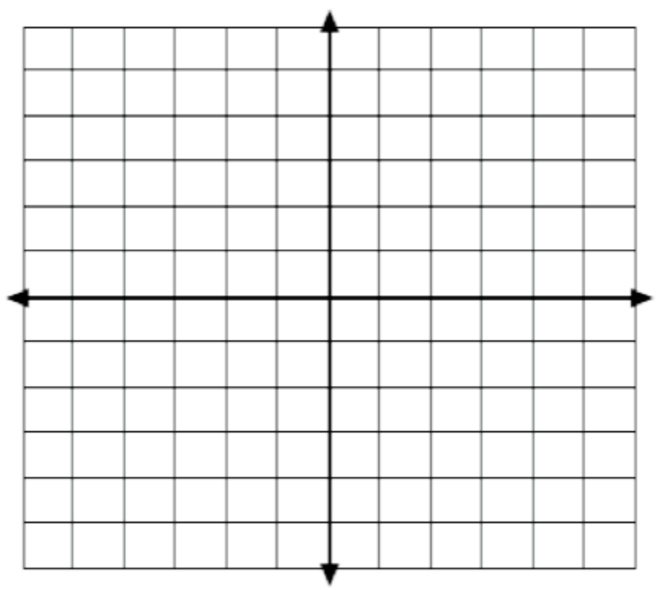
Try: 1a) Graph $4x + 2y \geq 10$

b) Determine if the point (1, 5) is a part of the solution.



2a) Graph $5x - 20y < 0$

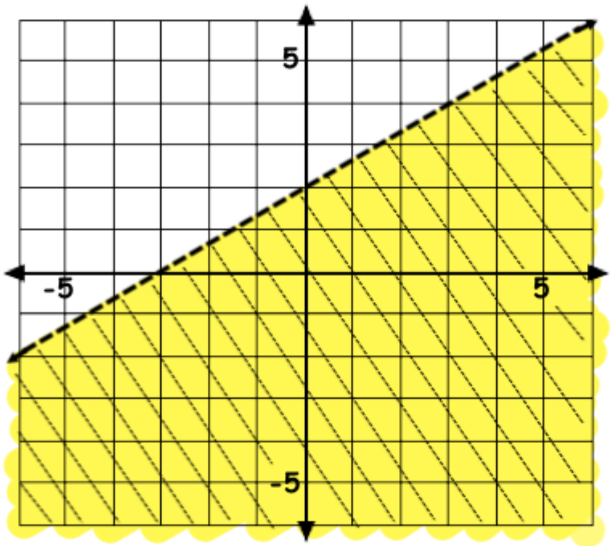
b) Determine if the point (-4, -1) is a part of the solution.



Example 2

Write an Inequality Equation Given Its Graph

Write an inequality equation to represent the graph.



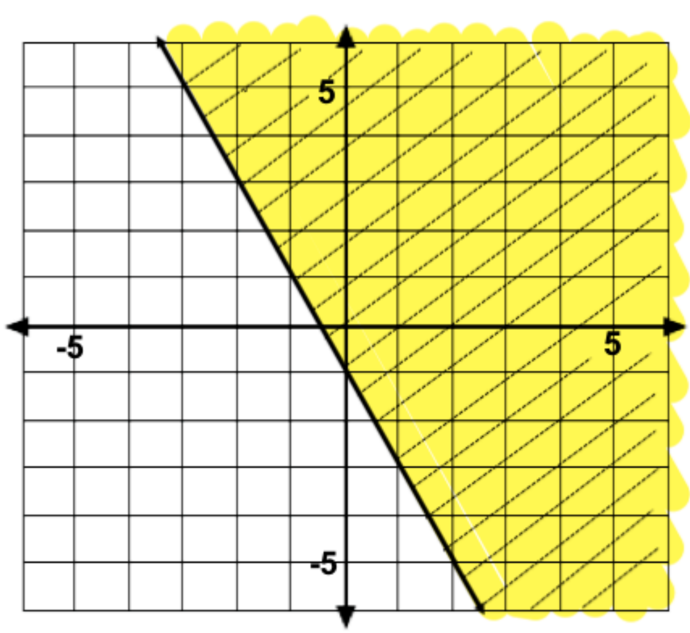
- write the equation in slope-intercept form, $y = mx + b$
- y-intercept is 2. So, $b = 2$
- from that point ⟶ use $\frac{rise}{run}$ to get slope $m = \frac{2}{3}$

- because the line is broke it's either $<$ or $>$. Since the shading is going down it's $<$.

The inequality equation is:

$$y < \frac{2}{3}x + 2$$

Try: Write an inequality equation to represent the graph.



Example 3

Write and Solve an Inequality

Suppose that you are constructing a tabletop using aluminum and glass. The most that you can spend on materials is $50. Laminated safety glass cost $60/m²$, and aluminum costs $1.75/ft. You can choose the dimensions of the table and the amount of each material used. Find all possible combinations of materials sufficient to make the tabletop.

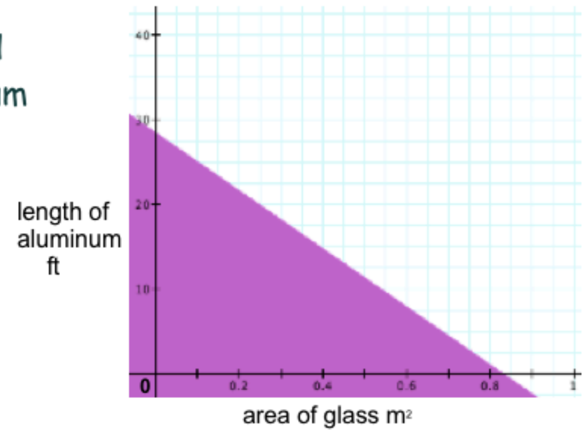
Solution

- let x represent the area of glass used and y represent the length of aluminum used. $60x + 1.75y \leq 50$
- solve for y in terms of x

$$1.75y \leq -60x + 50$$

$$y \leq \frac{-60x}{1.75} + \frac{50}{1.75}$$

- graph using your graphing calculator



Topic 2

Example 1

Solve Quadratic Inequalities - One Variable

Solve: a) $x^2 - 2x - 3 \leq 0$

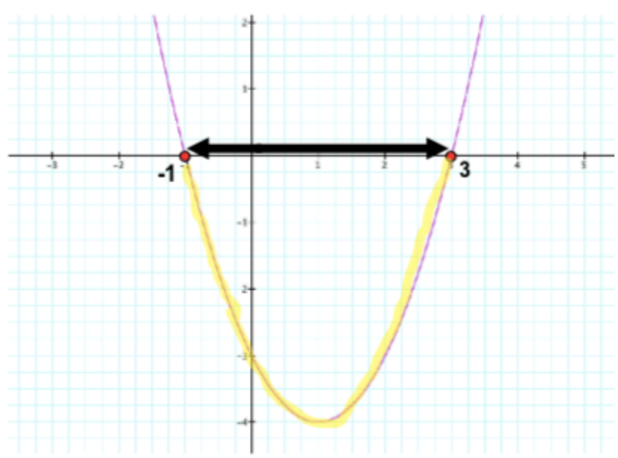
Solution:

- graph the function $f(x) = x^2 - 2x - 3$.

- indicate the roots (x-intercepts).
- highlight the part(s) of the function that are below zero.

The highlighted part is between -1 and 3, thus, the solution is:

$$\{x \mid -1 \leq x \leq 3, x \in R\}$$



b) $x^2 + x - 6 > 0$

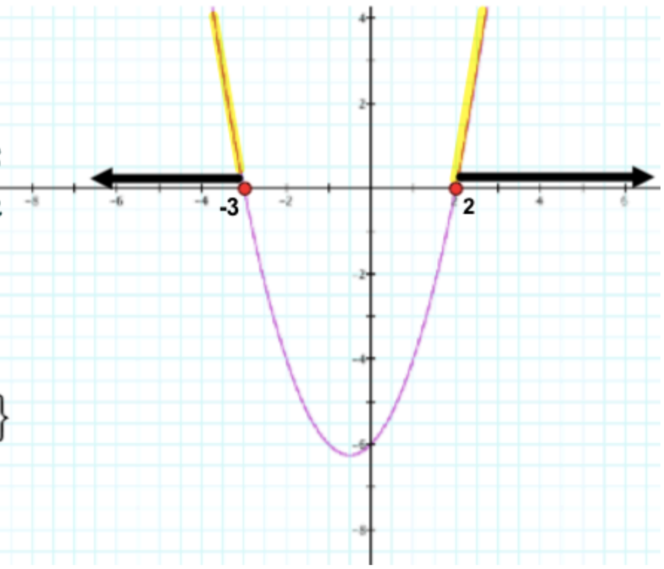
Solution:

- graph the function $f(x) = x^2 + 6x - 6$.
- indicate the roots (x-intercepts).
- highlight the part(s) of the function that are below zero.

The highlighted parts are going to the lesser numbers from -3 [$x < -3$], and to the greater numbers from 2 [$x > 2$].

Thus, the solution is:

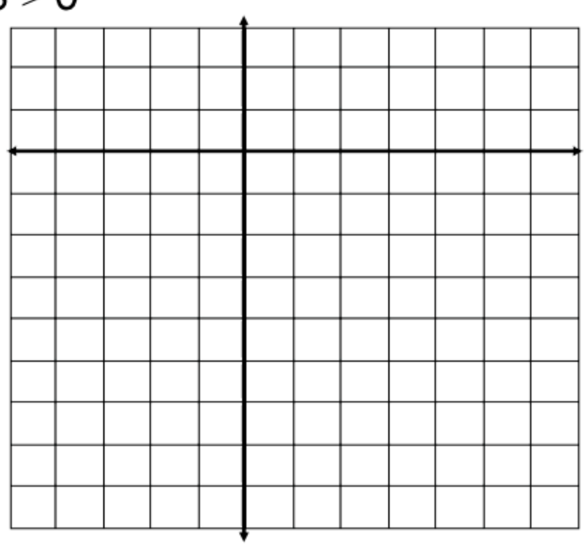
$$\{x \mid x < -3 \ or \ x > 2, x \in R\}$$



c) $2x^2 - 7x > 3$

Same steps as previous two questions, however, you must move the 3 to the left side of equation so you have $f(x) > 0$. → $2x^2 - 7x - 3 > 0$

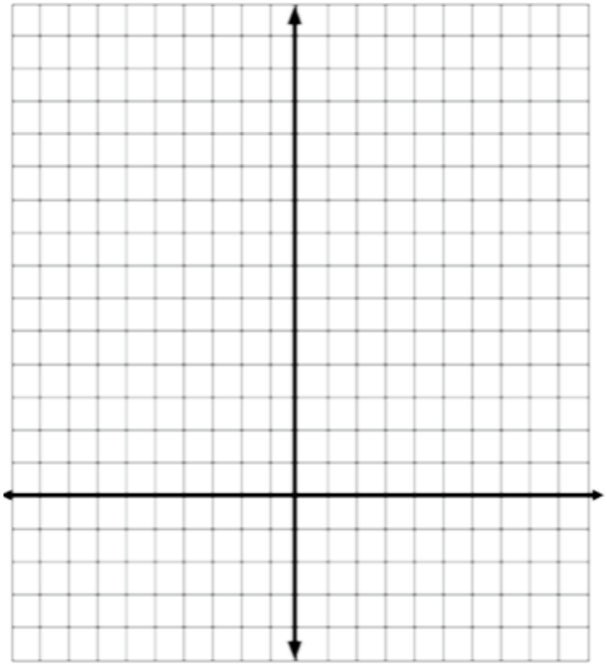
Hint: Use Graph. Calc. to find the roots.

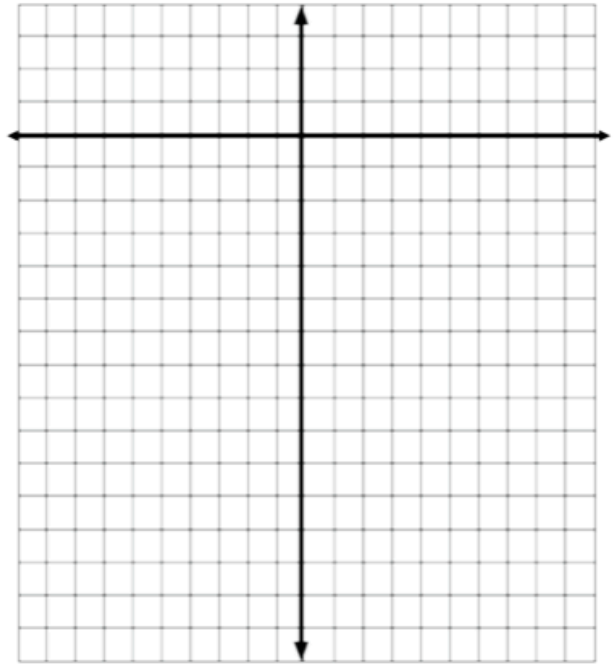


Try: **Solve:**

a) $-x^2 + 3x + 10 < 0$

b) $x^2 - 4x \geq 10$





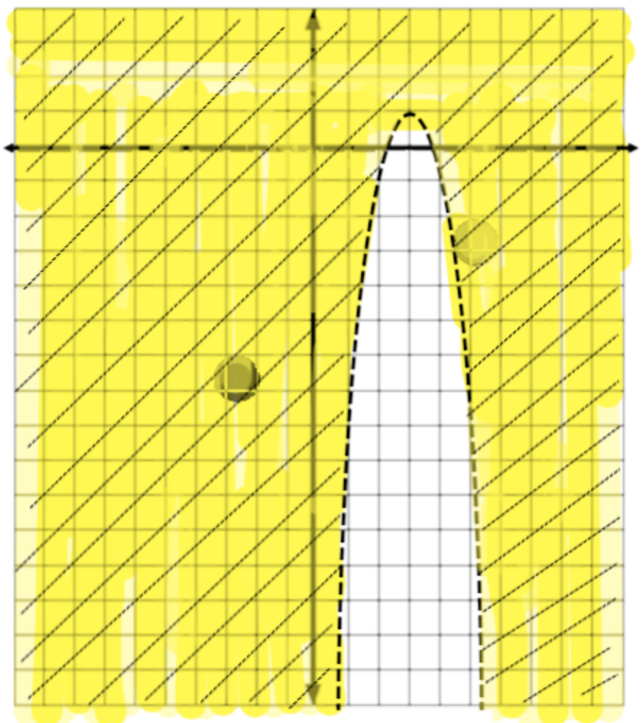
c) A baseball is thrown from a height of 1.5 m.
The inequality $-4.9t^2 + 17t + 15 > 0$ models the time, t, in seconds, that the baseball is in flight.
During what time interval is the baseball in flight?

Topic 3

Example 1

Graph a Quadratic Inequality in Two Variables

a) Graph $y > -2(x - 3)^2 + 1$

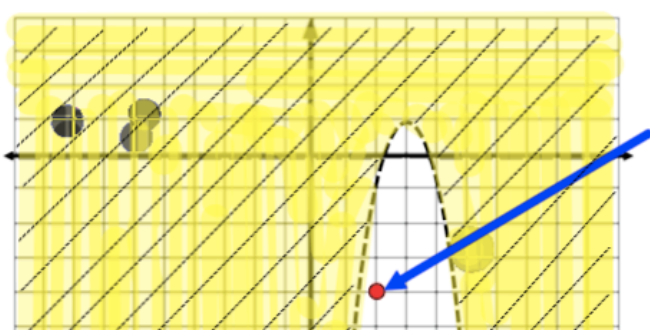


- graph the parabola
- since the inequality symbol is >, draw the parabola as a broken line
- test a point (0, 0) to show where the shading is - within or outside the parabola

Left Side	Right Side
y	$= -2(x - 3)^2 + 1$
$= 0$	$= -2(0 - 3)^2 + 1$
	$= -18 + 1$
	$= -17$

· the point (0, 0) satisfies the inequality, so shade the region outside the parabola

b) Determine if the point (2, -4) is a solution.

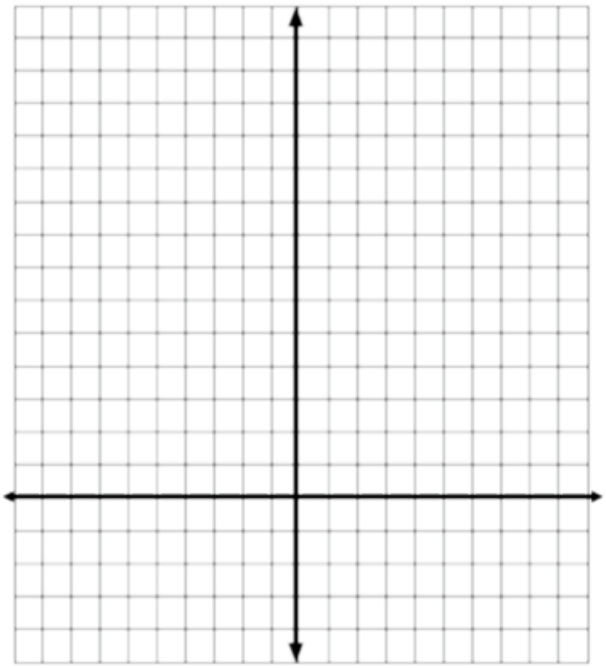


Solution:

No. The point (2, -4) is not in the shaded region.

Try: a) Graph $y > (x - 4)^2 - 2$

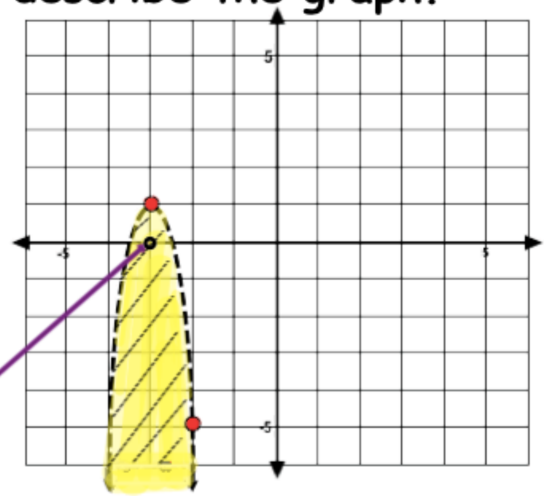
b) Determine if the point (2, 1) is a solution.



Example 2

Write an Inequality Equation

Write an inequality equation to describe the graph.



- select two points on the parabola, the vertex (-3, 1), a point (-2, -5)
  (p = -3, q = 1; x = -2, y = -5)
- use $y = a(x - p)^2 + q$ to get the equation

$-5 = a(-2 + 3)^2 + 1$

$-5 = a + 1$

$-6 = a$

$y = -6(x + 3)^2 + 1$

- broken line indicates $<$ or $>$
- pick a point in the shaded region (-3, 0)

$y = -6(x + 3)^2 + 1$

$0 = -6(-3 + 3)^2 + 1$

$0 < 1$

This is the correct inequality sign.

Solution: $y < -6(x + 3)^2 + 1$

Try:

Write an inequality equation to describe the graph.