

FOUNDATIONS & PRE-CALCULUS 10

Seminar Notes Learning Guides 6 & 7

**FACTORS &
PRODUCTS**

Topic 1

Factors and Multiples of Whole Numbers

Example 1 Determining the Prime Factors of a Whole Number

Write the prime factorization of 3300.

CHECK YOUR UNDERSTANDING

Try: Write the prime factorization of 2646.

Example 2 Determining the Greatest Common Factor

Determine the greatest common factor of 138 and 198.

CHECK YOUR UNDERSTANDING

Try: Determine the greatest common factor of 126 and 144.

Example 3

Determining the Least Common Multiple

Determine the least common multiple of 18, 20, and 30.

CHECK YOUR UNDERSTANDING

Try: Determine the least common multiple of 28, 42, and 63.

Example 4

Solving Problems that Involve Greatest Common Factor and Least Common Multiple

- a) What is the side length of the smallest square that could be tiled with rectangles that measure 16 cm by 40 cm? Assume the rectangles cannot be cut. Sketch the square and rectangles.
- b) What is the side length of the largest square that could be used to tile a rectangle that measures 16 cm by 40 cm? Assume that the squares cannot be cut. Sketch the rectangle and squares.

Topic 2

Perfect Squares, Perfect Cubes, and Their Roots

Example 1 Determining the Square Root of a Whole Number

Determine the square root of 1296.

Example 2 Determining the Cube Root of a Whole Number

Determine the cube root of 1728.

Example 3 Using Roots to Solve a Problem

A cube has volume 4913 cubic inches. What is the surface area of the cube?

CHECK YOUR UNDERSTANDING

Try: A cube has volume 12 167 cubic feet. What is the surface area of the cube?

Topic 3

Multiplying Polynomials

Example 1 Using the Distributive Property to Multiply Two Polynomials

Expand and simplify.

a) $(2h + 5)(h^2 + 3h - 4)$ b) $(-3f^2 + 3f - 2)(4f^2 - f - 6)$

Example 2 Multiplying Polynomials in More than One Variable

Expand and simplify.

a) $(2r + 5t)^2$ b) $(3x - 2y)(4x - 3y + 5)$

Example 3 Simplifying Sums and Differences of Polynomial Products

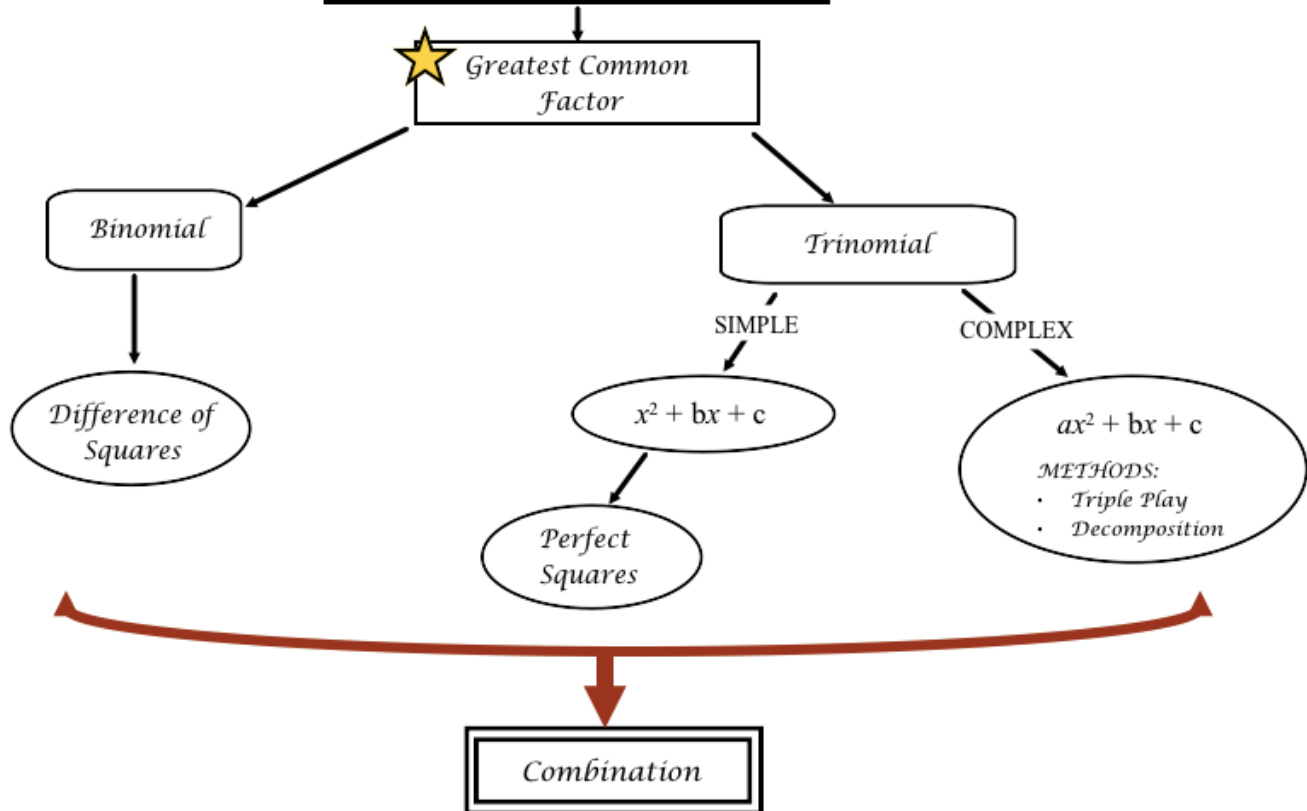
Expand and simplify.

a) $(2c - 3)(c + 5) + 3(c - 3)(-3c + 1)$

b) $(3x + y - 1)(2x - 4) - (3x + 2y)^2$

LEARNING GUIDE 7

Factoring Polynomials



Topic 1

Common Factors of a Polynomial

Example 1 Using Algebra Tiles to Factor Binomials

Factor each binomial.

a) $6n + 9$

b) $6c + 4c^2$

CHECK YOUR UNDERSTANDING

Try: Factor each binomial.

a) $3g + 6$ b) $8d + 12d^2$

Example 2 Factoring Trinomials

Factor the trinomial $5 - 10z - 5z^2$.

Verify that the factors are correct.

CHECK YOUR UNDERSTANDING

Try: Factor the trinomial

$$6 - 12z + 18z^2$$

Verify that the factors are correct.

Example 3 Factoring Polynomials in More than One Variable

Factor the trinomial. Verify that the factors are correct.

$$-12x^3y - 20xy^2 - 16x^2y^2$$

Topic 2

Polynomials of the Form $x^2 + bx + c$

First of all you'll need to review **Multiplying Polynomials...** this will help you check to see if your answer is correct or not.

Example 1 Multiplying Two Binomials

Expand and simplify.

a) $(x - 4)(x + 2)$

b) $(8 - b)(3 - b)$

Example 2 Factoring Trinomials

Factor each trinomial.

a) $x^2 - 2x - 8$

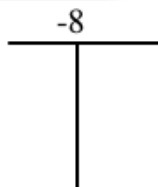
b) $z^2 - 12z + 35$

Set-up: $(x - \quad)(x + \quad)$

Think: $\square + \square = -2$

$\square \times \square = -8$

Make T-Chart
all products of 8



Verify that your
factors are correct.

CHECK YOUR UNDERSTANDING

Try: Factor each trinomial.

a) $x^2 - 8x + 7$

b) $a^2 + 7a - 18$

Example 3 Factoring a Trinomial Written in Ascending Order

Factor: $-24 - 5d + d^2$

CHECK YOUR UNDERSTANDING

Try: Factor: $-30 + 7m + m^2$

Topic 3

Polynomials of the Form $ax^2 + bx + c$

Example 4

Factoring a Trinomial by Decomposition

Decomposition: Factoring $ax^2 + bx + c$ when $a \neq 1$

Example: Factor $3x^2 + 11x + 6$

we look for 2 #'s that add to give the middle term (11)

2#'s that multiply to give the product of the first and last term $(3)(6) = 18$

add to get 11 and multiply to get 18 would be 9 and 2

DECOMPOSITION

Step 1: Find the two terms

The 2 terms would be $2x$ and $9x$

Step 2: We RE-WRITE our original expression with the factors in it

$$3x^2 + 11x + 6 = 3x^2 + 9x + 2x + 6$$

Step 3: Now we common factor by the first 2 terms

$$3x^2 + 9x + 2x + 6$$

$$= \underline{3x^2 + 9x} \quad | \quad + 2x + 6$$

$$= 3x(x + 3) \quad | \quad + 2x + 6$$

Step 3: Figure out the factors of the last two terms

We know the last two terms MUST have a factor that is the SAME as the 1st two terms

$$3x(x + 3) + ?(x + 3) \quad \text{SO what to multiply } (x + 3) \text{ by to get } +2x + 6 \text{ ?????}$$

the answer is 2 since $2(x + 3) = +2x + 6$

Step 4: Re-write as two factors

$$\begin{aligned} &3x(x + 3) + 2x + 6 \\ &= 3x(x + 3) + 2(x + 3) \\ &= (3x + 2)(x + 3) \end{aligned}$$

We can do this if we think of $3a + 4a = (3 + 4)a$
 Thus, $3(x + y) + 4(x + y) = (3 + 4)(x + y)$
 Thus $a(x + y) + b(x + y) = (a + b)(x + y)$

$$(3x + 4)(2x - 1)$$

Step 4: Foil Check

Triple Play: Factoring $ax^2 + bx + c$ when $a \neq 1$

Example: Factor $3x^2 + 11x + 6$

Step 1: Setup $\frac{(3x \quad)(3x \quad)}{3}$

Step 2: Now multiply the first # by the last #, then set-up a T chart
 $3 \times 6 = +18$

+18		
1	18	= 18
2	9	= 18
3	6	= 18

Put the 2 numbers into the top bracket slots with correct signs

★ Remember, these two numbers must add to the middle # and multiply to last #

Step 3: Divide out denominator $\frac{(3x + 2)(3x + 9)}{3}$

Answer: $(3x + 2)(x + 3)$

★ Watch out, sometimes you must divide n conquer to get rid of the denominator

Step 4: Foil Check

Factor.

a) $3s^2 - 13s - 10$

b) $6x^2 - 21x + 9$

CHECK YOUR UNDERSTANDING

Try: Factor.

a) $8p^2 - 18p - 5$

b) $24h^2 - 20h - 24$

Topic 4

Factoring Special Polynomials

Example 1

Factoring a Perfect Square Trinomial

Factor each trinomial. Verify by multiplying the factors.

a) $4x^2 + 12x + 9$

b) $4 - 20x + 25x^2$

CHECK YOUR UNDERSTANDING

Try: Factor each trinomial. Verify by multiplying the factors.

a) $36x^2 + 12x + 1$

b) $16 - 56x + 49x^2$

Example 2 Factoring Trinomials in Two Variables

Factor each trinomial. Verify by multiplying the factors.

a) $2a^2 - 7ab + 3b^2$ b) $10c^2 - cd - 2d^2$

Example 3 Factoring a Difference of Squares

Factor each binomial.

a) $25 - 36x^2$ b) $5x^4 - 80y^4$

CHECK YOUR UNDERSTANDING

Try: Factor each binomial.

a) $81m^2 - 49$

b) $162v^4 - 2w^4$

Example 4 Factoring a Trinomial with a Common Factor and Binomial Factors

Factor.

$$-4t^2 - 16t + 128$$

CHECK YOUR UNDERSTANDING

Try: Factor.

$$-5h^2 - 20h + 60$$