

# **FOUNDATIONS & PRE-CALCULUS 10**

## **Seminar Notes** **Learning Guides 12, 13 & 14**

**LINEAR  
FUNCTIONS**

## Topic 1

# Slope of a Line

## Construct Understanding

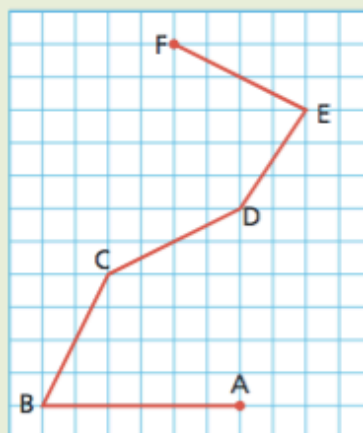
$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

### TRY THIS

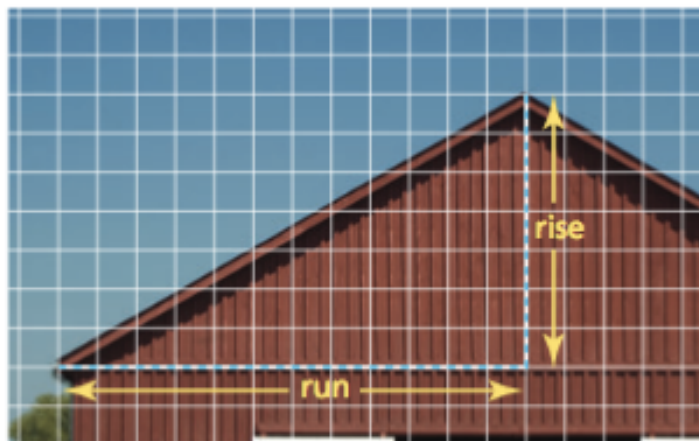
Work with a partner.

This diagram shows different line segments on a square grid.

- Think of a strategy to calculate a number to represent the steepness of each line segment.
- Which is the steepest line segment? How does your number show that?
- Which segment is the least steep? How does its number compare with the other numbers?



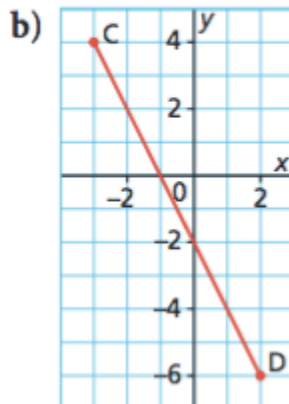
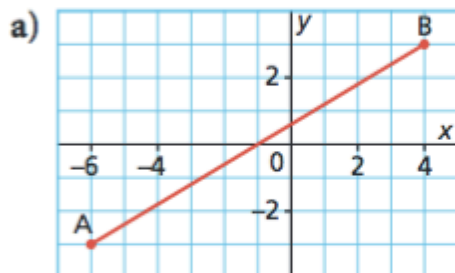
The steepness of a roof is measured by calculating its **slope**.



What is the steepness of this roof?

### Example 1 Determining the Slope of a Line Segment

Determine the slope of each line segment.

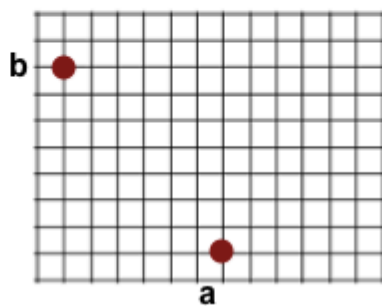


### Example 2 Drawing a Line Segment with a Given Slope

Draw a line segment with each given slope.

a)  $\frac{7}{5}$

b)  $-\frac{3}{8}$



### Example 3 Determining Slope Given Two Points on a Line

$$\text{Slope } (m) = \frac{y_2 - y_1}{x_2 - x_1}$$

Determine the slope of the line that passes through  $C(-5, -3)$  and  $D(2, 1)$ .

#### CHECK YOUR UNDERSTANDING

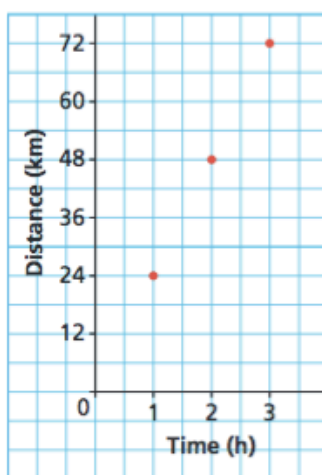
**Try:** Determine the slope of the line that passes through  $E(4, -5)$  and  $F(8, 6)$ .

### Example 4 Interpreting the Slope of a Line

#### CHECK YOUR UNDERSTANDING

Yvonne recorded the distances she had travelled at certain times since she began her cycling trip along the Trans Canada Trail in Manitoba, from North Winnipeg to Grand Beach. She plotted these data on a grid.

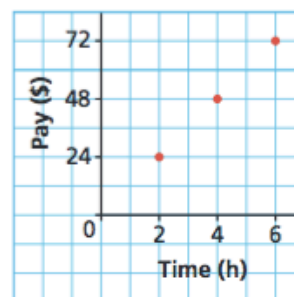
Graph of a Bicycle Ride



- What is the slope of the line through these points?
- What does the slope represent?
- How can the answer to part b be used to determine:
  - how far Yvonne travelled in  $1\frac{3}{4}$  hours?
  - the time it took Yvonne to travel 55 km?

**Try:** Tom has a part-time job. He recorded the hours he worked and his pay for 3 different days. Tom plotted these data on a grid.

Graph of Tom's Pay



- What is the slope of the line through these points?
- What does the slope represent?
- How can the answer to part b be used to determine:
  - how much Tom earned in  $3\frac{1}{2}$  hours?
  - the time it took Tom to earn \$30?

## Topic 2

# Slopes of Parallel and Perpendicular Lines

### Example 1 Identifying Parallel Lines

Line GH passes through  $G(-4, 2)$  and  $H(2, -1)$ . Line JK passes through  $J(-1, 7)$  and  $K(7, 3)$ . Line MN passes through  $M(-4, 5)$  and  $N(5, 1)$ . Sketch the lines. Are they parallel? Justify the answer.

### Example 2 Examining Slopes to Compare Lines

Line PQ passes through  $P(-7, 2)$  and  $Q(-2, 10)$ .

Line RS passes through  $R(-3, -4)$  and  $S(5, 1)$ .

- a) Are these two lines parallel, perpendicular, or neither?  
Justify the answer.

**Example 3****Identifying a Line Perpendicular to a Given Line**

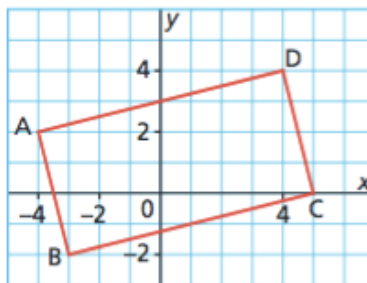
- a) Determine the slope of a line that is perpendicular to the line through  $E(2, 3)$  and  $F(-4, -1)$ .
- b) Determine the coordinates of  $G$  so that line  $EG$  is perpendicular to line  $EF$ .

**CHECK YOUR UNDERSTANDING**

- Try:** a) Determine the slope of a line that is perpendicular to the line through  $G(-2, 3)$  and  $H(1, -2)$ .
- b) Determine the coordinates of  $J$  so that line  $GJ$  is perpendicular to line  $GH$ .

**Example 4****Using Slope to Identify a Polygon**

$ABCD$  is a parallelogram. Is it a rectangle? Justify the answer.



# LEARNING GUIDE 13, 14

## Topic 1

## Slope-Intercept Form of the Equation for a Linear Function

### Example 1

#### Writing an Equation of a Linear Function Given Its Slope and $y$ -Intercept

The graph of a linear function has slope  $\frac{3}{5}$  and  $y$ -intercept  $-4$ .  
Write an equation for this function.

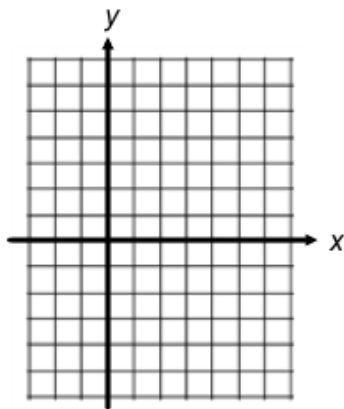
#### CHECK YOUR UNDERSTANDING

**Try:** The graph of a linear function has slope  $-\frac{7}{3}$  and  $y$ -intercept  $5$ .  
Write an equation for this function.

### Example 2

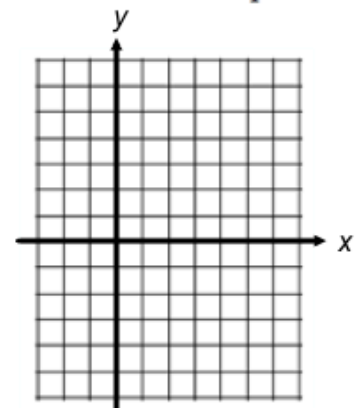
#### Graphing a Linear Function Given Its Equation in Slope-Intercept Form

Graph the linear function with equation:  $y = \frac{1}{2}x + 3$



#### CHECK YOUR UNDERSTANDING

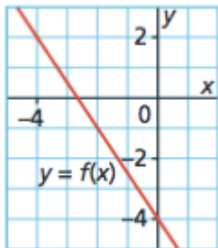
**Try:** Graph the linear function with equation:  $y = -\frac{3}{4}x + 2$



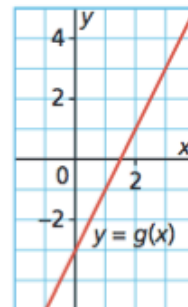
**Example 3****Writing the Equation of a Linear Function Given Its Graph**

Write an equation to describe this function.

Verify the equation.

**CHECK YOUR UNDERSTANDING**

**Try:** Write an equation to describe this function. Verify the equation.

**Example 4****Using an Equation of a Linear Function to Solve a Problem**

The student council sponsored a dance. A ticket cost \$5 and the cost for the DJ was \$300.

- Write an equation for the profit,  $P$  dollars, on the sale of  $t$  tickets.
- Suppose 123 people bought tickets. What was the profit?
- Suppose the profit was \$350. How many people bought tickets?
- Could the profit be exactly \$146? Justify the answer.



## Topic 2

# Slope-Point Form of the Equation for a Linear Function

### Slope-Point Form of the Equation of a Linear Function

The equation of a line that passes through  $P(x_1, y_1)$  and has slope  $m$  is:

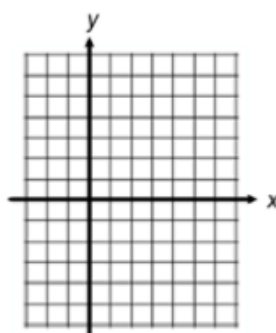
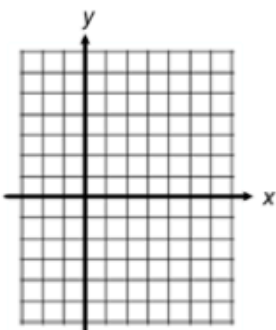
$$y - y_1 = m(x - x_1)$$

### Example 1 Graphing a Linear Function Given Its Equation in Slope-Point Form

- a) Describe the graph of the linear function with this equation:

$$y - 2 = \frac{1}{3}(x + 4)$$

- b) Graph the equation.



### CHECK YOUR UNDERSTANDING

- Try:** a) Describe the graph of the linear function with this equation:

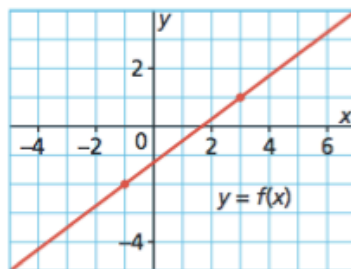
$$y + 1 = -\frac{1}{2}(x - 2)$$

- b) Graph the equation.

### Example 2 Writing an Equation Using a Point on the Line and Its Slope

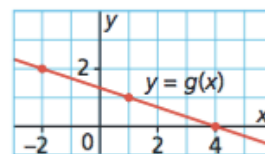
- a) Write an equation in slope-point form for this line.

- b) Write the equation in part a in slope-intercept form. What is the  $y$ -intercept of this line?



### CHECK YOUR UNDERSTANDING

- Try:** a) Write an equation in slope-point form for this line.



- b) Write the equation in part a in slope-intercept form. What is the  $y$ -intercept of this line?

### Example 3 Writing an Equation of a Linear Function Given Two Points

The sum of the angles,  $s$  degrees, in a polygon is a linear function of the number of sides,  $n$ , of the polygon. The sum of the angles in a triangle is  $180^\circ$ . The sum of the angles in a quadrilateral is  $360^\circ$ .

- Write a linear equation to represent this function.
- Use the equation to determine the sum of the angles in a dodecagon. ( 12 sided polygon )

#### CHECK YOUR UNDERSTANDING

**Try:** A temperature in degrees Celsius,  $c$ , is a linear function of the temperature in degrees Fahrenheit,  $f$ . The boiling point of water is  $100^\circ\text{C}$  and  $212^\circ\text{F}$ . The freezing point of water is  $0^\circ\text{C}$  and  $32^\circ\text{F}$ .

- Write a linear equation to represent this function.
- Use the equation to determine the temperature in degrees Celsius at which iron melts,  $2795^\circ\text{F}$ .

### Example 4 Writing an Equation of a Line That Is Parallel or Perpendicular to a Given Line

Write an equation for the line that passes through  $R(1, -1)$  and is:

- parallel to the line  $y = \frac{2}{3}x - 5$
- perpendicular to the line  $y = \frac{2}{3}x - 5$

#### CHECK YOUR UNDERSTANDING

**Try:** Write an equation for the line that passes through  $S(2, -3)$  and is:

- parallel to the line  $y = 3x + 5$
- perpendicular to the line  $y = 3x + 5$

## Topic 3

# General Form of the Equation for a Linear Relation

### General Form of the Equation of a Linear Relation

$Ax + By + C = 0$  is the general form of the equation of a line, where  $A$  is a whole number, and  $B$  and  $C$  are integers.

### Example 1 Rewriting an Equation in General Form

Write each equation in general form.

a)  $y = -\frac{2}{3}x + 4$

b)  $y - 1 = \frac{3}{5}(x + 2)$

### CHECK YOUR UNDERSTANDING

**Try:** Write each equation in general form.

a)  $y = -\frac{1}{4}x + 3$

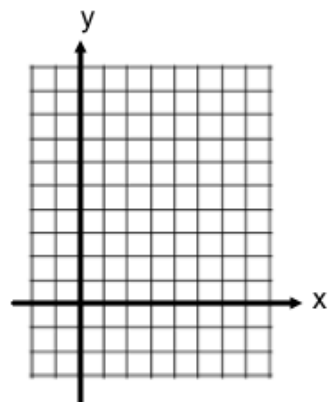
b)  $y + 2 = \frac{3}{2}(x - 4)$

### Example 2 Graphing a Line in General Form

a) Determine the  $x$ - and  $y$ -intercepts of the line whose equation is:  $3x + 2y - 18 = 0$

b) Graph the line.

c) Verify that the graph is correct.

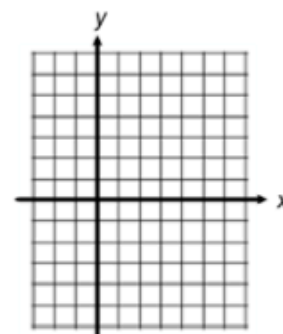


### CHECK YOUR UNDERSTANDING

**Try:** a) Determine the  $x$ - and  $y$ -intercepts of the line whose equation is:  $x + 3y + 9 = 0$

b) Graph the line.

c) Verify that the graph is correct.



**Example 3****Determining the Slope of a Line Given Its Equation in General Form**

Determine the slope of the line with this equation:

$$3x - 2y - 16 = 0$$

**CHECK YOUR UNDERSTANDING**

**Try:** Determine the slope of the line with this equation:

$$5x - 2y + 12 = 0$$

## Make Connections

Now you know three ways to write an equation:

1. **Slope-Intercept Form** >  $y = mx + b$
2. **Slope-Point Form** >  $y - y_1 = m(x - x_1)$
3. **General Form** >  $Ax + By + C = 0$

There is one more way: **Standard Form** >  $Ax + By = C$

**Example 4****Writing Equations in Various Form**

<b>Slope-Point</b>	<b>Slope Intercept</b>
<b>General</b>	<b>&amp; Standard</b>

**A. Given (-3, -4) and (1, 0).**


**B. Given a line perpendicular to  $y = -\frac{2}{3}x + 2$  and passes through point (2, 4).**
