PRE-CALCULUS 11

Seminar Notes

Learning Guides 7 & 8

QUADRATIC EQUATIONS

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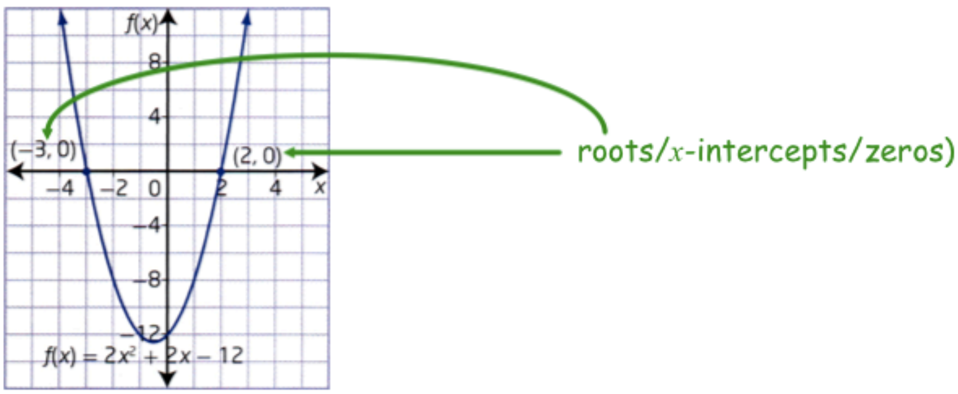
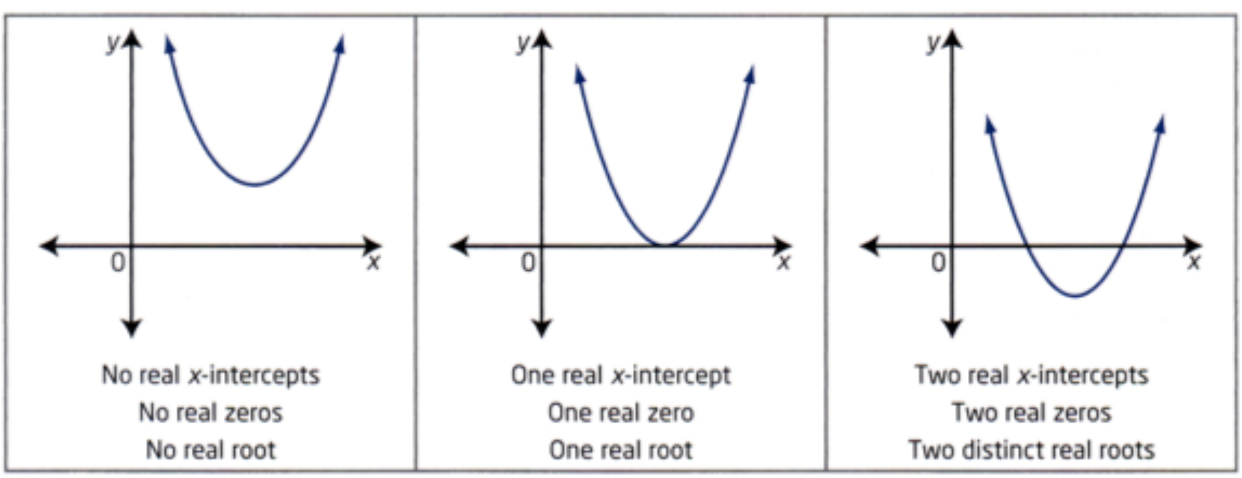
Topic 1 Quadratic Equations

Quadratic equations of the form $ax^2 + bx + c = 0$ can be solved by graphing the corresponding function, $f(x) = ax^2 + bx + c$.

Solutions are called the **roots** (or x-intercepts or zeros)



Quadratic equations can have no solutions, one solution or two solutions.

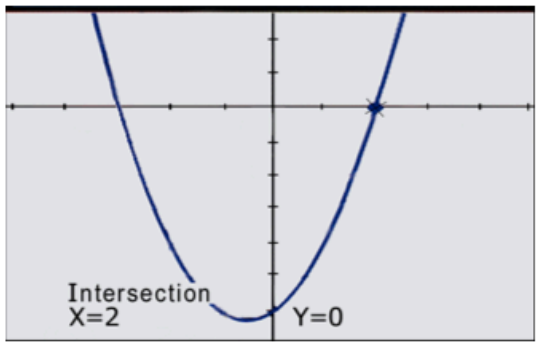


Example 1

Solving Quadratics by Graphing

Example: What are the roots of the equation $2x^2 + 2x - 12 = 0$

1. Enter in the graphing calculator as $y = 2x^2 + 2x - 12$
2. Adjust window if necessary
3. Find one of the x-intercepts (2nd TRACE 5 and ENTER 3 times)
4. Find the 2nd x-intercept as above, but make sure you move the cursor to the other side of the vertex before pressing enter
5. The roots are (2, 0) and (-3,0)



Check

1. Substitute $x = 2$ and $x = -3$ into the original equation

$$2x^2 + 2x - 12 = 0$$
$$2(2)^2 + 2(2) - 12 = 0$$
$$8 + 4 - 12 = 0$$
$$0 = 0$$

$$2x^2 + 2x - 12 = 0$$
$$2(-3)^2 + 2(-3) - 12 = 0$$
$$18 - 6 - 12 = 0$$
$$0 = 0$$

Both solutions are correct

Try: Determine the roots of each quadratic equation

a) $x^2 - 6x + 9 = 0$

b) $3x^2 - 7x + 6 = 0$

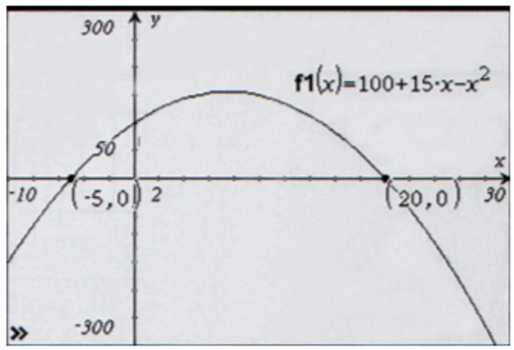
Example 2

Solving a problem with Quadratics

Example: The manager of a clothing store is investigating the effect that raising or lowering dress prices has on the daily revenue from dress sales. The function $R(x) = 100 + 15x - x^2$ gives the store's revenue R from dress sales, in dollars, where x is the price change in dollars. What price change will result in no revenue?

When there is no revenue, $R(x) = 0$. To answer the question, find the zeros.

1. Graph the equation. Adjust the window settings until you can see the vertex and the x-intercepts
2. Use the trace function to find the x-intercepts



3. The roots are (-5, 0) and (20,0)

Check

Substitute $x = -5$ and $x = 20$ into the original equation

$100 + 15x - x^2 = 0$	$100 + 15x - x^2 = 0$
$100 + 15(-5) - (-5)^2 = 0$	$100 + 15(20) - (20)^2 = 0$
$100 - 75 - 25 = 0$	$100 + 300 - 400 = 0$
$0 = 0$	$0 = 0$

Both solutions are correct

A price decrease of $\$5$ or an increase of $\$20$ will both result in no revenue from dress sales

Try: **The manager at Suzie's Fashions has determined that the function $R(x) = 600 - 6x^2$ models the weekly revenue, R, in dollars from sweatshirts as the price changes, where x, is the price change, in dollars. What price change will result in no revenue?**

Topic 2 Factoring Quadratic Equations

Example 1

Factor Quadratic Expressions

Example: Factor each of the following:

a) $x^2 + 4x - 5$ factors of -5 add to 4

$(x+5)(x-1)$

5	-1

b) $2x^2 - 2x - 12$ Factor out GCF

$2(x^2 - x - 6)$ factors of -6 add to -1

$2(x-3)(x+2)$

-6	1
-3	2

Triple Play

c) $2x^2 - x - 3$

multiply

$2x^2 - x - 3$ factors of -6 add to -1

-6	1
-3	2

$(2x-3)(2x+2)$

2

$(2x-3)(x+1)$

Decomposition

factors of -6 add to -1

-6	1
-3	2

$2x^2 - x - 3$

$2x^2 + 2x - 3x - 3$

$(2x^2 + 2x)(-3x - 3)$

$2x(x+1) - 1(x+1)$

$(x+1)(2x-1)$

d) $4x^2 - 81$ ← Ask yourself: can square root both

$(2x+9)(2x-9)$

Try: Factor each of the following:

a) $3x^2 + 3x - 6$ b) $\frac{1}{2}x^2 - x - 4$ c) $0.49j^2 - 36k^2$

Example 2

Factor Polynomials of Quadratic Form

Example: Factor each polynomial

a) $12(x+2)^2+24(x+2)+9$ — treat $x+2$ as a single variable, $r=x+2$

$12r^2+24r+9$ — substitute r for $x+2$

$3(4r^2+8r+3)$ — factor as before

$\frac{3(4r+6)(4r+2)}{4}$

$\frac{3(4r+6)(4r+2)}{2\times 2}$

$3(2r+3)(2r+1)$

$3[2(x+2)+3][2(x+2)+1]$ — substitute $x+2$ for r

$3(2x+4+3)(2x+4+1)$

$3(2x+7)(2x+5)$

b) $9(2t+1)^2-4(s-2)^2$

each term is a perfect square making it a difference of squares

$[3(2t+1)-2(s-2)][3(2t+1)+2(s-2)]$ — factor like a difference of squares

$(6t+3-2s+4)(6t+3+2s-4)$

$(6t-2s+7)(6t+2s-1)$

Try: Factor each of the following:

a) $-2(n+3)^2+12(n+3)+14$

b) $4(x-2)^2-0.25(y-4)^2$

Example 3

Solve Quadratic Equations by Factoring

Example: Determine the roots of each quadratic equation. Verify your solutions.

a) $x^2 + 6x + 9 = 0$ — factor trinomial

$(x+3)(x+3) = 0$

$(x+3) = 0$ or $(x+3) = 0$ — for the quadratic equation to equal 0, one of the factors must equal 0

$x = -3$ $\quad$ $x = -3$

Check

Substitute $x = -3$ into the original equation

$$x^2 + 6x + 9 = 0$$
$$(-3)^2 + 6(-3) + 9 = 0$$
$$9 - 18 + 9 = 0$$
$$0 = 0$$

b) $2x^2 - 9x - 5 = 0$ — factor trinomial

$$\frac{(2x-10)(2x+1)}{2} = 0$$

$$(x-5)(2x+1) = 0$$

$x - 5 = 0$ or $2x + 1 = 0$ — for the quadratic equation to equal 0, one of the factors must equal 0

$x = 5$ $\quad$ $2x = -1$

$x = -\frac{1}{2}$

Check

Substitute $x = 5$ and $x = -1/2$ into the original equation

$$2x^2 - 9x - 5 = 0$$
$$2(5)^2 - 9(5) - 5 = 0$$
$$50 - 45 - 5 = 0$$
$$0 = 0$$

$$2x^2 - 9x - 5 = 0$$
$$2\left(-\frac{1}{2}\right)^2 - 9\left(-\frac{1}{2}\right) - 5 = 0$$
$$\frac{1}{2} + \frac{9}{2} - 5 = 0$$
$$0 = 0$$

Example 4

Write a Quadratic Model Function

Example: A rectangle field has dimensions $x + 4$ and $3x - 10$, where x is measured in metres. The area of the field is 4840 m^2.

a) Write a equation to model the situation.
b) Solve for x.

Example 5

Write a Quadratic Model Function

Example: Two whole numbers that differ by 5. The sum of their squares is 53.

a) What are the two numbers.

LEARNING GUIDE 8

Topic 1

The Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The quadratic formula is used to find the roots of quadratic equations of the form $ax^2 + bx + c = 0$

Example 1

Use the Quadratic Formula to Solve Quadratic Equations

Example: Use the quadratic formula to solve $9x^2 + 12x = -4$

1. Write $9x^2 + 12x = -4$ in standard form, $ax^2 + bx + c = 0$

   $9x^2 + 12x + 4 = 0$ $\quad a = 9,\ b = 12,$ and $c = 4$

2. Substitute the values for a, b, and c into the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-12 \pm \sqrt{12^2 - 4(9)(4)}}{2(9)}$$

$$x = \frac{-12 \pm \sqrt{144 - 144}}{18}$$

$$x = \frac{-12 \pm \sqrt{0}}{18}$$

$$x = \frac{-12}{18}$$

$$x = -\frac{2}{3}$$

Check

Substitute $x = -\frac{2}{3}$ into the original equation

$$9x^2 + 12x = -4$$

$$9\left(-\frac{2}{3}\right)^2 + 12\left(-\frac{2}{3}\right) = -4$$

$$4 - 8 = -4$$

$$-4 = -4$$

Try: Determine the roots of the each equation to the nearest hundredth and verify your solution.

a) $3x^2 + 5x - 2 = 0$

b) $\frac{t^2}{2} - t - \frac{5}{2} = 0$

Example 2

Select a Strategy to Solve a Quadratic Equation

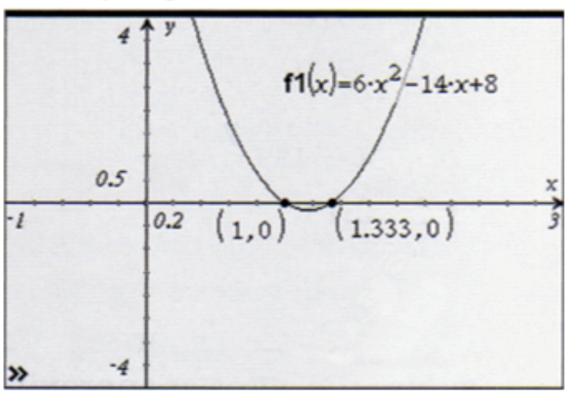
Example: Solve $6x^2 - 14x + 8 = 0$ by:

a) graphing

b) factoring

c) using the quadratic formula

a) Graphing



1. Graph the related function, $y = 6x^2 - 14x + 8$
2. Remember to graph $Y_2 = 0$
3. The x-intercepts are (1, 0) and about (1.33, 0)

b) Factoring

$6x^2 - 14x + 8 = 0$

$3x^2 - 7x + 4 = 0$

$(3x - 4)(x - 1) = 0$

$(3x - 4) = 0$ $(x - 1) = 0$

$3x = 4$

$x = \frac{4}{3}$ or $x = -1$

c) Quadratic Formula

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-14) \pm \sqrt{(-14)^2 - 4(6)(8)}}{2(6)}$

$x = \frac{14 \pm \sqrt{196 - 193}}{12}$

$x = \frac{14 \pm \sqrt{4}}{12}$

$x = \frac{14 \pm 2}{12}$

$x = \frac{14 + 2}{12}$ or $x = \frac{14 - 2}{12}$

$x = \frac{16}{12}$ $x = \frac{12}{12}$

$x = \frac{4}{3}$ $x = 1$

Topic 2

The Discriminant

The discriminant lets you determine the nature of the roots for a quadratic equations of the form $ax^2 + bx + c = 0$. It is the expression $b^2 - 4ac$ which is under the radical sign in the quadratic formula.

$b^2 - 4ac > 0$ 2 distinct real roots

$b^2 - 4ac = 0$ 2 equal real roots (one distinct real root)

$b^2 - 4ac < 0$ NO real roots

Example 1

Use the Discriminant to Determine the nature of the Roots

Example: Determine the nature of the roots of $-2x^2 + 3x + 8 = 0$.
Verify your answer with your calculator.

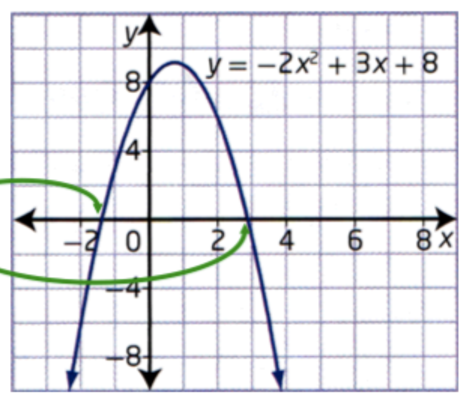
$b^2 - 4ac$

$(3)^2 - 4(-2)(8)$

$9 + 64$

73 ← Greater than 0, so there are 2 distinct real roots.

Graphing confirms 2 real roots.



Try: Determine the nature of the roots for each equation and verify your solution.

a) $2x^2 - 8x = -9$

b) $3x^2 - 4x + \frac{4}{3} = 0$