PRE-CALCULUS 11

Seminar Notes

Learning Guides 3 & 4

REAL NUMBERS, POWERS & FACTORING

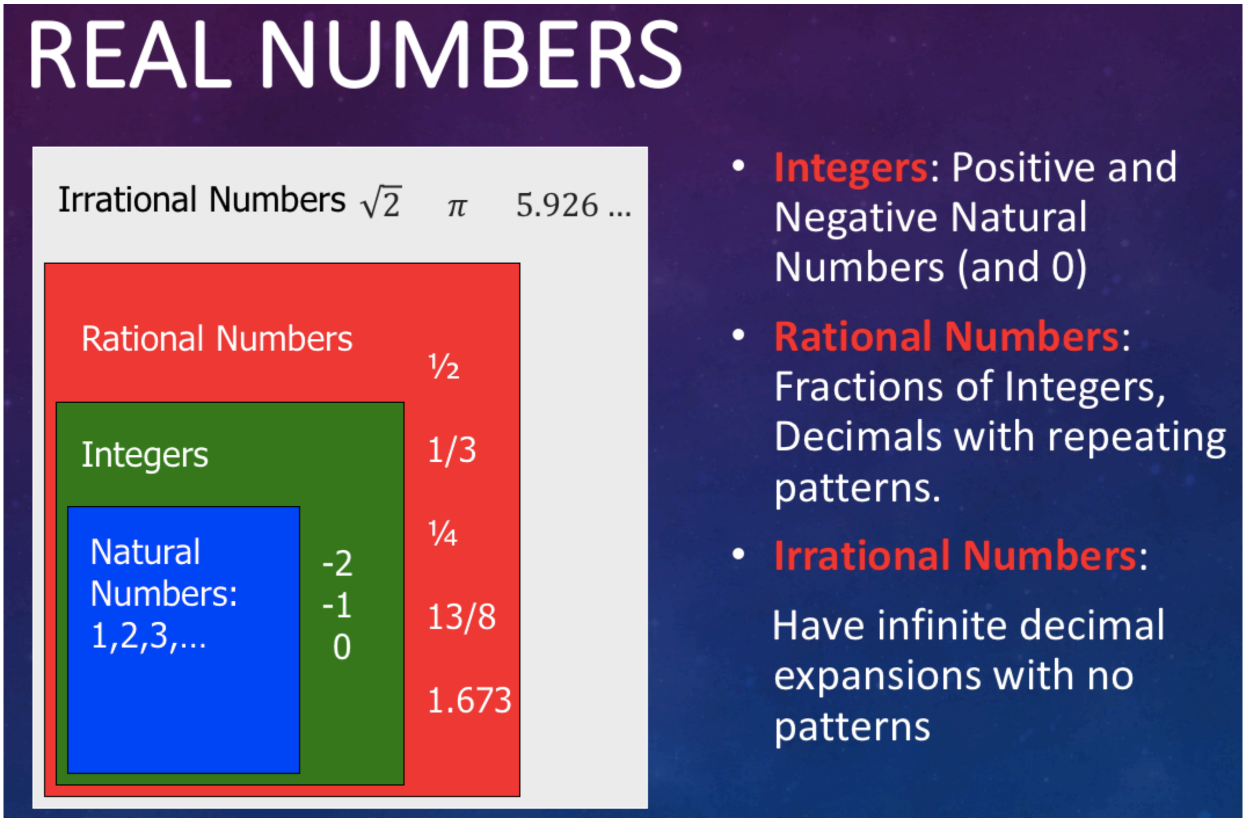
Frances Kelsey Secondary School – 2019/20

KEEPIN' IT REAL

Topic 1

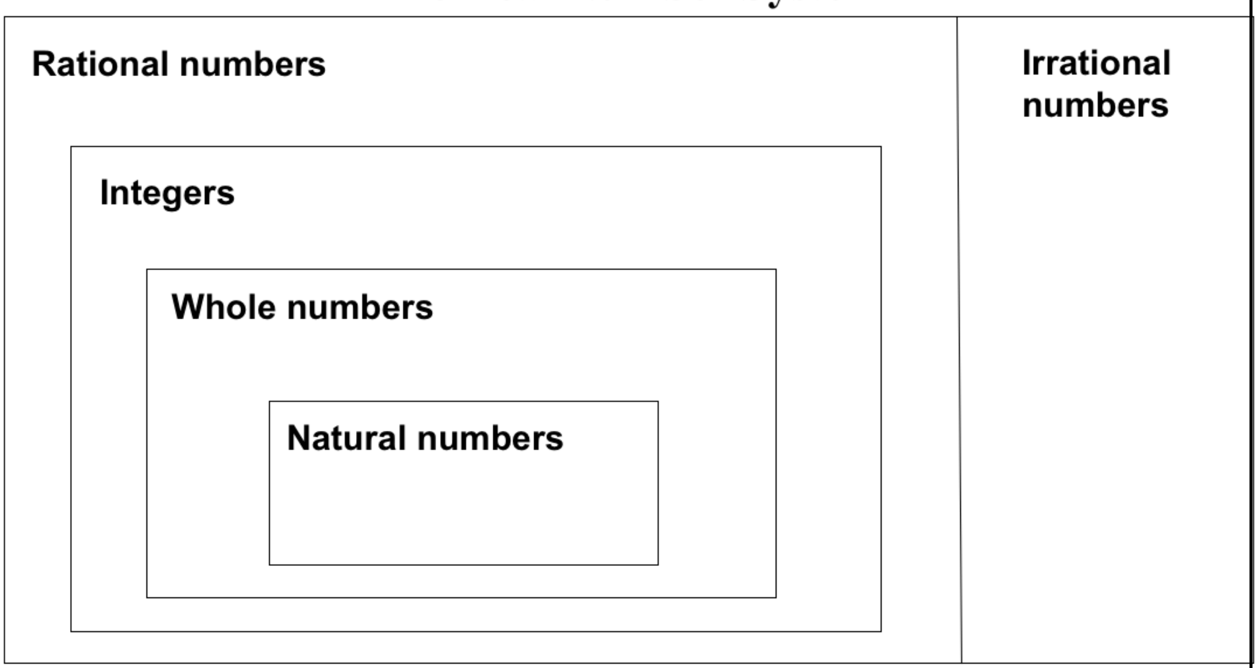
THE REAL NUMBER SYSTEM

WITH A PARTNER, LIST EXAMPLES OF AS MANY DIFFERENT **TYPES** OF NUMBERS AS YOU CAN THINK OF.



Now from the list that you generated with a partner, properly classify them below:

The Real Number System



Let's Recap:

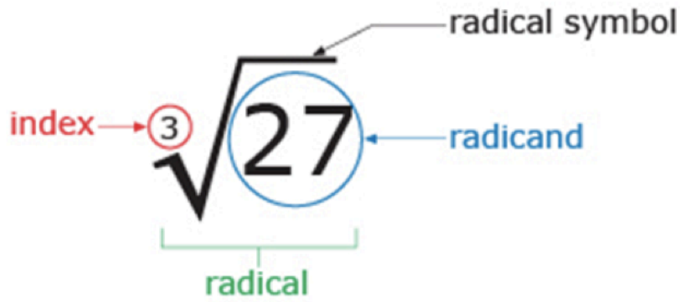
The Real Number System

- The **natural numbers** are the **counting numbers,** without _______.
- **Whole numbers** include the natural numbers and ____________.
- **Integers** include all whole numbers and their ________________.
- **Rational numbers** are real numbers that can be written as a _____________ where a and b are integers and $b \neq 0$. Any rational number can be represented as a terminating or a repeating ___________.
- **Irrational numbers** are any real numbers that are not _________.

Topic 2

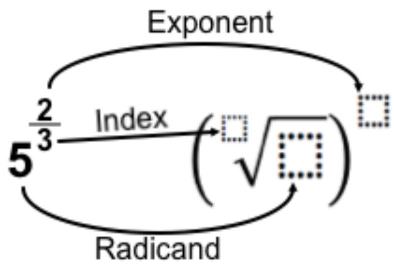
Roots and Radicals

❖A ***radical*** is used to indicate a root.



Example 1

Writing from Rational Exponents to Radical Form



$= \left(\sqrt[3]{5}\right)^2$ *or* $\sqrt[3]{5^2}$

Try: $8^{\frac{1}{5}} =$ $\qquad$ $(3x)^{\frac{4}{5}} =$

$-4x^{\frac{3}{5}} =$ $\qquad$ $3^{-\frac{5}{2}} =$

Example 2

Now write from Radical Form to Rational Exponents

$$\sqrt[4]{3^5} \longrightarrow 3^{\frac{5}{4}}$$

Try:

$\sqrt[7]{2^4}$

$\left(\sqrt{5}\right)^9$

$\dfrac{1}{\sqrt[4]{x}}$

Example 3 Evaluate Radicals

$$8^{\frac{2}{3}} = \left(\sqrt[3]{8}\right)^2$$

$$(2)^2 = 2$$

Try:

1. $16^{\frac{1}{4}}$
2. $144^{\frac{1}{2}} - 27^{\frac{1}{3}}$
3. $81^{\frac{3}{2}}$
4. $64^{\frac{7}{6}}$

Topic 3 Rational Exponents

Example 1

Simplify. All variables represent nonnegative numbers.

$\sqrt[4]{a^4 b^{20}}$

$$\begin{aligned}
\sqrt[4]{a^4 b^{20}} &= \left(a^4 b^{20}\right)^{\frac{1}{4}} && \textit{Definition of } b^{\frac{1}{n}}. \\
&= \left(a^4\right)^{\frac{1}{4}} \cdot \left(b^{20}\right)^{\frac{1}{4}} && \textit{Power of a Product Property} \\
&= \left[a^{4 \cdot \frac{1}{4}}\right] \cdot \left[b^{20 \cdot \frac{1}{4}}\right] && \textit{Power of a Power Property} \\
&= \left(a^1\right) \cdot \left(b^5\right) && \textit{Simplify exponents.} \\
&= ab^5
\end{aligned}$$

Try:

Simplify. All variables represent nonnegative numbers.

$\sqrt[4]{x^4 y^{12}}$

$\left(x^6 y^4\right)^{\frac{1}{2}} \sqrt{y^2}$

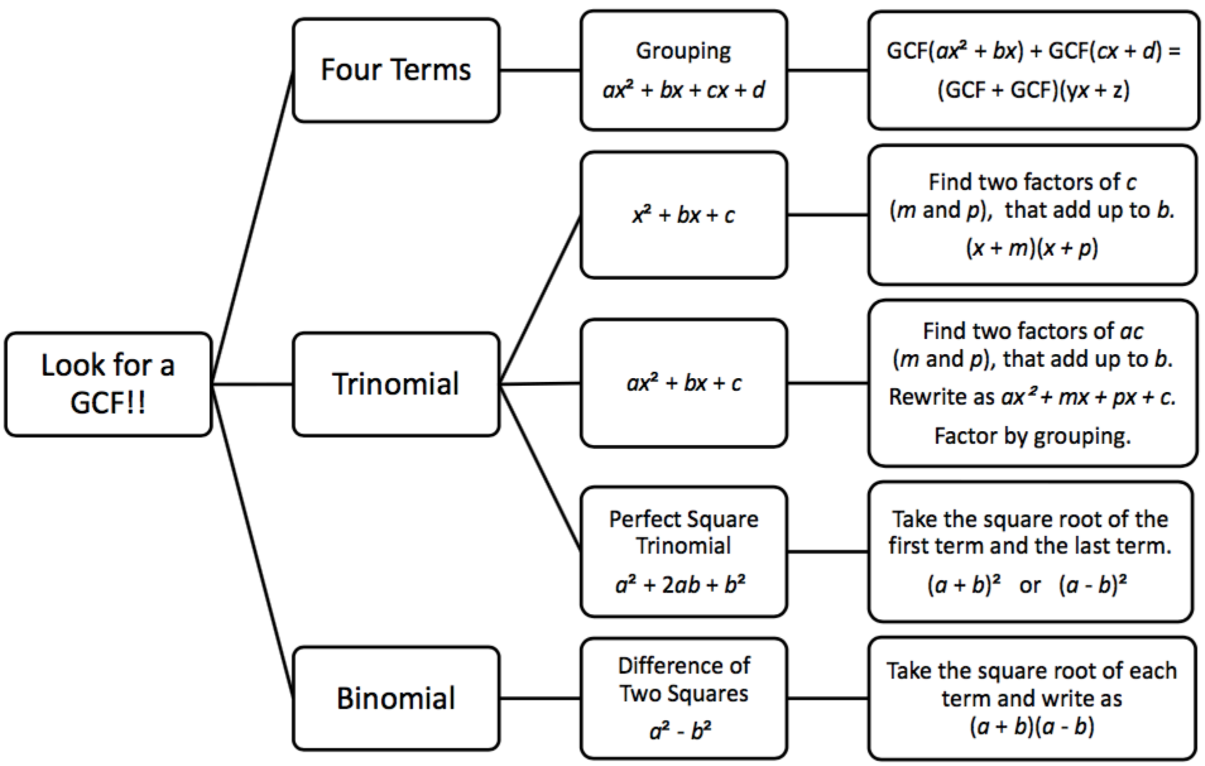
$\dfrac{\left(xy^{\frac{1}{2}}\right)^2}{\sqrt[5]{x^5}}$

$\dfrac{\left(x^2 y^{-\frac{2}{5}}\right)^{\frac{1}{3}}}{x^{\frac{2}{3}} y^{\frac{1}{2}} \cdot x^{-\frac{1}{5}} y^{\frac{2}{3}}}$

LEARNING GUIDE 4

Factoring

Have a good look at this "Factoring Flow Chart" …to be successful follow it starting with the all-important LOOKING FOR GCF FIRST!!!



Factoring Polynomials

Topic 1

Factoring Using GCF:

To factor using a GCF, take the greatest common factor (GCF), for the numerical coefficient. When choosing the GCF for the variables, if all terms have a common variable, take the ones with the lowest exponent.

Example: $9x^4 + 3x^3 + 12x^2$

GCF: Coefficients = 3
Variables (x) = x^2

GCF = $3x^2$
Next, you just divide each monomial by the GCF!
Answer = $3x^2(3x^2 + x + 4)$
Then, check by using the distributive property!

Try: $6x^3 - 4x + 8x$

$4x^4y^2 - 20x^3y^3 - 14x^3y^4$

Topic 2

Factoring Simple Trinomials (Case I):

Case I is when there is a coefficient of 1 in front of your variable^2 term (x^2).

<u>You have two hints that will help you:</u>

1) When the last sign is addition, both signs are the same and match the middle term.
2) When the last sign is subtraction, both signs are different and the larger number goes with the sign of the middle term.

<u>Examples:</u>

<u>Hint #1:</u>	<u>Hint #2:</u>
$x^2 – 5x + 6$	$x^2 + 5x – 36$
$(x -\ )(x -\ )$	$(x -\ )(x +\ )$
Find factors of 6, w/ sum of 5.	Find factors of 36 w/ difference of 5.
$(x – 3)(x – 2)$	$(x – 4)(x + 9)$
CHECK USING FOIL	CHECK USING FOIL

Sign Pattern for the Binomials

Trinomial Sign Pattern		Binomial Sign Pattern
+	+	(+)(+)
-	+	(-)(-)
-	-	1 plus and 1 minus
+	-	1 plus and 1 minus

Try:

$x^2 + 8x +16$ $\qquad$ $x^2 – 3x – 18$

$x^2 - 7x + 10$ $\qquad$ $x^2 + 8x – 20$

Topic 3

Factoring Trinomials (Case II):

Decomposition: Factoring $ax^2 + bx + c$ when $a \neq 1$

Example: Factor $3x^2 + 11x + 6$

we look for 2 #'s that add to give the middle term (11)

2#'s that multiply to give the product of the first and last term (3)(6) = 18

add to get 11 and multiply to get 18 would be 9 and 2

DECOMPOSITION

Step 1: Find the two terms

The 2 terms would be $2x$ and $9x$

Step 2: We RE-WRITE our original expression with the factors in it

$3x^2 + 11x + 6 = 3x^2 + 9x + 2x + 6$

Step 3: Now we common factor by the first 2 terms

$3x^2 + 9x + 2x + 6$

$= 3x^2 + 9x \;\; + 2x + 6$

$= 3x(x + 3) \;\; + 2x + 6$

Step 3: Figure out the factors of the last two terms

We know the last two terms MUST have a factor that is the SAME as the 1st two terms

$3x(x + 3) \quad + \,?\,(x + 3)$ SO what to multiply $(x + 3)$ by to get $+2x + 6$?????

the answer is 2 since $2(x + 3) = +2x + 6$

Step 4: Re-write as two factors

$3x(x + 3) \quad + 2x + 6$

$= 3x(x + 3) \quad + 2\,(x + 3)$

$= (3x + 2)(x + 3)$

> We can do this if we think of $3a + 4a = (3 + 4)a$
>
> Thus, $3(x + y) + 4(x + y) = (3 + 4)(x + y)$
>
> Thus $a(x + y) + b\,(x + y) = (a + b)(x + y)$

$(3x + 4)(2x - 1)$

Step 4: Foil Check

Factoring Trinomials (Case II another option to factor):

<u>Triple Play</u>: Factoring $ax^2 + bx + c$ when $a \neq 1$

<u>Example:</u> Factor $3x^2 + 11x + 6$

Step 1: Setup $\dfrac{(3x\quad)(3x\quad)}{3}$

Step 2: Now multiply the first # by the last # , then set-up a T chart

$3 \times 6 = +18$

+18			
1	18	= 19	
2	9	= $\boxed{11}$	middle #
3	6	= 9	

Put the 2 numbers into the top bracket slots with correct signs

☆ Remember, these two numbers must add to the middle # and multiply to last #

Step 3: Divide out denominator $\dfrac{(3x + 2)(3x + 9)}{3}$

Answer: $(3x + 2)(x + 3)$

☆ Watch out, sometimes you must divide n conquer to get rid of the denominator

Step 4: Foil Check

Try: $2x^2 + 5x - 12$ $3x^2 - x - 10$ $4x^2 - 8x + 3$

Topic 4

Factoring Perfect Squares:

They could be Binomials or Trinomials…but you must recognize that the first and last term MUST be perfect squares. (ex. 1, 4, 9, 16, 25, … or x^2, x^4, x^6, …)

Example 1: Factor $4x^2 + 12x + 9$

Step 1: Ask yourself…"Is the first and last terms Perfect?" In this example yes, 4 and 9.

Step 2: Now set up two brackets with the square root of each place in the brackets.

$$(2x \quad 3)(2x \quad 3)$$

Step 3: To figure out the signs follow this pattern.

$$\_\_ + \_\_ + \_\_ = (\_\_ + \_\_)(\_\_ + \_\_)$$
$$\_\_ - \_\_ + \_\_ = (\_\_ - \_\_)(\_\_ - \_\_)$$

$$= (2x + 3)(2x + 3)$$

Step 4: Then, check by using the distributive property!

Try: $x^2 - 36$ $\qquad$ $9x^2 - 16$ $\qquad$ $x^2 + 16x + 64$

Topic 5

Factoring Completely:

When asked to factor completely, you will have to use a combination of the methods that we have used previously. **GCF, GCF, GCF!**

Example 1: Factor $10x^2 - 22x + 4$

Step 1: Factor out GCF **2** $(5x^2 - 11x + 2)$

Step 2: Now look at what you have left in the bracket…ask yourself if you can factor it.
Look to see if it is a = 1, a > 1, or perfect squares.
In this example above it is a > 1 factoring.

$$2\left[\frac{(5x-1)(5x-10)}{5}\right]$$

$$= 2(5x-1)(x-2)$$

Step 3: Then, check by using the distributive property!

Try: $2x^2 + 5x - 12$ $5x^3 + 30x^2 + 45x$

Topic 6

Factoring Grouping:

Example 1: Factor by grouping: $x^3 + 7x^2 + 2x + 14$

Step 1: Group the first two terms together and then the last two terms together.

$$x^3 + 7x^2 + 2x + 14 =$$
$$(x^3 + 7x^2) + (2x + 14)$$

**Two groups of two terms*

Step 2: Factor out a GCF from each separate binomial.

$$(x^3 + 7x^2) + (2x + 14) =$$
$$x^2(x + 7) + 2(x + 7)$$

***Factor out an x squared from the 1st ( )**

**Factor out a 2 from the 2nd ( )*

Step 3: Factor out the common binomial.

$$x^2(x + 7) + 2(x + 7) =$$
$$(x + 7)(x^2 + 2)$$

**Divide (x + 7) out of both parts*

Step 4: CHECK YOUR ANSWER

Try: $4v^3 - 12v^2 - 5v + 15$ $\qquad$ $24p^3 + 15p^2 - 56p - 35$