PRE-CALCULUS 11

Seminar Notes

Learning Guides 16 & 17

TRIGONOMETRY

Frances Kelsey Secondary School – 2019/20

Trig. Review

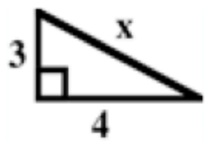
Basic Trigonometry

To find a side:

$$a^2 + b^2 = c^2$$

1. By Pythagorous

Example:



1. Write formula $a^2 + b^2 = c^2$ and substitute $3^2 + 4^2 = c^2$
2. Solve $9 + 16 = c^2$

$$25 = c^2$$

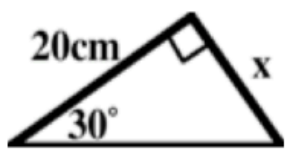
$$\sqrt{25} = c$$

$$5 = c$$

SOH CAH TOA

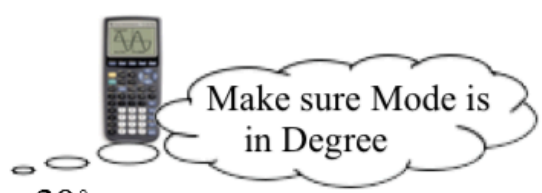
2. By Trigonometry.

Example:



1. Determine the correct ratio (sin, cos or tan) - here we use tan because we are using the opposite and adjacent sides.
2. Write out ratio:

$$\tan = \frac{opp}{adj}$$



3. Substitute:

$$\tan 30^\circ = \frac{x}{20}$$

or

$$\frac{\tan 30^\circ}{1} = \frac{x}{20}$$

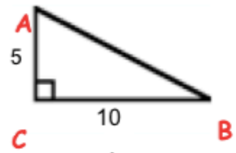
4. Cross multiply to solve:

$$x = \tan 30^\circ \times 20 = 11.6 \text{ cm}$$

To find an Angle

1. By Trigonometry

Example: Find angle B



1. Determine the correct ratio (sin, cos or tan) - here we use tan because we are using the opposite and adjacent sides.

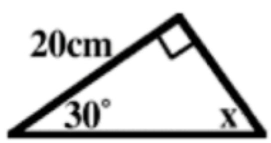
2. Write out ratio: $\tan = \frac{opp}{adj}$

3. Substitute: $x = \tan B = \frac{5}{10} = 0.5$

4. Solve using $\tan^{-1}$: $B = \tan^{-1}(0.5) = 26.6^\circ$

2. Using the Sum of Angles in a Triangle

Example: Find angle x

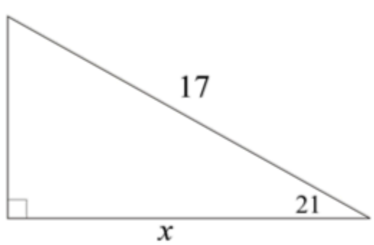


1. All 3 angles sum to 180° - we know 2 of the angles are 30° and 90°

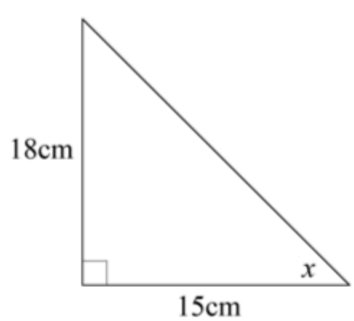
2. x = 180 - 90 - 30 = 60°

Try: Find the indicated side or angle

a)

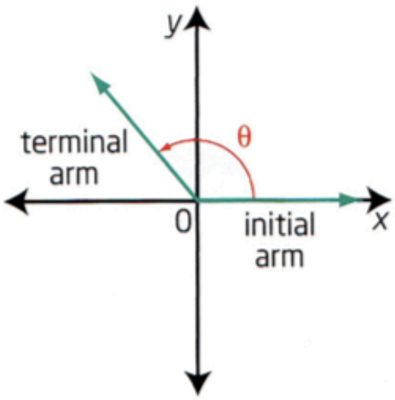


b)



Topic 1

Angles in Standard Position



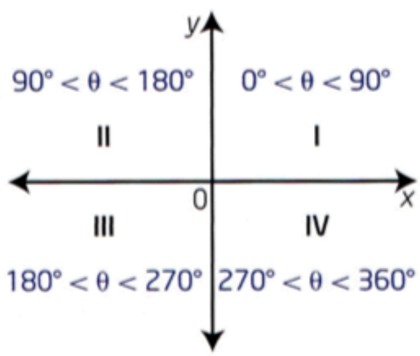
An angle is in standard position when:

1. it's vertex is at the origin

and

2. the initial arm is on the positive x-axis

*Angles in standard position are always measured counter-clockwise from the initial arm

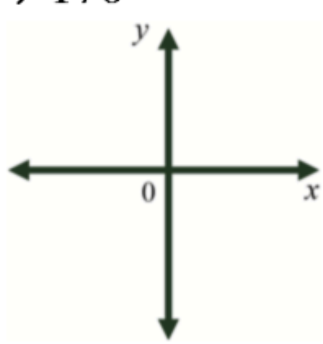


Angles in standard position are shown on the Cartesian plane. The x-axis and y-axis divide the plane into 4 quadrants

Example 1

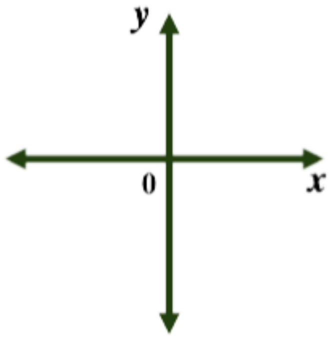
Sketch an Angle in Standard Postion, $0^\circ \le \theta \le 360^\circ$

Example: a) 170°

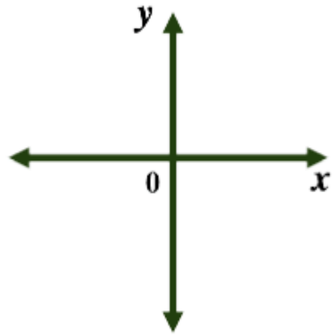


Try: **Sketch each Angle in Standard Postion, and state the quadrant in which the terminal arm lies.**

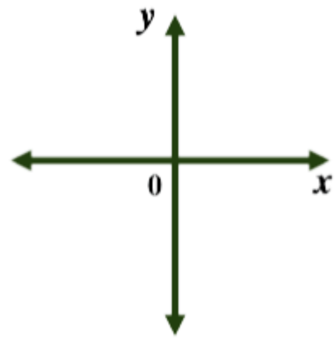
a) 37°



b) 320°



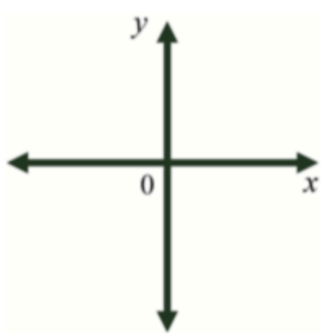
c) 245°



Example 1 - Part b

Directions: Are defined as a measure either east or west of north or south

Example: Show N40°W as an angle in standard position

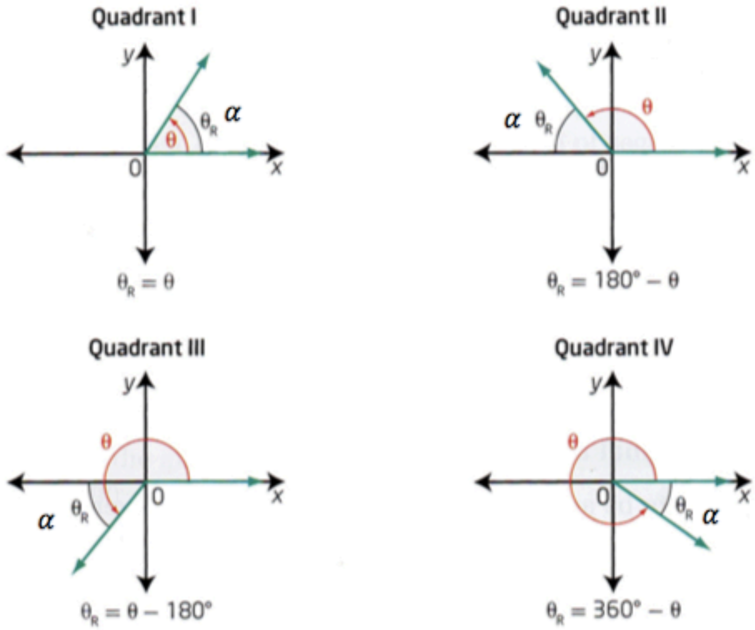


1. Start at North and go 40° toward west

Reference Angles

A Reference angle (θ_R) or α is:

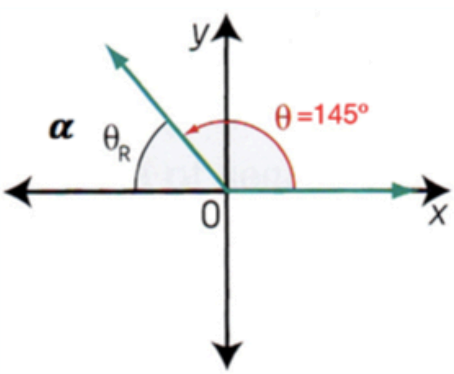
1. the acute ($\leq 90^\circ$) angle between the terminal arm and the x-axis
2. always positive



Example 2

Determine a Reference Angle

Example: $\theta = 145^\circ$



1. sketch angle
2. place the reference angle - find the shortest distance back to the x-axis
3. calculate reference angle

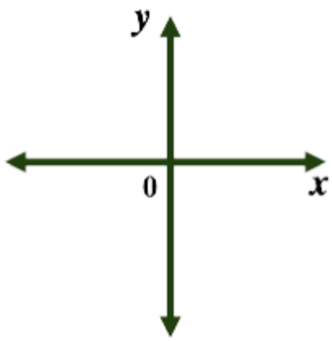
$$\theta_R = 180^\circ - \theta$$

$$\theta_R = 180^\circ - 145^\circ$$

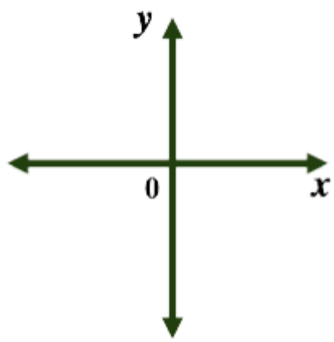
$$\theta_R = 35^\circ$$

Try: Determine the Reference Angle for each angle in standard postition

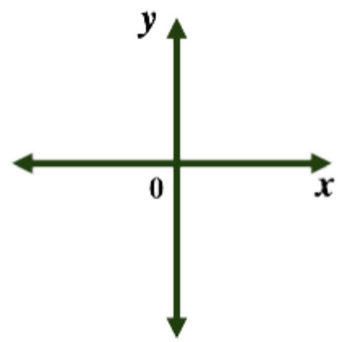
a) 79°



b) 243°



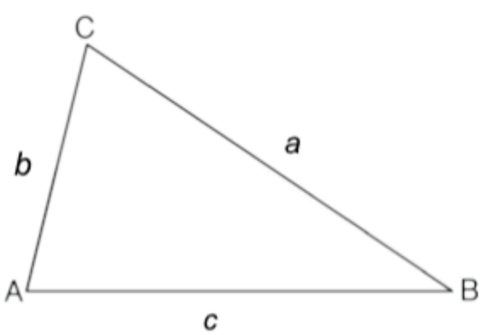
c) 317°



Topic 2

Sine Law

Sine Law is a relationship between the sides and angles of **any** triangle.

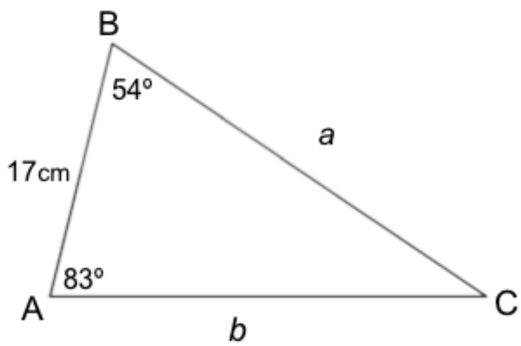


$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Example 1

Determine an Unknown Side Length

Example: Find the length of side b.



1. Calculate $\angle C$

$\angle C = 180^\circ - 83^\circ - 54^\circ$

$\angle C = 43^\circ$

2. $\frac{\sin C}{c} = \frac{\sin B}{b}$

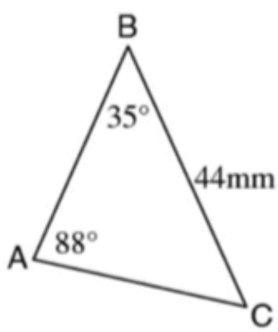
$\frac{\sin 43^\circ}{17} = \frac{\sin 54^\circ}{b}$

$b = \frac{\sin 54^\circ \times 17}{\sin 43^\circ} = 20.166...$

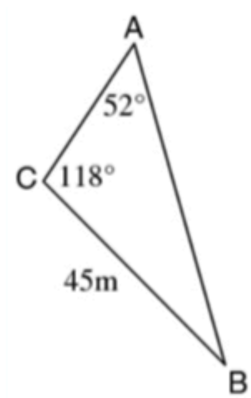
$b = 20.2$ cm

Try: Find the length of side c in each of the following triangles.

a)



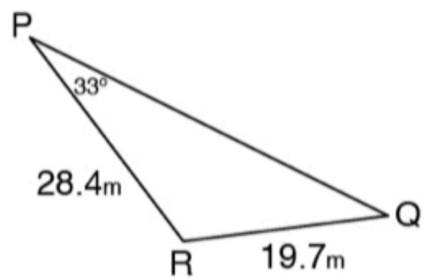
b)



Example 2

Determine an Unknown Angle Measure

Example: In ΔPQR, $\angle P = 33^\circ$, $p = 19.7$m, and $q = 28.4$m. Find the measure of $\angle R$, to the nearest degree.



1. Find $\angle Q$ using sine law

$$\frac{\sin 33^\circ}{19.7} = \frac{\sin Q}{28.4}$$

$$\sin Q = \frac{\sin 33^\circ \times 28.4}{19.7}$$

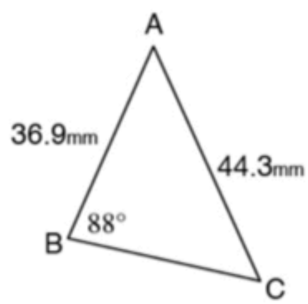
$$\angle Q = \sin^{-1}\left(\frac{\sin 33^\circ \times 28.4}{19.7}\right) = 51.73... \approx 52^\circ$$

2. Find $\angle R$ using sum of $\angle$'s in a triangle

$$\angle R = 180^\circ - 33^\circ - 54^\circ = 95^\circ$$

Try: Find the measure of $\angle A$, to the nearest degree, in each of the following triangles.

a)

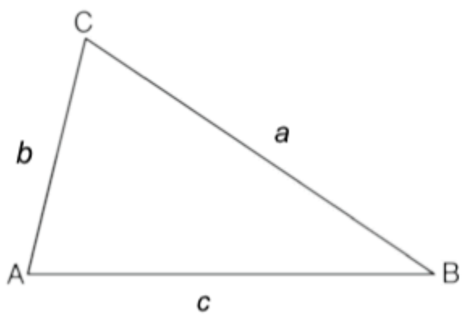


b) In ΔABC, $\angle B = 63^\circ$, $b = 25.5$ cm and $c = 17.3$cm

LEARNING GUIDE 17

Topic 1 **Cosine Law**

Sine Law is the relationship between the cosine of an angle and the lengths of the three sides of any triangle.



$$c^2 = a^2 + b^2 - 2bc\cos C$$

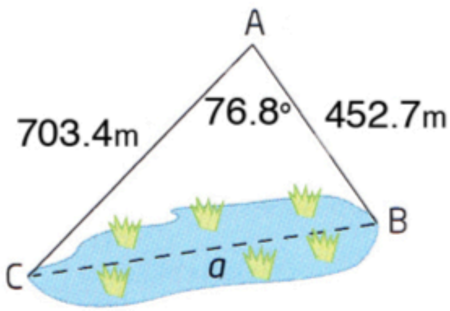
$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Example 1

Determine an Distance (Side Length)

Example: A surveyor measures the distance to one end of a lake as 703.4m. The distance to the other end is 452.7m and the angle between the two is 76.8°. Find the length of a lake.



1. Sketch a diagram
2. Use cosine law

$$a^2 = b^2 + c^2 - 2bc\cos A$$

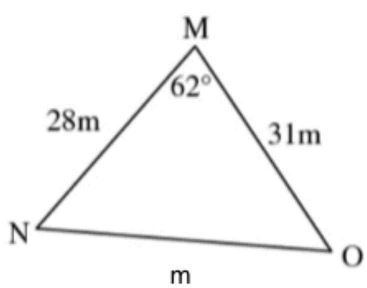
$$a^2 = (703.4)^2 + (452.7)^2 - 2(703.4)(452.7)\cos 76.8^\circ$$

$$a^2 = 554281.6894$$

$$a = \sqrt{554281.6894} = 744.5009... = 744.5m$$

Try: Find the length of the indicated side in each of the following triangles, to the nearest tenth.

a)

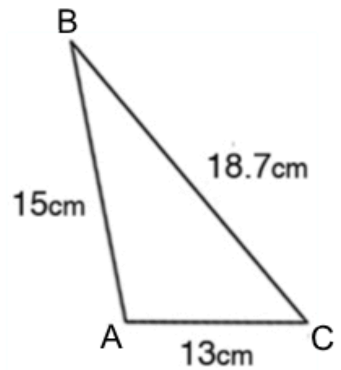


b) In ΔABC, $\angle B = 115^\circ$, $a = 9$cm and $c = 8$cm. Find b.

Example 2

Determine an Angle

Example: A triangular brace has side lengths of 15cm, 18.7cm and 13cm. Find the measure of the angle opposite the 15cm side.



1. Sketch a diagram
2. Use cosine law

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$(15)^2 = (18.7)^2 + (13)^2 - 2(18.7)(13)\cos C$$

$$225 - 349.69 - 169 = -486.2\cos C$$

$$-293.69 = -486.2\cos C$$

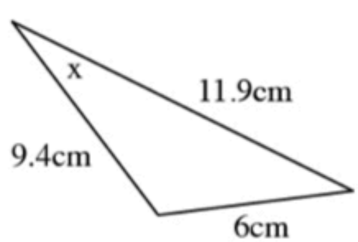
$$\frac{-293.69}{-486.2} = \cos C$$

$$\cos^{-1}\left(\frac{-293.69}{-486.2}\right) = \angle C$$

$$53^\circ = \angle C$$

Try: Find the measure of the indicated angle in each of the following triangles, to the nearest tenth.

a)



b) In ΔABC, $a = 9\text{m}$, $b = 18\text{m}$ and $c = 21\text{m}$. Find $\angle A$.

Example 3

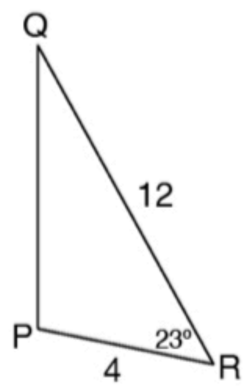
Solve a Triangle

Example: In ΔPQR, $p=12$, $q=4$, and $\angle R=23^\circ$. Find the length of the unknown side and the measure of the other 2 angles.

1. Sketch a diagram
2. Use cosine law to find r

$$r^2=p^2+q^2-2pq\cos R$$
$$r^2=12^2+4^2-2(12)(4)\cos 23^\circ$$
$$r=\sqrt{71.6315...}$$
$$r=8.46354...$$

3. Use cosine law to find $\angle$P



4. $p^2=q^2+r^2-2qr\cos P$

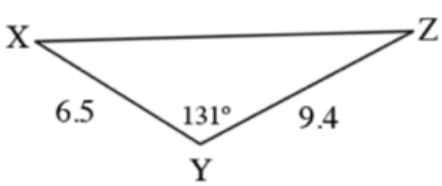
$$(12)^2=(4)^2+(8.453)^2-2(4)(8.453)\cos P$$
$$144-16-71.631=(-67.708)\cos P$$
$$\frac{56.368}{-67.708}=\cos P$$
$$\cos^{-1}\left(\frac{56.368}{-67.708}\right)=P$$
$$146.4^\circ=P$$

5. Find $\angle$Q using sum of $\angle$'s in a triangle

$$\angle Q=180^\circ-146.4^\circ-23^\circ$$
$$\angle Q=10.6^\circ$$

Try: Solve the following triangles. Round your answers to the nearest tenth.

a)



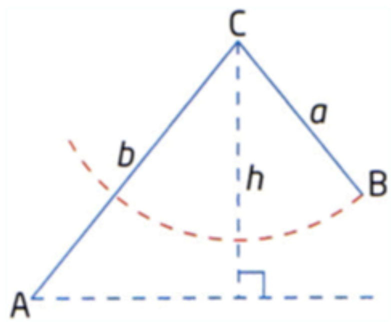
b) In ΔABC, $a=9$, $b=7$ and $\angle C=33.6^\circ$

Topic 2 **The Ambiguous Case**

If you are given two sides and an angle opposite one of those sides (ASS), the ambiguous case may occur. There are 3 possibilities:

1. no triangle exists with the given measures - NO SOLUTION
2. one triangle exists with the given measures - 1 SOLUTION
3. two distinct triangles exist - 2 SOLUTIONS

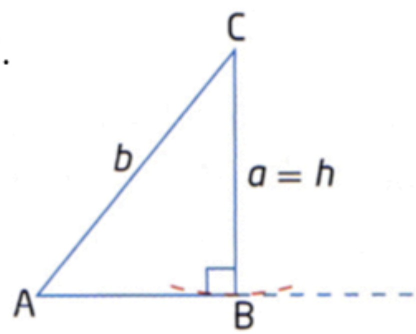
1.



$a < h$

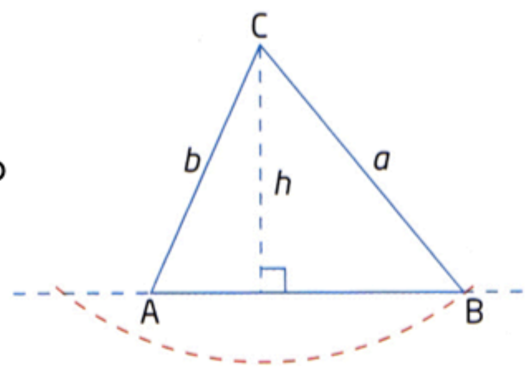
no solution - the sides don't meet

2.

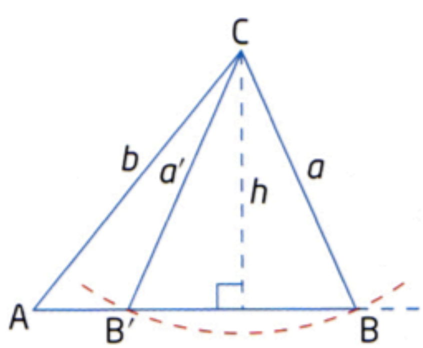


$a = h$ or $a \geq b$

one solution



3.



$h < a < b$

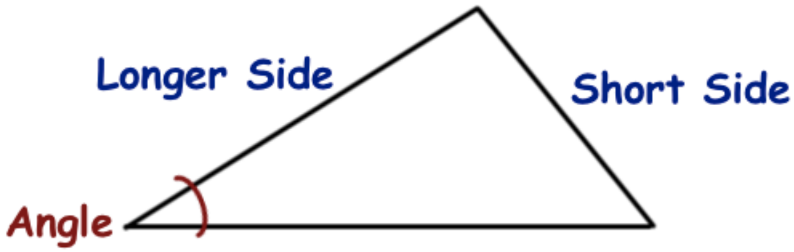
two solutions

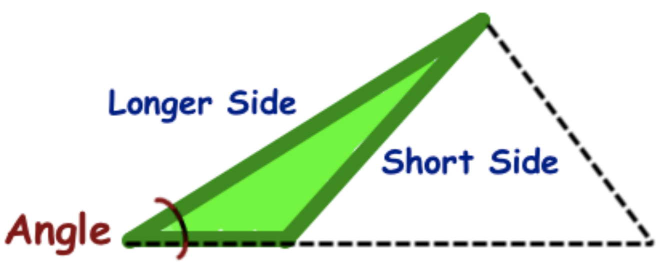
The Ambiguous Case

If you are given information that is an **"ASS"**

Angle (acute∠, Side, Side)

1st - Ambiguous Triangle Template:

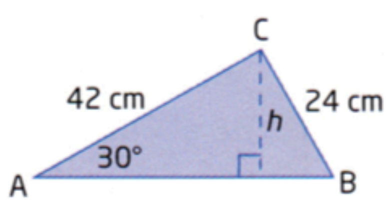




Example 3

Sine Law in an Ambiguous Case

Example: In ΔABC $\angle A = 30^\circ$, $a = 24$ cm, and $b = 42$ cm. Determine the measures of all other sides and angles



1. Sketch possible triangle
2. Find the height (h)

$\sin A = \frac{h}{b}$

$h = b\sin A$

$h = 42\sin 30^\circ$

$h = 21$

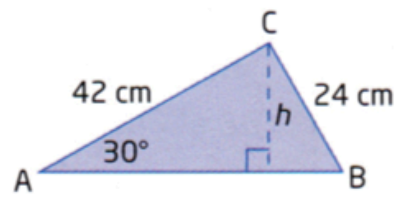
a > h, so there are 2 possible triangles

Example 3 cont.

Triangle 1:

3. Solve for $\angle B$ using sine law

$$\frac{\sin B}{b} = \frac{Sin A}{a}$$
$$\frac{\sin B}{42} = \frac{Sin 30^\circ}{24}$$
$$\sin B = \frac{42 Sin 30^\circ}{24}$$
$$\angle B = \sin^{-1}\left(\frac{42 Sin 30^\circ}{24}\right)$$
$$\angle B = 61.044... = 61^\circ$$

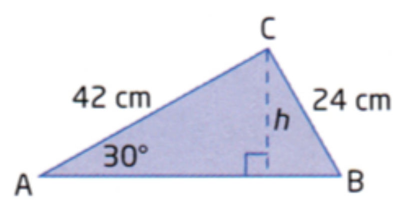


4. Find $\angle C$ (sum of angles in a Δ)

$$\angle C = 180^\circ - 61^\circ - 30^\circ = 89^\circ$$

5. Use sine law to find side c

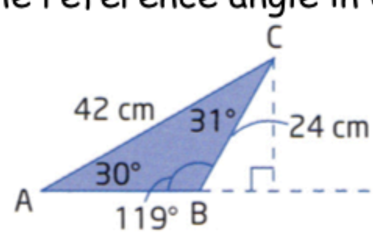
$$\frac{c}{\sin 89^\circ} = \frac{24}{\sin 30^\circ}$$
$$c = \frac{24 \sin 89^\circ}{\sin 30^\circ}$$
$$c = 47.992... = 48$$



Triangle 2:

1. Solve for $\angle B$ using 61° as the reference angle in quadrant II

$$\angle B = 180^\circ - 61^\circ$$
$$\angle B = 119^\circ$$



2. Find $\angle C$ (sum of angles in a Δ)

$$\angle C = 180^\circ - 119^\circ - 30^\circ$$
$$\angle C = 31^\circ$$

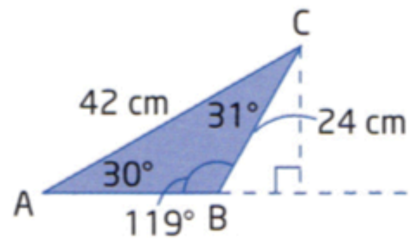
Example 3 cont.

3. Use sine law to find side c

$$\frac{c}{\sin 31^\circ} = \frac{24}{\sin 30^\circ}$$

$$c = \frac{24 \sin 31^\circ}{\sin 30^\circ}$$

$$c = 24.721... = 25$$



Try: In triangle ABC, $\angle A = 21^\circ$, $a = 12\text{m}$ and $b = 17\text{m}$. Determine the measures of all other sides and angles

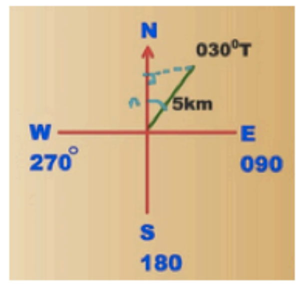
Topic 3

Bearings:
True and Conventional

True Bearing:

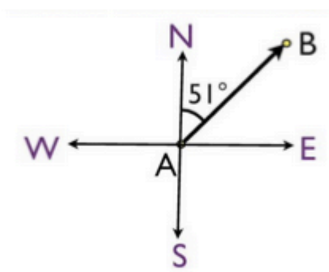
Is the direction to an object from a point measured clockwise from true north.

Example: Bearing of 030°



Conventional Bearing:

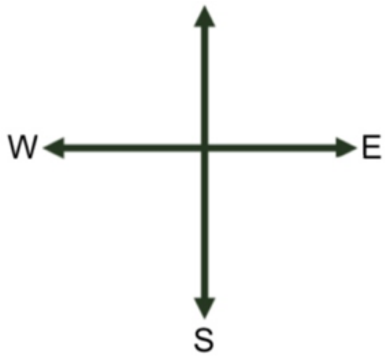
Is a direction point to an object stated as the number of degrees east or west of the north-south line.



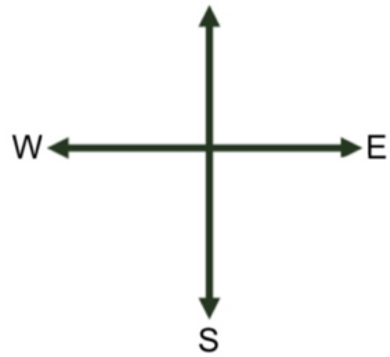
Example: N 51°E

Try: **Sketch each Conventional Bearing AND convert it into a True Bearing AND an angle in standard position.**

S45°E

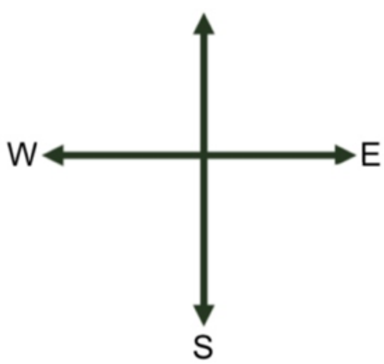


N80°W



Try: **Sketch each True Bearing AND convert it into a Conventional Bearing AND an angle in standard position.**

105°



220°